

Lattice Studies of Baryons

D.G. Richards

Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, VA 23606, USA

E-mail: dgr@jlab.org

Abstract. This talk describes progress at understanding the properties of the nucleon and its excitations from lattice QCD. I begin with a review of recent lattice results for the lowest-lying states of the excited baryon spectrum. The need to approach physical values of the light quark masses is emphasized, enabling the effects of the pion cloud to be revealed. I then outline the development of techniques that will enable the extraction of the masses of the higher resonances, and describe how such calculations provide insight into the structure of the hadrons. Finally, I discuss direct probes of the quark and gluon structure of baryons through the lattice measurement of the moments of quark distributions and of Generalized Parton Distributions.

1. Introduction

Spectroscopy is a powerful tool for uncovering the important degrees of freedom of a physical system and the interaction forces between them. The spectrum of QCD is very rich: conventional baryons (nucleons, Δ , Λ , Ξ , Ω , and so on) and mesons (π , K , ρ , *etc.*) have been known for nearly half a century, but other, higher-lying exotic states, such as glueballs, hybrid mesons and hybrid baryons bound by an excited gluon field, and ‘multi-quark’ states, consisting predominantly of four or five quarks in the case of mesons and baryons respectively, have proved more elusive, partly because our theoretical understanding of such states is insufficient, making their identification difficult.

Interest in excited baryon resonances in particular has been sparked by experiments dedicated to mapping out the N^* spectrum in Hall B at the Thomas Jefferson National Accelerator Facility (JLab); evidence for the possible existence of a strangeness $S = 1$ $qqqq\bar{q}$ pentaquark state, discussed in many talks at this workshop, has provided a further incentive for a detailed understanding of baryon resonances.

Much of our current understanding of conventional and excited hadron resonances comes from QCD-inspired phenomenological models. For conventional baryons, the extensive calculations by Isgur, Karl, and Capstick within a non-relativistic quark model[1, 2, 3] remain influential. However, there are a growing number of resonances which cannot be easily accommodated within quark models. States bound by an excited gluon field, such as hybrid mesons and baryons, are still poorly understood. The natures of the Roper resonance and the anomalously light $\Lambda(1405)^-$ remain controversial. Experiment shows that the first excited positive-parity spin-1/2 baryon lies below the lowest-lying negative-parity spin-1/2 resonance, a fact which is difficult to reconcile in quark models. The question of the so-called “missing” baryon resonances is still unresolved: the quark model predicts many more states[2, 3] than are currently known. Compared to the large number of positive-parity states, there are only a few low-lying negative-parity resonances. A quark-diquark picture of baryons predicts a sparser spectrum[4]. Various bag and soliton

models have also attempted to explain the baryon mass spectrum. The search for the $S = 1$ pentaquark states was spurred by a chiral soliton model. While most models expect the lightest pentaquarks to have positive parity [5, 6], a light, narrow isotensor state of negative parity can be accommodated within the quark model [7].

Given the current intense experimental efforts in spectroscopy in general, and of baryons in particular, the need to predict and understand the baryon spectrum from first principles is clear; lattice QCD calculations provide the means of undertaking such *ab initio* studies. The aim is not merely to obtain a set of masses for the states, but also to gain insight into the quark and gluon structure of the states and to understand the relevant degrees of freedom; this latter aspect will be an important emphasis of this talk.

A more direct measure of the quark and gluon structure of hadrons is provided through the parton distributions as determined in Deep Inelastic Scattering (DIS), and more generally through the Generalized Parton Distributions (GPD's). The determination of these quantities is the subject of intense experimental effort which needs to be matched by corresponding lattice investigations. Lattice QCD calculations are restricted to studying the moments of these distributions, and there has been substantial recent progress which I will review briefly at the end of the talk.

The layout of the remainder of this talk is as follows. In the next section, I will review lattice results for the lowest-lying baryon states composed of quarks with masses around that of the strange quark. I will then detail the importance of correctly including the effects of the pion cloud, and describe recent progress at achieving that goal. The next section will outline the recent development of techniques for a comprehensive study of the masses of the higher excitations, and plans to implement these techniques. Finally, I will present recent results on the moments of GPD's and of structure functions before giving my conclusions.

2. Recent results for the baryon spectrum

The computation of the masses of the lowest-lying states has long been a benchmark calculation of lattice QCD since it provides a direct comparison with well-known experimental quantities; a recent review is provided in [8]. The computation is in principle straightforward:

- (i) Choose an interpolating operator \mathcal{O} that has a good overlap with P , the state of interest,

$$\langle 0 | \mathcal{O} | P \rangle \neq 0,$$

and ideally a small overlap with other states having the same quantum numbers.

- (ii) Form the time-sliced correlation function

$$C(t) = \sum_{\vec{x}} \langle \mathcal{O}(\vec{x}, t) \mathcal{O}^\dagger(\vec{0}, 0) \rangle.$$

- (iii) Examine the behavior of the correlator at large Euclidean time

$$C(t) = \sum_P \frac{|\langle 0 | \mathcal{O} | P \rangle|^2}{2m_P} \exp -m_P t, \quad (1)$$

yielding the mass of the lightest state.

However, a precise comparison with experiment requires control and understanding of the systematic uncertainties: the extrapolation of the lattice volume $V \rightarrow \infty$, the control over discretisation errors by taking the lattice spacing $a \rightarrow 0$, and finally, and most delicately, the chiral extrapolation in the quark mass from the values at which the computations are performed to the physical quark masses.

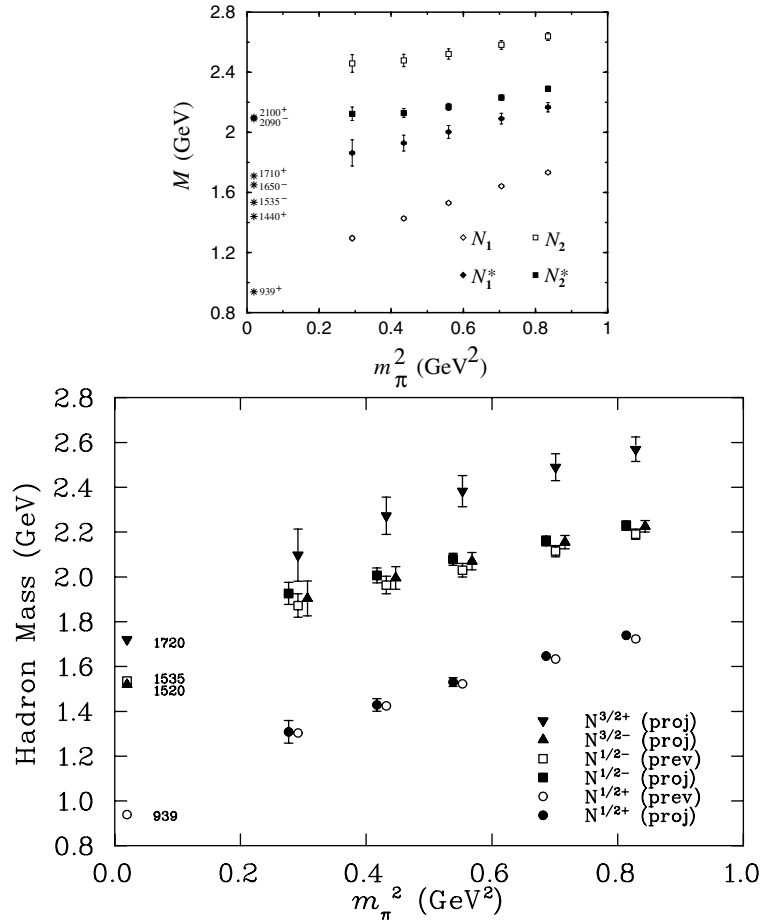


Figure 1. The left-hand plot shows of the lightest positive- and negative-parity nucleon resonances using two independent interpolating operators [9], while the right-hand plot shows also the spin-3/2 and spin-1/2 masses of both parities [10], obtained using the FLIC fermion action.

Whilst the benchmark calculations reviewed in [8] are in full QCD, most of the more exploratory studies described below are in the quenched approximation. The use of the quenched approximation introduces a systematic uncertainty of around 10% for most light-hadron quantities. The observation of excited resonances in lattice QCD is invariably more demanding than that of the ground states since, for hadrons composed of light quarks, the signal-to-noise ratio for excited-state correlators decreases with increasing excitation energy. None-the-less, there has recently been a flurry of activity aimed at computing the excited nucleon spectrum, and in particular the masses of the lightest spin-1/2 and spin-3/2 states of both parities [11, 12, 13, 9, 10, 14, 15]. These calculations employ a variety of fermion discretisation, each has quarks with masses around that of the strange quark, uses local interpolating operators, and each finds a spectrum broadly in line with quark-model expectations $m_N < m_{N^{1/2-}} < m_{N'}$, where N , $N^{1/2-}$ and N' are the nucleon, its parity partner, and first radial excitation of the nucleon, the so-called “Roper” resonance, respectively. This is illustrated in Figure 1. In particular, none of these calculations reveal evidence of two of the more puzzling observations in the nucleon spectrum, the anomalously light Roper resonance, and a light $\Lambda(1405)^-$.

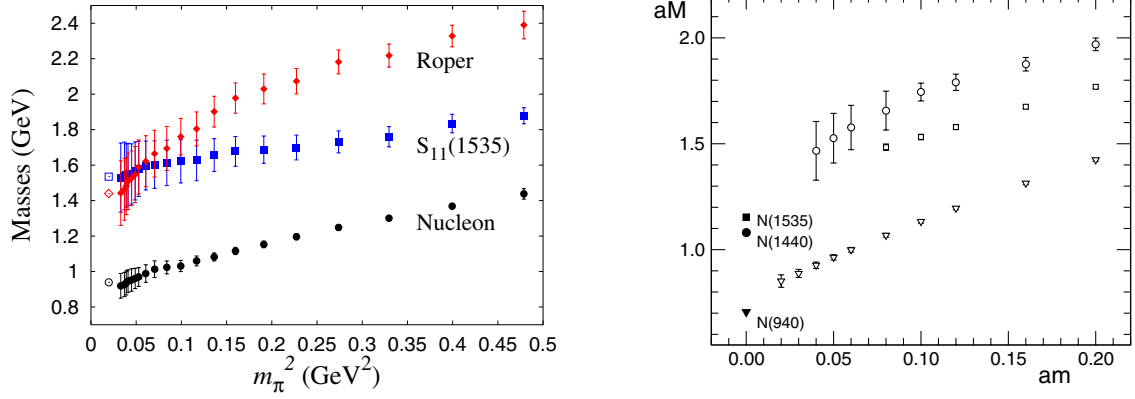


Figure 2. The left-hand plot shows the masses of the nucleon, its parity partner, and the first radial excitation of the nucleon obtained from a calculation using overlap fermions at a lattice spacing $a = 0.2$ fm [19]. The right-hand plot is a calculation of the masses using a variational approach[20]

A crucial realization in recent years has been that QCD at the physical values of the light-quark masses is very different from that for quarks with masses around that of the strange quark mass because of the rôle played by the pion cloud, and the resultant non-analytic behavior with the quark mass. Several groups have embarked on programs aiming at enabling fits to the lattice data so as to correctly incorporate this behaviour, both within full QCD and quenched QCD[16, 17].

In concert with this understanding has been the advent of fermion actions, satisfying the Ginsparg-Wilson relation[18], possessing an exact analogue of chiral symmetry at a finite lattice spacing, and the development of the computational resources required to exploit these actions. Thus quark masses approaching the physical light-quark masses are now within reach. A particular realization of the action is through overlap fermions, which admit the use of shifted-mass inverters allowing the simultaneous calculation of propagators at a range of quark masses. A Bayesian fit to the standard nucleon interpolating operator obtained in the quenched approximation to QCD using overlap fermions, with pion masses as low as 180 MeV, is reported in ref. [19]. The masses of the nucleon, its parity partner and the first radial excitation of the nucleon are shown in Figure 2, revealing an inversion of the ordering of the states at light pion masses; such a result is consistent with a picture of the experimentally observed $N(1440)$ Roper as indeed a simple three-quark resonance.

3. Higher excited resonances and variational methods

A comprehensive picture of resonances requires that we go beyond a knowledge of the ground state mass in each channel, and obtain the masses of the lowest few states of a given quantum number. This we can accomplish through the use of variational methods[21, 22]. Rather than measuring a single correlator $C(t)$, we determine a matrix of correlators

$$C_{ij}(t) = \sum_{\vec{x}} \langle \mathcal{O}_i(\vec{x}, t) \mathcal{O}_j^\dagger(\vec{0}, 0) \rangle,$$

where $\{\mathcal{O}_i; i = 1, \dots, N\}$ are a basis of interpolating operators with given quantum numbers. We then solve the generalized eigenvalue equation

$$C(t)u = \lambda(t, t_0)C(t_0)u$$

Table 1. The number of times n_Γ^J that the irreducible representation Γ of O occurs in the reduction of the irreducible representation J of $SU(2)$.

J	$n_{G_1}^J$	$n_{G_2}^J$	n_H^J
1/2	1	0	0
3/2	0	0	1
5/2	0	1	1
7/2	1	1	1
9/2	1	0	2

to obtain a set of real eigenvalues $\lambda_n(t, t_0)$. At large Euclidean times, these eigenvalues then delineate between the different masses

$$\lambda_n \longrightarrow e^{-M_n t} + \mathcal{O}(e^{-M_{n+1} t}).$$

The eigenvectors u are orthonormal with metric $C(t_0)$, and a knowledge of the eigenvectors can yield information about the partonic structure of the states.

An early attempt to use these methods to extract the first radial excitation of mesons and baryons employed operators constructed from a local, and from a spatially extended source[23]. A more recent application has been to the calculation of the mass of the Roper resonance, using spatially smeared, gauge-invariant three-quark operators of varying widths[20]. Though the authors are unable to approach the very light pion masses attained in reference [19], they find the behavior of the masses of the states with pion mass is at least suggestive of the level crossing observed in the Bayesian analysis, as shown in Figure 2. Furthermore, the resulting eigenstates are consistent with a radial node in the wavefunction.

Recently, the LHP Collaboration has developed the techniques to enable the construction of baryon interpolating operators that can be easily be extended to include multi-quark operators, and those with excited glue[24, 25]. This is delicate, since the cubic symmetry of the lattice admits only three double-valued, irreducible representations (IR's), G_1 , G_2 and H , of dimensions 2, 2 and 4 respectively[26, 27]. The irreducible representations J of the continuum group $SU(2)$ are reducible under the cubic group O ; the number of times n_Γ^J that each of these reducible representations occurs in the irreducible representation Γ of O is shown in Table 1. States with $J > 5/2$ lie in irreducible representations containing states with lower spins, and furthermore, for a given J , the different degrees of freedom can lie in different irreducible representations. The masses of the components in these distinct IR's will agree only in the continuum limit. Furthermore, an implicit assumption in previous lattice studies is the increase in ground-states masses with increasing spin; I will comment further on this below. Parity is easily incorporated, yielding the group O_h , with the corresponding IR's gaining the labels g and u for positive parity and negative parity respectively.

The starting point for the operator construction is a basis of gauge-invariant terms of the form

$$\Phi_{\alpha i; \beta j; \gamma k}^{ABC} = \varepsilon_{abc} (\tilde{D}_i^{(p)} \tilde{\psi})_{a\alpha}^A (\tilde{D}_j^{(p)} \tilde{\psi})_{b\beta}^B (\tilde{D}_k^{(p)} \tilde{\psi})_{c\gamma}^C, \quad (2)$$

where A, B, C indicate quark flavor, a, b, c are color indices, α, β, γ are Dirac spin indices, $\tilde{\psi}$ indicates a smeared quark field, and $\tilde{D}_j^{(p)}$ denotes the p -link covariant displacement operator in the j -th direction; the quark fields are smeared using a three-dimensional gauge-covariant Laplacian. These gauge-invariant operators are now combined into elemental operators having the appropriate flavor structure. The remaining step is to apply group-theoretical projections

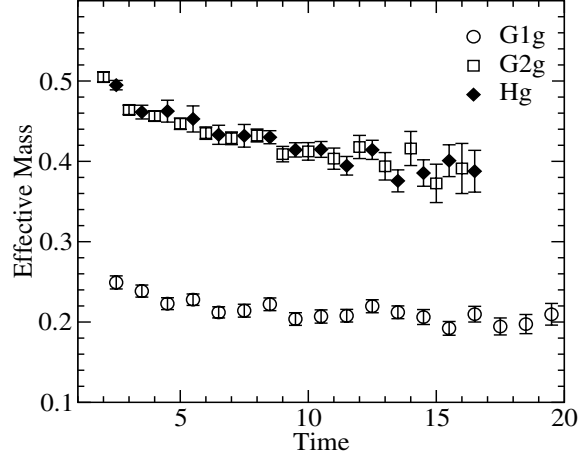


Figure 3. Ground-state effective masses for the each of the IRs G_{1g} , H_g and G_{2g} , where the subscript g denotes positive parity[28].

to obtain operators which transform irreducibly under all lattice rotations and reflection symmetries:

$$B_i^{\Lambda\lambda F}(t) = \frac{d_\Lambda}{g_{O_h}} \sum_{R \in O_h} D_{\lambda\lambda}^{(\Lambda)}(R)^* U_R B_i^F(t) U_R^\dagger, \quad (3)$$

where Λ refers to an O_h IR, λ is the row in the IR, g_{O_h} is the number of elements in O_h , d_Λ is the dimension of the Λ IR, $D_{mn}^{(\Lambda)}(R)$ is a Λ representation matrix corresponding to group element R , and U_R is the quantum operator which implements the symmetry operations.

An initial, exploratory study exploiting this formalism has recently been performed[28], at only a single value of the quark mass around that of the strange quark, and at a single value of the lattice spacing $a \simeq 0.1$ fm. Figure 3 shows the effective masses, defined by

$$M_{\text{eff}}(t) = \ln C(t)/C(t+1)$$

for the lowest-lying states for each of the three, positive-parity IR's. The apparent coincidence of the effective masses in the G_{2g} and H_g channels, containing spins $J = 5/2, 7/2, \dots$ and $J = 3/2, 5/2, \dots$ respectively, suggests that the usual identification of a mass extracted from H with the mass of a state of spin $3/2$ is somewhat premature. Indeed, experimentally the lowest-lying $I(J^P) = 1/2(5/2^+)$ state, the $N(1680) F_{15}$, is comparable in mass with the lowest-lying $I(J^P) = 1/2(5/2^+)$ state, the $N(1720) P_{13}$. Further lattice studies are essential to definitively identify the quantum numbers of baryon states, requiring the determination of the masses of several states in each channel, and the behavior of these masses in the approach to the continuum limit.

I have emphasized that determinations of the spectrum can provide insight into the structure of states, as well as their masses. Thus future lattice studies of baryons will need to include not only the simple three-quark fields introduced above, but also interpolating fields sensitive to excited glue ('exotics'), to molecular states, and, most saliently in this workshop, pentaquark or multi-quark states[29]. Perhaps most importantly, these states become unstable with decreasing pion mass, requiring more sophisticated, and computationally demanding, analysis.

4. Structure functions and generalized parton distributions

A direct probe of the structure of nucleons is provided through the measurements of form factors, structure functions, and generalized parton distributions. The parton distribution

functions measured in deep inelastic scattering (DIS) contain information about the fractional momentum carried by partons in the infinite-momentum frame. Generalized parton distributions (GPD's)[30, 31, 32] extend this description to include information about the transverse distribution of the partons within a nucleon, and furthermore provide a formalism that encompasses both DIS and the elastic form factors. Experimental information about GPD's comes from Deeply Virtual Compton Scattering (DVCS) that forms an essential component of the current and future JLab program.

These distributions are related to the matrix elements between nucleon states of the (non-local) light-cone operator

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi} \left(\frac{-\lambda n}{2} \right) \gamma \cdot n \mathcal{P} e^{-ig \int_{-\lambda/2}^{\lambda/2} d\alpha n \cdot A(\alpha n)} \psi \left(\frac{\lambda n}{2} \right), \quad (4)$$

where n is a unit vector along the light code. The use of a Euclidean lattice precludes the direct measurement of the matrix elements of these operators, and hence of a direct determination of the functional form of the structure functions and GPD's. However, the moments of the distributions with respect to the light-cone momentum fraction can be related to the matrix elements of local operators. For the flavor-non-singlet, unpolarized distributions these are

$$\mathcal{O}_q^{\{\mu_1 \mu_2 \dots \mu_n\}} = \bar{\psi} \gamma^{\{\mu_1} i D^{\mu_2} \dots D^{\mu_n\}} \psi. \quad (5)$$

In the case of DIS, the moments are simply related to the forward matrix elements:

$$\langle PS | \mathcal{O}_q^{\{\mu_1 \mu_2 \dots \mu_n\}} | PS \rangle = \int_0^1 dx x^n (q(x) + (-1)^{n+1} \bar{q}(x)) P^{\{\mu_1} \dots P^{\mu_n\}}, \quad (6)$$

where P and S are the nucleon momentum and spin respectively.

The off-forward matrix elements, between hadrons with momenta P and P' , are related to the moments of the generalized parton distributions $H(x, \xi, t)$ and $E(x, \xi, t)$, where $\Delta = P - P'$, $t = \Delta^2$ and $\xi = -n \cdot \Delta/2$. Specifically, for the case $\xi = 0$

$$\begin{aligned} \int dx x^{n-1} H(x, 0, t) &= A_{n,0}(t) \\ \int dx x^{n-1} E(x, 0, t) &= B_{n,0}(t). \end{aligned} \quad (7)$$

In particular, the lowest moments correspond to the familiar form factors $A_{10}(t) = F_1(t)$ and $B_{10}(t) = F_2(t)$. Analogous expressions, $\tilde{E}(x, \xi, t)$ and $\tilde{H}(x, \xi, t)$ together with the corresponding moments, can be constructed in polarized case.

The first lattice calculations were detailed in references [33] and [34]. These calculations, performed in full QCD, employed quark masses in the region of the strange quark mass. Nevertheless they allowed important insight into the structure of the nucleon, such as the angular momentum carried by the quarks within the nucleon, and the transverse distribution of quarks as $x \rightarrow 1$. A compendium of the results is given in the talk of Dru Renner[35].

In this talk, I will give a simple example of the insight into nucleon structure accessible through lattice QCD. The quark distribution can be considered in a representation in which the longitudinal structure is described in terms of the momentum fraction x whilst the transverse structure is described in terms of an impact parameter \vec{b}_\perp , corresponding to the transverse displacement of the quark relative to the ‘‘center’’ of the nucleon[38]. Thus we have

$$\begin{aligned} H(x, 0, \Delta_\perp^2) &= \int d^2 b_\perp q(x, b_\perp) e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp} \\ A_{n,0}(-\Delta_\perp^2) &= \int d^2 b_\perp \int dx x^{n-1}. \end{aligned} \quad (8)$$

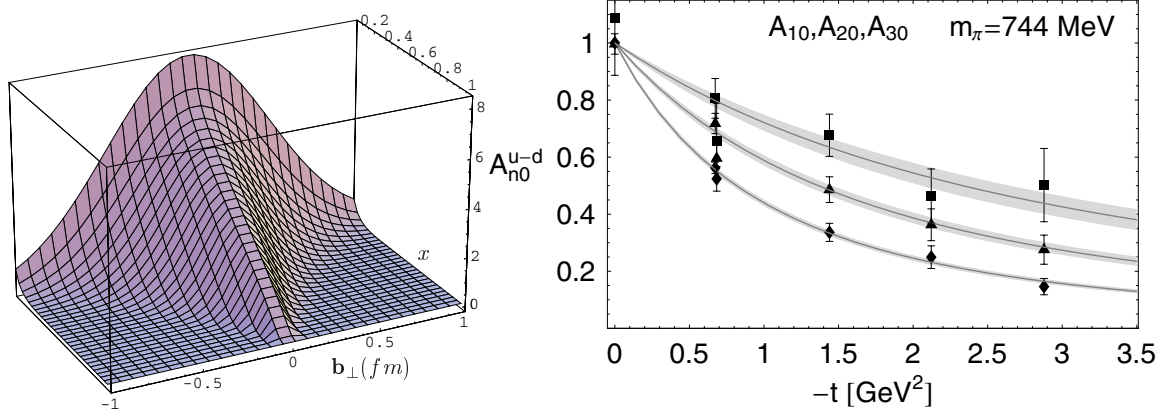


Figure 4. The left-hand plot shows the impact-parameter-dependent parton distribution within a simple model[36]. The right-hand plot shows the flavor-non-singlet moments as discussed in the text[37].

The expectation is that, in the limit $x \rightarrow 1$, where the quark carries an increasing fraction of the longitudinal momentum, the transverse distribution should become narrower. The higher moments in eqn. 7 are sensitive to increasingly larger values of x , and thus we would expect a correspondingly milder fall-off with $-t$. The lattice data do indeed support this picture, as illustrated in Figure 4; note that only moments differing by two correspond to moments of the same physical quantity.

The need to extend these calculations to physical values of the light quark masses is very apparent. A long-standing problem for lattice studies of structure functions was the apparent disagreement between lattice determinations of the momentum fraction carried by valence quarks in a proton, and the phenomenological values. An important development was the realization that this discrepancy could be resolved by correctly including the non-analytic behavior in the approach to the chiral limit[39, 40].

The first steps in this program have been taken by the LHP Collaboration[41], adopting a hybrid approach of computing domain-wall valence quarks on dynamical configurations generated using $2 + 1$ flavors of Asqtad sea quarks[42]; this has enabled pion masses as light as 360 MeV to be attained. The first results are already encouraging, in that the value for the nucleon axial charge, g_A , a quantity known to be somewhat sensitive to finite-volume effects[43], at the lightest value of the pion mass, and on the largest volumes, approaches the experimental value, as shown in Figure 4.

Future work as part of this project will include the computation of the higher moments of the GPD's, as well as quantities such as the N to Δ transition form factors, providing information on the deformation of the nucleon beyond that accessible through measurements of nucleon quantities alone. The next generation of calculations will employ a fully consistent formalism between the sea and valence quarks, and extend the range of accessible quantities to include flavor-singlet distributions.

5. Conclusions

In this talk, I have tried both to review recent lattice QCD calculations of hadron spectroscopy and structure, and to indicate future directions. The ground-breaking theoretical developments enabling computations at light quark masses are being matched by commensurate computational resources, germinating in the US from the Department of Energy's SciDAC Initiative. Studies

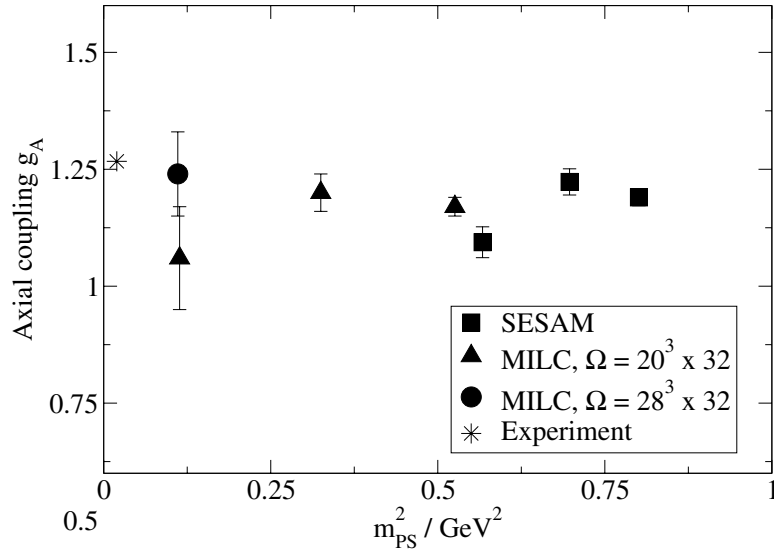


Figure 5. The nucleon axial charge, g_A , vs. m_{π}^2 showing the approach to the experimental value at decreasing pion mass and increasing volume[41].

of hadron structure are now approaching the regime where the physics of the pion cloud should be manifest. A more complete picture of the spectrum of QCD in both the meson and baryon sectors will follow the development of improved hadronic operators, together with the use of variational methods. Thus lattice studies will be in a position both to guide and interpret the exciting experimental program in hadronic physics.

Acknowledgments

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