Assesment of Multifrctality Using Levy Stability Index in Relativistic Nuclear Collisions

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Introduction

Large density fluctuations in rapidity density distributions were observed in high-energy interactions of cosmic rays [1] as well as in accelerator experiments [2, 3]. Such fluctuations may be viewed as reflection of a possible phase transition from ordinary matter to quark-gluon plasma, predicted by the QCD to occur in high energy nuclear collisions. The occurrence of non-statistical fluctuations in phase space observed in high energy hadronic collisions has generated considerable interest for investigating the mechanism of multiparticle production [1-4] in relativistic nuclear collisions.

Multifractality has recently become a major focal point of theoretical and experimental investigations in heavy-ion physics [5-6] for understanding the exact dynamics of multiprticle production. Also, relevant information about the occurrence of fluctuations in relativistic A-A collisions may be disentangled by studying the phenomenon of multifractality likely to be observed in these collisions [7-8]. The advantage of multifractal analysis lies in the fact that it allows an extension of our study to the negative moments also, whereas the factorial moments are defined for only positive integral values of the order of moments. Several authors have suggested quite different approaches to investigate the nature of the fractal dimensions. However, Hwa [5] was the first to provide an attractive formalism, based on G_q moments, for investigating multifractality. Fractals [9] are self-similar objects of non-integral dimensions and they fall into two categories: geometrically self-similar or uniform fractals and non-uniform fractals which are called multifractals.The fundamental characteristic of multifractality is that the scaling properties may be different for different regions of the system.

It may be stated that a self-similar fractal system can be characterized by an important parameter the Levy stability index μ [10]. Levy index gives an idea about the behavior of elementary fluctuations at the tail of pseudorapidity distribution. It may be stressed that μ = 0 means monofractality, μ<1 correspond to the so-called " calm singularity" while μ>1 indicates " wild singularity". Hence μ is also taken to be a measure of the degree of multifractality and it estimates cascading rate in self-similar branching process as well.

The Levy index analysis helps to classify intermittency regimes due to different kinds of phase transitions taking place during the cascading process. The conditions: (i) 0<μ<1, correspond to thermal phase transition and (ii) μ>1 correspond to nonthermal phase transition. A small but non-zero μ would not discard the possibility of quark-gluon plasma mixed with a cascading decay.

In this paper we have studied levy stability index, μ and multifractality for one dimensional charged particle density distribution using Hwa's moments for experimental, FRITIOF and HIJING generated data on 14.5A GeV/c ^{28}Si-AgBr interactions. We have calculated different parameters of multifractality. The observations are compared in view of a thermal and nonthermal phase transition and the presence of multifractal structure for both experimental and simulated data sets.

Mathematical Approach to Multifractal Analysis

As pointed out earlier, for separating the dynamical and statistical fluctuations and to study self-similar cascade process, it is necessary to suppress the effect of the statistical component. For this purpose, the modified G_q moments, as suggested by Hwa and Pan [5-6] are used:

$$G_q^m = \sum_{j=1}^{M\prime} P_j^q \, \theta(n_j - q) \qquad (1)$$

where summation is carried out over non-empty bins, M`only and $\theta(n_j - q)$ represents the usual step function which has been added to the old definition of G-moment in order to filter the statistical noise. According to the theory of multifractality, a self-similar particle emission process should exhibit a power-law behavior of the form:

$$G_q{}^{m} \propto M^{-t_q^m} \qquad (2)$$

where $t_q{}^{m}$ represents the modified mass exponent which can be obtained from Fig. 1. The dynamical component $\langle G_q \rangle$ can be determined from the following relation:

$$\langle G_q \rangle^{dyn} = \langle G_q \rangle / \langle G_q^{stat} \rangle M^{1-q} \qquad (3)$$

One can obtain from Eqs. 2 and 3

$$t_q^{dyn} = t_q - t_q^{stat} + q - 1 \qquad (4)$$

The fractal index t_q^{dyn} is related to the generalized fractal dimension D_q^{dyn} through the following equation:

$D_q^{dyn} = t_q^{dyn}/(q-1)$ (5)

The anomalous fractal dimensions, d_q is defined as

$d_q = 1 - D_q^{dyn}$ (6)

The intermittency indices ϕ_q are related to the anomalous fractal dimensions as:

$\phi_q = (q-1)\, d_q$ (7)

where q is the order of moment and we can further write intermittency index as:

$\beta_q = \phi_q / \phi_2 = (q-1)\, d_q / d_2$ (8)

β_q is related to Levy index, μ by [11]

$\beta_q = q^\mu - q / 2^\mu - 2$ (9)

Results and discussion

Displayed in Fig. 1 are the variations of $\ln\langle G_q^m \rangle$ with ln M for the experimental and simulated data. This figure shows a linear behavior of exactly the same type as predicted by Eqn. (2) indicating self-similarity. A quite similar behaviour in $\ln\langle G_q^m \rangle$ versus lnM plots for the experimental, FRITIOF and HIJING generated data are clearly visible in Fig. 1. However, it can be observed from the figure that the values of $\langle G_q^{stat} \rangle$ are relatively smaller than the corresponding values of $\langle G_q^m \rangle$, corresponding to higher values of q. The fractal exponent t_q^m of the power-law behavior is obtained by least square fitting of the data points. Similarly t_q^{stat} has been calculated for the Monte Carlo data sets and t_q^{dyn} has been calculated using Eqn. (4). With the help of t_q^{dyn} we have also calculated D_q^{dyn} using Eqn. (5). We have extracted β_q using Eqn. (8). Fig. 2 shows the variations of β_q with q for both experimental and simulated data sets using G-moment and F_q moment analysis. The data for F_q moment analysis has been obtained [12]. To determine the values of Levy index, μ we have used Eqn. (9). It is observed from the figure that the parameter β_q increases with q for all the data sets for both the technique. The errors shown in the figure are standard statistical errors. The values of μ using G-moment and F_q moment analysis for experimental, FRITIOF and HIJING generated data are found to be 1.05±0.02, 1.86±0.12, 2.23±0.1, 1.79±0.05, 1.28±0.07 and 2.31±0.17. The values of μ for HIJING generated data are > 2 for both the methods. However, the values of μ are quite different for different data sets. This means degree of multifractality β_q are different for experimental FRITIOF, and HIJING data sets. Now, μ>1 for both experimental and FRITIOF data sets for both the technique indicating the presence of nonthermal phase transition at low energy 14.5A GeV/c ^{28}Si-AgBr interactions.

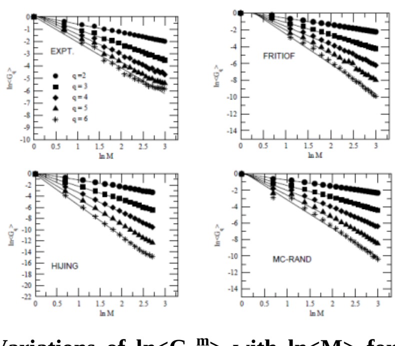

Fig. 1 Variations of $\ln\langle G_q^m \rangle$ with $\ln\langle M \rangle$ for the experimental and simulated data for ^{28}Si-AgBr interactions on 14.5A GeV/c

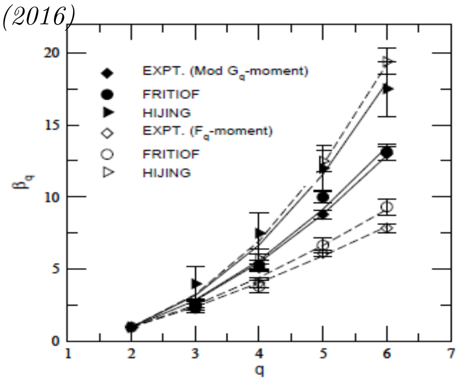

Fig. 2. Variations of β_q with q for experimental and simulated data using G-moment and F_q moment[12]

Conclusions

The values of μ obtained interms of F_q moment are gretater than unity for both experimental and FRITIOF data sets and the findings support the presence of non-thermal phase transition. The values of μ for experimental data is around 1 using G-moment analysis but considering error bar it is difficult to comment about nature of phase transtion. However, both the mehtods revelas the presence of non-thermal phase transition for experimental and FRITIOF data sets. Moreover, the different values of μ are obtained for both experimental and simulated data sets indicating the degree of multifractality, β_q are different for both the data sets using the approaches of G-moment and F_q moment.

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