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Erratum: Supermembranes and domain walls in $\mathcal{N}=1$, $D=4$ SYM

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1 $|k|=N$ case

In subsection 6.3 of the paper it has not been taken into account that the domain wall solutions for $|k|=N$ found therein do not satisfy the non-negativity of the contribution to the domain wall tension of the energy density of the scalar field $s(x)$. So these solutions should be discarded.

Namely, in the $|k|=N$ case the vacuum on the left and the right hand side of the membrane is the same (modulo N) and the tension of the whole system is zero. Nevertheless, the membrane can break at least half of the supersymmetry. The BPS equations (6.8) and (6.11) in the paper reduce to

$$\frac{|s|}{\Lambda^3}\left(\ln\frac{\Lambda^3}{|s|}+1\right)=-\frac{|s|}{\Lambda^3}\tan\left(\beta-\alpha+\frac{2\pi n}{N}\right)\left(\beta-\frac{2\pi k}{|k|}\Theta(x^3)\right)+\frac{\cos(\alpha-\frac{2\pi n}{N})}{\cos(\beta-\alpha+\frac{2\pi n}{N})} \tag{1.1}$$

and

$$\frac{\dot\beta}{9\rho N\Lambda}=-\cos\left(\beta-\alpha+\frac{2\pi n}{N}\right)\left(\frac{|s|}{\Lambda^3}\right)^{\frac{1}{3}}+\left(\frac{|s|}{\Lambda^3}\right)^{-\frac{2}{3}}\cos\left(\frac{2\pi n}{N}-\alpha\right). \tag{1.2}$$

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As we pointed out around eq. (6.10), for the case $|k| = N$ the values of α are not a priori restricted. So we may try to choose an appropriate value using the following reasoning. From the form (5.14) of the Veneziano-Yankielowicz part of the action we read that the contribution of the VY field $s(x)$ to the domain wall tension is always non-negative

$$\begin{aligned} T_s &= \int \mathrm{d}^3\xi \, dx^3 \left(K_{s\bar{s}} \dot{s}\dot{\bar{s}} + \frac{1}{K_{s\bar{s}}} \hat{W}_s \bar{\hat{W}}_{\bar{s}} \right) \\ &= \int \mathrm{d}^3\xi \left(2\,\mathrm{Im} \left((\hat{W}_{+\infty} - \hat{W}_{-\infty}) e^{-\mathrm{i}(\alpha-\pi)} \right) - \frac{1}{4\pi} |ks(0)| \right) \geq 0. \end{aligned} \tag{1.3}$$

Since in the case $|k| = N$ the first term in the second line of the above equation vanishes, the second term must be zero and hence on the membrane $|ks(0)| = 0$. This can only be consistent with eq. (1.1) if

$$\cos\left(\alpha - \frac{2\pi n}{N}\right) = 0 \qquad \Longrightarrow \qquad \alpha - \frac{2\pi n}{N} = \frac{\pi}{2} + \pi m, \qquad m \in \mathbb{Z}\,.$$

With this choice of α the equations (1.1) and (1.2) reduce to

$$\frac{|s|}{\Lambda^3} \left(\ln \frac{\Lambda^3}{|s|} + 1 \right) = \frac{|s|}{\Lambda^3} \cot\beta \left(\beta - \frac{2\pi k}{|k|} \Theta(x^3) \right) \tag{1.4}$$

and

$$\frac{\dot{\beta}}{9\rho N \Lambda} = (-)^{m+1} \sin\beta \left(\frac{|s|}{\Lambda^3} \right)^{\frac{1}{3}} . \tag{1.5}$$

The above equations do not have continuous solutions with the required choice of the asymptotic conditions for $s(x^3)$, i.e. $s|_{-\infty} = \Lambda^3 e^{2\pi \mathrm{i} \frac{n}{N}}$, $s|_{-\infty} = \Lambda^3 e^{2\pi \mathrm{i} \frac{(n \pm N)}{N}}$ and $|s(0)| = 0$.