

Figure 3: (left) The conceptual figure of the Wien-filter. (right) The effective length plot of magnetic and electric field for muon spin flip ($\alpha = 180$ [deg])

v_z is the longitudinal velocity of muon beam, E_x and B_y are electric and magnetic field excited by the Wien-filter. This condition is called as “Wien condition”. From the Thomas-Bargmann-Michel-Telegdi (T-BMT) equation [10, 11], the spin rotation angle α of the beam can be calculated as Eq.2.

$$\alpha = \frac{LeB_y}{\gamma m_0 v_z} = \frac{LeE_x}{\gamma m_0 v_z^2} \quad (2)$$

L is the effective length of fields, e is the elementary charge, γ is the Lorentz factor, m_0 is the mass of particle (muon). From these equations, the parameters of the fields are determined such as in the next section.

DESIGN OF THE WIEN-FILTER

This section discuss about the design of the Wien-filter. Figure 3 (right) shows the effective length dependence of the magnetic and electric fields when the input beam energy is the RFQ output (340 keV, $\beta \sim 0.08$) and the spin rotation angle is 180 deg. To maintain the bunch structure after RFQ, the effective length of the fields is set as 500 mm. In this case, the magnetic and electric fields are 0.18 [T] and 4.3 [kV/mm]. The emittance growth is important in the Wien-filter design. The emittance growth caused from the two components: 1) The beam momentum dispersion, 2) The difference of the electric and magnetic forces along the beam direction. The design is conducted such as this components should be reduced.

Emittance Growth From The Initial Beam

To understand the reference emittance growth in the SR, the emittance growth caused from the initial beam is simulated by Geant4 [12–15]. In this simulation, the SR is located after RFQ. Then, the RFQ output beam is used for the SR input beam. However, the large twiss parameter α cause the big momentum dispersion which effects the emittance growth. For this reason, the RFQ beam is transferred as the twiss parameter α is zero.

That profile is shown as Fig.4 (top). The momentum distribution of the RFQ output beam is shown as the Fig.5. This momentum dispersion causes the emittance growth in the Wien-filter. To evaluate the emittance growth caused from the initial beam condition, this simulation set the SR fields condition as the fields is 4.3 [kV/mm] and 0.18 [Tesla]

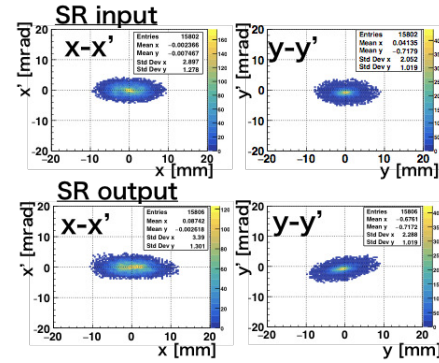


Figure 4: (top) SR input beam as the RFQ output beam after transferred. (bottom) SR output beam in the ideal case of fields.

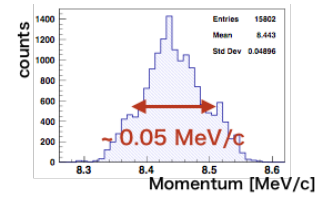


Figure 5: The momentum distribution of the RFQ output beam

in only the effective length region (500 mm). The muon beam profile after this region is shown as the Fig.4 (bottom).

From these beam profiles, the normalized RMS emittance is evaluated as Table 2. The emittance growth in $x - x'$ plane is +22%. This value is the reference for the design of the Wien-filter magnet and electrodes; the emittance growth after the realistic model should be close to the this reference emittance value.

Design Of The Magnet And Electrodes

The magnet is designed such as that the uniformity of the magnetic field is enough for the low emittance muon beam. The design and analysis is performed by Opera3D [16]. The yoke length is set as 500 mm and the other parameters and magnetic field distribution is shown as Fig.6. Based on this magnetic field distribution, the electrode shape is optimized so that the electric field distribution along the z-direction is same as that of magnetic field. For this purpose, the electrodes is designed as that edges is curved such as shown in Fig. 7. By tuning the parameters of electrodes, the difference of the Lorentz force from the electric and magnetic fields can be reduced about 5% shown as Fig. 8.

Table 2: The normalized RMS emittance before/after SR. The unit is [π mm mrad].

	Input	Output	Growth
$\epsilon_{n,x,rms}$	0.296	0.362	+22%
$\epsilon_{n,y,rms}$	0.167	0.167	0%

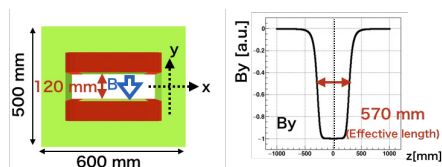


Figure 6: (left) The designed magnet in Opera3D. (right) The magnetic field distribution along z axis.

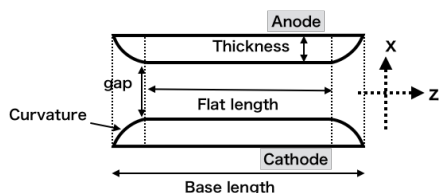


Figure 7: The concepts of the curved electrodes design.

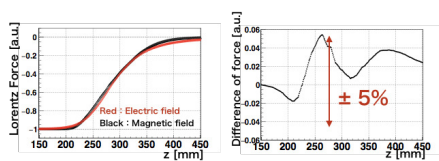


Figure 8: (left) The distribution of the electric and magnetic fields at the edges. (right) The difference of the electric and magnetic fields.

Emittance Growth By The Model

The emittance growth is evaluated by the current model. This input beam is the transferred RFQ output beam shown as Fig.4. Figure 9 shows the spin distribution after the SR. From this result, the normalized field is correct to flip the muon beam spin. Therefore, the emittance growth is evaluated with this normalized fields.

The beam profile after the Wien-filter model is shown as the Fig.10. From this profile, the emittance is evaluated as Table 3. The emittance growth is +64% in x - x' plane and +56% in y - y' plane.

These emittance growth in the current model is about 2-3 times greater than in the ideal case. To study this origin, the case of the no additional electric fields (E_y, E_z) is simulated. The beam profile of this simulation is shown as Fig.11. And

Table 3: The normalized RMS emittance before/after the current model. The unit is [π mm mrad].

	Input	Output	Growth
$\varepsilon_{n,x,rms}$	0.296	0.484	+64%
$\varepsilon_{n,y,rms}$	0.167	0.260	+56%

Table 4: The normalized RMS emittance before/after the current model without the additional electric fields (E_y, E_z). The unit is [π mm mrad].

	Input	Output	Growth
$\varepsilon_{n,x,rms}$	0.296	0.435	+47%
$\varepsilon_{n,y,rms}$	0.167	0.171	+2%

the emittance is shown as Table 4. The emittance growth is +47% in x - x' plane and +2% in y - y' plane. From this result, the emittance growth in the current model is mainly caused from the additional electric fields.

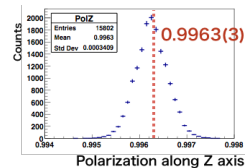


Figure 9: The polarization distribution along Z. The initial polarization is set as -1.

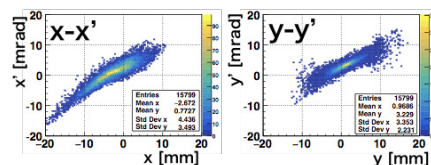


Figure 10: The beam profile after the current model.

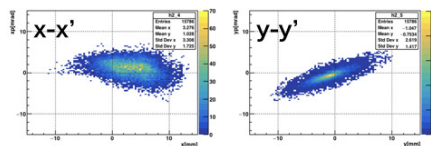


Figure 11: The beam profile after the current model without the additional electric field(E_y, E_z).

SUMMARY AND PROSPECTS

In the J-PARC muon g -2/EDM experiment, the muon beam spin can be flipped at the low-momentum section of the muon linac. By taking the dataset of the muon spin flip, we can take the spin flip analysis and the systematic error depend on time could be reduced. The Wien-filter is considered as a spin rotator.

The design of the Wien-filter is conducted such as that the difference of magnetic and electric fields is small. For this purpose, the electrodes edges are curved so that the electric field is same distribution as the magnetic field. As a result of this design, the emittance growth can be reduced to +64% in x - x' plane and +56% in y - y' plane.

This emittance growth is about three times greater than in the ideal fields case. If we can exclude the additional electric field, the emittance growth will be reduced to +47% in x - x' plane and +2% in y - y' plane. This value is comparable of the emittance growth by the each acceleration section. Therefore, we will tune the Wien-filter model and plan the demonstration with real muon beam.

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