INFRARED PHENOMENOLOGY OF ULTRAVIOLET PHYSICS BEYOND THE STANDARD MODEL

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Contents

Acknowledgments **iv**

1 Introduction **1**

2 Collider signals of compactification **6**

2.1 Introduction and Summary 6

2.2 Phenomenology 11

2.2.1 The photino lagrangian 11

2.2.2 Photini signatures at the LHC 15

2.2.3 Collider Bounds on String Photini 22

2.3 Cosmology of String Photini 23

2.3.1 Photino *qua* LSP 24

2.3.2 Photino decay into a nonthermal sector 25

2.3.3 Photino decay into a thermal sector 26

2.4 Light Photini from Gauge Mediation 27

2.4.1 Astrophysical signatures of light photini 30

2.5 Discussion 31

2.5.1 Origin of mixing 31

2.5.2 The scale of SUSY breaking 34

2.6 Conclusions 35

3 Goldstini Variations **36**

3.1 Introduction 36

3.2 Goldstini Masses in Supergravity 44

3.3 Warped and Sequestered Goldstini 46
3.3.1 Warped goldstini 5D 50
3.3.2 Sequestered goldstini in 4D 53
3.4 Multiple Goldstini and Modulini from ISS Sectors 56
3.4.1 ISS models at low energies 56
3.4.2 ISS modulini masses in Supergravity 60
3.4.3 Sub-leading corrections 62
3.5 Couplings and Phenomenology 64
3.5.1 Couplings in the warped/conformally-sequestered case 65
3.5.2 Distinguishing ISS SUSY-breaking 66
3.6 Conclusions 69

4 Dynamical supersymmetry breaking & flavor 71

4.1 Introduction 71
4.1.1 General strategy 73
4.2 A Simple Model 75
4.2.1 Basic scheme 75
4.2.2 Example 76
4.2.3 A landscape of simple models 79
4.3 SQCD with an adjoint 81
4.3.1 The electric theory 81
4.3.2 The magnetic dual 83
4.3.3 Metastable SUSY breaking 85
4.3.4 More general superpotential perturbations 91
4.4 Single-sector SUSY breaking 92
4.4.1 MSSM generations from composites 93
4.4.2 Sparticle spectrum 96
4.4.3 Supersymmetric flavor 99
4.5 Models 101
4.5.1 A model with approximate universality 101
4.5.2 A model with decoupling 103

4.5.3 Composite models with direct gauge mediation 106
4.6 Concluding remarks 109
4.6.1 Comparison to other explanations 109

5 Superconformal Models of Flavor **114**
5.1 Introduction 114
5.2 Flavor hierarchy from flavor anarchy 116
5.2.1 Standard Model flavor physics 117
5.2.2 Superconformal flavor physics 119
5.2.3 Constraints on superconformal sectors 121
5.3 R charges from a-maximization 123
5.3.1 Accounting for accidental $U(1)$s 126
5.4 Simple models with $SU(N)$ 128
5.4.1 SQCD with an adjoint 128
5.4.2 A ten-centered model 130
5.4.3 A more democratic model 131
5.4.4 Coupling to the adjoint Higgs 133
5.5 Simple models with $Sp(2N)$ 134
5.5.1 A ten-centered model 134
5.5.2 A more democratic model 136
5.6 Discussion 137
5.6.1 Saturating the unitarity bound 137
5.6.2 Decoupling 141
5.7 Conclusion 141

A Various goldstini variations **144**
A.1 Modulini masses from Supergravity 144
A.2 Masses to all orders in f 146
A.3 Decay widths 147

B Constraints on single-sector flavor **149**
B.1 Constraints from Flavor Changing Neutral Currents 149

B.1.1 Constraints on single-sector models 151
B.1.2 Constraints from $K^0 - \bar{K}^0$ 152
B.1.3 Constraints from other processes: $B^0 - \bar{B}^0$, $D^0 - \bar{D}^0$, $b \to s\gamma$, and $\mu \to e\gamma$ 153
B.1.4 Constraints from tachyonic stop mass 153
B.2 Unification 154
B.3 Models with less extra matter 156

C R-charges for Superconformal Flavor **159**
C.0.1 $SU(N)$ with $F = 10$ and A^3 superpotential 160
C.0.2 $SU(N)$ with $F = 10$ and A^4 superpotential 160
C.0.3 $SU(N)$ with $F = 11$ and A^3 superpotential 162
C.0.4 $SU(N)$ with $F = 10$, A^3 superpotential, and marginal coupling to Σ 163
C.0.5 $Sp(2N)$ with $2F = 10$ and A^3 superpotential 163
C.0.6 $Sp(2N)$ with $2F = 10$ and A^4 superpotential 164
C.0.7 $Sp(2N)$ with $2F = 12$ and A^3 superpotential 165

Bibliography **166**

List of Tables

5.1 Transformation properties of matter fields in SQCD with an adjoint . 129
5.2 Embedding of a ten-centered model with $F = 10$ 130
5.3 Matter content for $SU(N)$ theory with $F = 11$ 132
5.4 Meson decomposition under $SU(5)$ 132
5.5 Transformation properties of matter fields in $Sp(2N)$ with an antisymmetric tensor 134
5.6 Embedding of a ten-centered $Sp(2N)$ model with $2F = 10$ 135
5.7 $SU(5)$ embedding of a democratic $Sp(2N)$ model with $2F = 12$ . . . 136
5.8 Meson decomposition of under $SU(5)$ 136

C.1 R charges for $SU(N)$ theory with $F = 10$ flavors and cubic adjoint superpotential 160
C.2 R charges for $SU(N)$ theory with $F = 10$ flavors and quartic adjoint superpotential 161
C.3 R charges for $SU(N)$ theory with $F = 11$ flavors and cubic superpotential 162
C.4 R charges for $SU(N)$ theory with $F = 10$ flavors, cubic superpotential, and coupling to $SU(5)$ adjoint 163
C.5 R charges for $Sp(2N)$ theory with $2F = 10$ flavors and cubic antisymmetric superpotential 164
C.6 R charges for $Sp(2N)$ theory with $2F = 10$ flavors and quartic antisymmetric superpotential 164
C.7 R charges for $Sp(2N)$ theory with $2F = 12$ flavors and cubic superpotential 165

List of Figures

2.1 Different decay channels for both the LOSP decay into photini and interphotini transitions: via Z, Higgs, and sfermion. 16

2.2 The existence of multiple photini states lighter than the bino – and mixing with MSSM neutralinos via the bino – may modify MSSM cascade decay chains to the LSP. 18

2.3 The LHC signatures of multiple photini states at the LHC as a function of the photino-bino mixing ϵ_i and mass splittings δm_i. 19

3.1 A schematic example of multiple sequestered SUSY-breaking. In this setup there are two SUSY breaking sectors: The first sector is a SUSY breaking sector with one F-term of magnitude f which couples to the SSM with an effective mediation scale Λ. This sector needn't preserve an R-symmetry and could thus generate gaugino masses. The second sector is an R-symmetry preserving $SU(N_c)$ ISS sector where all non-zero F-terms are of magnitude yf, implying this sector has an overall effective-SUSY-breaking-scale of $\sqrt{N_c}yf$. The ISS sector couples to the SSM with a mediation scale of $\sqrt{x}\Lambda$. The overall effective SUSY breaking scale that determines the gravitino mass is $f_{eff} = f\sqrt{1 + N_c y^2}$. If $y \ll 1$, N_c goldstini, ζ, and $N_c(N_c - 1)$ modulini, χ, all arise from the ISS sector, as shown in Section 4, while the longitudinal mode of the gravitino dominantly arises from the first sector. 40

3.2 A diagram depicting a subset of the possible spectra. The left panel shows the SSM LOSP, the gravitino and goldstini/modulini from two ISS sectors, one at the end of a warped throat (so with mass spectrum at $2f_\omega/\omega$ as shown in Section 3.1), and one just gravitationally sequestered from the SSM. The right panel shows a possible spectrum where the SSM LOSP is lighter than $2m_{3/2}$, but, however, could still decay to a goldstino originating in a conformally sequestered (or warped) sector, here chosen not to be of ISS type, so there is only a single goldstini state, and no modulini. An interesting variant of this scenario occurs if the anomalous dimension of the SUSY-breaking field satisfies $\gamma_X > 1$, in which case LOSP decays could occur to a goldstino which is lighter than the gravitino. The resulting collider and cosmological phenomenology can depend strongly on which of these patterns is realised. Unlike for the various goldstini, decays to the modulini of a hidden ISS sector depend on the couplings in that sector, and are thus not guaranteed. Three different sectors are shown in order to elucidate a range of possibilities, though any number ≥ 2 of independent SUSY-breaking sectors implements the goldstini scenario. 42

3.3 LOSP decays to the gravitino, goldstini and modulini of a hidden ISS sector. The modulini masses are bounded below by $2m_{3/2}$ and the observation of such a decay pattern would provide strong support for the physical realisation of the ISS mechanism of SUSY breaking. . . . 68

5.1 Cartoon of energy scales. We assume the gauge group G is approximately conformal in the energy range $\Lambda_* < \mu < \Lambda$; that the fields charged under G decouple from the Standard Model around $M_{GUT} \sim \Lambda_*$; and that any Standard Model Landau poles lie at a scale $\Lambda_L > \Lambda$ (and ideally $> M_{Pl}$). In principle, fields charged under G may be responsible for breaking supersymmetry at a scale M_{SUSY}. 121

B.1 Constraints on first and second generation sfermion masses. Light gray region ruled out by $K-\bar{K}$-mixing. (a) Dark gray region ruled out by tachyonic stops at the weak scale ($m_{\tilde{t}}(1\,\mathrm{TeV}) < 0$). We assumed $m_{\tilde{g}} =$ 500 GeV, $m_{\tilde{t}}(100\,\mathrm{TeV}) = 1$ TeV. Note that the stop mass constraint disappears completely for the $m_{\tilde{f}_1}$ and $m_{\tilde{f}_2}$ mass range shown when $m_{\tilde{t}}(100\,\mathrm{TeV}) \sim 3$ TeV, which is roughly the expected value in the models under consideration. (b) Dark gray region ruled out by stops being too light at the weak scale to give a Higgs mass above LEP limits ($m_{\tilde{t}}(1\,\mathrm{TeV}) < 1000$ GeV, assuming trilinear coupling is negligible). We assumed $m_{\tilde{g}} = 500$ GeV, $m_{\tilde{t}}(100\,\mathrm{TeV}) = 1.6$ TeV. 155

Chapter 1

Introduction

The unreasonable effectiveness of the Standard Model as an explanation for physics below the weak scale is legendary; its theoretical architecture and ~ 20 parameters suffice to describe most measured particle physics observables with great accuracy. At the same time, the motivations for pushing beyond its framework are manifold. There is a pressing need to explain both the origin of Standard Model parameters themselves and the nature of observed phenomena lying beyond their scope. The motives are both theoretical and empirical in character; theoretical considerations motivate us to seek solutions for, e.g., the hierarchy problem (namely, the many orders of magnitude separating the scales of gravity and electroweak symmetry breaking); the smallness of the strong CP angle; the structure of Standard Model flavor; and the apparent unification of gauge couplings. Empirical observations compel us to explain, among other things, the measured degree of baryon asymmetry; the nature and structure of dark matter; and the small (but finite!) value of the cosmological constant. And then, of course – in addition to all this – there is the failure of the Standard Model to incorporate a theory of quantum gravity, the apparent non-renormalizability of which poses exceptional theoretical challenges. The energy scales at which solutions to these problems may arise range from the very low (around 4×10^{-3} eV in the case of the cosmological constant) to the very high (around the Planck scale, $\sim 10^{19}$ GeV, in the case of quantum gravity).

Many of these motivations for physics beyond the Standard Model may ultimately be addressed by the same tools of quantum field theory that describe the Standard Model itself. But matters are thornier for – at the very least – the hierarchy problem, the cosmological constant problem, and the quantum gravity "problem", the answers to which seem to require not merely new applications of field-theoretic tools, but rather the genesis of an entirely new theoretical architecture. Perhaps the most promising developments in this direction over the last several decades have been string theory and its natural descendent, supersymmetry (SUSY). String theory provides, to date, our best hope of formulating an ultraviolet-complete theory of quantum gravity. But consistent critical string theories must be formulated in ten dimensions; reproduction of the four-dimensional world of our observations requires that the six "extra" dimensions be compactified.[1] If string theory is responsible for the structure of the Standard Model or the resolution of the cosmological constant problem, the compactification manifold must necessarily be topologically complex. Such topological complexity may give rise to additional fields and sectors beyond the Standard Model, providing a window into physics of the ultraviolet even when the compactification scale is very high.

Supersymmetry – a symmetry relating bosons and fermions – is a natural consequence of string theory that may persist at energies well below the scale of compactification (although it is possible to have supersymmetry without evident string theory, and string theory without evident supersymmetry). In the case of the Standard Model, such supersymmetry would be manifest in the form of supersymmetric partners for all known particles. The nonobservation of supersymmetric partners at presently-explored energies suggests that supersymmetry must be broken at or above the weak scale. If supersymmetry is spontaneously broken near the weak scale, it would provide a natural resolution to the hierarchy problem by ensuring cancellation between divergent contributions to the Higgs mass. The masses of supersymmetric partners would likewise lie naturally at the weak scale, making evidence of supersymmetry apparent at the LHC. As such, weak-scale supersymmetry is one of the most attractive candidates for physics beyond the Standard Model. But even apart from

[1] Of course, one may also consider non-critical string theory in fewer than ten dimensions, but this lies somewhat outside our scope of consideration.

its potential role in stabilizing the Higgs mass, supersymmetry enables the study of strongly-coupled or otherwise intractable dynamics owing to a rich structure of dualities and exact results – making it a tremendous tool of both phenomenological and theoretical appeal.

Much of the physics entailed by string theory and supersymmetry is expected to lie at very high energies, well beyond the scales probed by colliders or ancillary experiments. If we are to have any hope of understanding the fundamental structure of reality, it is therefore instrumental to look for new ways in which the dynamics of inaccessible energies may be made apparent at more accessible ones. In this thesis, we will explore ways in which ultraviolet physics motivated by the shortcomings of the Standard Model may give rise to new signatures and phenomena in the infrared. We will find that physics at high energies often leaves its imprint – sometimes directly, often obliquely – on observables at lower energies, from which we may extract signals accessible at the LHC and other experiments.

In Chapter 2, we will study a potential (weak-scale) observational consequence of string theoretic realizations of the Standard Model. String theories with topologically complex compactification manifolds suggest the simultaneous presence of many unbroken $U(1)$ gauge symmetries without any light matter charged under them. The gauge bosons associated with these $U(1)$'s do not have direct observational consequences. However, in the presence of low energy supersymmetry the gauge fermions associated with these $U(1)$'s, the so-called "photini", mix with Standard Model gauginos and extend the MSSM neutralino sector. This leads to novel signatures at the LHC. Observation of a plenitude of photini at the LHC would be evidence that we live in a string vacuum with a topologically rich compactification manifold. This work was done in collaboration with Asimina Arvanitaki, Savas Dimopoulos, Sergei Dubovsky, and John March-Russell; and was published in [21].

In Chapter 3, we will study a related scenario involving a multiplicity of observable fields arising from realistic string compactification. Topologically complex compactification manifolds imply the existence of multiple sectors beyond the Standard Model, many of which may independently inhabit nonsupersymmetric minima. Multiple independent supersymmetry breaking sectors lead to multiple pseudo-goldstone fermions – would-be "goldstinos" – changing both supersymmetric collider phenomenology and cosmology. Motivated by considerations arising from the complexity of realistic string compactifications, we argue that many of the independent SUSY-breaking sectors should be conformally sequestered or situated in warped (Randall-Sundrum-like) throats, in which case significant changes to the previous $2m_{3/2}$ prediction for goldstini masses arise. If the sequestered hidden sector is, as is likely, a metastable SUSY-breaking sector of the general Intriligator-Seiberg-Shih (ISS) class then multiple goldstini can originate from within a single sector, along with many supplementary 'modulini', all with masses of order $m_{3/2}$. All of these fields couple to the MSSM through the 'Goldstino Portal'. Collider signatures involving SSM sparticle decays can provide strong evidence for both warped-or-conformally-sequestered sectors, and of the ISS mechanism of metastable SUSY breaking. Along with photini, the Goldstino Portal gives another potential window to the physics of the hidden sectors of string theory. This work was done in collaboration with John March-Russell and Matthew McCullough, and has been submitted for publication; it appears in preprint form in [47].

In Chapter 4, we shift gears somewhat to consider what the implications of supersymmetry and extended dynamics might be for Standard Model flavor. One exceptionally attractive picture for physics beyond the Standard Model arises if the fermions of the Standard Model are, in fact, composites of a strongly-interacting sector whose dynamics spontaneously breaks supersymmetry. Such single-sector compositeness addresses both Standard Model and supersymmetric flavor problems, as well as the origin and mediation of supersymmetry breaking. Using the tools of metastable SUSY breaking, we construct calculable models with low-energy supersymmetry where the flavor hierarchy is generated by quark and lepton compositeness, and where the composites emerge from the same sector that dynamically breaks supersymmetry. The observed pattern of Standard Model fermion masses and mixings is obtained by identifying the various generations with composites of different dimension in the ultraviolet. These "single-sector" supersymmetry breaking models give rise to various spectra of soft masses which are, in many cases, quite distinct from what is commonly found in models of gauge or gravity mediation. This work was done in collaboration with Rouven Essig, Sebastian Franco, Shamit Kachru, and Gonzalo Torroba; and was published in [46].

In Chapter 5, we study a closely related set of ideas, in which the Standard Model fermions are all elementary but couple to a supersymmetric, near-conformal sector over a range of energies. The observed hierarchy of fermion masses and mixings may be generated by renormalization group flow induced by these couplings. If the conformal sector is supersymmetric, these effects are rendered calculable by a combination of superconformal symmetry and a technique called a-maximization. The viability of such models depends on whether they generate the observed fermion mass hierarchy before the Standard Model gauge couplings hit a Landau pole. We construct a variety of simple vector-like models of superconformal flavor, including both ten-centered and democratic variations. We discuss in detail the subtleties of applying the a-maximization procedure to determine anomalous dimensions of Standard Model fields, and ultimately we find that a wide range of models based on $SU(N)$ or $Sp(2N)$ SQCD with fundamental and adjoint matter are viable theories of superconformal flavor. This work has been submitted for publication; it appears in preprint form in [45].

Chapter 2

Collider signals of compactification

2.1 Introduction and Summary

String theory is a mathematically successful and beautiful theory of quantum gravity. However, as is natural to expect for any theory of quantum gravity given the enormous value of the relevant energy scale, $M_{Pl} \simeq 10^{19}$ GeV, testing string theory at experimentally accessible energies is challenging. Two major qualitative predictions of string theory are supersymmetry (SUSY) and extra spatial dimensions. The weak hierarchy problem suggests that at least one of these phenomena may be accessible to observations at TeV energies.

The discovery of large extra dimensions at the LHC would certainly open a spectacular window into string dynamics. Here we concentrate on a more challenging scenario in which the weak hierarchy problem is solved by low energy SUSY, but the size of extra dimensions is very small (for instance, Planck or GUT scale). Although the very discovery of low energy SUSY would provide strong support for the string framework, it is natural in this case to ask whether further evidence for string theory at low energies may exist.

The usual characteristic signature of extra dimensions—excited Kaluza–Klein (KK) modes—is unavailable for small extra dimensions, as massive KK modes are too heavy to be produced. However, realistic string theory constructions typically result in extra-dimensional manifolds with rich and non-trivial topology. One way to characterize the topological complexity of the manifold is by enumerating closed sub-manifolds(cycles) of different dimensionality that cannot be deformed one into another—the so-called (co)homology classes. This is a natural generalization of the way in which orientable closed two-dimensional surfaces can be characterized by the number of handles. Realistic compactifications in string theory typically involve manifolds with a large number of cycles—from several hundreds to 10^5. The reason for this is simple combinatorics—generically there are many non-equivalent ways to embed a lower dimensional surface in a reasonably non-trivial six-dimensional manifold. For instance, the simplest Calabi–Yau space—a six torus—has six 1- and 5-cycles, fifteen 2- and 4-cycles, and twenty 3-cycles.

Interestingly, the topological complexity of a compactification manifold leaves imprints in the spectrum of KK zero modes, even if the size of the extra dimensions is arbitrarily small. To understand how this happens, let us recall another intrinsic feature of string theory: the presence of a new kind of gauge field, in the guise of antisymmetric tensor fields (forms) of various rank. In four space-time dimensions an antisymmetric second rank tensor (2-form) $B_{\mu\nu}$ is equivalent to a massless scalar field, while higher rank forms are non-dynamical. This changes in higher dimensions, where higher rank antisymmetric tensor fields can be both dynamical and different from the scalar (0-form), and vector (1-form) fields. Higher rank antisymmetric forms play a crucial role in the Green–Schwarz mechanism for anomaly cancellation in string theory and are related to the presence of extended objects in the theory such as strings and branes. Just as a vector field is coupled to the world-line of a charged particle, higher rank forms are coupled to the world-volumes of extended objects. Of particular interest in what follows are the Ramond–Ramond (RR) forms $C_{2,4}$ of type IIB theory of rank 2 and 4, or $C_{1,3}$ of type IIA theory of rank 1 and 3; the extended objects charged under these fields are D-branes [142].

The crucial property of antisymmetric tensor fields is that upon compactification they give rise to *many* KK zero modes, labeled by the independent cycles of the internal manifold [84]. Interestingly, the number of zero modes depends only on the topology of extra dimensions, but not on their absolute size. Indeed, zero modes are scale free, so that their number cannot depend on a dimensionful parameter.

Consequently, zero modes provide a probe of extra dimensions even in the limit where their size is tiny. The discovery of a large number of particles with similar properties whose presence is hard to motivate within a strictly 4-dimensional theory would be evidence for the existence of extra dimensions with complicated topology[1]. For instance, every independent 4-cycle Σ_i^4 in type IIB string theory gives rise to an ultra-light[2] (pseudo)scalar field with axion-like couplings, defined as an integral of C_4 over the 4-cycle. More generally, every independent n-cycle gives rise to a scalar KK zero mode in the presence of a rank n form.

One of these pseudoscalar fields may play the role of the QCD axion [152], while others may be observed by a number of cosmological and astrophysical experiments in the next decade [22]. It is worth keeping in mind that these string axions may acquire a high-scale mass in a number of ways (e.g., due to the presence of branes wrapping the corresponding cycles or from fluxes; they may also be projected away by orientifold planes). However, the strong CP problem suggests that at least one of these fields survives at low energies. Given the large number of independent cycles on a typical compactification manifold, it would be strange if only one of them gave rise to the light axion, thereby leading to the expectation of a plenitude of ultra-light axion-like particles— the "string axiverse" [22].

String axions are (pseudo)Goldstone bosons and cannot have any renormalizable couplings with the fields of the Standard Model. All their interactions are suppressed by the compactification scale, so there is no opportunity to observe string axions in conventional collider experiments. However, these string axions are not the only matter suggested by a topologically-complex compactification manifold. The main point of the current paper is that in the string axiverse with low energy SUSY it is natural to expect another plenitude of particles with weak scale masses that *can* be observed at the LHC.

The reason is that an antisymmetric form of rank n gives rise also to massless vector fields, labeled by the independent cycles of dimension $(n-1)$. As in the scalar case, these vectors are defined as integrals of the form over the corresponding cycle.

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[1] Replication of the Standard Model generations may already be a hint supporting this logic.

[2] Being massless at the perturbative level, these fields acquire a mass due only to non-perturbative effects.

For instance, in type IIB theory each of the 3-cycles Σ_i^3 makes it possible to define a 4d vector field

$$A_\mu^i = \int_{\Sigma_i^3} C_4 \tag{2.1.1}$$

by taking three of the four-form indices along the directions of the cycle. Moreover, each 4d vector field A_μ^i inherits a gauge symmetry from the underlying 10d abelian gauge symmetry of the RR field $C_4 \to C_4 + d\Lambda_3$, so that the end result is a plethora of 4d $U(1)$ gauge fields.

As with string axions, these fields may acquire a high mass from fluxes, or may be projected away by orientifold planes. However, as before, there is no reason for this to occur with all such vector fields and it is therefore natural to expect a plenitude of massless $U(1)$ fields in the string axiverse. It is interesting to note that essentially the same ingredients—cycles and form fields—give rise to the string landscape of vacua that motivates fine-tuning of the vacuum energy. Successful scanning of the vacuum energy suggests the presence of at least several hundreds of cycles (giving rise to the famous $10^{\text{few}\times 100}$ vacua of string theory), thus providing an additional motivation for the plenitude of axions/photons.

An important property of the string RR $U(1)$ fields is that there are generically no light states charged under them. The reason is that the only objects charged under RR forms are non-perturbative D-brane states, so that particle states charged under RR $U(1)$'s arise from D-branes wrapping the corresponding cycles. These states have masses above the string scale apart from the exceptional case of vanishingly small cycle volumes.

As a result, at low energies the RR $U(1)$'s interact with the Standard Model fields—which themselves arise from light perturbative string states—either through higher-dimensional operators unobservable at colliders, or through renormalizable kinetic mixing terms with the hypercharge $U(1)_Y$[3]. In the presence of light particles charged under additional $U(1)$'s such kinetic mixing would be strongly constrained from astrophysics and laboratory searches for millicharged particles [54, 55, 65, 134]. However, as discussed above, such light millicharged states are absent for RR $U(1)$'s

[3] As observational consequences are the main focus of this paper, we postpone the discussion of the origin of the mixing in string theory in the RR case until section 2.5.

and these constraints do not pertain. As a consequence of the absence of light charged states, kinetic mixings with RR photons can be removed by field redefinition in the low energy theory without introducing any physical effects apart from changing the value of the hypercharge gauge coupling. Consequently, massless RR vector fields *per se* do not provide a useful observational window into extra dimensions.

However, the situation becomes significantly more interesting in the presence of low energy SUSY. In this case massless RR photons are accompanied by their light fermionic superpartners—photini. Unlike vectors, RR photini acquire masses of order the gravitino mass $m_{3/2}$ as a result of SUSY breaking. If the dominant source of SUSY breaking for the MSSM also comes from the gravity mediation, then these photini masses are of the same order as the MSSM soft masses. This is the most interesting case for the LHC, and therefore will remain our primary focus in what follows. Another possibility is that the dominant source for the communication of SUSY breaking to the MSSM comes from gauge mediation, so that RR photini are much lighter than the MSSM superpartners.

As a consequence of a non-trivial photini mass matrix, the mixing of RR photini with the bino cannot be rotated away and has observable effects as we discuss in Section 3.5. For the purposes of LHC phenomenology, the significant result of this mixing is the extension of the MSSM neutralino sector by a plenitude of new states mixed with the bino through the gaugino mass matrix. This leads to a variety of possible signatures depending on the amount of mixing and the size of the inter-photini mass splittings, including extended supersymmetric cascades with high lepton and jet multiplicities arising from inter-photini transitions; displaced vertices from Lightest Ordinary Supersymmetric Particle (LOSP) decays or inter-photini transitions; cascades ending with different photini escaping the detector leading to multiple reconstructed masses for the invisible particle; and if the LOSP is charged so it stops, out-of-time decays of the LOSP to photini, with the possibility of the produced photini varying from event to event. Combinations of these signatures can also coexist.

Finally we emphasize that these photini signatures can occur for any set of $U(1)$'s that kinetically mix with hypercharge and do not possess light charged states, not just the photini of RR $U(1)$'s [69, 98]. Such multiple hidden $U(1)$'s are not uncommon in string theory and can arise from a variety of sources–for example, isolated branes not intersecting with the branes that realize the SM sector. If the isolated brane possess only vector-like matter—the more common case—the matter can get a large positive mass–squared leaving an unbroken $U(1)$ with no surviving light charged states.

Of course, the existence of a plenitude of (possibly very) weakly coupled photini may pose challenges for conventional cosmology. In Section 2.3 we consider the potential constraints on photini masses and mixings from cosmological considerations, as well as the various means by which these constraints may be obviated. In Section 2.4 we turn to the case of light photini in theories with gauge mediated SUSY breaking. Although the prospective signatures of such light photini at the LHC are less promising, their decays may give rise to observable astrophysical signals.

2.2 Phenomenology

2.2.1 The photino lagrangian

Let us now turn to the 4d effective theory arising from kinetic mixing between visible and hidden gauge sectors. It has been well known for many years [94] that theories with multiple $U(1)$ gauge symmetries may admit kinetic mixings among the different $U(1)$'s. Consider, for simplicity, the case of two such symmetries, $U(1)_a \times U(1)_b$. For the typical case of interest, $U(1)_a$ is a visible-sector gauge symmetry such as hypercharge $U(1)_Y$, while $U(1)_b$ is some hidden-sector gauge symmetry. In the basis in which the interaction terms have the canonical form, the pure gauge part of the Lagrangian can be written as

$$\mathcal{L}_{\rm gauge} \;=\; -\frac{1}{4}\,F^{\mu\nu}_{(a)}F_{(a)\mu\nu} - \frac{1}{4}\,F^{\mu\nu}_{(b)}F_{(b)\mu\nu} + \frac{\epsilon}{2}\,F^{\mu\nu}_{(a)}F_{(b)\mu\nu}\ . \tag{2.2.1}$$

where ϵ parametrizes the kinetic mixing between the two $U(1)$s. In a supersymmetric theory, such a Lagrangian generalizes to [57]

$$\mathcal{L}_{\rm gauge} \;=\; \frac{1}{32}\int d^2\theta\ \{W_aW_a + W_bW_b - 2\epsilon W_aW_b\} \tag{2.2.2}$$

where W_a and W_b are the chiral gauge field strength superfields for the two gauge symmetries (e.g., $W_a = \bar{D}^2 D V_a$ for the $U(1)_a$ vector superfield V_a). To bring the pure gauge portion of the Lagrangian to canonical form, we may shift the hidden-sector gauge field via

$$V_b^\mu \;\to\; V_b'^\mu = V_b^\mu - \epsilon V_a^\mu \tag{2.2.3}$$

so that $W_b \to W_b' = W_b - \epsilon W_a$. This renders the gauge Lagrangian diagonal,

$$\mathcal{L}_{\text{gauge}} \;=\; \frac{1}{32}\int d^2\theta \;\{W_a W_a + W_b' W_b'\}\;, \tag{2.2.4}$$

and shifts the visible-sector gauge coupling by an amount

$$g_a \to g_a/\sqrt{1-\epsilon^2}\;. \tag{2.2.5}$$

If there are no light states charged only under $U(1)_b$, then the above field redefinition produces no change in the interactions of states charged only under $U(1)_a$. Thus the theory is relatively uninteresting in the absence of light hidden-sector charged states; the hidden sector photon decouples entirely, the only remnant being the shift in the hypercharge gauge coupling [95]. The success of supersymmetric gauge coupling unification, if assumed to be non-accidental, then indicates that $\sum_i \epsilon_i^2 \lesssim 0.01$, where the sum runs over all $U(1)$'s with which hypercharge mixes.

However, the hidden sector gaugino λ_b may not decouple so readily when supersymmetry is broken. Although the $U(1)_b$ gauge boson may be decoupled by field redefinitions, the gaugino λ_b still mixes with the visible sector via off-diagonal terms in the gaugino mass matrix [98]. These remnant interactions between hidden-sector gauginos and visible-sector states provide indications of the hidden-sector gauge symmetry even in the absence of light states charged directly under $U(1)_b$.

Motivated by the appearance of many hidden-sector $U(1)$s arising from dimensional reduction of RR forms, let us now consider n hidden-sector $U(1)$s kinetically mixed with the Standard Model hypercharge $U(1)_Y$. The gauge bosons A_μ^i mix among themselves and with the hypercharge gauge boson B_μ via kinetic mixing, while the photini $\tilde{\gamma}_i$ mix among themselves and with the bino $\tilde{B}$ via both kinetic mixing and off-diagonal terms in the gaugino mass matrix. The structure of this mixing is determined by (among other things) the details of supersymmetry breaking, the geometry of the internal manifold, and induced mixing between brane and bulk gauge supermultiplets.

The gauge kinetic terms may be rendered canonical by hidden-sector field redefinitions analogous to those discussed above. In the absence of light charged states, the canonically normalized $U(1)$ gauge fields A_μ^i and their D-terms decouple entirely. The only remnant impact on the hypercharge gauge boson B_μ is a shift in the hypercharge gauge coupling, which may have implications for unification when the mixings are large.

The interesting physics lies in the photini. Despite the decoupling of the hidden-sector $U(1)$ gauge bosons, we crucially retain mixing in the gaugino mass matrix. The mixings between the photini and MSSM gauginos are encoded in the Lagrangian terms

$$\delta\mathcal{L} \supset iZ_{IJ}\lambda_I^\dagger \partial\!\!\!/ \lambda_J + m_{IJ}\lambda_I\lambda_J \tag{2.2.6}$$

where here I, J run across the bino $\tilde{B}$ and n photini $\tilde{\gamma}_i$; the Z_{IJ} encode arbitrary kinetic mixing, while the m_{IJ} are generated when supersymmetry is broken. As with the gauge kinetic terms, the gaugino kinetic terms may be diagonalized via field redefinitions so that $Z_{IJ} \to \delta_{IJ}$ and $m_{IJ} \to m'_{IJ}$. In particular, if the kinetic terms can be made canonical by the transformation $\lambda_I \to \lambda'_I = P_{IJ}^{-1}\lambda_J$, then $m'_{IJ} = P_{IK}^\dagger m_{KL} P_{LJ}$. It bears mentioning that if the original mass mixing terms are strictly proportional to the gauge kinetic term, then the mass mixing in the canonical basis vanishes. The persistence of mixing among gauginos requires that SUSY-breaking gaugino masses are not exactly proportional to the gauge kinetic mixing matrix, which has implications for the precise mechanism by which supersymmetry is broken and communicated to the gauginos. Moreover since the final physical mixing among the gauginos depends on the mass matrix mixing, the gauge-coupling unification constraint on the amount of kinetic mixing with hypercharge does not limit the size of the mixing among gauginos.

To study the neutralino mass eigenstates, we may diagonalize the gaugino mass matrix $\mathbf{m}$ via $\mathbf{m}_D = \mathbf{f}^*\mathbf{m}\mathbf{f}^{-1}$, where $\mathbf{f}$ is a unitary matrix. The mass eigenstate neutralinos $\tilde{N}_I$ may then be written as

$$\tilde{N}_I = f_{IJ}\lambda_J \tag{2.2.7}$$

where $I, J = 1, ..., n+4$ runs over the four MSSM neutralinos and the n photini; f_{IJ} are the components of the matrix $\mathbf{f}$, and $\lambda_I = (\tilde{B}, \tilde{W}, \tilde{H}_d, \tilde{H}_u, \tilde{\gamma}_1, ..., \tilde{\gamma}_n)$ are the gauge eigenstate gauginos with canonical kinetic terms.

When mixings are large, there is no particular distinction among neutralinos; every neutralino mass eigenstate is an admixture of MSSM gauginos and photini. In the limit of small mixing, however, the neutralinos decompose into mostly-MSSM and mostly-photino states. Consequently, we may think of the $\tilde{N}_a\,(a = 1, ..., 4)$ as mostly-MSSM neutralinos, and the $\tilde{N}_i\,(i = 5, ..., n+4)$ as mostly-photino neutralinos. For the sake of clarity we will henceforth concern ourselves primarily with the case of small mixings, though in principle a broad range of hidden-visible mixings may arise.

In the limit of small mixing, the components in $\mathbf{f}$ decompose accordingly: the coefficients f_{ab} are akin to those of the conventional MSSM neutralino matrix and depend principally on the parameters $m_Z, \tan\beta, \mu, m_1, m_2$. The coefficients f_{i1}, in turn, encode mixing between the hidden-sector photini and the bino. For simplicity, we will henceforth write $f_{i1} \equiv \epsilon_i$. It is this mixing that gives rise to interactions between hidden-sector photini and the fields of the MSSM. It is important to emphasize that these ϵ_i are not identical to the original kinetic mixing terms $\epsilon_i W_i W_Y$; they incorporate additional $\mathcal{O}(m_i/m_B)$ factors from the diagonalization of kinetic terms and the gaugino mass matrix.

In the limit of small ϵ_i, the mixings between photini and the higgsinos (and wino) are of order $f_{i(2,3,4)} \simeq f_{1(2,3,4)}\epsilon_i$, and may be parametrically smaller than the photino-wino mixing by MSSM mixings. Lastly, the coefficients f_{ij} correspond to mixings among the various photini, and vary from $\sim 10^{-3} - 1$ depending on the range of cycle areas and their intersection properties.[4]

[4]The mixings among photini are dictated by the gauge kinetic coupling matrix for the RR fields; at tree level this takes the form $\mathbf{Z}_{RR} \propto \mathbf{A}\mathbf{C}^{-1} + i\mathbf{C}^{-1}$, where the matrices $\mathbf{A} = \int_{CY} \beta \wedge *_6\, \alpha$ and $\mathbf{C} = \int_{CY} \beta \wedge *_6\, \beta$ are integrals over the Calabi-Yau of the three-forms α, β comprising the cohomology

2.2.2 Photini signatures at the LHC

The mixings between the bino and hidden-sector photini give rise to interactions between the mostly-photino neutralinos $\tilde{N}_i$ and the fields of the MSSM. The LHC signatures of these interactions depend on the mixing parameters and on the photini mass spectrum. Given the absence of low energy fields charged under RR $U(1)$'s, gravity mediation is the dominant source of the photini soft SUSY-breaking masses. For the remainder of this section we will assume that gravity mediation is also the dominant source of SUSY breaking for the fields of the MSSM. In this case the photini masses are of the same order as the MSSM soft masses; this is the scenario with the richest possible phenomenology. Another plausible scenario—in which the dominant source of the MSSM soft masses is gauge mediation, so that all the photini are much lighter than the MSSM superpartners—will be discussed in Section 2.4.

Given the expected multiplicity of the photini, on statistical grounds we may expect several (or perhaps many) of them to be lighter than the Lightest Ordinary Supersymmetric Particle (LOSP). This gives rise to interesting LHC signatures due to LOSP decays into photini and subsequent interphotini transitions. For definiteness, in the formulae below we will concentrate on the scenario wherein the LOSP is an MSSM neutralino; it is straightforward to generalize to other cases. This does not bring any qualitatively new features except for the smallest values of bino-photini mixing, in which case the charge of the LOSP becomes particularly significant. For these small mixings, a neutral LOSP escapes from the detector before decaying, while a charged (or colored) LOSP may stop in the detector due to electromagnetic interactions and produce a late decay signature out-of-time with collisions.

When the LOSP is an MSSM neutralino, photini production and subsequent inter-photini decays are dominated by the following three interactions:

1. Transitions through the Z-boson via couplings of the form $\tilde{N}_I \tilde{N}_J Z$.

2. Transitions through the neutral Higgs h via couplings of the form $\tilde{N}_I \tilde{N}_J h$.

basis dual to the three-cycles [106]. These matrices generally possess off-diagonal entries with values set by the geometrical moduli of the compactification.

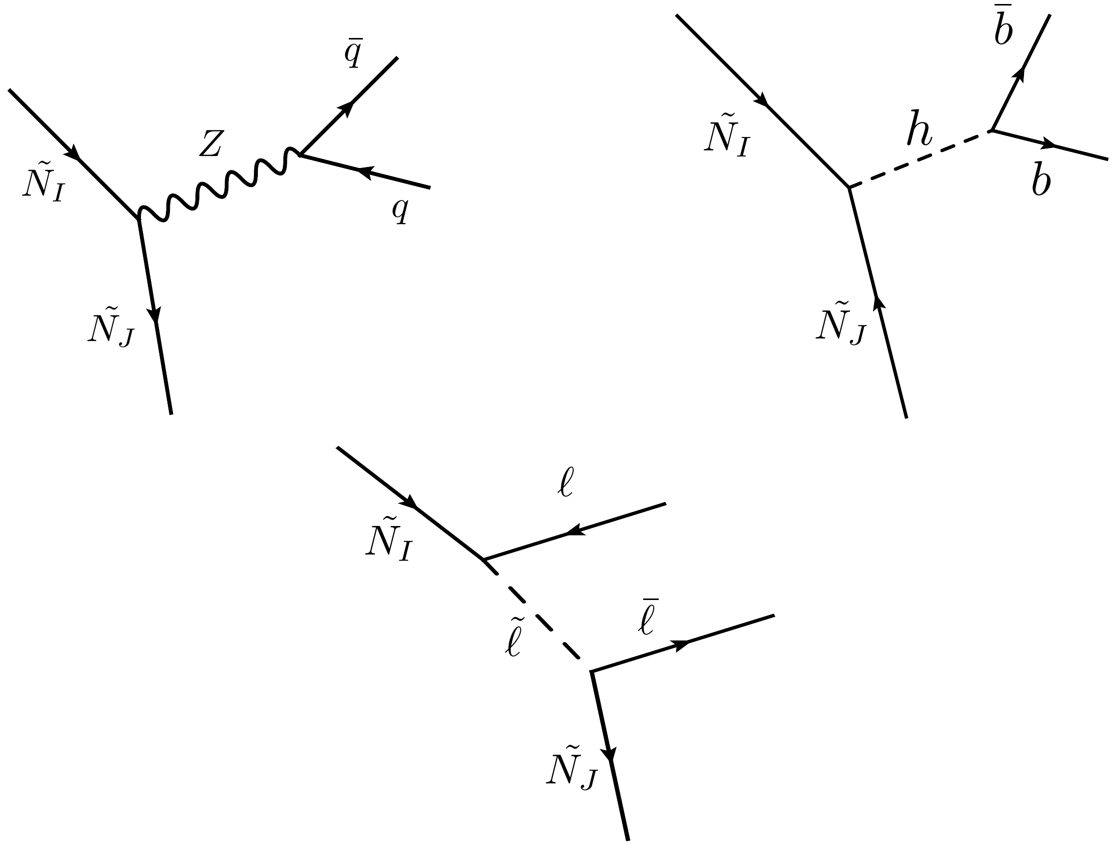

Figure 2.1: Different decay channels for both the LOSP decay into photini and inter-photini transitions: via Z, Higgs, and sfermion.

3. Transitions through intermediate squarks and sleptons via couplings of the form $\tilde{N}_I q\tilde{q}$ and $\tilde{N}_I l\tilde{l}$.

There may also be decays occurring via Standard Model photon emission, but such processes are suppressed by an additional loop factor and subdominant for a wide range of MSSM parameters [88]. Though suppressed relative to the interactions discussed above, processes involving photon emission may constitute another noteworthy signature at the LHC.

The decay rate $\Gamma^Z_{IJ}(\tilde{N}_I \to \tilde{N}_J + f\bar{f})$ via Z-boson emission is parametrically of order

$$\Gamma^Z_{IJ}(\tilde{N}_I \to \tilde{N}_J + f\bar{f}) \simeq \frac{1}{192\pi^2}\frac{\alpha_W}{c_W^2}\,|-f_{I3}f^*_{J3} + f_{I4}f^*_{J4}|^2\,\frac{(\delta m)^5}{m_Z^4}\,BR(Z \to f\bar{f}) \quad (2.2.8)$$

where $\delta m = m_I - m_J$ is the mass splitting between neutralinos (which we have assumed to satisfy $\delta m < m_Z$; for larger splittings, the Z boson is produced on-shell and two-body phase space dominates). In general, one expects decay chains ending in mostly-photino neutralinos to begin with the production of a mostly-MSSM neutralino. For the process $\tilde{N}_a \to \tilde{N}_j$, this corresponds to a lifetime of order

$$\tau_Z(\tilde{N}_a \to \tilde{N}_j) \simeq 10^{-13}\text{ s } \times \left(\frac{10^{-2}}{\epsilon}\right)^2 \left(\frac{1}{\eta}\right)^4 \left(\frac{10\text{ GeV}}{\delta m}\right)^5 \quad (2.2.9)$$

when $a = 1,2$ – i.e., $\tilde{N}_a$ is mostly-bino or mostly-wino. Here the factor $\eta \sim \mathcal{O}(m_Z/\mu) \sim \mathcal{O}(m_Z/m_{1,2})$ parametrizes the degree of mixing between MSSM gauginos, and may vary from $\sim 0.1 - 1$ depending on the size of SUSY-breaking soft masses. When $a = 3,4$ – i.e., $\tilde{N}_a$ is mostly-higgsino – the lifetime is of order $\sim \eta^2\tau_Z(\tilde{N}_{1,2} \to \tilde{N}_j)$. The lifetime for transitions $\tilde{N}_i \to \tilde{N}_j$ between mostly-photino neutralinos is similarly given by $\tau_Z(\tilde{N}_i \to \tilde{N}_j) \simeq \epsilon^{-2}\,\tau_Z(\tilde{N}_{1,2} \to \tilde{N}_j)$.

The decay rate $\Gamma^h_{IJ}(\tilde{N}_I \to \tilde{N}_J + f\bar{f})$ via the Higgs h goes like

$$\Gamma^h_{IJ}(\tilde{N}_I \to \tilde{N}_J + f\bar{f}) \simeq \frac{1}{192\pi^3}\,|-Q_{IJ}\sin\alpha - S_{IJ}\cos\alpha|^2\,\frac{(\delta m)^5}{m_h^4}\,BR(h \to f\bar{f}) \quad (2.2.10)$$

where $Q_{IJ} = \frac{1}{2}[f_{I3}(f_{J2} - t_W f_{J1}) + f_{J3}(f_{I2} - t_W f_{I1})]$, $S_{IJ} = \frac{1}{2}[f_{I4}(f_{J2} - t_W f_{J1}) + f_{J4}(f_{I2} - t_W f_{I1})]$, and α is the usual angle of rotation between the neutral Higgs mass eigenstates.

This corresponds to a lifetime for $\tilde{N}_a \to \tilde{N}_j$ of order

$$\tau_h(\tilde{N}_a \to \tilde{N}_j) \simeq 10^{-12}\text{ s } \times \left(\frac{10^{-2}}{\epsilon}\right)^2 \left(\frac{1}{\eta}\right)^2 \left(\frac{10\text{ GeV}}{\delta m}\right)^5 \left(\frac{m_h}{150\text{ GeV}}\right)^4 \quad (2.2.11)$$

for $a = 1,2$, with η as above. As before $\tau_h(\tilde{N}_{3,4} \to \tilde{N}_j) \sim \eta^2\tau(\tilde{N}_{1,2} \to \tilde{N}_j)$, and $\tau_h(\tilde{N}_i \to \tilde{N}_j) \sim \epsilon^{-2}\,\tau(\tilde{N}_{1,2} \to \tilde{N}_j)$.

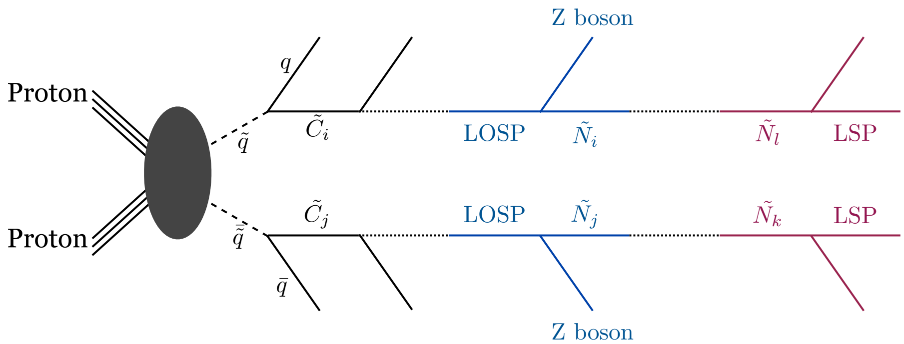

Figure 2.2: The existence of multiple photini states lighter than the bino – and mixing with MSSM neutralinos via the bino – may modify MSSM cascade decay chains to the LSP.

The decay rate via a sfermion goes like

$$\Gamma^{\tilde{l}}_{IJ}(\tilde{N}_I \to \tilde{N}_J + f\bar{f}) \simeq \frac{\alpha_W^2}{48\pi}\left| f^*_{I2} + t_W f^*_{I1} - \frac{m_l}{m_W c_\beta} f^*_{I3}\right|^2 \times \left| f^*_{J2} + t_W f^*_{J1} - \frac{m_l}{m_W c_\beta} f^*_{J3}\right|^2 \frac{(\delta m)^5}{m^4_{\tilde{f}}} \tag{2.2.12}$$

which corresponds to a lifetime for $\tilde{N}_a \to \tilde{N}_j$ of order

$$\tau_{\tilde{l}}(\tilde{N}_a \to \tilde{N}_j) \simeq 10^{-12}\ \text{s}\ \times \left(\frac{10^{-2}}{\epsilon}\right)^2 \left(\frac{10\ \text{GeV}}{\delta m}\right)^5 \left(\frac{m_{\tilde{l}}}{150\ \text{GeV}}\right)^4 \tag{2.2.13}$$

for $a = 1, 2$. In the case $a = 3, 4$, we have instead $\tau_{\tilde{l}}(\tilde{N}_{3,4} \to \tilde{N}_j) \sim \eta^{-2}\tau(\tilde{N}_{1,2} \to \tilde{N}_j)$ (unless $\tan\beta$ is large, in which case $\tau_{\tilde{l}}(\tilde{N}_{3,4} \to \tilde{N}_j) \sim \tau(\tilde{N}_{1,2} \to \tilde{N}_j)$). Transitions between mostly-photino neutralinos are again simply $\tau_{\tilde{l}}(\tilde{N}_i \to \tilde{N}_j) \sim \epsilon^{-2}\,\tau(\tilde{N}_{1,2} \to \tilde{N}_j)$. The dominant decay via sfermion exchange depends sensitively on sfermion spectroscopy; in general one expects sleptons to be lighter than squarks, thereby predominantly producing leptonic final states.

All three production mechanisms lead to parametrically similar rates. Although the decay rate via sfermion exchange is reduced at larger sfermion masses, at the same time two other channels are also being suppressed by the MSSM neutralino mixing parameter η, which is smaller for the heavier MSSM spectrum.

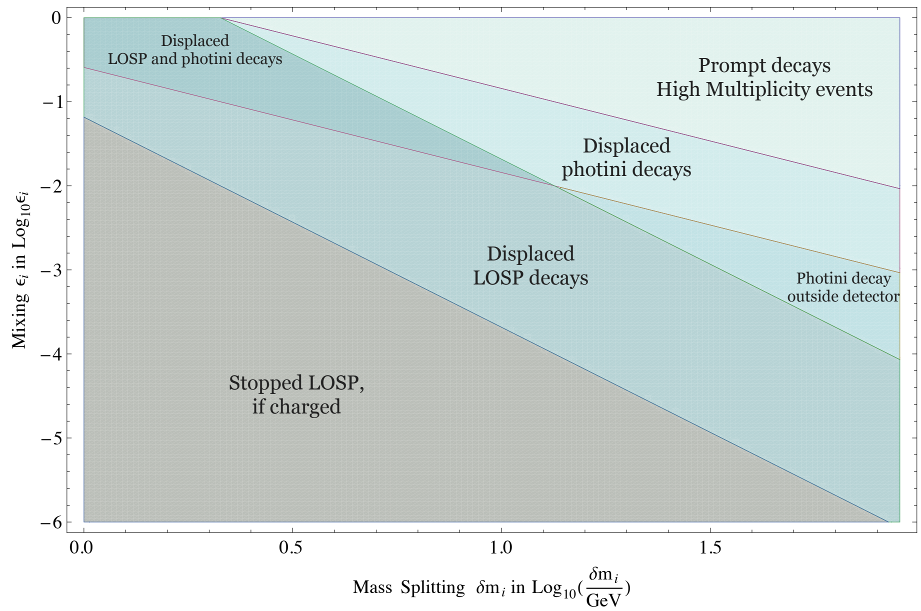

Figure 2.3: The LHC signatures of multiple photini states at the LHC as a function of the photino-bino mixing ϵ_i and mass splittings δm_i.

These processes may drastically modify MSSM particle cascades at the LHC, which no longer end at the MSSM LOSP (see Fig. 2.2). Depending on the values of the mixing parameters ϵ_i and mass splittings δm, photini give rise to several (potentially coexisting) classes of signatures illustrated in Fig. 2.3.

First, the usual supersymmetric cascades of the MSSM may both become longer and give rise to larger lepton multiplicities. These effects arise due to both decays of LOSP to photini and transitions among photini, either of which may happen promptly for large enough mixings and mass splittings. The branching ratios of Standard Model states produced during these transitions depend on which of the above-mentioned three decay channels dominates. In particular, decays via the Z or (especially) sleptons will increase the lepton multiplicity of these cascades.

Second, for smaller values of the mixing parameters and mass splittings one may see displaced vertices either from interphotini transitions, LOSP decays, or potentially both.

Furthermore, if interphotini transitions are too slow to be observed inside the detector it may be the case that multiple photini are discovered during the process of mass reconstruction. In particular, if the rates of LOSP decay into several different photini are competitive, but the rate of interphotini transitions are sufficiently slow, cascades may end with photini of different masses escaping the detector. One will then find that the observed kinematical distributions cannot be fitted by assuming a single value of the mass for the invisible particle at the end of the cascade.

For the smallest values of mixings, the signatures depend sensitively on the charge of the LOSP. Neutral LOSPs may exit the detector before decaying, resulting in rather pedestrian MSSM signatures. Charged LOSPs, however, may stop in the detector and decay out of time with collisions. As in the previous case, the interesting feature of these decays is that the mass of the invisible particle produced by late decays may vary from event to event, posing the same challenges for mass determination. This may be especially interesting for two body decays with otherwise straightforward kinematic edges, such as $\tilde{l} \to lN_i$.

It is worth stressing that some of these signatures may coexist, and the actual combination of signatures that will be observed depends on the details of the photini spectrum and mixings. For instance, if all mixing parameters are around $\sim 10^{-3}$ and the neutralino spectrum is somewhat dense, such that the mass difference between LOSP and the lightest photini is less then $\sim$ 30 GeV ("Displaced LOSP decays" region in Fig. 2.3), then one will observe both displaced vertices from the LOSP decays and cascade decays ending in a multiplicity of invisible particles with different masses.

Another interesting possibility is that the mixing is relatively large, $\epsilon_i \gtrsim 0.05$, and the mass splittings between the LOSP and some of the photini are quite substantial, $\gtrsim$ 50 GeV. This scenario – corresponding to the "Prompt decays..." region in the upper right corner in Fig. 2.3 – gives rise to longer prompt cascades, while a few of the lightest photini have smaller splittings (e.g., in the "Displaced photini decays" region in Fig. 2.3) and produce displaced vertices.

Displaced vertices from LOSP decays or interphotini transitions are likely to provide the most striking and immediate indication of multiple photini. However, to check the distinctive feature of the axiverse—photini multiplicity reflecting the topological complexity of the underlying compactification—requires accurate photini mass determination. Furthermore, in some cases the mass determination of the invisible particle(s) becomes the only way to distinguish this scenario from the MSSM at the LHC. Such is the case for, e.g., the "Photini decay outside detector" region in Fig. 2.3 corresponding to $\sim 10^{-3}$ mixing, in which the interphotini decays cannot be observed but prompt LOSP decays are assured by the significant splitting between the LOSP and photini.

The significant possibility of invisible final states with different masses motivates further development of mass determination techniques and their adaptation to cases in which the two decay chains in Fig. 2.2 are not identical. Of particular interest are chains in which the masses of the two final invisible particles are different, as would be the case if transitions between the lightest photini happen outside the detector. This generalization is straightforward for some of the existing mass-determination techniques such as the polynomial (e.g., [37, 110])) and the endpoint methods (e.g., [93]), but may require more work in other cases such as the $MT2$ method (e.g. [30, 122]) and its progeny (such as [36]). On the positive side, mass determination is more efficient for longer cascades, which are to be expected in the multi-photini scenario. Ultimately, it appears realistic to expect mass determination techniques at the LHC to distinguish photini states down to splittings of $5 \div 10$ GeV.

The LHC phenomenology of the multi-photini scenario may be spectacular, with many leptons and displaced vertices at the end of the MSSM cascades, but it requires dedicated collider study to determine how effective ordinary SUSY searches and kinematic techniques may be in determining the parameters of this new sector. In particular, care is required to distinguish the existence of a multi-photino sector from, e.g., NMSSM models with many singlinos. Only a measurement of the couplings between the different neutralino states (at the ILC, for example) will eventually reveal that photini couple through the bino, while for singlinos it is the Higgs that provides the bridge of communication to the MSSM.

2.2.3 Collider Bounds on String Photini

There appear to be surprisingly few collider bounds on the existence of light string photini. The customary LEP bound on the lightest neutralino mass comes from experimental limits on chargino masses and GUT relations between gaugino masses, the relaxation of which leaves few constraints on the mass and mixings of light neutralinos [64]. Potential bounds on the parameters of photini states can come either from precision measurements of the Z width or direct production, since there are no light states with RR $U(1)$ charge.

The best current bounds on photino production come from LEP direct search limits on processes like $e^+e^- \to \tilde{N}_1\tilde{N}_2$. The model-independent bounds from LEP OPAL searches at $\sqrt{s} = 208$ GeV constrain $\sigma(e^+e^- \to \tilde{N}_1\tilde{N}_2) \times BR(\tilde{N}_2 \to Z\tilde{N}_1) \lesssim 70$ pb for $m_1+m_2 < 208$ GeV [1]. This amounts to a relatively weak constraint on photino masses and mixings for all but the largest values of ϵ; the bounds are negligible even for $\epsilon \sim 1$ provided sufficiently heavy sleptons and small higgsino-photino mixing.

Precision electroweak observables may provide another probe of string photini. Among other quantities, the invisible Z width is a sensitive probe of additional light states. If there are N photino states that the Z boson can decay to, the contribution to its decay width is given by [92]

$$\delta\Gamma_Z \simeq \frac{G_F}{6\sqrt{2}\pi} m_Z^3 \sum_{i,j=1}^{N} (\epsilon_i\epsilon_j)^2 (f_{i4}f_{j4}^* - f_{i3}f_{j3}^*)^2 \sim \frac{G_F}{6\sqrt{2}\pi} N^2\epsilon^4\eta^4 m_Z^3 \tag{2.2.14}$$

$$\sim 0.03 \text{ MeV} \times N^2 \left(\frac{\epsilon}{0.1}\right)^4 \left(\frac{\eta}{1}\right)^4 \tag{2.2.15}$$

Given that the invisible Z decay width has been measured with an error of 1.5 MeV [10], photini states that are lighter than half the Z mass, i.e. 45 GeV, are constrained to have a combined mixing with the Higgsinos smaller than

$$\epsilon\eta \lesssim \frac{0.3}{\sqrt{N}}, \tag{2.2.16}$$

which is relevant only in the case where there are many photini lighter than $m_Z/2$

with $\mathcal{O}(1)$ mixing to the Standard Model.

Although there are many other potential constraints from existing Standard Model parameters (including, e.g., corrections to the W mass, $\sin^2\theta_W$, EDMs, muon $g-2$, and rare meson decays), such constraints are no stronger than the relatively weak bounds discussed above.

2.3 Cosmology of String Photini

The cosmological implications of multiple photini coupled to the MSSM through the hypercharge portal may be problematic (cf. [98]). Even if inflation does not reheat these states directly, they will be thermalized by MSSM interactions provided $\epsilon_i \gtrsim 10^{-6}$. If a photino is the LSP, it will generically exceed the observed dark matter relic abundance by a prohibitive amount.

A photino LSP $\tilde{\gamma}$ may freeze-out while nonrelativistic for mixings of $\epsilon \gtrsim 10^{-3}$. However, in this case the photino will be overabundant by a factor of $\sim \epsilon^{-4}$ (an unfortunate consequence of the convenient fact that weak interactions alone may produce the observed dark matter relic abundance). On the other hand, for $\epsilon < 10^{-3}$ their interactions will freeze-out while the photini are relativistic, so that the photino LSP will dominate over SM radiation at $T \sim m_{\tilde{\gamma}}$ for $10^{-6} < \epsilon < 10^{-3}$. When $\epsilon \lesssim 10^{-6}$, the photini do not reach thermal equilibrium with the MSSM, but out-of-equilibrium photino production will nonetheless overclose the universe with photini by an amount $\propto \left(\frac{\epsilon}{10^{-11}}\right)^2$. Clearly, some mechanism is necessary to dilute the photini overabundance for a vast range of mixings.[5] Of course, these constraints are far from ironclad. In what follows we will see how the challenges of photino cosmology may be overcome in a variety of ways.

[5] It is worth noting that the massless photons of these hidden $U(1)$s remain cosmologically irrelevant (provided they are not direct products of the inflaton's decay), since there are no light states charged under them and gravitational interactions alone will not lead to their overproduction.

2.3.1 Photino *qua* LSP

As observed above, the freeze-out of a nonrelativistic photino LSP generally leads to an overabundance of order ϵ^{-4}. However, it is nonetheless possible to obtain a suitable photino relic abundance from conventional freeze-out in a proscribed region of parameter space. For sufficiently large values of ϵ, coannihilation with MSSM higgsinos may lead to a freeze-out relic abundance compatible with observations.

Higgsino dark matter is well known to yield low relic abundance due to its efficient annihilation into gauge bosons and coannihilation with charginos. If the photino LSP is sufficiently close in mass to the higgsino (i.e., provided $(m_{\tilde{H}} - m_{\tilde{\gamma}})/m_{\tilde{\gamma}} \lesssim T_f/m_{\tilde{\gamma}} \sim 5\%$) it may coannihilate efficiently at freeze-out with an appreciable abundance of higgsinos. The coannihilation cross section scales as ϵ^2, and the resultant photino relic abundance is approximately $\Omega_{\tilde{\gamma}} h^2 \simeq 0.1 \left(\frac{10^{-1}}{\epsilon}\right)^2 \left(\frac{\mu}{100 \text{ GeV}}\right)^2$. A similar scenario may arise by coannihilation with the stau, again provided a correlation between masses within $\sim 5\%$.

Of course, the overproduction of photino dark matter for $\epsilon \lesssim 0.1$ may be ameliorated if the photini are themselves never in thermal equilibrium with the MSSM. Scattering processes that maintain photini in thermal equilibrium become inefficient below $\epsilon \lesssim 10^{-6}$. However, even if they are not in thermal equilibrium, an appreciable abundance of photini may still be generated via interactions of MSSM particles in the thermal bath. The resulting relic abundance from thermal production is relatively insensitive to the reheating temperature (as the photini couple to the MSSM via renormalizable interactions), and scales as $\Omega_{\tilde{\gamma}} h^2 \simeq 0.1 \left(\frac{m_{\tilde{\gamma}}}{100 \text{ GeV}}\right) \left(\frac{\epsilon}{10^{-11}}\right)^2$. Even photini that do not reach thermal equilibrium will be prohibitively overproduced by thermal production for all but the smallest mixings.

However, if the reheating temperature T_R following a period of entropy production is below the photino freeze-out temperature T_f, then the relic abundance may be significantly reduced [81]. While it is possible for the primary period of inflation to end with such low T_R, it would be difficult to account for baryogenesis or the observed cosmological density perturbations. A more palatable cosmological history might involve a second phase of weak-scale [145] or thermal [131] inflation at lower energies. Such cosmologies reconcile a low T_R with baryogenesis and density perturbations, and may be further required to resolve any additional moduli problems. The upper bound on T_R required to avoid excess thermal production is imposed by the requirement that MSSM superpartners not reach thermal equilibrium after reheating.

Of course, another possibility is simply that an MSSM neutralino is the LSP. Such a scenario is not unreasonable if the gauginos all obtain SUSY-breaking masses from a single source, since RG running may lower the masses of MSSM gauginos relative to those of the photini. In this case, all the conventional considerations for MSSM neutralino relic abundance still pertain. Such a scenario leads to unpromising photino signatures at the LHC for all but the largest values of ϵ; only for $\epsilon \gtrsim 0.1$ and $m_{\tilde{\gamma}} \sim m_{LSP}$ would the hidden-sector photini be expected to appear in sparticle cascades if a photino is not the LSP.

2.3.2 Photino decay into a nonthermal sector

The photino overabundance problem may also be ameliorated if the photini decay to a lighter R-parity odd state that was never in thermal equilibrium and does not dominate the energy density of the universe. In order for this to occur, it is necessary both for the photini to decay before their energy density dominates over radiation, and for the mass m_{LSP} of the R-parity odd particle to be sufficiently small. For $\epsilon \gtrsim 10^{-3}$ and $10^{-6} \gtrsim \epsilon \gtrsim 10^{-11}$ these requirements suggest that the lightest photino decay rate is $\Gamma > H(T_{eq}/\epsilon)$ in the former case and $\Gamma > H(T_{eq}\epsilon^{1/2}/10^{-11/2})$ in the latter case, where $T_{eq} \sim 1$ eV is the temperature at matter-radiation equality and $\frac{m_{LSP}}{m_{\tilde{\gamma}}} < \frac{1}{\epsilon^4}$ and $\frac{m_{LSP}}{m_{\tilde{\gamma}}} < \frac{\epsilon^2}{10^{-22}}$, respectively. If the decay involves any MSSM particles, the lifetime must not exceed one second in order to preserve successful BBN predictions. Finally, for $10^{-3} \gtrsim \epsilon \gtrsim 10^{-6}$ the lightest photino decouples while relativistic, so that we require $\Gamma < H(m_{\tilde{\gamma}})$ and $m_{LSP} < 0.1$ eV.

A promising candidate for such an R-parity odd particle may be an axino $\tilde{a}$ that couples to photini through interactions of the form $\frac{\alpha}{4\pi f_a}\tilde{a}\tilde{\gamma}_i\sigma_{\mu\nu}F_i^{\mu\nu}$. The mixing of the photini to the bino implies a decay channel $\tilde{\gamma}_i \to \tilde{a} + \gamma$, so that the lifetime has to be faster than 1 sec – i.e., $\frac{\alpha}{4\pi f_a} < 10^{-13}$ GeV^{-1}. The axino mass is naturally $\sim m_{3/2}$; lighter masses require a no-scale SUSY-breaking scenario that itself may be spoiled by radiative corrections. Even in the no-scale case, the very coupling that induces photino decay generates an irreducible one-loop contribution to the axino mass of order $m_{\tilde{a}} \sim \frac{m_{\tilde{\gamma}_i}}{16\pi^2}\left(\frac{\alpha}{4\pi f_a}\right)^2\Lambda^2 \sim 10^{-8} m_{\tilde{\gamma}_i}\left(\frac{\Lambda}{f_a}\right)^2$, where Λ is the smaller of the SUSY and the PQ breaking scale. As a result, the axino is unlikely to solve the photini overabundance problem for $\epsilon < 10^{-2}$.

2.3.3 Photino decay into a thermal sector

Finally, the photini overabundance problem may be solved if photini can decay before BBN into a sector (hidden or visible) that is in thermal equilibrium with the primordial plasma at the time of the decay. A hidden sector of this genre may naturally arise from a distant stack of D-branes on the compactification manifold. To make the photino decay possible it should contain, e.g., a pair of R-parity even chiral superfields $h, \bar{h}$ charged under the hidden sector $U(1)$ group. Then the photino decay will proceed through the mixing of the RR photini with the hidden sector neutralino. The μ-term that determines the mass of the fermionic components $\psi_{h,\bar{h}}$ must be in the range ~ 1 MeV$\div$1 GeV, in which case the lightest photino may decay into a scalar-fermion pair $\chi_h \to h\psi_{\bar{h}}$ through the gauge interactions. Since the scalar h is likely to acquire a significant soft mass from SUSY breaking, we also require a superpotential Yukawa interaction allowing the scalar to decay into a pair of hidden fermions. A toy example of such a hidden sector with the required properties would be a mirror sector with the MSSM field content but a somewhat smaller μ-term. The precise decay time depends on the mixing parameters, hidden sector Yukawas and scalar masses, but naturally happens before BBN since it proceeds through renormalizable interactions.

If the hidden photon is massless (or light, with a mass of order the hidden fermion masses) and mixes with the SM photon with the mixing parameter $\gtrsim 10^{-8}$, the hidden sector will remain in thermal equilibrium with the MSSM until the hidden fermions freeze out. The lower bound on the hidden fermion mass comes from the requirement that this freeze-out occurs before BBN. This scenario is just a supersymmetric version of the usual paraphoton scenario with the hidden fermions as light millicharged particles; existing bounds on millicharged particles leave a large region in the mass/mixing parameter space for this scenario to work.

The presence of the extra hidden sector may or may not have a significant impact on LHC phenomenology. An interesting scenario may arise if the rate for photino decay into the hidden sector is faster than the rate for transition between photini, while the MSSM neutralino decays preferentially into the RR photini. In this case, the displaced vertices due to the interphotini transitions are absent, but multiple missing final states with different masses still serve as a signature of the axiverse.

An equally viable scenario would involve the MSSM itself as the thermal sector, where the decay proceeds into Standard Model fermions through R-parity breaking operators. In lieu of exact R-parity, another anomaly-free discrete symmetry such as baryon triality could forbid dimension-four and -five operators leading to the proton decay while allowing the lepton-violating interactions LLE, QLD. The strongest bounds on some of the R-parity violating Yukawas in this case come from the neutrino masses at the level 3×10^{-6} (while some of the new Yukawas are practically unconstrained). Again, depending on the values of the new Yukawas and mixing parameters, these new interactions may either eliminate the LHC signatures of photini (e.g., if the new Yukawas are large and the would-be MSSM LSP decays immediately) or leave them completely unchanged (e.g., if all new Yukawas are at the level $10^{-5} \div 10^{-6}$, and the mixing between photini is at the level $\epsilon \sim 10^{-2}$).

2.4 Light Photini from Gauge Mediation

Thus far we have focused largely on theories where both MSSM fields and RR photini gain weak-scale soft masses from conventional gravity mediation. However, it is entirely possible that the primary communication of SUSY breaking to the Standard Model occurs through gauge interactions. Since the messengers of gauge mediation are assuredly not charged under the RR $U(1)$ gauge groups, gravitational effects are the sole source of photino masses and mixings; the natural value of photino masses is then $m_i \sim m_{3/2}$. Preserving the successful flavor-blindness of gauge mediation suggests that gravity-mediated contributions to soft scalar masses-squared are no more than one part in one thousand. On the other hand, sparticle mass limits require the messenger masses to exceed ~ 10 TeV. Taken together, this implies that the gravitino and photino masses in a gauge-mediated scenario may be expected to range from $m_{3/2}, m_i \sim 0.1$ eV $\div$ 1 GeV.

When the photino masses are particularly small, the bino-photino mixing is diminished further by the ratio of masses so that $f_{i1} \simeq \epsilon_i \frac{m_i}{m_B}$. This suggests that the effective mixing given by f_{i1} may be significantly smaller than the intrinsic mixing ϵ_i; for a gauge-mediated scenario, the expected range of photino masses implies $10^{-12} \lesssim f_{i1} \lesssim 10^{-2}$ for $\epsilon_i = 1$.

The LHC signatures of very light photini may differ from those of their heavier brethren. Interphotini transitions are suppressed by a factor of $\left(\epsilon \frac{m_{\tilde{\gamma}}}{m_B}\right)^4$ and thus quite unlikely to produce observable particle cascades. The same smallness of effective mixing between light photini and the MSSM does, however, increase the likelihood of displaced vertices. The lifetime for the decay of an MSSM neutralino LOSP to a light photino scales as

$$\tau(\tilde{N}_a \to \tilde{N}_i) \sim 10^{-8} \text{ s} \times \left(\frac{10^{-2}}{\epsilon}\right)^2 \left(\frac{1 \text{ MeV}}{m_i}\right)^2 \left(\frac{100 \text{ GeV}}{m_a}\right)^3 . \tag{2.4.1}$$

Such decay of a neutral LOSP in the detector will result in displaced vertices and missing energy from photini escaping the detector. In this case, however, the mass splittings among photini are far to small to be resolved with available mass resolution, so that all indications of multiplicity are lost. The lifetime may also be sufficiently long for the neutral LOSP to escape from the detector entirely before decaying, resulting in no deviations from the conventional MSSM phenomenology.

When SUSY breaking is communicated by gauge mediation, it is quite likely that the MSSM LOSP is charged – as occurs frequently with the stau for lower values of the messenger scale. The case of a charged LOSP remains exceptionally interesting for even the longest of lifetimes, as the LOSP is likely to stop in the detector due to electromagnetic interactions before decaying out of time into photini.

Naturally, the LHC signatures of light photini bear a superficial resemblance to those of a conventional gravitino LSP in theories of gauge mediation. Indeed, the rates for decays into photini and the gravitino may be competitive. The relative rates scale as

$$\frac{\Gamma(\tilde{N}_a \to \tilde{N}_a + ...)}{\Gamma(\tilde{N}_a \to \tilde{G} + ...)} \sim \frac{1}{12\pi}\frac{\alpha_W}{c_W^2}\epsilon_i^2 \frac{m_i^2 m_{3/2}^2 M_P^2}{m_B^2 m_Z^4} \tag{2.4.2}$$

where we have assumed a mostly-bino neutralino LOSP. This suggests that neutral LOSP decay to photini dominates over decays to the gravitino when $\epsilon_i \frac{m_i^2}{m_W^2} \gtrsim 10^{-15}$ (e.g., for $\epsilon_i \gtrsim 10^{-5}$ when $m_i \sim$ MeV – a wide range of parameters).

Discriminating between the two cases is largely a matter of branching ratios. For example, photon production via $\tilde{N}_a \to \gamma + \tilde{G}$ is the dominant channel for the decay of a bino-like neutralino into a gravitino, while decays involving Z or Higgs are suppressed by factors of $(1 - m_Z^2/m_a^2)^4$ and $(1 - m_h^2/m_a^2)^4$, respectively. In contrast, the decay of a bino-like neutralino into photini proceeds dominantly via the Z or Higgs, while the decay into (Standard Model) photons $\tilde{N}_a \to \gamma + \tilde{N}_i$ is suppressed by an additional loop factor.

The cosmology of such light photini is, as one might expect, somewhat delicate. Even if the intrinsic mixing ϵ is large, the effective mixing is bound to be significantly smaller. Consequently, if a photino is the LSP there is little hope of attaining an appropriate relic abundance from freeze-out. Indeed, if the photini ever achieve thermal equilibrium with the MSSM, they will generically freeze-out while relativistic and remain subject to the usual constraints on hot relics. A more likely scenario is that these photini never achieve thermal equilibrium (as is the case, e.g., for $\epsilon \lesssim 10^{-1}$ when $m_{\tilde{\gamma}} \sim$ MeV), though they may be overproduced by scattering in the thermal bath. Owing to the smallness of the effective mixing, however, the resulting relic abundance may be suitable for a far greater range in ϵ; for light photini the abundance from thermal production is approximately

$$\Omega_{\tilde{\gamma}} h^2 \simeq 1.0 \times \left(\frac{\epsilon}{10^{-3}}\right)^2 \left(\frac{m_{\tilde{\gamma}}}{1 \text{ MeV}}\right)^3 . \tag{2.4.3}$$

In cases where the photini are overproduced, the mechanisms discussed in Sec. 2.3 may still be effective, albeit at significantly lower scales. In any case, the longevity of the MSSM LOSP may also be a problematic source of late decays; for too small values of ϵ (e.g., $\epsilon \lesssim 10^{-6}$ for $m_{\tilde{\gamma}} \sim$ MeV), the LOSP decay may spoil the successful predictions of Standard Model BBN. A long-lived charged or colored LOSP would be further constrained by the CMB and heavy element searches. The decay of the gravitino itself into photini and hidden-sector photons is relatively uninteresting owing to the lightness of the gravitino, occurring with a lifetime far exceeding the age of the universe: $\tau(\tilde{G} \to \tilde{\gamma}_i + \gamma_i) \sim 10^{23} \left(\frac{m_{\tilde{G}}}{1 \text{ MeV}}\right)^3$ s. In this case, the longevity of the gravitino implies that the usual gauge mediation constraints on gravitino cosmology must be respected, even though the gravitino is not the LSP.

If, instead, the gravitino is the true LSP, very little changes; decays of the MSSM LOSP still occur preferentially into photini and Standard Model fields for a wide range of parameters. The decay of the lightest photino into a gravitino and hidden sector photon is extremely slow, also on the order of $\tau(\tilde{N}_i \to \tilde{G} + \gamma_i) \sim 10^{23} \left(\frac{m_{\tilde{G}}}{1 \text{ MeV}}\right)^3$ s. Both the lightest photino and the gravitino are cosmologically long-lived, and conventional considerations regarding their abundances and impact on Standard Model cosmology still apply[6].

Another potential cosmological bound on light photini with masses $\lesssim$ 30 MeV may come from supernova cooling via photino-strahlung. The pair-production of light photino states is highly suppressed, however, and readily satisfies constraints from SN1987a [63] due to the small effective mixing, $\epsilon \frac{m_{\tilde{\gamma}}}{m_B} \lesssim 10^{-5}$. For example, the so-called "Raffelt criterion" – that exotic cooling processes do not alter the observed neutrino signal provided their emissivity is sufficiently small – requires $\dot{\mathcal{E}} \lesssim 10^{19}$ ergs/g/s. For light photini, the emissivity of photino-strahlung via slepton exchange scales as $\dot{\mathcal{E}} \sim 10^{19} \left(\frac{100 \text{ GeV}}{m_{\tilde{e}}}\right)^4 \left(\frac{\epsilon\,(m_{\tilde{\gamma}}/m_B)}{0.1}\right)^4$ ergs/g/s, consistent with observations for $\epsilon \frac{m_{\tilde{\gamma}}}{m_B} \lesssim 10^{-1}$.

2.4.1 Astrophysical signatures of light photini

Although it is important that no decays occur around the time of BBN, transitions among light photini may be slow enough to occur on cosmologically interesting

[6]Note, however, that both in this and the previous cases, some of the constraints will be modified, because gravitinos are not being produced from the LOSP decays, that lead instead to the photini production.

timescales. The production of Standard Model particles during interphotini transitions may be observable and, moreover, well-suited to explain observed astrophysical anomalies associated with MeV-scale physics. The 511 keV excess associated with $e^+ + e^-$ annihilations recently measured by the INTEGRAL satellite [105, 114, 115] may be explained by just such transitions. It is crucial that electrons and positrons produced by the decay of a dark matter particle not be injected with more than a few MeV of energy in order to fit existing gamma ray backgrounds [32], a constraint easily satisfied by transitions among photini with masses and splittings of order $\sim$ MeV. The photon flux measured by INTEGRAL may be accounted for by a dark matter particle of mass m and abundance Ω decaying into (among other things) $e^+ + e^-$ with a lifetime [50]

$$\tau_{INT} \sim 10^{26} \left(\frac{\Omega}{0.2}\right) \left(\frac{1 \text{ MeV}}{m}\right) \text{ s.} \tag{2.4.4}$$

Amusingly, the interphotini decays via, e.g., off-shell Z emission occur with a lifetime

$$\tau(\tilde{N}_i \to \tilde{N}_j + e^+ + e^-) \sim 10^{23} \left(\frac{10^{-5}}{\epsilon}\right)^4 \left(\frac{1 \text{ MeV}}{\delta m}\right)^5 . \tag{2.4.5}$$

Consequently, transitions among light photini may account for the INTEGRAL signal with $\epsilon \sim 10^{-5}$ for $m_{\tilde{\gamma}}, \delta m \sim$ MeV, for which the abundance from thermal production is expected to be $\Omega h^2 \simeq 10^{-4}$. This assumes, of course, that the gravitino is not the LSP or that (invisible) decay rates into a gravitino LSP are slower than interphotini transitions.

2.5 Discussion

2.5.1 Origin of mixing

For hidden $U(1)$'s realized as either perturbative heterotic string states [57] or as gauge excitations of D-branes of type-II string theory [3] the kinetic mixing with $U(1)_Y$, if absent at tree level, arises by a process that directly generalizes the classic calculation of Holdom [94]. For example in the type-II case stretched open string states with one end on the SM brane stack and the other on the hidden D-brane lead to massive states charged under both $U(1)$'s, and a one loop open string diagram then, in general, generates kinetic mixing [3]. An interesting feature of the D-brane calculation is that it can also be interpreted as a tree-level exchange of a bulk closed string state between the two stacks, and both the NS-NS two–form B_2 and for Dp-branes the RR p-form C_{p-1} lead to mixing. The open string description is most useful for small separations between the stacks, while for large separation the supergravity approximation to the closed string computation is more appropriate and allows the treatment of both warped compactifications and those with fluxes. The resulting mixing is model dependent, ranging in size from $\mathcal{O}(1)$ in the case of tree-level mixing, to in the loop-generated case a one-loop factor down to exponentially suppressed values if the compactification is warped, or if the mediating fields are massive, e.g., due to fluxes.

From the effective field theory point of view kinetic mixing among $U(1)_Y$ and the RR $U(1)$ is also allowed. However, one may worry that there might be a subtlety arising from the absence of perturbative string states carrying RR charges.

Indeed, as already summarized the conventional mechanism giving rise to mixing between $U(1)$ gauge bosons relies on integrating out heavy bi-fundamental fields charged under both gauge groups. However, substantial mixing between hypercharge $U(1)_Y$ and RR gauge fields cannot be generated in this way. Indeed, as explained above, there are no states charged under RR fields with masses below the string scale. Moreover, the only states carrying RR charges are non-perturbative D-branes states, which should be thought of as solitonic states from the viewpoint of the hypercharge $U(1)_Y$ (which itself typically arises as a conventional perturbative string state). One may expect that loops involving such solitonic states are exponentially suppressed.[7]

Nevertheless, the mixing between hypercharge $U(1)_Y$ and RR gauge fields can be generated directly at the level of the string-scale supergravity effective action. One example of a situation giving rise to such a mixing was discussed in [106]. Namely, one considers a D7 brane in the type IIB theory that wraps a four-cycle with a non-contractible loop inside. In the presence of a non-trivial Wilson line along this loop a

[7] Note that in a theory with gravity there must exist Reissner-Nordstrom black holes charged under both $U(1)$'s. Once again the contribution to kinetic mixing from integrating out these bi-fundamental states is expected to be exponentially small.

mixing between the D7's perturbative $U(1)$ gauge field and RR $U(1)$ may arise from the world-volume Chern-Simons action

$$\int_{\mathrm{D7}} C_4 \wedge F \wedge F \ .$$

The size of this mixing is controlled by the corresponding Wilson loop; this term takes the form of kinetic mixing when self-duality conditions are imposed on the C_4. Perhaps an even simpler example of such kinetic mixing arising from the Chern-Simons action occurs in the case of a D5 brane wrapping a two-cycle in type IIB string theory [85]. The term of interest appears in the Chern-Simons action from the pull-back of the RR form to the world-volume of the D5 brane,

$$\int_{\mathrm{D5}} \zeta \, dC_4 \wedge F \ .$$

Here ζ is a (4d) complex scalar modulus parameterizing deformations of the D5 brane. Once again, this interaction takes the form of kinetic mixing when self-duality conditions are imposed on the C_4.

In these examples the D-branes serve as portals giving rise to a mixing between perturbative and RR gauge sectors. The D-brane may be either directly a part of the brane configuration giving rise to the Standard Model gauge group, or belong to the hidden sector and acquire a mixing with the hypercharge $U(1)$ at one loop in the conventional way.

Kinetic mixing between hidden and visible $U(1)$'s then begets mass mixing; the gaugino mass matrix descends from the gauge kinetic mixing matrix when supersymmetry is broken. Properly speaking, the full gauge kinetic matrix for both photini and MSSM gauginos depends on, e.g., complex structure moduli z_k (in the IIB case; the same role is played by Kähler moduli in the IIA case). The mass matrix arises when the complex structure moduli are replaced by their F-term expectation values, so that $m_{IJ} \propto F_{z_k} \partial_{z_k} Z_{IJ}(z_l)$. In general, the F terms of the various complex structure moduli are expected to vary, so that the mass matrix $\mathbf{m}$ is not strictly proportional to the kinetic mixing matrix $\mathbf{Z}$. Likewise, the size of mass mixings may exceed the size of kinetic mixings, so that $\epsilon \sim \mathcal{O}(1)$ mixings in the gaugino mass matrix may remain consistent with perturbative gauge coupling unification.

This discussion strongly suggests, that the effective field theory expectation is correct; there is no obstruction for the mixing between RR gauge fields and hypercharge, just as there is no obstruction for mixing between two D-brane $U(1)$'s. Nevertheless, it is worth keeping in mind that, to the best of our knowledge, there is no explicit example of string theory vacuum supporting this point. For instance, it turns out that the D7 mechanism above doesn't give rise to non-zero mixing for the toric Calabi-Yau's (the only explicitly studied example), due to apparently accidental cancelation.

We don't think this is a reason to worry that the phenomenology discussed in this paper may be disfavored—it appears likely that the lack of explicit examples is just a reflection of the well-known fact, that constructing explicit string vacua with stabilized moduli is hard. At the very least, as explained in the introduction, extra $U(1)$ factors with no light charged states may come also from hidden D-branes, rather than from RR fields. Rather, we consider this theoretical problem as a motivation for further studies of the plausible sources and sizes of the mixing.

2.5.2 The scale of SUSY breaking

Throughout this paper we have focused on the observational consequences of string photini in a conventional low energy SUSY scenario. However, such photini may also be observed at the LHC in a high-scale scenario such as split supersymmetry [15, 16, 80]. In this case, scalar superpartners are heavy and inaccessible at the LHC, but fermions remain at the weak scale due to chiral symmetries. In this scenario string photini masses are likely to remain at the weak scale due to the same R-symmetry that keeps gauginos light, rendering photini observable at the LHC. The phenomenology of split SUSY events involving the direct production of non-colored superpartners remains quite similar to the case of low energy SUSY. The main difference between photini signatures of split supersymmetry and conventional SUSY is the absence of interphotini transitions through an intermediate slepton, because all sleptons are now very heavy.

A particularly interesting feature of split SUSY is that the gluino is very long lived, since all its decay channels go through a heavy intermediate squark. This results in a spectacular signature due to delayed decays of gluinos stopped in the detector. Such a signal persists in the presence of string photini, but an interesting new feature is that the wide range of photini signatures discussed earlier – in particular, cascades and displaced vertices – may now appear out-of-time in the decays of stopped gluinos.

2.6 Conclusions

A string-theoretic universe with small extra dimensions is often thought to leave few explicit signatures at low energies – and particularly few signatures accessible at the LHC. Here we have seen, however, that the topological complexity of compactification manifolds in string theory suggests the presence of many unbroken $U(1)$'s without light charged states. Despite the decoupling of the photons associated with these $U(1)$'s, contact with the Standard Model may still arise due to mixing between the photini and MSSM bino in the presence of low energy supersymmetry. This mixing gives rise to a broad range of novel signatures at the LHC, including displaced vertices and cascade decays from both LOSP decays to photini and interphotini transitions, as well as multiple reconstructed masses for particles exiting the detector. Such signatures pose new challenges to existing techniques for event reconstruction and mass determination at the LHC. Should a plenitude of photini be observed at the LHC in this fashion, it would provide compelling infrared evidence for a topologically rich string compactification in the ultraviolet.

Chapter 3

Goldstini Variations

3.1 Introduction

If the Standard Model is UV completed by string theory – consistent with the hypothesis of supersymmetry (SUSY) – the topological complexity of realistic compactification manifolds suggests the existence of many additional sectors sequestered from the fields of the Standard Model. The dimensional reduction of form fields may result in a proliferation of light axion-like scalar fields [23, 24], or weak-scale abelian vector fields and their superpartners [21], which can dramatically alter standard cosmological, astrophysical, and collider phenomenology. Moreover, the presence of stacks of spacetime-filling branes may lead to nonabelian gauge sectors with fundamental matter in the four-dimensional theory. The mere observation that such supersymmetric nonabelian gauge theories possess metastable SUSY-breaking vacua [99] suggests that supersymmetry may be broken in these different (purely field-theoretic) sectors. Furthermore, there are numerous additional ways in which supersymmetry may be broken by intrinsically stringy objects – e.g., nonsupersymmetric flux backgrounds or the presence of both D- and anti D-branes in the compactification manifold. On a topologically complex compactification manifold with various nonabelian gauge sectors, fluxes, branes, and antibranes, it is not unreasonable to expect a rich variety of SUSY-breaking dynamics to coexist. Thus, the existence of multiple (likely metastable) SUSY-breaking sectors is not merely a theoretical novelty, but rather a well-motivated consequence of physics in the ultraviolet.[1]

Historically, however, the study of SUSY breaking and its phenomenology has focused on a single sector additional to the Supersymmetric Standard Model (SSM), whose dynamics give rise to a nonsupersymmetric ground state. Recently it has been shown [39, 40] that relaxing this assumption to include multiple sources of SUSY breaking can lead to interesting and appealing scenarios in which the conventional phenomenology of single-sector SUSY breaking is significantly modified, similar to the way in which multiple photini can also alter conventional SUSY phenomenology [21]. In this paper we wish to extend the results of [39, 40] with an eye towards the underlying physical context in which multiple SUSY breaking is likely to arise.

The mediation of this multiple-sector supersymmetry breaking to the Standard Model may occur in any of the customary ways, leading to weak-scale soft masses and the usual successes of the SSM. However, even if the fundamental interactions between these sectors and the SSM are Planck-suppressed, the multiple breaking of supersymmetry gives rise to less-suppressed couplings between additional 'goldstini' and SSM fields. In this fashion, the existence of new sectors with otherwise-unobservable couplings to the SSM may be revealed via what we may call the 'Goldstino Portal'. In this sense the goldstini and their companions are further distinguished from moduli, whose masses are likewise around $m_{3/2}$ but whose couplings are Planck-suppressed.

Specifically, in [40] it was argued that the presence of multiple sequestered sectors that break SUSY spontaneously gives rise to multiple 'goldstini' of mass $2m_{3/2}$ in addition to the true global goldstino which provides the extra degrees of freedom of the gravitino of mass $m_{3/2}$. It was further shown [39,40] that such a set-up can lead to exciting new signatures at the LHC which could confirm not only the validity of the supergravity framework but also the presence of multiple sequestered SUSY breaking sectors, providing indirect, but striking, evidence for complexity of the string compactification. Subsequently it was shown that this scenario can also lead to solutions of the cosmological problems with a heavy gravitino LSP [39].

[1] Indeed, the fact that cosmological evolution preferentially populates the metastable vacua of SQCD rather than the supersymmetric vacua [6, 48, 71] provides a strong argument that the mere existence of multiple (reheated) nonabelian gauge sectors with light fundamental matter implies the existence of simultaneous SUSY breaking in multiple sectors.

These considerations come with two caveats. The first is purely experimental; the smallness of flavour-changing neutral currents (FCNCs) and other signs of Standard Model flavour violation imply that the flavour-violating contributions to SSM soft masses from all SUSY-breaking sectors must necessarily be small. In particular, this requires that all SUSY breaking communicated via intrinsically flavour-violating mediation mechanisms such as (the non-anomaly-mediated part of) gravity mediation must be at most one thousand times smaller than that communicated via flavour-preserving mechanisms. Although this is possible if all such contributions to SUSY breaking are conveniently small to begin with, it seems much more plausible that the smallness of flavor violation arises from locality and warping [144] or conformal sequestering [126, 129]. Once again, this is a well-motivated consequence of physics in the ultraviolet. Sequestering is known to arise readily in the presence of strongly warped backgrounds such as warped throats, e.g., type IIB string theory [107], and the ubiquity of warped throats on realistic compactification manifolds is well-known [60, 78, 91]. The pairing of multiple SUSY breaking and sequestering via warped throats is suggested by more than just FCNC considerations alone; the very existence of multiple goldstini requires it as multiple *unsequestered* SUSY breaking sectors simply lead to one ur-breaking of supersymmetry. But if sequestering and multiple SUSY breaking are so closely intertwined, it is then natural to consider what implications sequestering may have on the spectrum and phenomenology of the resulting goldstini. In particular, we will argue below that warping and sequestering lead to substantial deviations from the goldstino mass prediction of $2m_{3/2}$, and that the spectrum of goldstini – and resulting collider phenomenology – are richer than previously thought.

The second consideration is largely theoretical. Weak-scale supersymmetry in the SSM favours *dynamical* means of SUSY breaking in order to explain the hierarchy between the Planck and SUSY-breaking scales [154]. In turn, dynamical SUSY breaking in general requires a SUSY-breaking sector to possess a rich set of gauge dynamics and fields. It is therefore instrumental to consider whether common classes of dynamical SUSY-breaking theories might modify or alter the goldstino spectrum, perhaps by the presence of additional light states. At the very least, the interactions required to stabilize SUSY-breaking vacua frequently alter the corresponding goldstino mass. Moreover, we will argue that it is quite common that a *single* dynamical SUSY-breaking sector gives rise to *multiple faux goldstini*. Such additional states may then couple to MSSM fields through the Goldstino Portal, and their observation would shed further light on the nature of the supersymmetry breaking sector(s).

In short: the potential observability of multiple SUSY breaking has been well established. However, it is instrumental to ask whether the additional physics that naturally accompanies multiple SUSY breaking may enrich and expand the goldstino spectrum and phenomenology.

In particular, in Section 3.2 we compute the goldstino mass for a general class of effective supersymmetry breaking Lagrangians using the conformal compensator formalism. We will find that important corrections to the goldstino mass arise from the effects of stabilizing the SUSY-breaking vacuum. In Section 3.3 we show how the presence of warping, or conformal sequestering, significantly modifies the prediction of $2m_{3/2}$ for the goldstini masses. The discovery of such modifications would be 'smoking gun' for the presence of such dynamics in one or more sequestered hidden sectors. In Section 3.4 we study the particle content of a hidden Intriligator-Seiberg-Shih-type (ISS) [99] sector preserving a (discrete) R-symmetry. We show that such a sector would give rise to N_c goldstini and $N_c(N_c - 1)$ 'modulini' of mass $\geq 2m_{3/2}$ (in the absence of warping or conformal sequestering), where N_c is the number of colours in the asymptotically free UV gauge group. A simple example of this setup is schematically illustrated in Figure 3.1. Although such a discrete-R-symmetry-preserving sector is incapable of generating gaugino masses, in the context of multiple SUSY breaking sectors this poses no problem. In particular, since there is no phenomenological reason to require more than one of the independent SUSY-breaking sectors to break R-symmetry, and requiring all independent sectors to break R-symmetry severely constrains the number of possible string landscape vacua, we expect that our results on ISS-type sectors should apply to realistic theories of SUSY-breaking in the string landscape.[2]

[2]We note in passing that such R-symmetry-preserving sequestered SUSY breaking sectors can also lead to attractive phenomenological features, such as, *e.g.*, cosmologically acceptable thermal leptogenesis [39].

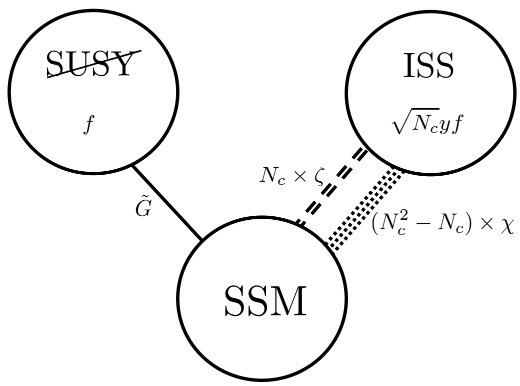

Figure 3.1: A schematic example of multiple sequestered SUSY-breaking. In this setup there are two SUSY breaking sectors: The first sector is a SUSY breaking sector with one F-term of magnitude f which couples to the SSM with an effective mediation scale Λ. This sector needn't preserve an R-symmetry and could thus generate gaugino masses. The second sector is an R-symmetry preserving $SU(N_c)$ ISS sector where all non-zero F-terms are of magnitude yf, implying this sector has an overall effective-SUSY-breaking-scale of $\sqrt{N_c} y f$. The ISS sector couples to the SSM with a mediation scale of $\sqrt{x}\Lambda$. The overall effective SUSY breaking scale that determines the gravitino mass is $f_{eff} = f\sqrt{1 + N_c y^2}$. If $y \ll 1$, N_c goldstini, ζ, and $N_c(N_c - 1)$ modulini, χ, all arise from the ISS sector, as shown in Section 4, while the longitudinal mode of the gravitino dominantly arises from the first sector.

Figure 3.2 provides a schematic illustration of the new possibilities that arise for the mass spectrum of goldstini/modulini. Such states are typically grouped into sets with a goldstino (or goldstini) at a lower limit point at $\beta m_{3/2}$ with modulini sitting relatively tightly spaced above this limit. For unsequestered or unwarped sectors, $\beta = 2$ (modulo potentially large corrections from stabilization of the SUSY-breaking vacuum as explained in Section 3.2). Otherwise, any value $2 \geq \beta \geq 0$ is possible, so some subset of the goldstini/modulini may be lighter then the gravitino, while the lightest observable-sector supersymmetric partner (LOSP) may either sit above all the goldstini and modulini, or may be in the middle of the spectrum of states. We emphasise that in theories with multiple sectors it is unreasonable to expect, indeed unlikely, for the LSP to reside in our sector.[3] The true LSP may be the gravitino, one of the limit point goldstini, or yet another state, such as a hidden photino.

In Section 3.5 we briefly discuss the couplings of the goldstini and modulini of our secnario, and among other topics, present a potential 'smoking gun' collider signature that can give evidence for the physical realisation of the ISS mechanism of SUSY-breaking.[4] In general the goldstini of multiple SUSY breaking sectors, including those within a hidden ISS sector, couple to SSM chiral multiplets through the Goldstino Portal as [40]

$$\mathcal{L}_{int} \supset \sum_{i=1}^{N} \sum_{a=1}^{N-1} \frac{\tilde{m}_i^2 V_{ia}}{f_i} \zeta_a \psi \phi^\dagger \tag{3.1.1}$$

where N is the total number of F-terms, f_i, in all sectors, $\tilde{m}_i^2$ is the soft mass contribution from the i'th hidden sector F-term, $\tilde{m}_i^2 = -f_i^2/\Lambda_i^2$, the effective mediation scale of the i'th hidden sector to the SSM is Λ_i, and V_{ia} is the rotation matrix that diagonalises the goldstini mass matrix.[5] The ζ_a are the $N-1$ goldstini mass eigenstates and the true global goldstino that forms the longitudinal component of the gravitino is the N^{th} eigenstate with zero mass in this basis. If we make the reasonable assumption that SUSY breaking from all sectors is not communicated in an identical way, *i.e.*, if

[3] We particularly thank Lawrence Hall for stressing the importance of this point to us.

[4] We will return to the detailed phenomenology of the goldstini/modulini and their effects on collider experiments and astrophysical and cosmological observations in a later work.

[5] To avoid confusion, note that the f_i have mass-dimension two.

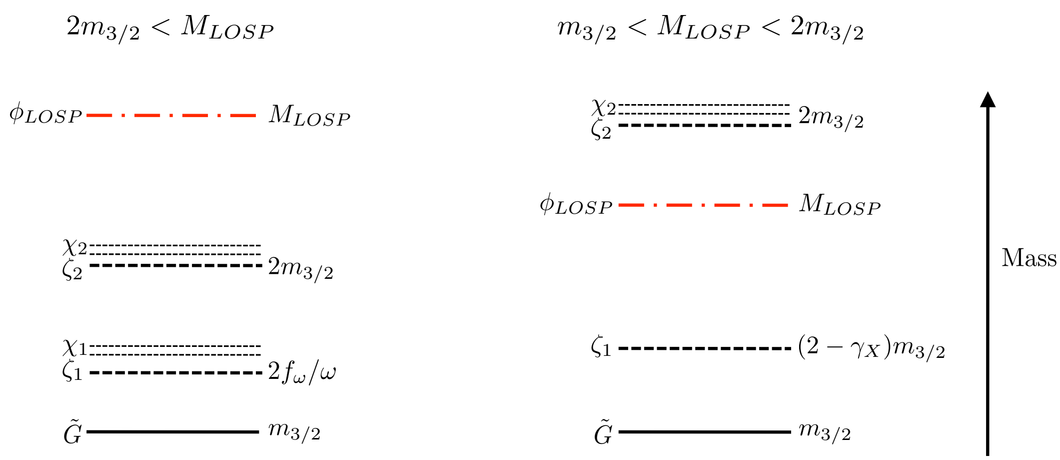

Figure 3.2: A diagram depicting a subset of the possible spectra. The left panel shows the SSM LOSP, the gravitino and goldstini/modulini from two ISS sectors, one at the end of a warped throat (so with mass spectrum at $2f_\omega/\omega$ as shown in Section 3.1), and one just gravitationally sequestered from the SSM. The right panel shows a possible spectrum where the SSM LOSP is lighter than $2m_{3/2}$, but, however, could still decay to a goldstino originating in a conformally sequestered (or warped) sector, here chosen not to be of ISS type, so there is only a single goldstini state, and no modulini. An interesting variant of this scenario occurs if the anomalous dimension of the SUSY-breaking field satisfies $\gamma_X > 1$, in which case LOSP decays could occur to a goldstino which is lighter than the gravitino. The resulting collider and cosmological phenomenology can depend strongly on which of these patterns is realised. Unlike for the various goldstini, decays to the modulini of a hidden ISS sector depend on the couplings in that sector, and are thus not guaranteed. Three different sectors are shown in order to elucidate a range of possibilities, though any number ≥ 2 of independent SUSY-breaking sectors implements the goldstini scenario.

not *all* Λ_i are equal, then couplings of the SSM to all goldstini are generated by the interaction of Eq.(3.1.1). Crucially, these couplings between the goldstini and SSM are parametrically stronger than gravitational, even if the effective mediation scale is $\sim M_P$. Moreover, these goldstini-SSM couplings distinguish the goldstini from other $m_{3/2}$-scale fermions such as, e.g., derivatively-coupled modulini, whose couplings to SSM states at a scale E are suppressed relative to those of goldstini by $E\Lambda_i/f_i$.[6]

For the field-theoretic breaking of global SUSY, it is reasonable to expect that the distribution of breaking scales is roughly log-flat since, assuming SUSY is unbroken at tree level, breaking only occurs via non-perturbative effects (modulo technicalities involving Fayet-Illiopoulos terms), which scan over an exponentially large range of scales as UV couplings and beta-function coefficients are linearly changed [102, 154]. In the context of multi-sector SUSY breaking, there should be a lower cutoff on this distribution of SUSY-breaking scales, implied by the (at least gravitational strength, *i.e.*, anomaly-mediated) communication of breaking from the dominant SUSY-breaking sector to the sub-dominant ones. If we require SUSY to solve the hierarchy problem, and we assume the dominant SUSY breaking sits at the intermediate scale, the lowest independent SUSY-breaking sector should have scale $\sim$ TeV. In Section 3.2 we will see how such a scenario leads to a strong modification of the goldstino mass when arising from a sector with such a very low SUSY-breaking scale.

In fact, in the landscape of string theory, one might naively expect 'tree-level' breaking due to the presence of fluxes or anti-D-branes in the vacuum not to be distributed at all scales, but instead concentrated at the string scale. Nevertheless, because of the presence of warped throats (caused by the back reaction from fluxes or branes), an approximately log-flat distribution of SUSY-breaking scales can still apply due to the approximately log-flat distribution of throat lengths expected in realistic string compactifications [60, 78, 91], and such structures are further motivated by the phenomenological necessity of conformal sequestering if SUSY is relevant to the

[6]Though this is true of conventional, derivatively coupled modulini, there are of course exceptions – for example, the fermionic components of moduli superfields involved in supersymmetry breaking, whose couplings to SSM states are goldstino-like. We particularly thank Joe Conlon for discussions on related issues.

solution of the hierarchy problem. In this context it is also noteworthy that anti-D-branes (or equivalent fluxes) sitting at the IR tip of one or more throats can have significant utility in the string landscape, as the presence of the anti-D-brane charges relaxes the tadpole constraints on the allowed vacua, and thus allows for a (quite possibly exponentially) larger landscape of vacua.

Having discussed our view of the overall scene in which the goldstini scenario is set and motivated, we now turn to our specific results, starting with the changes to the goldstini mass spectrum arising from the stabilization of SUSY breaking vacua.

3.2 Goldstini Masses in Supergravity

Perhaps the clearest way to study the goldstino mass spectrum in supergravity is through the use of the conformal compensator formalism [51, 52, 128]. The relevant physics may be captured by considering a single chiral superfield $X(y) = x(y) + \sqrt{2}\psi_X(y)\theta + f_X(y)\theta^2$ with a Polonyi-type superpotential and Kähler terms necessary for stabilizing the vacuum at finite $\langle x\rangle$. The Lagrangian is given by

$$\mathcal{L} = \int d^4\theta\, \phi^\dagger\phi \left(X^\dagger X - \frac{c(X^\dagger X)^2}{M^2} + ... \right) + \int d^2\theta \phi^3 f X + h.c. \tag{3.2.1}$$

where $\phi = \phi + f_\phi\theta^2$ is the conformal compensator and $c > 0$. Such a Lagrangian naturally arises as an effective description of SUSY breaking valid below the scale M (e.g., an O'Raifeartaigh model with fields of mass M)[7]. The quartic stabilizing term in the Kähler potential is absolutely necessary in the context of supergravity; its absence would induce a runaway to large field values. If X were the only source of supersymmetry breaking, we would identify ψ_X as the true longitudinal goldstino G that is eaten by the gravitino. Indeed, in this case the zero momentum equation of motion for x may be solved to yield $x = \frac{\psi_X^2}{2f_X}$. Thus in the far infrared we may write

[7]Note that here, for simplicity, we have assumed an R-symmetry is preserved. Inclusion of Kähler terms such as $c' X^\dagger X^3/M^2$ allow the study of R-breaking cases, with similar results to the R-preserving case.

X as a nonlinear superfield,

$$X = \frac{G^2}{2f_X} + \sqrt{2}G\theta + f_X\theta^2 \tag{3.2.2}$$

which corresponds to the usual nonlinear parameterization of the goldstino $G \equiv \psi_X$ [116].

Now let us consider the effects of multiple SUSY breaking on the fermion ψ_X. We assume the dominant contribution to SUSY breaking comes not from X, but from other sectors sequestered from X, so that $\langle f_\phi/\phi\rangle = m_{3/2}$. Clearly, it is now necessary to keep careful track of dependence on the conformal compensator. We may analyze the effects of SUSY breaking on X by going to the canonical basis via the rescaling $X \to X/\phi$ and solving the auxiliary equation of motion to find

$$f_X = -\frac{2c(f_\phi/\phi)|x|^2 x + 2cx^\dagger\psi_X^2 + fM^2}{M^2 - 4c|x|^2}. \tag{3.2.3}$$

By minimizing the resulting scalar potential for x, we may then extract the mass for the would-be goldstino $\eta \equiv \psi_X$,

$$m_\eta = 2m_{3/2}\left(1 - \frac{M^2 m_{3/2}^2}{2cf^2} + ...\right) \tag{3.2.4}$$

where additional correction terms are $\mathcal{O}\left(\frac{M^4 m_{3/2}^4}{c^2 f^4}\right)$. This expansion is valid in the regime $m_{3/2}^2/c \ll f^2/M^2 \lesssim m_{3/2}M_P$. One can see that as $\sqrt{f}$ (and M) approaches $m_{3/2}$ these corrections become significant and a different expansion is necessary. From a numerical study we find that for $\sqrt{f} < m_{3/2}$ these large corrections can drive the goldstini mass much smaller than $2m_{3/2}$. Such corrections are to be expected as in this case the SUSY-breaking communicated to a sector becomes larger than the breaking within the sector itself.

A few remarks are in order. As can be seen clearly in Eq.(3.2.4), the goldstino mass in a given SUSY-breaking sector depends on both the overall scale of SUSY breaking and the scales within the sector itself; the interplay of supersymmetry breaking and vacuum stabilization leads to important corrections to the goldstino mass. Of course, the generalization of this setup to N SUSY-breaking sectors is straightforward, resulting in N goldstini η_i; in the mass eigenbasis these become the eaten longitudinal goldstino and $N-1$ uneaten goldstini ζ_a (related to the η_i by $\eta_i = V_{ia}\zeta_a$, where V_{ia} is the rotation matrix that diagonalizes the goldstini mass matrix).

In this case it may seem that if one makes a unitary transformation such that there is only one Polonyi field $G = \sum_i f_i X_i / f_{eff}$ all other orthogonal combinations $\tilde{X}_i$ might remain massless by this derivation. However, in this new basis the stabilizing Kähler term will lead to mixed interactions between G and the other fields $\tilde{X}_i$. The non-zero vev of G then leads to masses for the fermionic components of $\tilde{X}_i$ and the same results are recovered.

In subsequent sections, we will often be interested in computing corrections to $m_\eta = 2m_{3/2}$ due to additional physics such as warping and sequestering. In such cases, for convenience we will dispense with the details of stabilization and instead carry out a naïve application of the nonlinear goldstino parameterization

$$X = \frac{\eta^2}{2f_X} + \sqrt{2}\eta\theta + f_X\theta^2 \tag{3.2.5}$$

for the would-be goldstino η given a Polonyi superpotential and canonical Kähler term for X. For free fields without warping this parameterization gives the leading result $m_\eta = 2m_{3/2}$, which omits the corrections due to stabilization but is valid in the limit $m_{3/2}^2/c \ll f^2/M^2$. Such a simplifying assumption will make the effects of new physics more transparent, with the understanding that corrections from stabilization have been suppressed.

Let us now turn to one such genre of new physics – the changes to the goldstini mass spectrum arising from the warping and/or conformal sequestering that can naturally occur in the string landscape.

3.3 Warped and Sequestered Goldstini

The observed smallness of FCNCs require that SUSY breaking communicated via flavour-violating mechanisms such as gravity mediation must necessarily be subdominant to flavor-preserving contributions. Absent some degree of unnatural tuning, this is most readily achieved by warping on an extra-dimensional space [144] or, in four dimensions (and essentially equivalent by AdS/CFT duality) sequestering by a conformal sector [126, 129]. Such sequestering is known to arise readily in string theory in the presence of strongly warped backgrounds [107]. But even apart from considerations of flavour, the persistence of multiple goldstini requires that different SUSY breaking sectors be sequestered from each other in a similar fashion; in fact, one should think of sequestering and multiple goldstini as inextricably intertwined. Given the effective dimensional transmutation brought about by warping, it is then natural to consider whether the scale of goldstino masses may be significantly modified if the additional SUSY breaking sector is at the bottom of a warped throat or in the far IR of a pseudo-conformal sector.

As before, the conformal compensator formalism may be used to clearly study the effects of warping or sequestering on the goldstino mass prediction, $m_\eta = 2m_{3/2}$. To get started, consider some number of chiral superfields X_i with Polonyi-type superpotentials and a sequestered Kähler potential. The relevant Lagrangian is

$$\mathcal{L} = \int d^4\theta \; \phi^\dagger \phi \sum_i (X_i^\dagger X_i + ...) + \int d^2\theta \; \phi^3 \sum_i \mu_i^2 X_i + \text{h.c.} \tag{3.3.1}$$

where, for simplicity, we have omitted terms necessary for stabilizing the SUSY breaking vacuum.[8] Here $\phi = \phi + f_\phi \theta^2$ is the appropriate conformal compensator, which, as we will see below, need *not* always be identified with the SUGRA conformal compensator. Rescaling $X_i \to X_i/\phi$ and expanding X_i in the nonlinear parameterization

$$X_i = \eta_i^2/2f_{X_i} + \sqrt{2}\eta_i\theta + f_{X_i}\theta^2 \tag{3.3.2}$$

[8] As mentioned above, this omits the leading corrections to m_η due to stabilization found in Eq.(3.2.4).

we obtain

$$\mathcal{L} \supset \int d^2\theta\, \phi^2 \sum_i \mu_i^2 X_i = -\frac{1}{2}\left(2\frac{f_\phi}{\phi}\right)\sum_i \eta_i^2 + \text{constant} \quad . \tag{3.3.3}$$

There are two salient details worth noting in this result. The first is that here we have assumed the X_i are free fields with canonical scaling dimension; as we will discuss below, the result changes significantly when the scaling dimensions of fields responsible for SUSY breaking differ from unity. The second is that the conformal compensator is ultimately responsible for setting the goldstino masses. The additional Goldstini of multiple SUSY breaking obtain masses of order $m_\eta = 2\frac{f_\phi}{\phi}$; this only corresponds to $m_\eta = 2m_{3/2}$ when $f_\phi/\phi = m_{3/2}$, which is not guaranteed to be the case, as we will shortly show.

Perhaps the simplest example of such deviations arise when the some of the chiral fields X_i possess scaling dimensions $\Delta_{X_i} > 1$, possibly at a conformal or near-conformal fixed point.[9] Such circumstances arise frequently in theories with conformal sequestering [107, 126, 129, 148], and more generally whenever SUSY breaking sectors are strongly coupled.

To see the effects of large anomalous dimensions more clearly, let us focus on the case of a single chiral superfield X with scaling dimension $\Delta_X \neq 1$ at a conformal fixed point. It is frequently the case that X is a component of a gauge invariant chiral operator of some interacting gauge theory, e.g., an $SU(2)$ theory with moduli space of gauge-invariant meson operators parametrized as

$$M = \begin{pmatrix} \epsilon Z & X \\ -X^T & \mathcal{O}(X^2/Z) \end{pmatrix} \quad . \tag{3.3.4}$$

Expanding around $Z \neq 0, X = 0$, conformal symmetry demands that the Kähler

[9] In order to compare results with those in [40], we will use the conventions $\Delta_X = 1 + \gamma_X$ to define the anomalous dimension γ_X in terms of the scaling dimension Δ_X. This corresponds to the choice $d \ln Z_X / d \ln \mu_R = -2\gamma_X$.

potential for X must be of the form

$$K = \phi^\dagger \phi (Z^\dagger Z)^{1/\Delta_Z} \left[1 + \frac{X^\dagger X}{(Z^\dagger Z)^{(\Delta_X/\Delta_Z)}} + ... \right] \quad , \tag{3.3.5}$$

where ϕ is the SUGRA conformal compensator with $\langle \phi \rangle = 1 + m_{3/2}\theta^2$. We assume there is also a superpotential Polonyi term

$$W = \phi^3 \mu^2 X \tag{3.3.6}$$

where the constant μ^2 has dimension $(3 - \Delta_X)$; additional Kähler terms are required, as usual, to stabilize the SUSY breaking vacuum of X. We can study the theory near the origin of moduli space in terms of redefined fields

$$\hat{Z} = \phi Z^{1/\Delta_Z} \quad \text{and} \quad \hat{X} = \phi^{\Delta_X} X / \hat{Z}^{\Delta_X - 1} \quad , \tag{3.3.7}$$

for which the Kähler potential is canonical (without any dependence on ϕ) and the superpotential term becomes

$$W \rightarrow \phi^{2-\gamma_X} \mu^2 \hat{Z}^{\gamma_X} \hat{X} \quad . \tag{3.3.8}$$

We are interested in the mass term for the goldstino component of $\hat{X}$. Utilizing the nonlinear parameterization of Eq.(3.3.3), we find, in the case of current interest,

$$\mathcal{L} \supset -\frac{1}{2}(2 - \gamma_X) m_{3/2} \eta^2 \quad , \tag{3.3.9}$$

from which we see the goldstino mass is

$$m_\eta = (2 - \gamma_X) m_{3/2} \quad , \tag{3.3.10}$$

in agreement with the perturbative result of [40]. The key point here is that γ_X need not be perturbative, so in principle the goldstino mass may range from 0 to $2m_{3/2}$ depending on the size of γ_X. For example, if the superpotential of the gauge theory at the interacting fixed point involved a marginal operator, $W \supset \mathrm{tr} M^2$, we would have $\gamma_X = 1/2$ and thus $m_\eta = \frac{3}{2} m_{3/2}$.

This is a simple example of our first point – that the mass of a goldstino coming from a chiral superfield X depends sensitively on the scaling dimension Δ_X. The smallness of FCNCs suggests that multiple SUSY-breaking sectors, if present, must be sequestered in order to avoid prohibitive flavour-violating contributions to soft masses. In four dimensions, this is most readily accomplished by conformal sequestering, in which case anomalous dimensions $\gamma_X \neq 0$ are generically expected.

Thus far our discussion has also assumed that the mediation of dominant SUSY breaking arises through the conventional SUGRA conformal compensator; as we will now argue, this, too, no longer holds in many situations where SUSY breaking fields are sequestered by conformal dynamics or warping in higher-dimensional spaces.

3.3.1 Warped goldstini 5D

In order to probe the effects of warping on goldstino masses, let us consider a toy model of warping in the form of a 5D supersymmetric Randall-Sundrum model [143]. While such constructions are perhaps not as realistic as those based on more complete warped throat solutions [112, 113], they nonetheless capture much of the relevant physics. To set notation, we take the 5th dimension to be compactified on an interval of length πr via a S^1/Z_2 orbifold, with metric

$$ds^2 = e^{-2kr|\theta|}\eta_{\mu\nu}dx^\mu dx^\nu + r^2 d\theta^2 \tag{3.3.11}$$

for $-\pi < \theta \leq +\pi$; the slope discontinuities at $\theta = 0, \pi$ signal the presence of 4D branes fixed by the orbifold boundary conditions. These branes mock up the resolved physics of the UV Calabi-Yau 'head', and IR throat 'tip' in the IR of the more realistic complete string solutions. As usual, the warp factor $e^{-2kr|\theta|}$ indicates that physical scales on the $\theta = \pi$ IR brane are redshifted relative to those on the $\theta = 0$ UV brane.

At energies below the mass of the lightest gravitational Kaluza-Klein (KK) mode, we may employ an effective 4D Lagrangian describing the physics of fields localized on UV and IR branes separated by a warped throat. The Lagrangian for this effective theory is [26, 128]

$$\mathcal{L} = -\frac{3M_5^3}{k}\int d^4\theta\left(\phi^\dagger\phi - \omega^\dagger\omega\right) + \int d^4\theta(\phi^\dagger\phi K_{UV} + \omega^\dagger\omega K_{IR}) \\ + \int d^2\theta(\phi^3 W_{UV} + \omega^3 W_{IR}) + h.c. \tag{3.3.12}$$

Here ϕ is the conformal compensator field and ω is the "warp factor" superfield,

$$\omega = \phi e^{-kT} \tag{3.3.13}$$

where $T = \pi r + ...$ is the radion superfield (in a horrible abuse of notation, we will write the warp factor superfield in terms of its scalar and auxiliary components as $\omega = \omega + f_\omega\theta^2$). The physics we are interested in will be encoded by Kähler and superpotential terms for a Polonyi field localized on the IR brane. The anomaly-mediated communication of supersymmetry breaking to fields localized in the IR arises via the warped conformal compensator ω, giving rise to supersymmetry breaking of order f_ω/ω.

Ultimately, the size of supersymmetry breaking seen by IR fields is determined by the stabilization mechanism fixing the expectation value of the radion, and hence the warp factor superfield ω. Although it is often the case that simple forms of radius stabilization lead to $\langle f_\omega/\omega\rangle \sim \langle f_\phi\rangle$ (as in, e.g., [128]), we will be interested in a much more general class of stabilization mechanisms.

Now let us consider the effects of this stabilization on supersymmetry breaking in the IR. Suppose that the field content in the IR includes one or more fields breaking supersymmetry (in addition to other sources of supersymmetry breaking in the UV). We may represent this locally by a Polonyi model for a field X, via a superpotential term $W_{IR} = \mu^2 X + ...$ and Kähler term $K_{IR} = X^\dagger X$ (along with the usual additional Kähler terms necessary to stabilize the potential). Assume now that the dominant contribution to supersymmetry breaking arises elsewhere on the manifold, so that X can be identified as a non-linear pseudo-goldstino field. To study the dynamics of X, we may rescale $X \to X/\omega$, which results in canonical Kähler terms for X and a superpotential

$$W_X = \omega^2 \mu^2 X \quad . \tag{3.3.14}$$

The resulting goldstino mass term is

$$\mathcal{L} \supset \omega f_\omega \frac{\mu^2}{f_X} \eta^2 = -\frac{1}{2}(2 f_\omega/\omega)\eta^2 \quad . \tag{3.3.15}$$

As expected, the mass for this IR-localized goldstino depends on the warped SUSY-breaking order parameter $\langle f_\omega/\omega \rangle$ rather than the UV order parameter $\langle f_\phi \rangle$.

What are the effects of warping on the goldstino mass spectrum? Clearly, in the case of no warping, $\omega = \phi = 1 + m_{3/2}\theta^2$ and hence $m_\eta = 2f_\omega/\omega = 2m_{3/2}$, consistent with the familiar result. Moreover, in the event that there is nontrivial warping but the stabilization mechanism yields $f_\omega/\omega \sim f_\phi = m_{3/2}$, we again obtain $m_\eta \simeq 2m_{3/2}$. However, this is far from the only possible outcome. Consider a stabilization superpotential of the form [127]

$$\mathcal{L} = \int d^2\theta (c_{UV}\phi^3 + c_{IR}\omega^3 + \epsilon \phi^{3-n}\omega^n) + h.c - f_{UV}^2[1 + \text{gravity terms}] \quad . \tag{3.3.16}$$

The first two terms can arise from constant superpotentials localized on the UV and IR branes; the third term requires a bulk gauge theory with some massive fundamental matter.

For $\epsilon \ll c_{UV}, c_{IR}$ and $n < 3$, the ϵ term contributes a vev to ω of order

$$|\langle \omega \rangle|^{4-n} = \frac{n(3-n)}{6} \left| \frac{\epsilon c_{UV}}{c_{IR}^2} \right| \ll 1 \quad , \tag{3.3.17}$$

with

$$\left| \frac{\langle f_\omega \rangle}{\langle \omega \rangle} \right| = \frac{|c_{IR}|}{M_P^2} |\langle \omega \rangle| \quad \text{and} \quad |\langle f_\phi \rangle| = \frac{|c_{UV}|}{M_P^2} = \frac{f_{UV}}{\sqrt{3} M_P} \quad . \tag{3.3.18}$$

Here the radion mass is of order $\langle f_\omega/\omega \rangle$, while the gravitino mass is of order $\langle f_\phi \rangle$. Significantly, the order parameter for SUSY breaking in the IR is parametrically suppressed relative to $\langle f_\phi \rangle$. Thus, in this case, the goldstino mass is

$$m_\eta = 2\frac{f_\omega}{\omega} = 2\frac{|c_{IR}|}{M_P^2} |\langle\omega\rangle| \ll 2m_{3/2} \quad . \tag{3.3.19}$$

Depending on the choice of stabilization parameters, this results in a goldstino mass ranging between $0 < m_\eta \le 2m_{3/2}$. The generalization to many Goldstini is straightforward; for $\eta \to \eta_i$ one need only take $\omega \to \omega_i$ and $f_\omega \to f_{\omega_i}$ in the case of multiple throats.

3.3.2 Sequestered goldstini in 4D

As one might expect, we can also see the effects of warping on goldstino masses in a strictly four-dimensional picture of conformal sequestering. In this situation the role of warping is played by the dynamics of a superconformal sector coupling to the IR fields. Following [127], for the sake of specificity we will focus on the case of a 4D $SU(2)$ SUSY gauge theory with 8 fundamentals P and superpotential

$$W = \lambda P^4 + \kappa P^2. \tag{3.3.20}$$

This theory flows to a conformal fixed point in the infrared, where the coupling λ is assumed to be marginal. At the conformal fixed point, the superconformal R-symmetry fixes the scaling dimension of P such that $\Delta_P = 3/4$; thus λ is dimensionless (marginal) and the coefficient κ has scaling dimension $\Delta_\kappa = 3/2$. The moduli space of gauge invariant operators can be parameterized as

$$PP = \begin{pmatrix} \epsilon Z & Y \\ -Y^T & \mathcal{O}(Y^2/Z) \end{pmatrix} \tag{3.3.21}$$

where $\epsilon = i\sigma_2$ is the antisymmetric tensor. Here Y is a 2×6 matrix of fields.

Conformal symmetry constrains the theory below the scale Z to have Kähler terms of the form

$$K = \phi^\dagger\phi(Z^\dagger Z)^{2/3}[1+\mathcal{O}(|Y|^2/|Z|^2)] \\ = \hat{Z}^\dagger\hat{Z}[1+\mathcal{O}(|\hat{Y}|^2/|\hat{Z}|^2)]] \tag{3.3.22}$$

where $\hat{Z} = \phi Z^{2/3}$ and $\hat{Y} = \phi Y/Z^{1/3}$. In terms of these variables, the superpotential becomes

$$W = \lambda\hat{Z}^3 + \kappa\hat{Z}^{3/2}\phi^{3/2} \quad , \tag{3.3.23}$$

which has the same form as our 5D Randall-Sundrum theory with $n = 3/2$ and $\hat{Z} \sim \omega$. Indeed, if we make the identifications $\hat{Z} \to M_P\omega$, $\lambda \to c_{IR}/M_P^3$, and $\kappa \to \epsilon/M_P^{3/2}$, we may reproduce all the details of the warped model in terms of a four dimensional conformal field theory.

Of course, we may consider a wide range of conformal field theories with various marginal operators at the conformal fixed point. In general, a superpotential

$$W = \lambda P^k + \kappa P^2 \tag{3.3.24}$$

leads, below the scale of Z, to an effective superpotential

$$W = \lambda\hat{Z}^3 + \kappa\hat{Z}^{6/k}\phi^{3-6/k} \tag{3.3.25}$$

where $n = 6/k$. The constraint $n > 3$ corresponds to $k > 2$; for $k > 2$, $\Delta_Z < 3$, which is eminently sensible in order that κ remain a relevant deformation. In any event, we need not commit to a specific conformal field theory; any dynamics with $\Delta_Z > 1$ may suffice.

Now let us consider the effects of sequestering on Goldstini coupled to the conformal sector. This corresponds to coupling the field $\hat{Z}$ (which is our stand-in for the warp superfield) to a field with a Polonyi term. First, consider the theory where the field X that breaks SUSY is a total composite of scaling dimension $\Delta_X = 3$. This is the four dimensional analog of a purely "IR-localized" field; the case of $\Delta_X < 3$

corresponds to a partially-localized field, which we will discuss momentarily.

Expanding around $Z \neq 0$ and $X = 0$, the Kähler potential below the scale Z is constrained by conformal symmetry to be of the form

$$K \supset \phi^\dagger \phi (Z^\dagger Z)^{1/\Delta_Z} \left[1 + \frac{X^\dagger X}{(Z^\dagger Z)^{(3/\Delta_Z)}} + ... \right] \tag{3.3.26}$$

We may thus define canonical fields

$$\hat{Z} = \phi Z^{1/\Delta_Z} \quad \text{and} \quad \hat{X} = \phi^3 X / \hat{Z}^2 \tag{3.3.27}$$

in terms of which the Kähler potential is canonical.

If the theory contains a Polonyi term for the candidate SUSY breaking field X, the superpotential is of the form

$$W = \phi^3 \mu^2 X \rightarrow \mu^2 \hat{Z}^2 \hat{X} \tag{3.3.28}$$

It is then a simple matter to compute the goldstino mass; the Lagrangian includes a term

$$\mathcal{L} \supset -\frac{1}{2} \left(2 \frac{f_Z}{\hat{z}} \right) \eta^2 \tag{3.3.29}$$

so that the goldstino mass is given by $m_\eta = 2 f_Z / \hat{z} \sim 2 f_\omega / \omega$, as expected from the results of the previous subsection. The stabilization mechanism for Z is simply the one considered earlier.

We may also consider the case where $1 \leq \Delta_X \leq 3$, i.e., the candidate SUSY breaking field has a large anomalous dimension but should not be interpreted as being completely localized on the brane; rather, it has a warped profile in the 5D picture corresponding to a bulk mass term.

Once again, expanding around $Z \neq 0$ and $X = 0$, the Kähler potential is constrained by conformal symmetry to take the form

$$K \supset \phi^\dagger \phi (Z^\dagger Z)^{1/\Delta_Z} \left[1 + \frac{X^\dagger X}{(Z^\dagger Z)^{(\Delta_X/\Delta_Z)}} + ... \right] \ , \tag{3.3.30}$$

with canonical fields given by $\hat{Z} = \phi Z^{1/\Delta_Z}$ and $X = \phi^{\Delta_X} X/\hat{Z}^{\Delta_X - 1}$, in terms of which the Kähler potential is canonical. The superpotential term for X thus takes the form

$$W = \phi^3 \mu^2 X \to \mu^2 \phi^{2-\gamma_X} \hat{Z}^{\gamma_X} \hat{X} \quad . \tag{3.3.31}$$

Carrying out the nonlinear parameterization for the goldstino, we find in this case a goldstino mass

$$m_\eta = (2 - \gamma_X) m_{3/2} + \gamma_X \frac{f_Z}{\hat{z}} \quad . \tag{3.3.32}$$

This result interpolates nicely between the results found in the limiting cases $\gamma_X = 0$ and $\gamma_X = 2$. In the former limiting case, we retrieve the physics of a free Polonyi field with no warping; in the latter limiting case, the physics of a fully sequestered Polonyi field where the scale of SUSY breaking is set not by f_ϕ, but f_ω/ω.

Thus far we have remained relatively agnostic about the detailed physics of supersymmetry breaking, but this, too, may have a significant impact on the spectrum of goldstini, as we will now see.

3.4 Multiple Goldstini and Modulini from ISS Sectors

The notion of multiple SUSY breaking sectors prompts us to consider how SUSY may be broken within each sector. The ISS models [99] demonstrate that SQCD with massive flavours exhibits a meta-stable SUSY breaking ground state. Further, the simplicity of such models would suggest that spontaneously broken SUSY is generic in SUSY field theory and in the landscape of string vacua. Therefore it is natural to consider, in the context of multiple SUSY breaking sectors, that some number may well be of the ISS type, without the addition of any of the singlets or deformations that are absent in the original ISS models, and that are needed only to break R-symmetries. Here we show that such a sector would give rise to multiple goldstini fields along with many more 'modulini' fields of mass $\geq 2m_{3/2}$.[10] These extra states could potentially lead to a smoking gun signature of an ISS hidden sector by determining missing energy in LOSP decays to the gravitino, goldstini and modulini.

3.4.1 ISS models at low energies

To illustrate the essential physics we concentrate on the classic ISS-model of SQCD with N_c colours and $N_c+1 \leq N_f < \frac{3}{2}N_c$ flavours in the free magnetic range [101, 149, 150]. The generalization to other gauge groups should be straightforward. A simple, intuitive understanding of why such an ISS sector gives rise to multiple goldstini fields comes from the fact that, in the far IR, it flows to multiple decoupled O'Raifeartaigh-like models as we now show.

Using Seiberg duality [149] the IR-free description of the theory is described by an $N_f \times N_f$ gauge singlet meson matrix Π_{ij} and N_f flavours of magnetic quarks φ_i and $\tilde{\varphi}_j$ in the fundamental (respectively anti-fundamental) of a $SU(\tilde{N} = N_f - N_c)$ magnetic gauge theory. This theory is weakly coupled at low energies and has a superpotential given by

$$W = h \text{ Tr } \left[\varphi \cdot \Pi \cdot \tilde{\varphi} - \mu^2 \cdot \Pi\right]. \tag{3.4.1}$$

We assume a generic, non-hierarchical, matrix μ^2_{ij} which can be diagonalized without loss of generality. Among other symmetries this theory exhibits a $U(1)_R$ symmetry where the φ fields have zero R-charge and Π has R-charge 2.

Considering the F-components of the meson superfields,

$$-F^\dagger_{\Pi_{ij}} = h\varphi_i \cdot \tilde{\varphi}_j - h\mu^2_{ij} \tag{3.4.2}$$

the first term in this matrix equation is of rank $N_f - N_c$ whereas the second term is of rank $N_f > N_f - N_c$, therefore it is impossible to have $f_{\Pi_{ij}} = 0$ for all $\{i, j\}$ and SUSY

[10] Purely for typographical clarity we ignore, throughout this section, the possibility of the warping or conformal sequestering considered in Section 3.3. We emphasise that typically the metastable SUSY-breaking of ISS-type studied in the present section should also come along with such sequestering dynamics, leading to the changes in overall goldstini (and modulini) mass scales and couplings explicated in Section 3.3.

is broken. This is the famous ISS 'rank condition'. The minimum of the potential is

$$V = \sum_{i}^{N_c} (h\mu_i^2)^2 \tag{3.4.3}$$

where μ_i^2 are the N_c smallest eigenvalues of μ_{ij}^2. This minimum occurs in field space

$$\Pi = \begin{pmatrix} Y & Z \\ \tilde{Z} & \Phi \end{pmatrix}, \quad \varphi = \begin{pmatrix} \varphi_0 + \chi, & \rho \end{pmatrix}, \quad \tilde{\varphi} = \begin{pmatrix} \tilde{\varphi}_0 + \tilde{\chi} \\ \tilde{\rho} \end{pmatrix}, \quad \mu^2 = \begin{pmatrix} \tilde{\mu}_0^2 & 0 \\ 0 & \mu_0^2 \end{pmatrix} \tag{3.4.4}$$

with $\varphi_{\mathbf{0}} \cdot \tilde{\varphi}_{\mathbf{0}} = \tilde{\mu}_0^2$. Also, Φ is an $N_c \times N_c$ matrix of fields, Y is $(N_f - N_c) \times (N_f - N_c)$, ρ is $N_c \times N_c$ and the dimensionality of the other terms is apparent from these assignments. Upon rewriting the superpotential in terms of these fields it splits into three pieces $W = W_1 + W_2 + W_3$ with

$$\begin{aligned} W_1 &= h \text{ Tr } [\rho \cdot \Phi \cdot \tilde{\rho} + \rho \cdot \tilde{Z} \cdot \tilde{\varphi}_0 + \varphi_0 \cdot Z \cdot \tilde{\rho} - \mu_0^2 \cdot \Phi] \\ &= -h \text{ Tr } [\mu_0^2 \cdot \Phi] + h \sum_{i=1}^{N_f - N_c} (\boldsymbol{\phi}_{1_i} \cdot \Phi \cdot \boldsymbol{\phi}_{2_i} + \tilde{\mu}_{0_i}(\boldsymbol{\phi}_{1_i} \cdot \boldsymbol{\phi}_{4_i} + \boldsymbol{\phi}_{2_i} \cdot \boldsymbol{\phi}_{3_i})) \end{aligned} \tag{3.4.5}$$

Here the $\boldsymbol{\phi}$ are N_c dimensional vectors, and the $\tilde{\mu}_{0_i}$ are the first $N_f - N_c$ diagonal components of the $\tilde{\mu}_0^2$ matrix. In the first line we recognise W_1 as an O'Raifeartaigh-like model and in the second line the fields $\rho, \tilde{\rho}, \tilde{Z}$, and Z have been written as matrices made up of row and column vectors to demonstrate explicitly how the superpotential W_1 decomposes into $N_f - N_c$ O'Raifeartaigh-like sectors. The remaining pieces of the superpotential are

$$W_2 = h \text{ Tr } [\chi \cdot Y \cdot \tilde{\chi} + \chi \cdot Y \cdot \tilde{\varphi}_0 + \varphi_0 \cdot Y \cdot \tilde{\chi}] \tag{3.4.6}$$

$$W_3 = h \text{ Tr } [\rho \cdot \tilde{Z} \cdot \tilde{\chi} + \chi \cdot Z \cdot \tilde{\rho}] \quad . \tag{3.4.7}$$

W_2 comprises a sector which doesn't break SUSY and contains massive chiral superfields along with the Goldstone superfields of the spontaneously broken symmetries. The Goldstone fields of the spontaneously broken $SU(N_f - N_c)$ are eaten by the gauge superfields through the supersymmetric Higgs mechanism. These SUSY-preserving fields are only coupled to the SUSY-breaking sector through the cubic terms in W_3 and can therefore be consistently neglected when considering the first sector.

It is clear that Φ remains massless at tree level, and the diagonal component of Φ contains the goldstino. The pseudo-moduli of this field become massive at one-loop level through their interactions with the heavy $\boldsymbol{\phi}$ fields and these masses can be calculated to all orders in the SUSY breaking parameters with the use of the Coleman-Weinberg potential [44]. However, as we would later like to embed this theory in SUGRA, and the Coleman-Weinberg approach is not manifestly supersymmetric, we choose instead to work in terms of the effective Kähler potential which arises when the heavy superfields are integrated out. This agrees with the Coleman-Weinberg potential to second order in the SUSY breaking F-terms and in the limit where SUSY is unbroken this is exact at one-loop.[11]

In general for a superpotential of the form

$$W = \frac{1}{2} M_{ij} \varphi_i \cdot \tilde{\varphi}_j \quad , \tag{3.4.8}$$

where M_{ij} includes mass terms and the pseudomoduli fields, the exact one-loop Kähler potential is given by

$$K^{(1)} = -\frac{1}{32\pi^2} \text{ Tr } \left[M^\dagger M \log \left(\frac{M^\dagger M}{|\Lambda|^2} \right) \right] \quad . \tag{3.4.9}$$

Reading off the matrix M from Eq.(3.4.5) one finds that the theory describing the light fields contained in Φ, after integrating out the heavy fields contained in ϕ, is described by the superpotential

$$W = h \text{ Tr } [\mu_0^2 \cdot \Phi] \quad , \tag{3.4.10}$$

and the effective Kähler potential $K_{eff} = K^{(0)} + K^{(1)}$, where $K^{(0)}$ is the canonical

[11] Including higher order corrections in the SUSY breaking parameters would necessitate including supercovariant derivates. The effective Kähler potential is sufficient for our needs.

Kähler potential, and $K^{(1)}$ is given by

$$K^{(1)} = -\frac{h^2}{32\pi^2}\sum_{i=1}^{N_f-N_c} \text{Tr}\left[2\left(2+\log\left(\frac{|\tilde{\mu}_{0_i}|^2}{\Lambda^2}\right)\right)\Phi^\dagger\cdot\Phi + \frac{1}{3|\tilde{\mu}_{0_i}|^2}(\Phi^\dagger\cdot\Phi)^2+...\right] \quad (3.4.11)$$

Here the ellipses denote higher order terms which we can ignore as we are studying the theory near the origin of field space, $\langle\Phi\rangle \ll \tilde{\mu}_0$. (The first logarithmic correction to the terms quadratic in Φ corresponds to one-loop wavefunction renormalization of the fields.) Eqs.(3.4.10) and (3.4.11) are sufficient for studying the low-energy phenomenology of the ISS model. One can see from the quartic term in the Kähler potential that, once the diagonal components of Φ develop F-terms, a scalar potential for all pseudo-moduli in Φ is generated, and in these (global) SUSY ISS models all scalars are stabilized at the origin $\langle\Phi\rangle = 0$.

Most importantly for our purposes, this low energy theory respects the R-symmetry detailed earlier, forbidding the fermions in Φ, hereafter called 'modulini', from gaining mass. This can also be understood by considering the ISS model before integrating out the massive fields: As there are more fermions with R-charge $Q_R = 1$ than with $Q_R = -1$, then, if the vacuum is R-symmetry preserving, not all fermions can obtain a Dirac mass, implying some remain massless.

One may worry that sub-leading corrections spoil this result. There exist corrections to the Kähler potential of the form $\delta K \sim$ Tr $[\Phi^\dagger\cdot\Phi]^2/|\Lambda|^2$ where Λ is the strong coupling scale of the theory. These corrections have interesting consequences when the theory is embedded in SUGRA, however as they respect the R-symmetry, we conclude that, in the global limit, they do not contribute to the modulini masses. There is also a non-perturbative explicit R-symmety-breaking superpotential term

$$W = N_c(h^{N_f}\Lambda^{-(3N_c-2N_f)}\det[\Pi])^{1/(N_f-N_c)} \quad (3.4.12)$$

generated by gaugino condensation [7, 14, 56, 101]. However it preserves a discrete R-symmetry subgroup larger than $\mathbb{Z}_2$, and thus the modulini remain protected from gaining a mass. We will return, in the next section, to a discussion of these operators in the context of SUGRA .

In summary, one sees that in the global SUSY limit the metastable supersymmetry-breaking vacuum of SQCD with N_c colours and $N_c + 1 \leq N_f < \frac{3}{2}N_c$ massive flavours flavours contains N_c^2 massless modulini (of which N_c are goldstini). In the next section we show that in local supersymmetry these modulini acquire a mass $\geq 2m_{3/2}$ (ignoring warping and/or conformal-sequestering).

3.4.2 ISS modulini masses in Supergravity

In SUGRA with spontaneously broken SUSY, one requires a constant term in the superpotential to cancel the cosmological constant from the non-zero scalar potential. This constant breaks any continuous R-symmetry and we would expect this to manifest itself in a hidden ISS sector by displacing the minimum of the scalar potential for the pseudo-moduli from the origin, and in turn generating masses for the modulini.

First, by considering Eq.(3.4.11) we see that when $\langle\Phi\rangle \neq 0$ modulini masses are indeed generated. As long as $\langle\Phi\rangle \ll \tilde{\mu}_0$ the dominant contribution comes from the quartic operator, higher order terms leading to subdominant corrections suppressed by higher powers of $\langle\Phi\rangle/\tilde{\mu}_0$. For $f_a \sim \mu \ll M_P^2$ for all a, we show in Appendix A.1 that the condition $\langle\Phi\rangle \ll \tilde{\mu}_0$ is satisfied. Thus our SUGRA analysis is valid whenever the scale of supersymmetry breaking is parametrically below the Planck mass.

Moreover, as also detailed in Appendix A.1, we find, for fields, X_i, with super- and Kähler-potentials of the form,

$$W = W_0 + f_a X_a \tag{3.4.13}$$

$$K = X_a X^\dagger_{a^\star} + \frac{1}{\mu^2} A_{ab^\star cd^\star} X_a X^\dagger_{b^\star} X_c X^\dagger_{d^\star} \quad , \tag{3.4.14}$$

and under the same conditions, that the fermion masses are given by

$$m_{ab} = 2m_{3/2} \left(A_{(ad^\star bl^\star)} (A_{(ij^\star kl^\star)} f_i f_{j^\star})^{-1} f_{d^\star} f_k - \frac{f_a f_b}{f_{eff}^2} \right) \tag{3.4.15}$$

once the goldstino direction has been rotated away. (The goldstino direction is the zero eigenvector of this mass matrix which can clearly be seen as $f_a m_{ab} = 0$.)

Armed with Eq.(3.4.15) we can apply these results to modulini from the ISS sector. In Section 3.4 we identified two quartic operators in the Kähler potential that may lead to modulini masses: Tr $[(\Phi^\dagger\Phi)^2]$ and Tr $[(\Phi^\dagger\Phi)]^2$, and in both cases the tensor A can be written in terms of the identity matrix. First we consider just the operator $\frac{1}{\mu^2}$ Tr $[(\Phi^\dagger\Phi)^2]$ and diagonalise the F-terms as in Section 3.4. In this basis one finds for the mass matrix

$$m_{ab,cd} = 2m_{3/2}\left(\frac{f_a^2+f_b^2}{2f_af_b}\delta_{ad}\delta_{bc} - \frac{f_af_c}{f_{eff}^2}\delta_{ab}\delta_{cd}\right) \tag{3.4.16}$$

where the f_a are the diagonal elements of the $h\mu_0^2$ matrix in the superpotential. We can now split Φ into two sets of fields to study the masses.

First, focusing on the diagonal elements, i.e. $a = b, c = d$, we find a mass matrix:

$$m_{aa,bb} = 2m_{3/2}\left(\delta_{ab} - \frac{f_af_b}{f_{eff}^2}\right) \tag{3.4.17}$$

which has N_c eigenvalues of $2m_{3/2}(1,1,1,...,1,0)$. The field with zero mass is the true goldstino field $G = f_i\Phi_{ii}/f_{eff}$ that mixes with the gravitino and is eaten leading to a gravitino of mass $m_{3/2}$. In the presence of multiple SUSY breaking sectors this field is in general a mixture of the goldstini from all sectors and from the ISS sector we would then expect N_c 'goldstini' fields, ζ, of mass $2m_{3/2}$.

Now considering the off-diagonal fields, i.e. $a \neq b, c \neq d$, we find that the only non-zero terms in the mass matrix have $c = b, d = a$ and are of the form

$$m_{ab,ba} = 2m_{3/2}\left(\frac{f_a^2+f_b^2}{2f_af_b}\right) \quad . \tag{3.4.18}$$

These fields in general have $m \geq 2m_{3/2}$, with a lower limit, $m = 2m_{3/2}$, in the case with all F-terms equal (again ignoring warping and/or conformal sequestering). These off-diagonal fields are the (now massive) modulini, χ, which accompany the goldstini.

In summary, in the context of multiple sequestered SUSY breaking sectors, from each meta-stable ISS-type SUSY-breaking sector one expects N_c goldstini of mass $2m_{3/2}$ and $N_c(N_c-1)$ modulini with mass $m \geq 2m_{3/2}$. We emphasise that this result is valid in the absence of extra singlet fields or other deformations of the global-SUSY-limit of the ISS sector that spoil the discrete R-symmetry outlined in Section 3.4.1. Nevertheless, as explained in the Introduction, our results are expected to apply to realistic theories of SUSY-breaking in the string landscape as there is no reason to require *all* of the independent SUSY-breaking sectors to break their discrete R-symmetries.

3.4.3 Sub-leading corrections

There are a number of operators that might alter these results. We first consider those we expect to arise within the ISS sector itself. Naïvely, a cause for concern is the fact that by using the effective Kähler potential we are omitting higher order terms in an expansion in $f/\mu^2 \lesssim 1$. We have, however, calculated the full one-loop diagram for the modulini masses, which includes these corrections to all orders and where the effects of SUGRA are included, and we find that the goldstini masses remain unaltered and the modulini masses remain bounded below by $2m_{3/2}$, so do not change the qualitative results from the previous section. The full results of this calculation are contained in Appendix A.2.

Next, as discussed in [99] and in Section 3.4.1, there are corrections due to the underlying microscopic theory of the form $\delta K \sim \text{ Tr }[\Phi^\dagger \cdot \Phi]^2/|\Lambda|^2$ where Λ is the strong coupling scale of the theory. As highlighted in [99] the effects from these operators are expected to be small as $|\Lambda| \gg \mu$. Including this operator we find that the masses of the fermions are altered slightly. In particular if we set all F-terms equal we find that for

$$K = \text{ Tr }[\Phi^\dagger \cdot \Phi] - \frac{a}{|\mu|^2} \text{ Tr }[(\Phi^\dagger \cdot \Phi)^2] - \frac{b}{|\Lambda|^2} \text{ Tr }[\Phi^\dagger \cdot \Phi]^2 \tag{3.4.19}$$

and the superpotential in Eq.(3.4.10) the fermion masses are

$$m = 2m_{3/2}\frac{1}{1+\frac{bN_c|\mu|^2}{a|\Lambda|^2}} \tag{3.4.20}$$

As $|\Lambda| \gg |\mu|$ then unless $b \gg a$ corrections from the microscopic theory are small (though possibly phenomenologically interesting). The sign of b is unknown and so these small corrections to the goldstini and modulini masses can potentially be positive or negative.

As mentioned earlier gauge interactions also lead to an explicit breaking of the R-symmetry through the generation of the low energy superpotential [7,14,56,99,101]

$$W = N_c (h^{N_f} \Lambda^{-(3N_c - 2N_f)} \det[\Pi])^{1/(N_f - N_c)} \quad . \tag{3.4.21}$$

Corrections due to this term should be small, though. First, this operator leads to a superpotential term proportional to $\Phi^{N_f/(N_f-N_c)}$; however we know that $N_f > 3(N_f - N_c)$ so the discrete R-symmetry remaining after the inclusion of this operator will forbid majorana masses for the modulini in Π in the global SUSY limit. On the other hand, once the theory is embedded in SUGRA, we know the R-symmetry is broken and this leads to vacuum expectation values for the scalar components of $\langle \Pi \rangle \sim m_{3/2}$ [2]. As with the corrections to the Kähler potential we would then expect this operator to lead to masses for the modulini. We can estimate these corrections as

$$\begin{aligned} \delta m &\sim N_c h^{N_f/(N_f-N_c)} \Lambda^{-(3N_c-2N_f)/(N_f-N_c)} \langle \Phi \rangle^{N_f/(N_f-N_c)} & (3.4.22) \\ &\sim m_{3/2} \left(\frac{m_{3/2}}{\Lambda} \right)^{(3N_c-2N_f)/(N_f-N_c)} & (3.4.23) \end{aligned}$$

and as $\Lambda \gg \mu \gg m_{3/2}$ and also $3N_c > 2N_f$, these corrections should be small unless $(3N_c - 2N_f)/(N_f - N_c) < 1$.

Other operators may arise from outside the ISS sector which lead to R-symmetry breaking and would modify these masses. Such scenarios have been discussed in detail in [40] and thus we direct the interested reader to this work for a through discussion. We note that if the ISS sector(s) is/are sequestered from other SUSY breaking sectors, and only couple to them via the SSM, then corrections to these masses should be at least a loop factor smaller than $m_{3/2}$. If the ISS sector only couples to the SSM and other SUSY breaking sectors gravitationally then we expect the masses not to deviate from the calculation above.

3.5 Couplings and Phenomenology

We now turn briefly to collider phenomenology. For definiteness throughout this section, we will take as a working assumption the set-up described in detail in Refs. [39, 40], namely, the goldstino and gravitino masses are $\geq \mathcal{O}(10\text{GeV})$, and the SUSY-breaking scales are such that the goldstini have comparable, or greater couplings to the SSM than the gravitino. In such a set-up all SUSY collider events will terminate in a cascade decay to the LOSP which may further decay to the goldstino or gravitino within the detector. Such an event can lead to striking signatures at the LHC [59] such as monochromatic electrons or muons in the case of a selectron or smuon LOSP [138]. The lifetime of the LOSP could be determined by observing decays of stopped LOSPs within the detector [25, 42] or within a proposed stopper detector [70, 89, 90] which could be constructed after the observation of long-lived charged particles. Further, when observing these decays it may be possible to determine the masses of the gravitino and goldstino using the methods discussed in [38, 41, 111]. Therefore, under these assumptions, it may be possible to measure the gravitino and goldstino masses and couplings to the SSM LOSP and we will take this to be the case throughout the remainder of this work.

3.5.1 Couplings in the warped/conformally-sequestered case

Let us begin with a few brief remarks on the coupling of warped and sequestered goldstini to the Standard Model. The couplings of the goldstini to Standard Model fields in this case come from interactions of the form

$$\mathcal{L} \supset \sum_i \frac{1}{\Lambda_i^2} \int d^4\theta X_i^\dagger X_i \Phi^\dagger \Phi \quad . \tag{3.5.1}$$

In the case of conformal sequestering (the situation is analogous for warping), large anomalous dimensions associated with the operator X_i lead to a suppression of the above operator at scales $E < \Lambda_i$ of order $(E/\Lambda_i)^{2\gamma_i}$ (if the operator $X_i^\dagger X_i$ corresponds to a conserved current there is no suppression). Assuming the exit from the conformal fixed point is controlled by SUSY breaking, this amounts to a suppression of order

$(\sqrt{f_{X_i}}/\Lambda_i)^{2\gamma_i}$ in the infrared. Ultimately, this suppression affects both the goldstino-SSM couplings as well as the contributions of this sector to SSM soft masses, such that the infrared interactions are still of the form

$$\mathcal{L} \supset \sum_i \frac{1}{\Lambda_i^2}\left(\frac{f_{X_i}}{\Lambda_i^2}\right)^{\gamma_i} \int d^4\theta X_i^\dagger X_i \Phi^\dagger \Phi = \sum_{i,a} \frac{\tilde{m}_i^2 V_{ia}}{f_i} \zeta_a \psi \phi^\dagger \quad . \tag{3.5.2}$$

Here the conformal suppression is simply absorbed into the soft mass $\tilde{m}_i$, which therefore may be significantly smaller than naive expectations. The principle effect of this is to further suppress the contributions of sequestered or warped sectors to both SSM soft masses and the relevant SSM-goldstino couplings. However, for *fixed* TeV-scale soft masses, the couplings of such goldstini to the SSM are still significantly stronger than gravitational. The couplings of, e.g., derivatively-coupled modulini in a warped/sequestered sector are suppressed relative to this by the usual factor $E/\tilde{m}$ at energy scale E, but not by additional factors from warping.

As mentioned in the Introduction, V_{ia} is the rotation matrix that diagonalises the goldstini mass matrix:

$$m_{ij} = 2m_{3/2}\left(\delta_{ij} - \frac{f_i f_j}{f_{eff}^2}\right) \quad , \tag{3.5.3}$$

where the ζ_a are the $N-1$ goldstini mass eigenstates and the true goldstino that forms the longitudinal component of the gravitino is the N^{th} eigenstate with zero mass in this basis. We see that as $\sum_i f_i V_{i,a\neq N} = 0$ then if SUSY breaking from all sectors is communicated in an identical way, i.e. all Λ_i are equal, then the goldstini couplings to the SSM would be zero, and we would only interact with the true goldstino that forms the longitudinal mode of the gravitino. However, it is a reasonable assumption that in general not all Λ_i are equal, and if even one of these effective mediation scales is different then couplings to all goldstini are generated.

3.5.2 Distinguishing ISS SUSY-breaking

An important question is how we could possibly distinguish if we were coupled to N_c goldstini from one hidden ISS sector, multiple goldstini from many different sectors or just one goldstino with a different effective SUSY breaking scale. We will see that making this distinction is in principle possible, however we will first consider some moral differences between these scenarios. First of all for a hidden ISS sector one would expect a larger coupling of goldstini to sfermion-fermion pairs than gaugino-gauge boson pairs. This would imply a hidden SUSY breaking sector that preserves an R-symmetry. Secondly the F-terms of a hidden ISS sector should be roughly the same magnitude whereas there is no a priori reason to expect the SUSY breaking scale in multiple sequestered sectors to be similar. Finally the SUSY breaking F-terms of a hidden ISS sector would be mediated in a similar way, and therefore couplings to goldstini arising within a single ISS sector should be of the same order of magnitude.

We illustrate the possibility of making this distinction with the example given in the Introduction of two sequestered SUSY breaking sectors which couple to the SSM differently as illustrated in Figure 3.1. Considering the decay of a scalar LOSP to the goldstini via the Goldstino Portal, if we ignore details of phase-space factors, the partial width for this process is:

$$\Gamma_{\phi^\dagger\to\zeta\psi} \simeq \frac{1}{16\pi} m_\phi \sum_{a=1}^{N-1} |C_a|^2 \tag{3.5.4}$$

where C_a is the dimensionless coupling of the goldstini to ϕ and ψ given in Eq.(3.5.2). As detailed in Appendix A.3, if the sfermion masses are generated through a Kähler potential term of the form

$$K_{soft} = \frac{\text{Tr }[\Phi^\dagger\cdot\Phi]\phi^\dagger\phi}{x\Lambda^2} \tag{3.5.5}$$

where ϕ is an MSSM field, we find that the respective decay widths to goldstini and

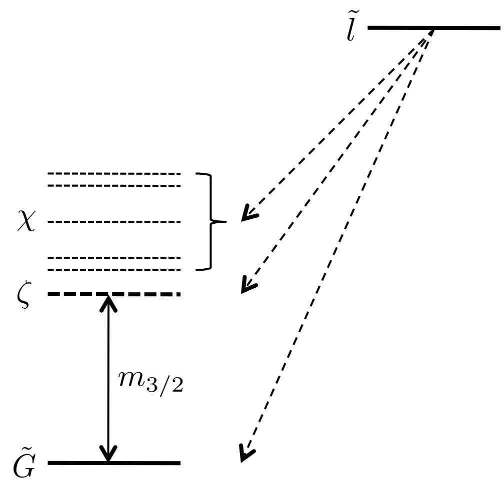

Figure 3.3: LOSP decays to the gravitino, goldstini and modulini of a hidden ISS sector. The modulini masses are bounded below by $2m_{3/2}$ and the observation of such a decay pattern would provide strong support for the physical realisation of the ISS mechanism of SUSY breaking.

the gravitino are:

$$\Gamma_{\phi^\dagger \to \zeta\psi} \simeq \frac{m_\phi}{16\pi}\left(\frac{(x-1)f}{x\Lambda^2}\right)^2 \frac{N_c y^2}{1+N_c y^2} \tag{3.5.6}$$

$$\Gamma_{\phi^\dagger \to G\psi} \simeq \frac{m_\phi}{16\pi}\left(\frac{f}{x\Lambda^2}\right)^2 \frac{(x+N_c y^2)^2}{1+N_c y^2} \tag{3.5.7}$$

As expected we see that in the limit $x \to 1$ the decay channel to goldstini vanishes and we recover the usual decay width to the gravitino. More importantly however we see that N_c from the ISS sector always appears in combination with y^2 which parameterises the overall scale of the SUSY breaking in the ISS sector. Therefore in this scenario with Goldstino Portal couplings alone one could not distinguish the N_c goldstini in an ISS sector from a SUSY breaking sector with one goldstino and a higher SUSY breaking scale.

However, there is in principle no reason for the couplings to be of the form in

Eq.(3.5.5) and the more general coupling

$$K_{soft} = \frac{B_{ijkl}\Phi^{\dagger}_{ij}\Phi_{kl}\phi^{\dagger}\phi}{x\Lambda^2} \tag{3.5.8}$$

where B takes some unknown values, allows not only the goldstini to couple to the SSM fields but the off-diagonal modulini fields also couple with similar strength. This arises through the non-zero F-terms of the SUSY breaking fields.[12] This coupling then allows for a 'smoking gun' collider signature of a hidden ISS sector as LOSP decays will occur to the goldstini and modulini of the hidden ISS sector. If not all F-terms are identical then the modulini masses are all greater than $2m_{3/2}$ (in the absence of warping or conformal sequestering), but bounded below by this value and the long-lived LOSP decay spectrum would be observed to be of the form depicted in Figure 3.3. This decay pattern would give strong support for the ISS mechanism of SUSY breaking if observed and the number of colours in the hidden ISS sector could, in principle, be deduced from the number of decay lines.

This signature is distinct from, say, a LOSP decaying to many gravitino-mass-scale moduli or modulini. This is because the couplings to the ISS sector particles are not simply Planck-scale but depend on a combination of the SUSY-breaking scale in the ISS sector and the messenger scale, which needn't necessarily bear any relation to the Planck scale. Therefore, although one would generically expect many $O(m_{3/2})$ mass particles in top down constructions such particles would not typically lead to the LOSP decay signatures as could arise from a hidden ISS sector.

3.6 Conclusions

As the LHC begins to make inroads in the exploration of the electroweak scale, it is timely to consider what experimental indications may be found regarding physics at

[12]We note that the coupling of hidden sector SUSY-preserving fields to the SSM through their interactions with the SUSY breaking fields can therefore be of the same strength as goldstino couplings to the SSM and this could have phenomenological consequences for other models of SUSY breaking. This mechanism of SUSY-preserving fields 'hitching' through the Goldstino Portal could be useful in scenarios where small renormalizable couplings to gauge-invariant combinations of SSM fields are desired.

much higher scales. Certainly the discovery of Standard Model superpartners would be a great breakthrough in itself, but there also exists the potential to learn much more about the mechanism of supersymmetry breaking and its communication to our sector. The observation of LOSP decays to a gravitino would tell us about the quantum nature of gravity [35], while the goldstini proposal [40] shows both that such an observation could be consistent with a standard cosmology [39] and that the observation of LOSP decays to additional goldstini would imply the existence of other sequestered SUSY breaking sectors. As we have argued, the existence of such SUSY breaking sectors is a natural consequence of compactification on a topologically complex manifold.

Considering the consequences of multiple sequestered SUSY breaking sectors in light of experimental constraints (e.g., FCNCs) and theoretical considerations (e.g., the structure of calculable models of dynamical SUSY breaking) leads to a surprisingly rich spectrum of fields whose masses range from $0-2\,m_{3/2}$ and whose interactions with Standard Model fields may be much stronger than the naive mediation scales suggest. The presence of such goldstini and modulini spanning a range of masses would significantly alter conventional supersymmetric phenomenology. Moreover, measuring their masses and couplings at the LHC would lend insight into not merely the mechanism of supersymmetry breaking, but also the existence and dynamics of additional sectors coupled to the Standard Model through the Goldstini Portal. In this fashion, various features of ultraviolet physics – warping, conformal dynamics, metastable supersymmetry breaking – may become evident in the infrared via the mass spectrum and interactions of goldstini and modulini.

Chapter 4

Dynamical supersymmetry breaking & flavor

4.1 Introduction

Two central mysteries in fundamental physics involve the discrepancy between G_{Fermi} and G_{Newton}, and the origin of the patterns in the quark and lepton Yukawa couplings. Supersymmetry is a well motivated candidate which addresses the first question. It is then natural to ask, can we find supersymmetric models of weak scale physics where both questions are answered simultaneously, and the dynamics that explains the weak scale also explains the texture of the fermion mass matrix?

One promising idea which could explain the structure of the Yukawa couplings is compositeness. If the first two generations of quarks and leptons are composites at some intermediate scale Λ, while flavor physics is generated at $M_{\text{flavor}} \gg \Lambda$, then the masses and mixings of the first two generations will be suppressed by the small parameter $\epsilon \equiv \Lambda/M_{\text{flavor}}$. The third generation should be elementary (external to the strong dynamics), because the top quark Yukawa coupling is $\mathcal{O}(1)$ and thus not suppressed. It was proposed in [17,130] that perhaps the strongly-coupled sector that is responsible for dynamical supersymmetry breaking could also generate the first two generations of quarks and leptons as composites of the same strong dynamics. Such "single-sector" models could give a simultaneous explanation of the Planck/weak hierarchy and the masses and mixings of Standard Model particles.

While this is an attractive idea, there were no calculable examples. Recently, using the fact that supersymmetric QCD has simple metastable vacua that exhibit dynamical supersymmetry breaking [103], calculable examples of such single-sector models were developed [72]. The simplest examples give rise to two composite generations, both arising from dimension two operators in the high energy theory. The natural texture of the matrix of masses and mixings is then of the form

$$\begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} . \tag{4.1.1}$$

In the models of [72], the first two generations of sparticles are parametrically heavier than the third generation sparticles.

It would be nice, however, to find other classes of calculable single-sector models where the mass matrix can take a more general form. For instance, if one of the generations arises from a dimension three operator in the high energy theory, while the other arises from a dimension two operator, one would expect a mass matrix of the slightly more appealing form

$$\begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix} . \tag{4.1.2}$$

Our goal in this paper is to explore the class of calculable single-sector models that can be constructed given the current state-of-the-art in models of dynamical supersymmetry breaking. We will find that models with this flavor structure – as well as models with additional parameters that give more general classes of mass matrices – can easily be constructed.

In the models of [17, 130], as well as the newer calculable models in [72], the composite generations not surprisingly couple more strongly to the SUSY-breaking order parameter than the elementary third generation (whose leading sfermion mass arises from gauge mediation, after weakly gauging the Standard Model subgroup of the global symmetry group of the SUSY-breaking theory). Therefore, one is led to phenomenology very reminiscent of the "more minimal" scenario advocated by Cohen, Kaplan and Nelson [43] (see also the earlier paper [58]), where the first and second generation sfermion masses are larger than those of the third generation. One of the surprises we shall find here is that in some of our models even some of the composites can have leading masses arising from gauge mediation and comparable to the third generation masses.

4.1.1 General strategy

Before we proceed to a detailed analysis, it is worth explaining the general strategy. One of the most elegant ideas for explaining the texture of Yukawas given by Eq. (4.1.2), which matches observation reasonably well, is to postulate that the first and second generations are secretly composite above some scale Λ, and in the high-energy theory their Yukawa couplings are then irrelevant operators. With a first and second generation emerging from operators whose dimensions in the UV are 3 and 2 (and an elementary third generation), one naturally gets the structure above, with the small parameter

$$\epsilon = \Lambda/M_{\rm flavor} \tag{4.1.3}$$

emerging from the suppression of irrelevant operators in the high-energy theory. For $\epsilon \sim 10^{-1}$, this is an excellent starting point for matching observations.

More concretely, consider an asymptotically free SQCD theory with gauge group G, fundamental quarks $(Q, \tilde{Q})$ and a field U in a 2-index tensor representation of the gauge group. We will call this the "electric theory", and its dynamical scale will be denoted by Λ.

A promising approach to constructing calculable models arises when the theory has an infrared dual description (the "magnetic theory") where the mesons $(QU\tilde{Q})$ and $(Q\tilde{Q})$ are weakly coupled. These are the fields that will produce the first and second generations. Generically, the IR theory will also contain magnetic quarks $(q, \tilde{q})$, and a field $\tilde{U}$ in a rank 2 tensor representation of the magnetic gauge group.

Furthermore, we imagine that there is some additional UV physics at a scale $M_{\rm flavor} > \Lambda$, responsible for generating the Yukawa couplings[1]

$$
\begin{aligned}
W_{Yuk} \quad \supset \quad & \frac{1}{M_{\rm flavor}^4}(QU\tilde{Q})H(QU\tilde{Q}) + \frac{1}{M_{\rm flavor}^3}(Q\tilde{Q})H(QU\tilde{Q}) + \\
& \frac{1}{M_{\rm flavor}^2}(Q\tilde{Q})H(Q\tilde{Q}) + \frac{1}{M_{\rm flavor}}(Q\tilde{Q})H\Psi_3 + \Psi_3 H \Psi_3 \ . \qquad (4.1.4)
\end{aligned}
$$

Rescaling the fields by appropriate powers of Λ so that they are canonically normalized gives a Yukawa matrix of the form (4.1.2).

In general, the mesons $(Q\tilde{Q})$ and $(QU\tilde{Q})$ contain more matter than just the first two Standard Model generations. It will be shown that some of the extra components of these fields together with the magnetic quarks yield a weakly coupled supersymmetry breaking model (as in [103]). In this effective description, supersymmetry breaking occurs through tree-level and one-loop interactions, while the supersymmetry breaking scale is generically an inverse loop factor above the electroweak scale.

The organization of the paper is as follows. In §4.2, we present the simplest model which naturally gives rise to two composite generations with a Yukawa matrix of more general type than (4.1.1). This model has two parameters in the flavor sector instead of one, and so while it can model observations quite well, it is perhaps less elegant than the more predictive structure in (4.1.2). Therefore, in §4.3, we move on to a class of models which give rise to the structure (4.1.2). A starring role is played by the metastable SUSY-breaking vacua of supersymmetric QCD with fundamental flavors and an additional adjoint chiral superfield. After discussing the asymptotically free electric theory and its infrared free magnetic dual, we find new metastable SUSY breaking vacua.

In §4.4 we show how this simple model naturally explains the flavor hierarchy and present the fermion and sparticle spectrum. We also discuss constraints on the sparticle spectra from flavor changing neutral currents (FCNCs). The simplest model is consistent with the constraints from FCNCs only in a small region of parameter

[1] The MSSM contains separate H_u and H_d fields, but we will simplify schematic equations of this sort by just denoting both Higgs fields by H throughout the paper.

space, and in §4.5 we present more general models that accommodate current bounds.

Finally, in §4.6, we present our conclusions, where we also briefly compare this method of explaining the Yukawa flavor pattern to other common explanations in the literature. Two appendices are devoted to a more careful discussion of FCNCs (Appendix B.1) and a discussion of gauge coupling unification and the existence of Landau poles (Appendix B.2). Since all of the models we study will typically have a lot of extra massive matter at very high scales, gauge coupling unification can be challenging; however, as explained Appendix B.3, one way to reduce the number of extra supermassive fields significantly is to abandon the requirement that the very massive extra matter fill out complete $SU(5)$ multiplets.

4.2 A Simple Model

4.2.1 Basic scheme

We will implement the pattern (4.1.2) in models with calculable dynamical supersymmetry breaking, with the composites arising from the SUSY breaking sector. However, because the models are rather complicated, it is useful to start by realizing a more modest goal. One could instead envision a model which generates the first and second generations as composites of *different* strongly coupled sectors. With both arising from, say, dimension two operators in the high-energy theory, the resulting Yukawa texture would be of the form

$$\begin{pmatrix} \epsilon^2 & \epsilon\delta & \epsilon \\ \epsilon\delta & \delta^2 & \delta \\ \epsilon & \delta & 1 \end{pmatrix} , \tag{4.2.1}$$

with

$$\epsilon = \Lambda_1/M_{\text{flavor}}, \quad \delta = \Lambda_2/M_{\text{flavor}} \ . \tag{4.2.2}$$

While this is less elegant than the first idea, we will see that it is quite simple to realize in practice. One can therefore compare the relative complexity of the model building required to realize the different textures and decide which seems more appealing. In fact, as we will see, the simplest class of models which realizes the texture (4.2.1) can also, by variation of parameters, realize the texture (4.1.2). So it is quite natural to consider both patterns.

4.2.2 Example

The most obvious way to make such a model is to simply combine two of the calculable single-sector models that produce a single composite generation which is dimension two in the UV theory, discussed in §4.1 of [72].

For instance, consider supersymmetric $SU(N_c)$ QCD with $N_c = 11$ and with $N_f = 12$ flavors of quarks $Q, \tilde{Q}$, and a common quark mass $m \ll \Lambda$. This theory has metastable vacua which are evident in the weakly coupled magnetic dual description [149], valid at energies $\ll \Lambda$. The magnetic dual is an $SU(N_f - N_c)$ gauge theory with N_f flavors of magnetic quarks $q, \tilde{q}$, and a meson Φ which transforms in the $(\mathrm{Adj} + \mathbf{1})$ of the $SU(12)$ flavor group but is a gauge singlet. The magnetic superpotential is

$$W = h\ \mathrm{Tr}(\Phi\tilde{q}q) - h\mu^2\ \mathrm{Tr}\Phi\ , \tag{4.2.3}$$

where the second term arises due to the mass deformation of the electric theory. Here,

$$\mu \sim \sqrt{m\Lambda} \tag{4.2.4}$$

and we can set $\Lambda_{\mathrm{magnetic}} = \Lambda$ (where the magnetic theory develops a Landau pole at $\Lambda_{\mathrm{magnetic}}$), so $h \sim 1$.

This theory breaks supersymmetry by the rank condition [99]; the magnetic quarks develop a vacuum expectation value (vev) which breaks the $SU(12)$ flavor symmetry to $SU(11)$, and $F_\Phi \neq 0$. We choose an embedding of the Standard Model $SU(5)$ into the $SU(12)$ flavor group such that:

$$\begin{aligned} Q &= (\mathbf{5} + \overline{\mathbf{5}} + \mathbf{1}) + \mathbf{1} \\ \tilde{Q} &= (\overline{\mathbf{5}} + \mathbf{5} + \mathbf{1}) + \mathbf{1} \end{aligned} \tag{4.2.5}$$

where the decomposition in parentheses indicates the embedding into $SU(11)$. The mesons of the magnetic theory can then be decomposed according to

$$\Phi = \begin{pmatrix} Y_{1\times 1} & Z^T_{1\times 11} \\ \tilde{Z}_{11\times 1} & X_{11\times 11} \end{pmatrix} , \tag{4.2.6}$$

with $Y, Z, \tilde{Z}$ and X transforming in the $\mathbf{1}, \overline{\mathbf{11}}, \mathbf{11}$, and $(\mathrm{Adj} + \mathbf{1})$ of $SU(11)$.

In terms of $SU(5)$ quantum numbers, X decomposes as

$$X = (\mathbf{10} + \bar{\mathbf{5}}) + \left[2 \times \mathbf{24} + \mathbf{15} + \overline{\mathbf{15}} + \overline{\mathbf{10}} + 2 \times \mathbf{5} + \bar{\mathbf{5}} + 3 \times \mathbf{1}\right] . \tag{4.2.7}$$

We see that there is an entire Standard Model generation, and additional matter which can be given a large mass (at the scale Λ) as in [72], by adding appropriate "spectators" to the QCD dynamics and deforming the superpotential by a suitable mass term:

$$W_3 = \lambda \sum_{\mathbf{R}} \left((Q\tilde{Q})_{\mathbf{R}} S_{\overline{\mathbf{R}}} \right) \tag{4.2.8}$$

with the sum over representations running over the representations in brackets in (4.2.7), and $S_{\overline{\mathbf{R}}}$ being spectators added in the appropriate *conjugate* representations. After recalling that the relation between the magnetic meson and $Q\tilde{Q}$ involves a power of Λ to canonically normalize the meson, one sees that this gives the unwanted matter masses of order $\lambda\Lambda$ which can be a very high scale. (We envision choosing Λ just below the GUT scale, for instance.)

The composite generation arising from X is obviously of dimension two in the high-energy theory, and therefore it will have Yukawa couplings suppressed by the ratio of scales $\Lambda/M_{\mathrm{flavor}}$. The scalars in X are pseudo-moduli which receive a calculable mass from loops in the magnetic theory, of order $h^2\mu$. Gauge mediation, with "messengers" coming both from the composite generation and some of the additional components of X and the magnetic quarks, will transmit masses of order $g_{SM}\mu$ to the other Standard Model generations [72].

It is then clear how to proceed to make a simple model which gives rise to the pattern of Yukawa couplings in (4.2.1), with two composite generations. Consider an $SU(N_{c,1}) \times SU(N_{c,2})$ gauge theory with $N_{f,1}$ flavors of quarks in the first gauge factor and $N_{f,2}$ in the second.

If we choose $N_{c,i} = 11$, $N_{f,i} = 12$, and independent quark masses m_i for the two sets of quarks, we end up with two copies of the previous model, with SUSY-breaking scales $\mu_{1,2} = \sqrt{m_{1,2}\Lambda_{1,2}}$.

Gauge invariance forbids any additional marginal or relevant couplings in the electric theory, so in fact the most generic renormalizable superpotential for the high-energy theory takes precisely the form we wish, though the choice of parameters $m_i \ll \Lambda_i$ is only technically natural and would need to be retrofitted [61] in an acceptable construction.

Adding now an elementary pair of Higgs bosons and an elementary third generation, we will find precisely the pattern of Yukawas in (4.2.1), with ϵ and δ as in (4.2.2).

The FCNC problems in this kind of model will be discussed in §5 and Appendix B. With heavy sparticle masses $\sim$ 20 TeV in the first two generations, moderate degeneracy is required. The soft masses of the first two generations will be controlled by $\mu_1/4\pi$ and $\mu_2/4\pi$; then the μ_i should be chosen to lie in the range $\sim$ 250 TeV to avoid prohibitive FCNCs.[2] Gauge mediated masses will then be dominated by the larger of these two scales. There will be 8 additional messenger pairs in the $\mathbf{5} + \overline{\mathbf{5}}$ of $SU(5)$, coming from the magnetic quarks and mesons in the two QCD sectors. Therefore, these models will have a Landau pole before the GUT scale.

In the discussion so far, we have not broken R-symmetry. We can incorporate R-breaking by adding a further superpotential deformation to the electric theory, $\Delta W_{el} \sim (Q\tilde{Q})^2$. This perturbation was studied in some detail in [66]. The perturbation to the magnetic dual theory is

$$W_4 = \frac{1}{2} h^2 \mu_\phi \ \mathrm{Tr}(\Phi^2) \ . \tag{4.2.9}$$

[2] This introduces a new coincidence problem: why are the masses generated by two unrelated sectors of strong dynamics relatively close to one another? We require μ_1 and μ_2 to be within roughly twenty percent of one another to avoid problems with FCNC; the relevant constraints on similar models will be discussed in great detail in §5 and Appendix B. We note that obtaining the two sectors from a single theory at higher energies, along the lines indicated in the next section, could ameliorate this coincidence problem.

This perturbation both explicitly breaks R-symmetry, and leads to a larger *spontaneous* breaking, as the $SU(11)$ singlet in X develops a vacuum expectation value. Also, after the addition of this coupling, we see that the composite generation no longer arises strictly from X – competition between the masses from (4.2.8) and (4.2.9) render it an admixture of the $\mathbf{10}+\overline{\mathbf{5}}$ from X, and one of the spectators. However, for $\mu_\phi \ll \Lambda$ it is largely composite, with the admixture of spectator suppressed by the small parameter μ_ϕ/Λ. To get acceptable gaugino masses we will set $h^2\mu_\phi$ to lie in the TeV range, while Λ is not far from M_{GUT}, so this ratio is negligibly small.

4.2.3 A landscape of simple models

We can derive the simple model in §4.2.2 by starting with a high-energy theory consisting of a single $SU(N_c)$ gauge group with N_f quark flavors together with an adjoint superfield U. The dynamics of this theory was studied in detail, in the presence of an adjoint superpotential, in [117–119]. Let us imagine that our theory has a superpotential

$$W \;=\; \frac{g_{k+1}}{k+1}\ \mathrm{Tr}\ (U^{k+1}) + \ldots + g_1\ \mathrm{Tr}\ (U) \;=\; \mathrm{Tr}\ (P_{k+1}(U))\ , \tag{4.2.10}$$

where $P_{k+1}(U)$ is a generic degree $k+1$ polynomial $P_{k+1} = \sum_{j=1}^{k+1} \frac{g_j}{j} U^j$, and g_1 should be interpreted as a Lagrange multiplier imposing the tracelessness constraint on U.

The classical vacua of this theory can be found by setting the eigenvalues of the $N_c \times N_c$ traceless matrix U equal to various roots of the equation

$$P'(x) = \sum_{j=0}^{k} g_j x^j = \prod_{i=1}^{k}(x-a_i) \;=\; 0. \tag{4.2.11}$$

Let us assume that P is sufficiently generic so that $a_i \neq a_j$ for $i \neq j$.

In the vacuum where N_i of the eigenvalues of U are equal to a_i, and a total of p different a_i appear as eigenvalues of U, the gauge group is broken as

$$SU(N_c) \to \prod_{i=1}^{k} SU(N_i) \times U(1)^{p-1} \tag{4.2.12}$$

where $\sum_i N_i = N_c$.

The classical low-energy physics is that of a product of SUSY QCD theories with N_f quark flavors, but it is clear that in the quantum theory the physics depends in detail on the precise values of the a_i, since e.g. $a_i - a_j$ determines the masses of charged off-diagonal components of the U field which serve as bi-fundamentals connecting the different gauge factors. As long as the k roots a_i in (4.2.11) are distinct, the adjoint superfield gives rise to no massless excitations in any of these vacua. Not all such partitions give rise to a theory with supersymmetric quantum vacua. For instance, if any of the $SU(N_i)$ factors has $N_i > N_f$, it suffers from a runaway to infinity in field space.

Now, deforming the high-energy theory by a small quark mass m for the N_f quark flavors (small compared to the effective adjoint mass in each vacuum), we obtain a landscape of vacua with different $SU(N_i)$ gauge factors, each with N_f quarks. The different SQCD sectors have different scales Λ_i, determined by matching scales at the value of the adjoint mass. In particular, the scale of the ith theory is determined in terms of the scale Λ of the original electric theory by

$$\Lambda_i^{3N_i - N_f} = \Lambda^{2N_c - N_f} g_{k+1}^{N_i} \prod_{j \neq i} (a_i - a_j)^{N_i - 2N_j}. \tag{4.2.13}$$

This implies that the supersymmetry breaking scale of each $SU(N_i)$ theory is determined in terms of the scale of the parent $SU(N_c)$ gauge theory, the quark mass m, and the pattern of symmetry breaking encoded in (4.2.11). We can then anticipate generating a variety of vacua starting from one high-energy gauge theory, giving rise to a discretuum of possible values of the parameters ϵ, δ in §4.2.1.

Interestingly, in §4.3, we will also obtain models with the texture (4.1.2) from this class of gauge theories with $k = 2$. So it is possible that one high-energy theory could give rise, in different vacua, to single-sector models which each have realistic phenomenology, with different explanations for the physics of flavor!

4.3 SQCD with an adjoint

The previous section explored a class of models giving a Yukawa matrix (4.2.1) based on two parameters ϵ and δ. The rest of the paper is devoted to constructing calculable models with a "dimensional hierarchy", where the first and second generations arise from composite fields of dimension 3 and 2, respectively, while the third generation (denoted by Ψ_3) and Higgs are elementary. Such models naturally give rise to the desired Yukawa texture (4.1.2) involving a single parameter ϵ.

We now focus on a study of the theory which appeared in §4.2.3: the electric gauge theory will be $SU(N_c)$ SQCD, with N_f quarks $(Q_i, \tilde{Q}_j)$, and a field U in the adjoint of the gauge group. While the analysis of §4.2.3 was concerned with large adjoint masses (such that the adjoint could be integrated out of infrared physics), henceforth we will be interested in the case where the adjoint mass is small and its dynamics remains important at low energies. This theory has been studied in detail in [117–119], and we start by reviewing their conclusions.[3]

4.3.1 The electric theory

We begin by specializing to the case where the adjoint has a general renormalizable superpotential

$$W_{el} = \frac{g_U}{3}\ \mathrm{Tr}\ U^3 + \frac{m_U}{2}\ \mathrm{Tr}\ U^2 + \lambda\ \mathrm{Tr}\ U\,. \tag{4.3.1}$$

This superpotential will not have any metastable SUSY breaking vacua, which requires additional perturbations discussed below in §4.3.3. Here ' Tr ' means a trace over the gauge indices, while ' Tr' will be used to indicate traces over flavor indices. λ is a Lagrange multiplier field, imposing $\mathrm{Tr}\ U = 0$. We denote the strong coupling scale by Λ. Calculability in the magnetic dual theory discussed below will require $m_U \ll \Lambda$. Higher dimensional operators $\mathrm{Tr}\ U^{k+1}$ with $k \geq 3$ are dangerously irrelevant and may influence IR physics if present; see [119] for a discussion of their effect. In general we will focus on theories with $k = 2$, albeit with some discussion of theories with $k \geq 3$ in subsequent sections.

[3] See e.g. [9] for a rather different construction of metastable vacua in SQCD with an adjoint.

The matter content with its gauge and anomaly free global symmetry quantum numbers is (for $m_U = 0$),

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_V$	$U(1)_R$
Q	$\square$	$\square$	1	1	$1-\frac{2}{3}\frac{N_c}{N_f}$
$\tilde{Q}$	$\overline{\square}$	1	$\overline{\square}$	-1	$1-\frac{2}{3}\frac{N_c}{N_f}$
U	Adj	1	1	0	$\frac{2}{3}$

A nonzero mass m_U breaks the R-symmetry. It will be useful to think of m_U as a background superfield with R-charge 2/3.

The superpotential has two critical points, a_1, a_2. The different classical vacua correspond to placing r_1 eigenvalues of U equal to a_1, and $r_2 = N_c - r_1$ eigenvalues equal to a_2. The gauge group is broken to

$$SU(N_c) \to SU(r_1) \times SU(r_2) \times U(1)\,. \tag{4.3.2}$$

Imposing the tracelessness condition $r_1 a_1 + r_2 a_2 = 0$, the critical points are[4]

$$a_1 = \frac{r_2}{r_1 - r_2}\frac{m_U}{g_U}\ ,\ a_2 = -\frac{r_1}{r_1 - r_2}\frac{m_U}{g_U}\,. \tag{4.3.3}$$

The low energy theory splits into two decoupled SQCD sectors with only fundamental matter (as long as $m_U \neq 0$). Quantum-mechanically, the vacua are stable if all $r_i \leq N_f$; therefore, a necessary condition for the theory to have a stable vacuum is $N_f \geq N_c/2$. N_f will also be restricted to $N_f < \frac{2}{3}N_c$ so that the magnetic theory is IR free. Summarizing, we will work in the range

$$\frac{N_c}{2} < N_f < \frac{2}{3}N_c \tag{4.3.4}$$

(The case $N_f = N_c/2$ is excluded because there are no magnetic quarks.)

An important role will be played by the two mesons

$$(M_1)_{ij} = \tilde{Q}_i Q_j\ \ ,\ \ (M_2)_{ij} = \tilde{Q}_i U Q_j\,, \tag{4.3.5}$$

[4]Vacua with $r_1 = r_2$ can only exist for $m_U = 0$. This case won't arise in our discussions.

where the gauge indices are contracted and suppressed. The moduli space is parametrized by these mesons and baryons (we refer the reader to [119] for their definition, which will not be needed here), modulo classical relations. Notice that in [119], the dimension 3 meson was defined as

$$M_2^{KSS} = \tilde{Q}\left(U + \frac{m_U}{2g_U}\right) Q\,. \tag{4.3.6}$$

The redefinition $U \to U_s = U + \frac{m_U}{2g_U}$ amounts to setting $m_U = 0$ and simplifies considerably the electric-magnetic duality discussion. However, we will work with the definition (4.3.5), where M_2 has classical scaling dimension 3, instead of being a linear combination of dimension 2 and dimension 3 fields. This simplifies the structure of the Yukawa couplings (4.2.1) when we later embed the first Standard Model generation inside M_2.

4.3.2 The magnetic dual

The magnetic dual theory consists of SQCD, with gauge group $SU(\tilde{N}_c = 2N_f - N_c)$ and strong coupling scale $\tilde{\Lambda}$, N_f quarks $(q, \tilde{q})$, one magnetic adjoint field $\tilde{U}$, and two gauge singlet fields corresponding to the mesons (4.3.5). The theory has a superpotential[5]

$$\begin{aligned} W_{mag} &= -\frac{g_U}{3}\ \mathrm{Tr}\ \tilde{U}^3 + \frac{N_c}{2\tilde{N}_c} m_U\ \mathrm{Tr}\ \tilde{U}^2 + \tilde{\lambda}\ \mathrm{Tr}\ \tilde{U} + \\ &+ \frac{g_U}{\hat{\Lambda}^2}\left(\frac{\tilde{N}_c - N_c}{2\tilde{N}_c}\frac{m_U}{g_U}\ \mathrm{Tr}(M_1 q\tilde{q}) + \mathrm{Tr}(M_1 q\tilde{U}\tilde{q}) + \mathrm{Tr}(M_2 q\tilde{q})\right)\,. \end{aligned} \tag{4.3.7}$$

The Lagrange multiplier $\tilde{\lambda}$ is introduced to impose $\mathrm{Tr}\ \tilde{U} = 0$.

The energy scale $\hat{\Lambda}$ appears because M_1 and M_2 are elementary, but have scaling dimensions 2 and 3, respectively. This dimensionful quantity is related to the electric

[5] We are dropping a constant term which depends only on g_U. This becomes important when trying to match the gauge invariants $\mathrm{Tr}\ U^n \to \mathrm{Tr}\ \tilde{U}^m$. Also, (4.3.7) differs slightly from the expression in [119]; this is due to the meson definitions (4.3.5) and (4.3.6).

(Λ) and magnetic ($\tilde{\Lambda}$) dynamical scales by

$$\Lambda^{2N_c-N_f}\tilde{\Lambda}^{2\tilde{N}_c-N_f}=\left(\frac{\hat{\Lambda}}{g_U}\right)^{2N_f}. \tag{4.3.8}$$

For $m_U=0$, the gauge and global (nonanomalous) symmetry transformations are

	$SU(\tilde{N}_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_V$	$U(1)_R$
q	$\square$	$\overline{\square}$	1	$\frac{N_c}{\tilde{N}_c}$	$1-\frac{2}{3}\frac{\tilde{N}_c}{N_f}$
$\tilde{q}$	$\overline{\square}$	1	$\square$	$-\frac{N_c}{\tilde{N}_c}$	$1-\frac{2}{3}\frac{\tilde{N}_c}{N_f}$
$\tilde{U}$	Adj	1	1	0	$\frac{2}{3}$
M_1	1	$\square$	$\overline{\square}$	0	$2-\frac{4}{3}\frac{N_c}{N_f}$
M_2	1	$\square$	$\overline{\square}$	0	$\frac{8}{3}-\frac{4}{3}\frac{N_c}{N_f}$

Notice the different R-charge of M_1 and M_2 (which can be read off directly in the electric theory). A nonzero mass m_U breaks the R-symmetry.

In the range (4.3.4), the magnetic theory is IR free and the Kähler potential can be expanded

$$K=\frac{1}{\alpha_1|\Lambda|^2}\ \mathrm{Tr}(M_1^\dagger M_1)+\frac{1}{\alpha_2|\Lambda|^4}\ \mathrm{Tr}(M_2^\dagger M_2)+\frac{1}{\alpha_3}\ \mathrm{Tr}(q^\dagger q+\tilde{q}\tilde{q}^\dagger)+\frac{1}{\alpha_4}\ \mathrm{Tr}\ (\tilde{U}^\dagger\tilde{U})+\ldots \tag{4.3.9}$$

where α_i are order one positive numbers and '...' include interaction terms. The canonically normalized mesons are

$$\Phi:=\frac{M_1}{\sqrt{\alpha_1}\,\Lambda}=\frac{\tilde{Q}Q}{\sqrt{\alpha_1}\,\Lambda}\ ,\quad \Phi_U:=\frac{M_2}{\sqrt{\alpha_2}\,\Lambda^2}=\frac{\tilde{Q}UQ}{\sqrt{\alpha_2}\,\Lambda^2}\,. \tag{4.3.10}$$

Similarly, replacing $q\to\sqrt{\alpha_3}q$, $\tilde{q}\to\sqrt{\alpha_3}\tilde{q}$ and $\tilde{U}\to\sqrt{\alpha_4}\tilde{U}$ gives canonical kinetic terms to the adjoint and magnetic quarks. Henceforth, only canonically normalized fields will be used.

The superpotential then becomes

$$\begin{aligned} W_{mag} &= \frac{\tilde{g}_U}{3}\ \mathrm{Tr}\ \tilde{U}^3+\frac{\tilde{m}_U}{2}\ \mathrm{Tr}\ \tilde{U}^2+\tilde{\lambda}'\ \mathrm{Tr}\ \tilde{U}+ \\ &+ \frac{h}{\Lambda}\left[c_1\tilde{m}_U\ \mathrm{Tr}(\Phi q\tilde{q})+c_2\ \mathrm{Tr}(\Phi q\tilde{U}\tilde{q})\right]+h\ \mathrm{Tr}(\Phi_U q\tilde{q})\,. \end{aligned} \tag{4.3.11}$$

The parameters introduced here are related to the previous ones by

$$\begin{aligned}\tilde{g}_U &:= -(\alpha_4)^{3/2} g_U \ , \ \tilde{m}_U := \frac{\alpha_4 N_c}{\tilde{N}_c} m_U \ , \ h := \sqrt{\alpha_2}\alpha_3 \frac{g_U \Lambda^2}{\hat{\Lambda}^2} \\ c_1 &:= \left(\frac{\alpha_1 \alpha_4}{\alpha_2}\right)^{1/2} \frac{N_c - N_f}{\tilde{g}_U N_c} \ , \ c_2 := \left(\frac{\alpha_1 \alpha_4}{\alpha_2}\right)^{1/2} \ , \ \tilde{\lambda}' := \sqrt{\alpha_4}\,\tilde{\lambda}\,. \end{aligned} \quad (4.3.12)$$

Also, $\tilde{m}_U \ll \Lambda$ is required for calculability in the magnetic theory (although in the opposite limit, $\tilde{m}_U \gg \Lambda$, the adjoint may be integrated out of the electric theory to produce the models of §4.2.3).

We end this analysis by pointing out the following interesting consequence of the duality. All the interactions between the meson Φ and the rest of the fields of the magnetic theory are suppressed by $1/\Lambda$. At energies $E \ll \Lambda$, Φ approximately decouples from the rest of the system. In particular, while the trilinear coupling between Φ_U and the magnetic quarks is order h, the corresponding interaction for Φ is only order $h\tilde{m}_U/\Lambda$. This difference can be understood as follows: When $\tilde{m}_U = 0$ the $U(1)_R$ symmetry presented before forbids a coupling $\Phi q \tilde{q}$. Turning on a nonzero mass and treating it as a spurion superfield, the only trilinear coupling allowed by R-symmetry is $(\tilde{m}_U/\Lambda)\Phi q \tilde{q}$.

4.3.3 Metastable SUSY breaking

The low energy theory (4.3.11) contains a massive adjoint $\tilde{U}$, magnetic quarks $(q, \tilde{q})$ interacting with a meson Φ_U, and an extra meson Φ whose interactions with the other fields are suppressed by $1/\Lambda$. The $(\Phi_U, q, \tilde{q})$ sector is very similar to the magnetic theory studied by Intriligator, Seiberg, and Shih (ISS) in [103], although the corresponding electric theories are quite different. For instance Φ_U is of dimension 3 in the UV, while the ISS meson has scaling dimension 2.

We focus on vacua with $\langle \text{ Tr } \tilde{U}^2 \rangle = 0$, corresponding to $r_1 = N_f$, $r_2 = N_c - N_f$ in (4.3.3). For this choice of parameters the magnetic gauge group is unbroken. In addition, to reduce the amount of additional matter (see §4.4), we choose $\tilde{N}_c = 1$ (for this choice the magnetic gauge group is trivial). Then there is also no magnetic adjoint, and the magnetic superpotential simplifies to

$$W_{mag} = c_1 h \frac{\tilde{m}_U}{\Lambda}\ \operatorname{Tr}(\Phi q \tilde{q}) + h\ \operatorname{Tr}(\Phi_U q \tilde{q})\,. \tag{4.3.13}$$

Importantly for the low energy physics, in this case there is an additional R-symmetry under which the mesons have charge 2, while the magnetic quarks have charge 0. This symmetry, which is anomalous, will be denoted by $U(1)'_R$. Once the Standard Model gauge group is embedded in the symmetry group of the theory, we will need to break $U(1)'_R$ in order to generate large enough gaugino masses.

In the low-energy theory, $\tilde{m}_U/\Lambda$ appears as a free parameter which determines how strongly the meson Φ couples to the magnetic quarks. For pedagogical purposes, we first restrict ourselves to the limit $\tilde{m}_U \ll \Lambda$, which simplifies the analysis considerably. While this limit can lead – for a careful choice of parameters – to a phenomenologically viable model that is not in conflict with current limits from FCNCs (see §4.5.1), larger values of $\tilde{m}_U$ (§4.5.2) or additional superpotential interactions (§4.5.3) are desirable.

In this weakly coupled description, a supersymmetry breaking vacuum is generated once a term $\operatorname{Tr}\Phi_U$ is added to the superpotential.[6] Following [66, 82], the $U(1)'_R$ symmetry will be broken by adding a small explicit breaking term proportional to $\operatorname{Tr}\Phi_U^2$. Furthermore, in order to avoid an exactly massless superfield, a mass term $\operatorname{Tr}\Phi^2$ is needed.

Summarizing, the superpotential including the minimal set of deformations required to construct a realistic model of SUSY breaking is

$$\begin{aligned} W_{mag} &= c_1 h \frac{\tilde{m}_U}{\Lambda}\ \operatorname{Tr}(\Phi q \tilde{q}) + \frac{1}{2} m_\Phi\ \operatorname{Tr}\Phi^2 \\ &+ \left[-h\mu^2\ \operatorname{Tr}\Phi_U + h\ \operatorname{Tr}(\Phi_U q \tilde{q}) + \frac{1}{2} h^2 \mu_\phi\ \operatorname{Tr}(\Phi_U^2) \right]. \end{aligned} \tag{4.3.14}$$

To facilitate the interpretation of the model, the fields and interactions that will be responsible of breaking supersymmetry have been collected inside square brackets. The deformation parameters m_Φ, μ and μ_ϕ should be parametrically smaller than

[6] We break SUSY predominantly with Φ_U because the interactions of Φ with the magnetic quarks are suppressed by $\tilde{m}_U/\Lambda \ll 1$. Other deformations are explored below.

the dynamical scale Λ so that microscopic corrections to the Kähler potential can be neglected.

Equation (4.3.14) is the full superpotential when $\tilde{N}_c = 1$. For $\tilde{N}_c > 1$, it is straightforward to add the adjoint and interactions described in (4.3.11); in this case, the formulas below are still valid in the vacuum $\langle\ \mathrm{Tr}\ \tilde{U}^2\rangle = 0$.

Foreseeing the use of this theory as a single-sector model of SUSY breaking, we point out that certain off-diagonal components of Φ_U and Φ will be identified with the first and second Standard Model generations. Of course, such components cannot have large vector-like supersymmetric masses via superpotential terms (4.3.14) that couple them to conjugate fields. The Standard Model composite generations will be made massless by introducing heavy spectator fields coupled to the unwanted conjugate fields. However, for now we will analyze the theory with superpotential (4.3.14) and no extra fields.

In the electric theory, the deformations added to (4.3.13) to arrive at (4.3.14) correspond to perturbing (4.3.1) by

$$\Delta W_{el} \sim \lambda_Q\ \mathrm{Tr}(QU\tilde{Q}) + \frac{1}{\Lambda_0}\ \mathrm{Tr}(Q\tilde{Q})^2 + \frac{1}{\Lambda_0^3}\ \mathrm{Tr}(QU\tilde{Q})^2 \tag{4.3.15}$$

where Λ_0 is some UV scale satisfying $\Lambda_0 \gg \Lambda$. In particular, the Yukawa interaction $\lambda_Q\ \mathrm{Tr}(QU\tilde{Q})$ in (4.3.15) gives rise to the SUSY-breaking source term $-h\mu^2\ \mathrm{Tr}(\Phi_U)$ appearing in (4.3.14). Thus μ is related to the parameters of the electric theory by

$$h\mu^2 := \lambda_Q\sqrt{\alpha_2}\Lambda^2\ ,\ \ \mu := \sqrt{\frac{\lambda_Q}{\alpha_3 g_U}}\hat{\Lambda}, \tag{4.3.16}$$

The parametric separation of scales $\mu \ll \Lambda$ required for calculability and metastability in the magnetic theory arises from the smallness of the dimensionless coupling λ_Q, as contrasted with the dimensionful quark mass m of [103]. Indeed, all the deformations introduced in (4.3.15) arise from marginal and irrelevant interactions in the electric theory. In particular, no small quark mass term $mQ\tilde{Q}$ will be needed. More general perturbations will be discussed momentarily.

Since μ_ϕ comes from an irrelevant operator in the electric theory, we naturally have $\mu_\phi \ll \mu$. The analysis then proceeds as in [66]. In the limit $\mu_\phi \to 0$ supersymmetry is broken at tree level by the rank condition, and Φ_U is stabilized at the origin due to one-loop effects. For finite $\mu_\phi \ll \mu$, the $U(1)'_R$ is explicitly broken and supersymmetric vacua appear at a distance μ^2/μ_ϕ from the origin. The SUSY breaking vacuum is displaced slightly from the origin and is still parametrically long-lived. The observation that the CW potential can generate a minimum when the potential slopes to a supersymmetric vacuum at tree-level was noticed in [67, 68]. Of course, there are also supersymmetric vacua at large values of Φ_U, whose existence crucially relies on (calculable) non-perturbative effects [7], but as in [9, 103] the longevity of the metastable vacuum here is guaranteed by the hierarchy $\mu/\Lambda \ll 1$. Finally, the theory possesses a large number of additional vacua labeled by the possible partitions (4.3.2) of the gauge group; stability of the vacuum with $\langle\ \mathrm{Tr}\ \tilde{U}^2\rangle = 0$ against potential transitions into such vacua may be guaranteed provided $\mu \ll \tilde{m}_U$, which is readily accommodated.

Let us now analyze the pattern of supersymmetry breaking in more detail. We parameterize the fields as

$$\Phi_U = \begin{pmatrix} Y_{U,\,\tilde{N}_c\times\tilde{N}_c} & Z^T_{U,\,\tilde{N}_c\times(N_f-\tilde{N}_c)} \\ \tilde{Z}_{U,\,(N_f-\tilde{N}_c)\times\tilde{N}_c} & X_{U,\,(N_f-\tilde{N}_c)\times(N_f-\tilde{N}_c)} \end{pmatrix}\,, \tag{4.3.17}$$

$$\Phi = \begin{pmatrix} Y_{\tilde{N}_c\times\tilde{N}_c} & Z^T_{\tilde{N}_c\times(N_f-\tilde{N}_c)} \\ \tilde{Z}_{(N_f-\tilde{N}_c)\times\tilde{N}_c} & X_{(N_f-\tilde{N}_c)\times(N_f-\tilde{N}_c)} \end{pmatrix}\,, \tag{4.3.18}$$

$$q^T = \begin{pmatrix} \chi_{\tilde{N}_c\times\tilde{N}_c} \\ \rho_{(N_f-\tilde{N}_c)\times\tilde{N}_c} \end{pmatrix}\,,\ \tilde{q} = \begin{pmatrix} \tilde{\chi}_{\tilde{N}_c\times\tilde{N}_c} \\ \tilde{\rho}_{(N_f-\tilde{N}_c)\times\tilde{N}_c} \end{pmatrix}\,. \tag{4.3.19}$$

We will not present the spectrum of this model in detail[7], but only focus on the fields ρ, $\tilde{\rho}$, Z_U, $\tilde{Z}_U$, Z, and $\tilde{Z}$. Integrating out these fields generates the (bosonic) Coleman-Weinberg (CW) potential, which in general is given by

$$V_{CW} = \frac{1}{64\pi^2}\,\mathrm{STr}\,M^4 \log\frac{M^2}{\Lambda^2_{\mathrm{cut}}}\,, \tag{4.3.20}$$

[7] We refer the reader to [66].

where M is the mass matrix of the fields being integrated out and Λ_{cut} is some high-energy cut-off [44]. The superpotential for the fields that generate the CW potential that will lift the tree-level runaway direction X_U is

$$W \supset h \begin{pmatrix} \rho & Z_U & Z \end{pmatrix} \begin{pmatrix} X_U & \chi & \frac{c_1 \tilde{m}_U}{\Lambda}\chi \\ \tilde{\chi} & h\mu_\phi & 0 \\ \frac{c_1 \tilde{m}_U}{\Lambda}\tilde{\chi} & 0 & m_\Phi \end{pmatrix} \begin{pmatrix} \tilde{\rho} \\ \tilde{Z}_U \\ \tilde{Z} \end{pmatrix} , \tag{4.3.21}$$

where $\chi\tilde{\chi}$ is given by (4.3.27). Since we take $\tilde{m}_U/\Lambda \ll 1$, the Z, $\tilde{Z}$ fields completely decouple from the $\rho, \tilde{\rho}, Z_U, \tilde{Z}_U$ sector. Moreover, the SUSY breaking field X_U couples in this limit only to the $\rho, \tilde{\rho}, Z_U, \tilde{Z}_U$ sector, and we can focus on the fermion mass matrix

$$M_f = h \begin{pmatrix} X_U & \chi \\ \tilde{\chi} & h\mu_\phi \end{pmatrix} . \tag{4.3.22}$$

The bosonic components of $\rho, \tilde{\rho}, Z_U, \tilde{Z}_U$ will have masses given by

$$M_b = \begin{pmatrix} M_f^\dagger M_f & -h^* F^*_{X_U} \\ -hF_{X_U} & M_f M_f^\dagger \end{pmatrix} , \text{ with } \; - F^*_{X_U} = h \begin{pmatrix} -\mu^2 + h\mu_\phi X_U & 0 \\ 0 & 0 \end{pmatrix} . \tag{4.3.23}$$

The analysis proceeds now exactly as in [66], and we may borrow the results from there. Near the origin of field space, the Coleman-Weinberg potential from integrating out $\rho, \tilde{\rho}, Z_U$, and $\tilde{Z}_U$ is

$$V_{CW} = m^2_{CW} |X_U|^2 + \ldots \tag{4.3.24}$$

where '...' refers to higher order interactions and mixings with X that can be neglected. The "CW mass" is

$$m^2_{CW} = b|h^2\mu|^2 \;\;,\;\; b = \frac{\log 4 - 1}{8\pi^2}\tilde{N}_c \,. \tag{4.3.25}$$

Combining (4.3.24) with the tree-level potential computed from (4.3.14),

$$V_{\text{tree}} = (N_f - \tilde{N}_c)| - h\mu^2 + h^2\mu_\phi X_U|^2, \tag{4.3.26}$$

we find

$$\langle hX_U \rangle \approx \frac{\mu^2 \mu_\phi^*}{b|\mu|^2 + |\mu_\phi|^2} \approx \frac{\mu_\phi^*}{b} \ , \ \ \langle \chi \tilde{\chi} \rangle \approx \mu^2 \tag{4.3.27}$$

and

$$|W_{X_U}| \approx |h\mu^2| \, . \tag{4.3.28}$$

Importantly for the low energy phenomenology, the vev of X_U is larger than μ_ϕ^* by the inverse loop factor $1/b \sim 16\pi^2$. Hence, the spontaneous breaking of the R-symmetry is parametrically larger than the explicit one, and gaugino masses can be sufficiently large. Corrections suppressed by $1/\Lambda$ have been neglected.

The field Φ is stabilized supersymmetrically,

$$W_\Phi = 0 \ , \ \langle X \rangle = 0 \ , \ \langle Y \rangle \approx -c_1 \frac{\tilde{m}_U}{\Lambda} \frac{h\mu^2}{m_\Phi} \, , \tag{4.3.29}$$

where we have neglected corrections of $\mathcal{O}(\mu_\phi \mu^2 \tilde{m}_U^3/(m_\Phi^2 \Lambda^3))$. From the F-term for the magnetic quarks, we find

$$\langle Y_U \rangle = -c_1 \frac{\tilde{m}_U}{\Lambda} \langle Y \rangle \, . \tag{4.3.30}$$

The rest of the fields are stabilized at the origin. The hierarchy $\mu_\phi \ll \mu \ll \Lambda$ ensures that the vacuum is parametrically long-lived against transitions into the various supersymmetric vacua [66]. The theory receives microscopic corrections controlled by $\tilde{m}_U/\Lambda$ and μ/Λ, which are parametrically suppressed compared to the IR effects we have discussed. At this order, it is consistent to set $\langle Y \rangle = \langle Y_U \rangle = 0$. Moreover, (4.3.14) implies that there are one-loop contributions mixing X and X_U,

$$V_{1-loop} \sim m_{CW}^2 \, \mathrm{Re} \left(\frac{\tilde{m}_U}{\Lambda} X^* X_U \right) \, . \tag{4.3.31}$$

This is negligible in the limit $\tilde{m}_U \ll \Lambda$. Finally, we note that the unbroken global symmetry is

$$SU(N_f - \tilde{N}_c) \times U(1) \, . \tag{4.3.32}$$

In §4.4, we will weakly gauge and identify a subgroup of $SU(N_f - \tilde{N}_c)$ with the

Standard Model gauge group. This will mean that part of the X_U, X, Z_U, $\tilde{Z}_U$, ρ, $\tilde{\rho}$, Z, and $\tilde{Z}$ will have Standard Model gauge charges. In particular, we will identify part of X_U and X with the first and second generation Standard Model fermions.

4.3.4 More general superpotential perturbations

Let us summarize what we have done so far:

1. We have constructed a metastable vacuum based on the (almost decoupled) sector $(\Phi_U, q, \tilde{q})$, by having superpotential terms that are linear and quadratic in Φ_U; see (4.3.14).

2. The extra meson Φ has been lifted by adding an appropriate mass term, which is naturally large in the magnetic theory once $U(1)'_R$ is broken. This sector is decoupled from the SUSY breaking sector at leading order in $\tilde{m}_U/\Lambda$. It is important to note that later on, we will re-couple part of this sector (one chiral generation), and decouple the rest via couplings to spectator fields.

3. In the metastable vacuum, the magnetic gauge group is completely Higgsed at the scale $\langle \chi\tilde{\chi}\rangle = \mu^2$. The magnetic adjoint $\tilde{U}$ is massive and its interactions with the rest of the fields are suppressed by $1/\mu$ and $1/\Lambda$. Or, in the case of $\tilde{N}_c = 1$, the magnetic theory has no adjoint to begin with, as explained around (4.3.14).

In the high-energy electric gauge theory, we have allowed only specific marginal and irrelevant operators (4.3.15). The aim of this subsection is to discuss what happens when more general deformations are allowed.

Adding a U^4 piece changes the chiral ring and introduces extra degrees of freedom in the low energy theory. The resulting low energy phenomenology will be analyzed in §4.5.3. On the other hand, adding U^n factors (with $n \leq 3$) to any superpotential term containing the mesons $(Q\tilde{Q})$ and/or $(QU\tilde{Q})$, modifies negligibly the low energy theory. This is because we are considering a vacuum where the magnetic adjoint does not have a vev, and it has suppressed couplings to the SUSY breaking sector.

We are thus left with the possibility of adding irrelevant operators up to dimension 6, formed from the two mesons. One possibly dangerous term arises from the dimension 5 operator $(\tilde{Q}Q)(\tilde{Q}UQ)$. This would result in a mixing between Φ and Φ_U in the low energy magnetic theory.

The full magnetic superpotential arising from marginal and irrelevant deformations of the electric superpotential, up to dimension 6, is then of the form

$$\begin{aligned} W_{mag} &= -h\mu^2 \ \mathrm{Tr}\,\Phi_U + \frac{1}{2}m_\Phi \ \mathrm{Tr}\,\Phi^2 + \Delta m \ \mathrm{Tr}\,\Phi\Phi_U + \alpha \ \mathrm{Tr}\,\Phi^3 + \\ &+ \frac{1}{2}h^2\mu_\phi \ \mathrm{Tr}\,\Phi_U^2 + c_1 h\frac{\tilde{m}_U}{\Lambda} \ \mathrm{Tr}(\Phi q\tilde{q}) + h \ \mathrm{Tr}(\Phi_U q\tilde{q})\,. \end{aligned} \tag{4.3.33}$$

The cubic term does not alter our analysis of the metastable vacuum near the origin of field space. Furthermore, as long as $(\Delta m)^2 \lesssim m_\Phi\mu_\phi$, the results of the previous subsection are approximately correct.

However, for $(\Delta m)^2 > m_\Phi\mu_\phi$, the computation of the metastable vacuum receives important corrections. In this range there is still a metastable vacuum, but now both Φ_U and Φ play a role in the SUSY breaking dynamics, and receive direct soft masses. This alternative will be explored, and exploited, in §4.5.2.

4.4 Single-sector SUSY breaking

The model of §4.3 with magnetic superpotential (4.3.14) will now be used to construct a "single-sector" SUSY breaking model in which some Standard Model generations are composite mesons of the strongly coupled electric theory. In §4.4.1, we discuss a simple embedding of the first and second generation Standard Model fermions into the mesons of the SUSY breaking sector. We show how this generates the desired fermion Yukawa matrix, (4.1.2), and thus naturally produces the observed flavor hierarchy. In §4.4.2, we estimate the parametric contributions to various sparticle masses. While the gaugino masses are generated from gauge mediation only, the sfermions may obtain a mass from gauge mediation or directly from the supersymmetry breaking sector (in particular, from the one-loop Coleman Weinberg potential).

Constraints on the sfermion masses from flavor-changing neutral currents (FCNCs) are discussed in §4.4.3. Although the sfermion masses are diagonal in the flavor basis in which the fermion Yukawa matrices take on the texture of (4.1.2), large off-diagonal sfermion mass terms may be generated after diagonalizing the fermion Yukawas. This can lead to large FCNCs unless the sfermion masses of first two generations are roughly the same (universal) or are both very heavy (decoupled). Successful model-building then amounts to finding various limits of the adjoint model that give rise to soft terms compatible with FCNC and other constraints. We will reserve a discussion of specific parametric limits and viable soft spectra for §4.5.

4.4.1 MSSM generations from composites

A simple single-sector SUSY breaking model can be constructed by embedding the first Standard Model generation inside the meson Φ_U and the second generation inside the meson Φ (the embeddings are described in detail below). The third generation will come from an additional elementary field, which we denote by Ψ_3. The fields Φ and Φ_U were defined in (4.3.10) but are reproduced here schematically for convenience:

$$\Phi_U \sim \frac{\tilde{Q}UQ}{\Lambda^2} \quad , \quad \Phi \sim \frac{\tilde{Q}Q}{\Lambda} \, . \tag{4.4.1}$$

While both Φ_U and Φ are dimension one fields at low energies in the magnetic theory, they are dimension three and two fields, respectively, in the UV electric theory. The fermion Yukawa couplings will be generated at high energies – at which the electric theory is weakly coupled – through couplings between the Standard Model fields contained inside $\tilde{Q}UQ$, $\tilde{Q}Q$, and Ψ_3 and an elementary Higgs field, H,

$$\begin{aligned} W_{Yuk} \supset \; & \frac{1}{M_{\text{flavor}}^4}(QU\tilde{Q})H(QU\tilde{Q}) + \frac{1}{M_{\text{flavor}}^3}(Q\tilde{Q})H(QU\tilde{Q}) + \\ & \frac{1}{M_{\text{flavor}}^2}(Q\tilde{Q})H(Q\tilde{Q}) + \frac{1}{M_{\text{flavor}}}(Q\tilde{Q})H\Psi_3 + \Psi_3 H \Psi_3. \end{aligned} \tag{4.4.2}$$

Here M_{flavor} denotes the "flavor scale" at which these terms are generated, and we have neglected $\mathcal{O}(1)$ dimensionless couplings. Since $\tilde{Q}UQ$, $\tilde{Q}Q$ and Ψ_3 are dimension three, two, and, one, respectively, the generated Yukawa couplings are suppressed by different powers of the flavor scale M_{flavor}. At low energies, this Yukawa superpotential becomes

$$\begin{aligned} W_{Yuk} \supset \; & \frac{\Lambda^4}{M_{\text{flavor}}^4}\Phi_U H \Phi_U + \frac{\Lambda^3}{M_{\text{flavor}}^3}\Phi H \Phi_U + \\ & \frac{\Lambda^2}{M_{\text{flavor}}^2}\Phi H \Phi + \frac{\Lambda}{M_{\text{flavor}}}\Phi H \Psi_3 + \Psi_3 H \Psi_3. \end{aligned} \tag{4.4.3}$$

Setting $\epsilon = \Lambda/M_{\text{flavor}}$ gives the following fermion Yukawa matrix (up to $\mathcal{O}(1)$ dimensionless couplings)

$$\begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \tag{4.4.4}$$

which will generate the desired flavor hierarchy for $\epsilon \sim 10^{-1}$. Note that it requires $\Lambda \sim 10^{-1} M_{\text{flavor}}$, so that the strong coupling scale of the electric theory cannot be too much below the "flavor" scale.

We now describe the embedding of the Standard Model fields inside the SUSY breaking mesons in more detail. To present our results in a compact way, an $SU(5)$ GUT notation will be adopted, but the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ can be easily used instead. The latter embedding will be explored in Appendix B.3 and has the advantage that it generates less additional heavy Standard Model charged matter that change the RG running of the Standard Model gauge couplings — in particular, Landau poles (which we discuss in Appendix B.2) can be pushed to much higher energy scales.

The minimal choice for the number of flavors and colors of the electric theory corresponds to

$$N_f = 12\ ,\ \tilde{N}_c = 1 \ \Rightarrow\ N_c = 23$$

The $SU(N_f = 12)$ global symmetry is broken to $SU(N_f - \tilde{N}_c = 11)$ by the vacuum expectation value $\chi\tilde{\chi} = \mu^2$ (see (4.3.27)). The Standard Model GUT group is a weakly gauged $SU(5)$ subgroup of $SU(11)$, with the following embedding of $SU(5)$

into $SU(12)$:

$$Q \sim (\mathbf{5} + \bar{\mathbf{5}} + \mathbf{1}) + \mathbf{1}\ , \tilde{Q} \sim (\bar{\mathbf{5}} + \mathbf{5} + \mathbf{1}) + \mathbf{1}, \tag{4.4.5}$$

where the representations in round brackets denote the embedding into $SU(11)$.

The mesons of the magnetic theory decompose as (see (4.3.17))

$$\Phi_U = \begin{pmatrix} Y_{U,\,1\times1} & Z^T_{U,\,1\times11} \\ \tilde{Z}_{U,\,11\times1} & X_{U,\,11\times11} \end{pmatrix}\ ,\ \Phi = \begin{pmatrix} Y_{1\times1} & Z^T_{1\times11} \\ \tilde{Z}_{11\times1} & X_{11\times11} \end{pmatrix}\ , \tag{4.4.6}$$

The fields $(Y_i, \chi, \tilde{\chi})$ fields are all singlets under the Standard Model gauge group, while X_U and X decompose as

$$(\mathbf{10} + \bar{\mathbf{5}}) + \left[2 \times \mathbf{24} + \mathbf{15} + \overline{\mathbf{15}} + \overline{\mathbf{10}} + 2 \times \mathbf{5} + \bar{\mathbf{5}} + 3 \times \mathbf{1}\right], \tag{4.4.7}$$

where the representations in round brackets will form the desired Standard Model fermions and the matter in square brackets represents additional matter that we will want to remove.

The unwanted matter can be removed by the addition of spectator fields $S_{\bar{\mathbf{R}}}$ for each representation $\mathbf{R}$ in square brackets (except the singlet piece $\mathrm{Tr}\,(X_U)$, which participates in supersymmetry breaking) and with superpotential couplings

$$\begin{aligned} W_{el} \ &\supset\ \lambda_{1\mathbf{R}} \sum_{\bar{\mathbf{R}}} S_{1\bar{\mathbf{R}}} (Q\tilde{Q})_{\mathbf{R}} + \lambda_{2\mathbf{R}} \frac{1}{\Lambda_0} \sum_{\bar{\mathbf{R}}} S_{2\bar{\mathbf{R}}} (QU\tilde{Q})_{\mathbf{R}} \\ \rightarrow W_{mag} \ &\supset\ \lambda_{1\mathbf{R}} \Lambda \sum_{\bar{\mathbf{R}}} S_{1\bar{\mathbf{R}}} X_{\mathbf{R}} + \lambda_{2\mathbf{R}} \frac{\Lambda^2}{\Lambda_0} \sum_{\bar{\mathbf{R}}} S_{2\bar{\mathbf{R}}} X_{U,\mathbf{R}}. \end{aligned} \tag{4.4.8}$$

The unwanted matter will now have masses of order Λ and Λ^2/Λ_0, where Λ_0 is some UV scale above Λ, as in (4.3.15). Note that in this case, we do not need to add the perturbations $(\ \mathrm{Tr}(\tilde{Q}Q))^2$ and $(\ \mathrm{Tr}(\tilde{Q}UQ))^2$ as in [66], which would have given mass to the fermionic components of X and X_U, since these components can be lifted by trilinear couplings with spectators as in (4.4.8) (of course, these perturbations could have been added without qualitatively changing the discussion).

We also include spectators that pair up with Z and $\tilde{Z}$, which are also charged under the Standard Model gauge group. It is worth briefly explaining why we can include spectators to remove the unwanted $Z, \tilde{Z}$ particles in this model, but not e.g. in the models of [72]. In ISS-like models, the Z and $\tilde{Z}$ are in the same multiplet as the magnetic meson that breaks SUSY by the rank condition, and they receive a tree-level SUSY-breaking mass. This is because they mix with the ρ components of the magnetic quarks, which obtain a mass from the $\tilde{q}\Phi q$ coupling in the magnetic superpotential. Therefore, they play an important role in the calculation of the one-loop Coleman-Weinberg potential, and altering the spectrum of $Z, \tilde{Z}$-mesons, even if it could be done without creating instabilities, would drastically affect the model. In this model, in contrast, there are two magnetic mesons, and only Φ_U is playing a role in the supersymmetry breaking, while Φ is almost a spectator to the dynamics. Therefore, the $Z, \tilde{Z}$ mesons play no role in the Coleman-Weinberg computations, and can be safely given a large mass of order Λ^2/Λ_0 from the coupling (4.3.15), or an even larger mass of order $\lambda\Lambda$ by adding appropriate spectators.

Once the chiral deformation (4.4.8) is turned on, the $(\mathbf{10} + \bar{\mathbf{5}})$ Standard Model fermions from X and X_U (see (4.4.6)) are massless to all orders in perturbation theory.[8] Non-zero masses with the appropriate flavor hierarchy can be generated with the superpotential in (4.4.2).

4.4.2 Sparticle spectrum

Having identified superfields of the Standard Model with various components of the mesons Φ and Φ_U, we may now make parametric estimates for the soft masses obtained by gauginos, sfermions, and the gravitino in the SUSY-breaking vacuum.

There are two possible contributions to the sparticle masses. One contribution can come from a direct coupling to SUSY breaking. This is the case for the composite first generation sfermions in X_U that obtain a (large) mass from the CW potential,

$$V_{CW} \sim m_{CW}^2 |X_U|^2 \ , \ m_{CW} \sim \sqrt{b} h^2 \mu. \tag{4.4.9}$$

[8] More precisely, due to the μ_ϕ perturbation the chiral fermions have a very small admixture with the spectators. This mixing is of order $(\mu_\phi \Lambda_0/\Lambda^2) \sim 10^{-14}$ in the range of interest $\mu_\phi \sim$ TeV, $\Lambda \sim M_{GUT}$, $\Lambda_0 \sim M_{Pl}$, and can be safely ignored.

The composite second generation sfermions arising from X have couplings to the SUSY breaking sector that are suppressed by the ratio $\tilde{m}_U/\Lambda$. For $\tilde{m}_U/\Lambda \ll 1$, the second generation sfermions obtain only a negligibly small mass from the CW potential, even though they are composites! The gauginos and third generation do not directly feel the SUSY breaking.

The second contribution to the sparticle masses comes from gauge mediation. After weakly gauging, for example, a $SU(5)$ or $SU(3)_C \times SU(2)_L \times U(1)_Y$ subgroup of the global $SU(N_f - \tilde{N}_c)$ symmetry as in (4.4.5), the fields $\rho, \tilde{\rho}, Z_U$, and $\tilde{Z}_U$ will be charged under the Standard Model gauge group and act as messengers of supersymmetry breaking to the sparticle sector. (We have seen in §4.3.3 and §4.4.1 that the fields Z and $\tilde{Z}$ can be decoupled from the SUSY breaking sector and given very heavy masses of $\mathcal{O}(\Lambda)$, so their interactions with the sparticle sector can be completely ignored.) The messenger masses may be computed from (4.3.22) and (4.3.23); we refer the reader to [66] for the details. Very roughly, at leading order the fermionic components have masses $\sim h\mu$, while the bosonic components have masses $\sim 0, h\mu$, and $2h\mu$; the massless bosons will acquire a mass $\sim g_{SM}\mu$ when the flavor group is gauged. In the Standard Model embedding of (4.4.5), we have $4 \times (\mathbf{5} + \bar{\mathbf{5}})$ messengers, so that gauge coupling unification is in principle possible (for a complete discussion of unification in these models, see Appendix B.2).

The gauge mediated two-loop contribution to the sfermion squared masses is parametrically given by

$$m_{GM}^2 \sim \left(\frac{g^2}{16\pi^2}\right)^2 \frac{(hF_{X_U})^2}{M^2}, \tag{4.4.10}$$

where g is a Standard Model gauge coupling, $F_{X_U} \sim h\mu^2$ is the SUSY breaking F-term of the field X_U, and $M \sim h\mu$ is a typical messenger mass. We have neglected a sum over Dynkin indices and $\mathcal{O}(1)$ numbers — the precise expression is much more complicated and will not be needed for our purposes. Schematically, the gauge-mediated contribution to sfermion soft masses is thus

$$m_{GM} \sim \frac{g^2}{16\pi^2} h\mu. \tag{4.4.11}$$

The gauge mediated contribution to the gaugino masses is given in [66]. The mass must be proportional to the R-symmetry breaking, which is dominated by the spontaneous breaking from the vev of $\langle hX_U \rangle \sim \mu_\phi/b$. We find

$$m_{\lambda_a} \sim \frac{g_a^2}{16\pi^2} \langle hX \rangle \sim g_a^2 \mu_\phi \,, \tag{4.4.12}$$

where g_a, $a = 1, 2, 3$, are the Standard Model $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge couplings, and we have again neglected $\mathcal{O}(1)$ numbers (as well as even smaller corrections of $\mathcal{O}(\mu_\phi/(16\pi^2))$ from the explicit R-symmetry breaking). Notice that the $1/b$ factor in the the spontaneous R-symmetry breaking vev, $X \sim \mu_\phi/b$, cancels the loop factor.

The gauge mediated contribution to the sfermion and gaugino masses are comparable if

$$\mu_\phi \sim \mu/(16\pi^2). \tag{4.4.13}$$

If we want gauge mediated masses of $\mathcal{O}(1 \text{ TeV})$, we need to choose (assuming $h \sim 1$ for now)

$$\mu_\phi \sim 1 \text{ TeV}, \quad \sqrt{F} \sim \mu \sim \mathcal{O}(100 - 200 \text{ TeV}), \tag{4.4.14}$$

so that the direct SUSY breaking contribution from the CW potential to the first (and possibly second) generation sfermions is

$$m_{CW} \sim 10 \text{ TeV}. \tag{4.4.15}$$

A more detailed analysis reveals that in order for the metastable vacuum to be long-lived (which requires $\mu_\phi \ll \mu$), the gauge mediated contribution to the colored sfermions cannot be much less than $\sim 2.5 - 3$ TeV, with the masses of the bino, wino, and gluino around 70 GeV, 130 GeV, and 500 GeV, respectively [66].

The gravitino mass in this theory is simply given by

$$m_{3/2} \sim \sqrt{\frac{N_f - \tilde{N}_c}{3}} \frac{h\mu^2}{M_P} \tag{4.4.16}$$

For the low SUSY breaking scale considered here, the gravitino is light and has a mass of

$$m_{3/2} \sim 10 \text{ eV}, \tag{4.4.17}$$

which makes it cosmologically quite safe [153].

4.4.3 Supersymmetric flavor

An essential challenge faced by single-sector models — and, indeed, by all models of supersymmetry breaking and mediation — is to generate a spectrum of soft masses compatible with observational constraints on flavor-changing neutral currents (FCNCs). In general, the soft masses for squarks and sleptons explored in §4.4.2 are not diagonal in the same basis as the fermion mass matrix, leading to potentially prohibitive FCNCs. But the virtue of *calculable* models of single-sector SUSY breaking and flavor is that phenomenologically viable spectra may be related directly to microphysical parameters of the theory, and viable models may be found as a function of such parameters. In light of the potential soft terms discussed above, it is thus natural to consider what ranges of ultraviolet parameters in the adjoint model give rise to supersymmetric soft spectra compatible with experimental constraints.

Absent any additional mechanism to generate alignment between the Yukawa matrices and sfermion soft masses, spectra compatible with FCNCs may arise from either approximate universality or decoupling. Universality – for which the sfermion mass matrices are proportional to the identity – suffices because the identity is diagonal in any basis, so that no sfermion mass mixing is generated when we rotate to the fermion mass eigenbasis. Although small deviations from universality are acceptable (and, indeed, inevitable given RG evolution of soft parameters to the weak scale), they must remain rather small compared to the overall scale of soft masses.

Decoupling, on the other hand, exploits the observation that sfermion contributions to FCNCs scale as high inverse powers of the sfermion mass, and vanish as the sfermion masses are taken to infinity. The size of the top Yukawa coupling implies that only the third generation of sfermions needs be near the weak scale to preserve the naturalness of weak-scale SUSY as a solution to the hierarchy problem. Fortunately, FCNC constraints are strongest for the first two generations of sfermions, so that flavor constraints and naturalness may be simultaneously satisfied by making the first two generations heavy while keeping the third generation light. This approach leads to "more minimal" [43, 58] models with an inverse hierarchy of sfermion masses. In such scenarios, the masses of the first two generations of sfermions are constrained by the two-loop sfermion contribution to the stop mass, which renders the stop tachyonic when $m_{\tilde{f}_1}, m_{\tilde{f}_2} \gtrsim 20$ TeV unless the high-scale stop mass is unnaturally large [18].

In the models considered here, sfermions of the first two generations may acquire SUSY-breaking soft masses directly, while all three generations acquire universal SUSY-breaking soft masses from gauge mediation. Barring additional superpotential terms mixing the mesons of the magnetic theory, these soft masses are all diagonal in the same basis as the non-diagonal Yukawa textures (4.1.2). If the gauge-mediated contributions are around $m_{GM} \sim 1 - 3$ TeV, the third generation of sfermions is light enough to roughly preserve the naturalness of electroweak symmetry breaking. It is then a question of how large the additional contributions to the first and second generations coming from m_{CW} must be in order to avoid FCNCs. In general, both must be $\gtrsim 5$ TeV with some degree of degeneracy; a detailed treatment of FCNC and other constraints on the sfermion spectrum is contained in Appendix B.1.

The great virtue of calculable single-sector models is that these flavor constraints may be related explicitly to the UV parameters of the theory. In the limit $\tilde{m}_U/\Lambda \ll 1$, only the first generation feels supersymmetry breaking directly. In general, such a spectrum – with sfermions of the first generation much heavier than those of the second and third – yields prohibitive contributions to FCNCs. However, if the coupling h is sufficiently small it is possible for such contributions to satisfy approximate universality given a certain degree of tuning. For larger values of $\tilde{m}_U/\Lambda$, both first and second generations obtain significant soft masses directly from SUSY breaking, realizing a calculable version of the "more minimal" scenario. This is perhaps the most natural spectrum of supersymmetry breaking in such theories, and (calculably) reminiscent of the dimensional hierarchy spectra in [130]. Finally, it is possible for all three generations to receive soft masses solely from direct gauge mediation if the chiral ring is extended slightly. These models naturally satisfy FCNC constraints via universality, but lose some of the single-sector appeal.

We will now detail these approaches in §4.5.

4.5 Models

In light of the potential soft terms described in §4.4.2 and the supersymmetric flavor constraints outlined in §4.4.3, let us now consider various limits of the adjoint theory that give rise to phenomenologically viable spectra. In §4.5.1 we will consider the simplest theory with approximate universality, which involves a simple embedding but requires some degree of tuning to satisfy FCNC constraints. In §4.5.2 we will consider models with the familiar inverse hierarchy of soft masses; these models readily satisfy flavor constraints but entail a slightly less minimal embedding of Standard Model fields. In §4.5.3 we expand the chiral ring of the adjoint model to include theories where all three generations obtain universal masses from direct gauge mediation. In this case, the composite field that breaks SUSY is distinct from those giving rise to Standard Model generations, but all the ingredients of SUSY breaking, mediation, and flavor are contained within the same gauge sector.

4.5.1 A model with approximate universality

We begin by exploring the simplest single-sector model that requires only the minimal Standard Model embedding of (4.4.7). Though admittedly not the most elegant model, this approach will illustrate some of the issues that will reappear in more elaborate alternatives.

In the limit $\tilde{m}_U/\Lambda \ll 1$, only the first generation feels supersymmetry breaking directly; the meson Φ in which the fields of the second generation are embedded remains approximately supersymmetric. Gauging the flavor symmetry then produces universal gauge-mediated masses for all three generations. From Eqs. (4.4.9) and (4.4.11), these respective soft masses are

$$m_{CW} \sim \frac{h}{4\pi}\, h\mu \;,\; m_{GM} \sim \frac{\alpha_g}{4\pi}\, h\mu \,, \tag{4.5.1}$$

where $\alpha_g = g_{SM}^2/4\pi$. The first generation thus obtains a mass-squared of $m_{\tilde{f}_1}^2 \sim m_{CW}^2 + m_{GM}^2$ while the second generation obtains a mass-squared of only $m_{\tilde{f}_2}^2 \sim m_{GM}^2$. For low sfermion masses where $m_{GM} \sim 3$ TeV, we need $m_{\tilde{f}_1}$ to be the same as $m_{\tilde{f}_2}$ within $\sim 2-5\%$ in order to avoid large FCNCs (see Appendix B). This requires the CW contribution to the first generation mass to be smaller than the gauge-mediated mass, which may be achieved only if $h \lesssim \alpha_g/4$. There is no reason for h to be so small, but it is interesting that tuning a single dimensionless coupling can help solve the problem from FCNCs. In this case, the direct SUSY breaking mass from the one-loop effective potential is much smaller than the gauge mediated mass, and the spectrum looks like a very minor deviation from that of standard gauge mediation.

One tension in the reasoning of the previous paragraph comes from the observation that $h \ll 1$ is in conflict with astrophysical constraints that imply a lower bound $h \gtrsim \mathcal{O}(1)$. Indeed, recall that in scenarios with a low scale of SUSY breaking and warm gravitino dark matter the gravitino mass has an upper bound of ~ 16 eV, which translates into a bound on the SUSY-breaking scale of [153]

$$V_{min}^{1/4} = |\sqrt{h}\,\mu| \lesssim 260\,\mathrm{TeV}\,. \tag{4.5.2}$$

Then fixing the stop mass in (4.5.1) gives a lower bound on h,

$$\sqrt{h} \gtrsim \frac{4\pi}{\alpha_g}\,\frac{m_{\tilde{t}}}{260\,\mathrm{TeV}} \sim \mathcal{O}(1)\,. \tag{4.5.3}$$

Of course, this bound may be obviated by large entropy production at late times.

Absent a cosmological solution, this tension may also be removed by the following simple modification. Let us allow two different μ parameters, $\mu_1 > \mu_2$,

$$W \supset -h\;\mathrm{Tr}(\mu^2 \Phi_U) = -h\mu_1^2\;\mathrm{Tr}\,Y_U - h\mu_2^2\;\mathrm{Tr}\,X_U\,. \tag{4.5.4}$$

(Notice that nothing forbids such different μ's once the global symmetry group is explicitly broken by weakly gauging the Standard Model subgroup.) By the rank condition, the VEV of χ is set by the largest μ_1,

$$\langle \chi \tilde{\chi} \rangle = \mu_1 \, .$$

On the other hand, the SUSY breaking scale is

$$|W_{X_U}| = |h\mu_2^2| \, .$$

In this more general setup, the direct and gauge-mediated masses become

$$m_{CW} \approx \frac{h}{4\pi} \frac{h\mu_2^2}{\mu_1} \quad , \quad m_{GM} \approx \frac{\alpha_g}{4\pi} \frac{h\mu_2^2}{\mu_1} \, . \tag{4.5.5}$$

The upper bound on the scale of SUSY breaking from the astrophysical bound on the gravitino mass now does not limit h, but rather

$$\mu_1 \lesssim \frac{\alpha_g}{4\pi} \frac{(260\,\mathrm{TeV})^2}{m_{\tilde{t}}} \, . \tag{4.5.6}$$

Then it is possible for h to be small enough to satisfy approximate universality. Although the tuning of h to accommodate FCNC constraints is somewhat arbitrary, it gives rise to a satisfactory spectrum of sfermions in the simplest embedding of Standard Model fields into the adjoint model.

4.5.2 A model with decoupling

A more familiar approach to viable single-sector SUSY breaking with a dimensional hierarchy is to adopt a decoupling solution in which the first- and second-generation sfermions are heavy. Indeed, this is the natural spectrum arising in adjoint models for finite values of $\tilde{m}_U/\Lambda$.

Just from the couplings in the superpotential (4.3.14), the SUSY breaking sector induces a soft mass for the second generation at one loop of order

$$m_{CW,2} \sim \left(\frac{N_c - N_f}{g_U} \frac{m_U}{\Lambda} \right) \frac{h}{4\pi} h\mu \tag{4.5.7}$$

where the factor inside the brackets comes from the fact that the interaction between Φ and the magnetic quarks is proportional to m_U/Λ, and the second factor is the usual CW mass (4.3.25). Order-one numerical factors coming from the precise matching (4.3.12) have been absorbed into g_U, and we have set $\tilde{N}_c = 1$. Recall that m_U and g_U are the mass and cubic coupling of the adjoint field U in the electric theory.

In our case, $(N_c - N_f) \sim \mathcal{O}(10)$ and g_U can be made smaller than one. By taking m_U/Λ small but finite (unlike the case $m_U/\Lambda \to 0$ of §4.3.3 and §4.5.1), it is possible to obtain

$$\frac{N_c - N_f}{g_U} \frac{m_U}{\Lambda} \sim \mathcal{O}(1)\,. \tag{4.5.8}$$

For $h \sim \mathcal{O}(1)$, the direct SUSY breaking mass contribution is larger than the gauge mediated effect,

$$m_{CW,2} \sim \frac{h}{4\pi}\, h\mu > \frac{\alpha_g}{4\pi}\, h\mu \tag{4.5.9}$$

and both first- and second-generation sfermions can be made much heavier than the stop.

There is, however, a small obstacle to this simple picture that needs to be overcome. From the superpotential (4.3.14), the magnetic quarks $q, \tilde{q}$ only couple to the linear combination

$$\frac{N_c - N_f}{g_U} \frac{m_U}{\Lambda} \Phi + \Phi_U \tag{4.5.10}$$

which gets a mass from the one-loop CW potential

$$V_{CW} \approx m_{CW}^2 \;\; \mathrm{Tr} \left[\left(\frac{N_c - N_f}{g_U} \frac{m_U}{\Lambda} X + X_U \right)^\dagger \left(\frac{N_c - N_f}{g_U} \frac{m_U}{\Lambda} X + X_U \right) \right]\,. \tag{4.5.11}$$

The orthogonal combination remains light. Therefore, at first glance it seems that the effect of increasing the coefficient m_U/Λ is simply to redefine which scalar acquires a one-loop mass and which scalar receives a mass only from gauge mediation. At the level of the sfermion mass matrices, however, this would generate large off-diagonal elements strongly constrained by FCNCs; such mixings would require prohibitively large sfermion masses $\gtrsim 100$ TeV to evade flavor constraints.

We can solve this problem by noticing that if the first generation sfermions ($\mathbf{10}+\bar{\mathbf{5}}$)

come from matrix elements $X_{U,ij}$ which are different from the matrix elements X_{kl} containing the second generation, then (4.5.11) will give independent masses to each of the Standard Model sfermions. In other words, both generations can come from the linear combination (4.5.10) albeit from different matrix elements, and both then acquire comparable one-loop masses.

For this, we need to be able to have two different $(\mathbf{10}+\bar{\mathbf{5}})$ inside each meson. The minimal choice corresponds to

$$N_f = 17 \;,\; N_c = 33$$

with the $SU(5)_{SM}$ embedding

$$Q \sim \mathbf{1} + [\mathbf{1} + \mathbf{5} + \mathbf{5} + \bar{\mathbf{5}}] \;,\; \tilde{Q} \sim \mathbf{1} + [\mathbf{1} + \bar{\mathbf{5}} + \bar{\mathbf{5}} + \mathbf{5}] \,.$$

Each of the mesons X and X_U contains two independent $(\mathbf{10}+\bar{\mathbf{5}})$'s, plus additional matter that is lifted by coupling it to spectator fields. The corresponding Standard Model generations are identified with orthogonal elements $\mathbf{10}+\bar{\mathbf{5}}$.

To ensure that this happens, the superpotential coupling Eq. (4.4.8) of the spectators to the appropriate matrix elements can be enforced by a discrete symmetry. For instance, we can consider a vector-like $\mathbb{Z}_2$, with charge assignments $Q \sim \mathbf{1}_+ + [\mathbf{1}_+ + \mathbf{5}_+ + \mathbf{5}_- + \bar{\mathbf{5}}_+]$, opposite charges for $\tilde{Q}$, and with U being odd. Introducing, in particular, $\overline{\mathbf{10}}_-$ and $\mathbf{5}_-$ spectators, the $\mathbf{10}_- + \bar{\mathbf{5}}_-$ mesons are lifted. Only the $\mathbf{10}_+ + \bar{\mathbf{5}}_+$ from each $Q\tilde{Q}$ and $QU\tilde{Q}$ survive – and these come from different matrix elements since U is odd. Notice that this discrete symmetry commutes with the global symmetry group left unbroken by the $SU(5)_{SM}$ embedding. Also, since $U \to -U$ is not a symmetry in the presence of a $\mathrm{Tr}\; U^3$ superpotential, its coefficient g_U has to be small in order for this analysis to be approximately correct. In practice, $g_U \lesssim \epsilon \sim \mathcal{O}(0.1)$ is required.

A fully realistic single-sector model satisfying the bounds from FCNCs is then possible, albeit with a slightly less minimal embedding of the Standard Model into composites of the strong dynamics. Let us consider a simple example. Take the messenger scale to be

$$M = h\mu \approx 250 \text{ TeV} . \tag{4.5.12}$$

Setting $h \sim \mathcal{O}(1)$, and $m_U/\Lambda \sim \mathcal{O}(0.01)$, the sfermion spectrum at the messenger scale is

$$m_{\tilde{f}1} \approx 20 \text{ TeV} \ , \ m_{\tilde{f}2} \approx 15 \text{ TeV} \ , \ m_{\tilde{f}3} \sim 1.5 \text{ TeV} \tag{4.5.13}$$

The gaugino masses are

$$m_\lambda \sim \mathcal{O}(1\,\text{TeV}) \text{ for } \mu_\phi \sim \mathcal{O}(1\,\text{TeV}) \tag{4.5.14}$$

and the metastable vacuum is parametrically long-lived. In this class of models, the number of messengers is $6 \times (\mathbf{5} + \bar{\mathbf{5}})$ so that perturbative unification is not possible. It would be interesting to find a model that unifies and where the first two generation sfermions have decoupled to the multi-TeV scale.

As a final remark connecting with the discussion in §4.3.4, when (4.5.8) is satisfied the field breaking susy and R-symmetry is a linear combination of Φ and Φ_U with order one coefficients – see Eq. (4.5.10). Turning on generic superpotential deformations $\Delta W_{el} = (Q\tilde{Q})^n (QU\tilde{Q})^m$, the properties of the metastable vacuum will be fixed by only the largest linear and quadratic meson terms. These have to satisfy the stability conditions found in [66], while other terms play a subleading role. Therefore the metastable vacuum will exist and be long-lived for quite generic superpotential deformations.

4.5.3 Composite models with direct gauge mediation

So far we have found models where both composite generation sfermions acquire soft masses from direct couplings to the SUSY breaking sector (see §4.2 and §4.5.2) or where the first generation gets a direct SUSY breaking mass, while the second predominantly obtains a mass from gauge-mediation (see §4.3.3 and §4.5.1). We saw that in order to satisfy FCNC constraints in the latter scenario, the one-loop SUSY breaking mass must be considerably suppressed relative to the gauge-mediated masses.

This limit suggests a slightly more general "single-sector" scenario in which SUSY-breaking still arises from strong dynamics of the $SU(N_c)$ gauge group, but all the soft masses come predominantly from gauge mediation. In this case, the flavor problem would be solved automatically due to the flavor-blindness of the gauge interactions. Though one might argue that this is no longer strictly a single-sector theory — SUSY-breaking is now external to the composites comprising Standard Model fields — such models still retain a pleasing amount of compactness. No new ingredients beyond the fields and interactions of the $SU(N_c)$ gauge theory are required, and all the messengers, SUSY-breaking fields, and Standard Model composites arise from the same dynamics. In this section we will present a simple deformation of the adjoint model possessing these properties.

Consider the adjoint model of §4.3, but allowing a U^4 term in the electric superpotential (the general U^k case has been studied in [119]),

$$W_{el} = \frac{1}{4}\frac{1}{\Lambda_U} \text{ Tr } U^4 + \frac{g_U}{3} \text{ Tr } U^3 + \frac{m_U}{2} \text{ Tr } U^2 \tag{4.5.15}$$

The magnetic dual has gauge group $SU(\tilde{N}_c = 3N_f - N_c)$, N_f magnetic quarks $(q, \tilde{q})$, a magnetic adjoint $\tilde{U}$, and three gauge singlets

$$M_1 = \tilde{Q}Q\ ,\ M_2 = \tilde{Q}UQ\ ,\ M_3 = \tilde{Q}U^2Q\,.$$

It will be useful to work in terms of the canonically normalized mesons,

$$\Phi_j = \frac{M_j}{\Lambda^j}\,,$$

up to order one numerical constants from the Kähler potential as in Eq. (4.3.10).

Again, we will focus on the case $\tilde{N}_c = 1$, for which the magnetic dual is a theory of weakly coupled hadrons with superpotential

$$W_{mag} = h \text{ Tr}(\Phi_3 q\tilde{q}) + \frac{2}{3}h^2 g_U \text{ Tr}(\Phi_2 q\tilde{q}) + \frac{1}{2}h^2\left(\frac{m_U}{\Lambda} - \frac{1}{9}hg_U^2\right) \text{ Tr}(\Phi_1 q\tilde{q}) \tag{4.5.16}$$

where $h = \Lambda_U/\Lambda$. In the limit

$$g_U \ll 1\,,\ \frac{m_U}{\Lambda} \ll 1\,, \tag{4.5.17}$$

the dimension 2 meson Φ_1 and the dimension 3 meson Φ_2 are almost decoupled from the rest of the low energy fields $(\Phi_3, q, \tilde{q})$.

These fields $(\Phi_3, q, \tilde{q})$ are then used to break SUSY in a by now familiar way. Adding the superpotential deformation

$$\Delta W_{el} \sim \frac{1}{\Lambda_0}\,(QU^2\tilde{Q}) + \frac{1}{\Lambda_0^5}\,(QU^2\tilde{Q})^2 \;\Rightarrow \Delta W_{mag} \sim -h\mu^2\ \operatorname{Tr}\Phi_3 + h^2\mu_\phi\ \operatorname{Tr}(\Phi_3)^2 \tag{4.5.18}$$

breaks SUSY by the rank condition, creates a metastable vacuum at a distance $\sim 16\pi^2\mu_\phi$ from the origin of Φ_3 space, and breaks the R-symmetry both explicitly and spontaneously (the latter dominating).

The first and second SM generations are identified with Φ_2 and Φ_1 respectively, with the third generation being elementary. In the limit (4.5.17), none of the composite generations participate directly in the SUSY breaking. Therefore the sfermion soft squared masses come predominantly from gauge mediation, involving the SUSY breaking fields $(q, \tilde{q})$ only at two loops. These contributions are flavor blind and hence there are no flavor problems.

It is quite surprising that calculable single-sector models exist where the composite soft masses come predominantly from direct gauge mediation. The gauge dynamics we have found is rich enough to provide marginal couplings (g_U and m_U/Λ in the example above) that control the strength of the direct SUSY breaking masses. It is possible to set these parameters to zero without changing the SUSY breaking scale and messenger masses. It would be interesting if this mechanism has an analog in single-sector models with gravity duals [33, 75, 133].

4.6 Concluding remarks

We have introduced and studied calculable models of single-sector SUSY breaking that have fully realistic Yukawa textures (implementing the dimensional hierarchy idea) and satisfy FCNC bounds, considerably improving earlier constructions [72]. The beauty of these constructions stems from the way in which the apparently intricate structure of the MSSM originates from a rather minimal, calculable gauge theory.

Our discussion focused primarily on a class of models based on SQCD with fundamental flavors and an adjoint. These theories possess composites of various dimensions, controlled by the adjoint superpotential, and exhibit a surprisingly wide range of interesting behaviors. In certain parametric limits they give rise to models in which first- and second-generation sfermions are heavy due to compositeness and decouple. Perhaps more unexpectedly, there are also models in this class where compositeness gives rise to realistic Yukawa matrices, but all sfermion masses come predominantly from gauge mediation and are thus universal.

The parametric limits presented here represent a fraction of the possible single-sector models that may emerge from theories of supersymmetric QCD with fundamental flavors and a rank-two tensor field. It would be useful to further explore the range of possible soft spectra that may be realized such theories. Moreover, the models we have considered suffer somewhat from a surfeit of extra matter charged under the Standard Model; it would certainly be compelling to find other examples of calculable theories with more efficient embeddings.

Of course, such single-sector theories are but one approach (among many) to addressing the Standard Model flavor problem. We conclude by comparing and contrasting our mechanism with other explanations for the Yukawa hierarchies which exist in different classes of models.

4.6.1 Comparison to other explanations

The earliest class of explanations, and probably the best explored, use the Froggatt-Nielsen idea [73]. Here, one introduces a new $U(1)$ symmetry, R, broken by the vev of a new scalar $\langle\phi_1\rangle$ which has charge $+1$. One assumes that all of the Standard Model fermions are exactly massless in the limit that R is unbroken - that is, one assigns different charges to their left and right-handed components. Finally, one assumes the existence of some very heavy set of fermions (with various values of R) at a scale $\langle\phi_0\rangle \gg \langle\phi_1\rangle$, whose mass is set by the expectation value of another R-neutral Higgs field ϕ_0. By assigning appropriate charges under R to the Standard Model fermions, one can then generate Yukawa couplings suppressed by different powers of $\epsilon = \langle\phi_1\rangle/\langle\phi_0\rangle$. Models which are broadly successful in accounting for flavor physics can emerge from this framework. Some of the most successful models have more than one small parameter. The scales involved are not very tightly constrained by data, so such models can account for observed physics and remain untestable in the foreseeable future.

An idea closely related to our own is to consider supersymmetric models where the MSSM generations interact with a strongly coupled SCFT (at least over some range of energies). If the MSSM Yukawa couplings receive different anomalous dimensions, this can provide an explanation of Yukawa hierarchies [137]. A recent exploration of this idea appears in [141]. We note that this is very similar to our mechanism; here, the large anomalous dimension comes from the fact that the MSSM fields are secretly composite and hence the Yukawa couplings are higher dimension operators above the compositeness scale Λ. In addition, our mechanism correlates this structure with the dynamics of SUSY breaking.

A much more recent class of interesting, field-theoretic ideas appears in [83]. These "Domino theories" are incompatible with conventional low-energy supersymmetry, but are otherwise an economical proposal for generating realistic Yukawa textures.

A very wide class of inter-related ideas uses the physics of extra dimensions:

• In superstring compactifications, e.g. those of the heterotic string, it is easy to find supersymmetric scenarios where the tree-level Yukawa couplings are related to topological invariants of the compactification manifold. These invariants often give some vanishing couplings, usually because the homology cycles on which some of the matter fields are localized do not intersect with the Higgs or with the other matter field in the relevant Yukawa coupling. In such a circumstance, the leading coupling is generated by world-sheet or space-time instanton effects, due to supersymmetric non-renormalization theorems. (The instanton is a non-local object in the internal dimensions, and can connect the disconnected homology cycles). In a topology where only the top quark Yukawa is present at tree level, this can provide an attractive explanation for the rough features of the fermion mass matrix. Note that this idea requires multiple parameters to match the observed spectrum, since each instanton action is *a priori* unrelated to the others; this idea also remains untestable until one reaches the compactification scale, which is typically $\sim M_{\rm GUT}$. Many modern variants of this idea also exist in brane-world scenarios involving D-branes in Type II string theories. For recent discussions in heterotic and type II models, see [34] and [97], for instance. Very recent work in the context of F-theory, where instantons do not play an important role in the attempts to explain flavor physics, is summarized in [53].

• In theories where the Standard Model gauge fields propagate in "thick" branes (e.g. live in flat extra dimensions which are not excessively large), one can obtain Yukawa hierarchies by localizing the matter fermions within these branes [20, 108, 109]. In these split fermion scenarios, there are parameters governing both the location of the fermions (and the Higgs scalars), and the thickness or form of their wavefunctions. In many ways, this is similar to the first scenario above. With a small set of such parameters, one can find acceptable scenarios. These models can be testable at the TeV scale, but need not be [125].

• In theories with warped (AdS-like) extra dimensions, with Standard Model gauge fields in the bulk, one can try to explain flavor by localizing fermions at different points along the radial direction of AdS [77, 96]. Such theories are dual to large N gauge theories [132]. Fields localized in the IR are composites of the CFT dynamics, while those localized in the UV are elementary fields external to the CFT. It can be of interest to have either an elementary Higgs (e.g. in a supersymmetric scenario where supersymmetry is broken at the end of the warped throat geometry), or a composite Higgs (e.g. in non-supersymmetric Randall-Sundrum scenarios). In the former case, the fermions localized at the IR end of the geometry (which are highly composite) will have the smallest Yukawa couplings, while in the latter case the highly composite fermions will have the largest Yukawa couplings. In such scenarios, like in the split fermion scenarios, there are again typically several parameters; they are now associated with the anomalous dimension of the CFT operator which couples the Standard Model fermion to the large N CFT. The non-supersymmetric scenarios of this sort are likely to be testable at the LHC due to the existence of charged, light KK modes coming from the TeV-scale end of the throat geometry. In the supersymmetric scenarios this scale is considerably higher, since it is associated with SUSY-breaking, and there may be no Standard Model charges visible at this scale in any case (since there is no need for the Standard Model gauge fields to have support in the entire warped geometry). In this general framework, there are in fact recent steps towards making holographic duals of models quite similar to the ones we have considered [33, 75, 133].

In several of these cases, there are clear implications for the physics of grand unification. In the Froggatt-Nielsen models, one must extend the GUT group by an additional $U(1)$ and add new matter multiplets at a high scale. This is not compatible with standard $SU(5)$ GUTs. In the cases with split or warped localized fermions, one has the normal difficulties associated with "explaining" unification as opposed to postulating it by tuning additional matter content (which is of course un-necessary in the MSSM). In particular in string theory realizations of the third scenario, it is challenging to avoid Landau poles, due to the large number of massive matter fields involving in typical constructions of the observable sector and the large N CFT (see e.g. §5 of [33]). The case with instanton-suppressed Yukawa couplings is naively compatible with unification, though it introduces new parameters and renders the apparent relations in e.g. (4.1.2) somewhat *ad hoc*.

The explanation of flavor in our single-sector models is most similar in spirit to the last extra-dimensional scenario we discussed, in the supersymmetric case with an elementary Higgs and small couplings for the highly composite fermions. The composites in our models are analyzed via Seiberg duality instead of using AdS/CFT duality, but both classes of models rely on compositeness to suppress Yukawa couplings. We are close to having models which avoid Landau poles, but the pile-up of extra matter fields at the scale Λ where the composite generations are generated remains an obstacle to making models with honest, weakly-coupled unification. Since our models involve at most one or two parameters in the flavor sector, they are quite competitive in terms of predictivity with all of the classes of scenarios enumerated above. The correlation between soft-terms and Yukawa couplings, evident in most of the single-sector models (with at least one and often both of the first two generations having large sparticle masses in most of the known classes of models), is a further prediction which is absent in the non-supersymmetric theories, in supersymmetric realizations of the Froggatt-Nielsen mechanism, and in the methods based on instanton calculus in supersymmetric string compactifications.

Chapter 5

Superconformal Models of Flavor

5.1 Introduction

The observed hierarchy of fermion masses and mixings remains one of the most puzzling features of the Standard Model; the masses of three generations of quarks and leptons range over more than five orders of magnitude. Yet the observed spectrum is not entirely random, but rather seems to reflect an underlying structure. The masses of subsequent generations are arrayed with nearly even spacing; intergenerational mixings exhibit a nearest-neighbor hierarchy; and the similarity of down-type quark and lepton masses relative to up-type quark masses is suggestive of a grand unified theory (GUT) at high energies.

It seems likely that a comprehensive ultraviolet completion of the Standard Model may feature some explanation for this apparent flavor structure. Perhaps the most common approach to such theories of flavor involves engineering the observed Yukawa textures directly, either through approximate symmetries or radiative corrections [62, 73, 83, 86, 123, 124]. A compelling alternative approach is to treat the Yukawa matrices as entirely anarchical, consistent with effective field theory, and generate the flavor hierarchy through wavefunction renormalization. In four dimensions, this may be readily accomplished by coupling the Standard Model to a sector with strong conformal dynamics [8, 76, 137, 141] or by assembling Standard Model fermions themselves as composites of some strong dynamics [17, 46, 72, 130]. The Yukawa matrices then acquire the desired hierarchical form in terms of the canonically-normalized low-energy degrees of freedom. In both cases, supersymmetry frequently plays a role, both in rendering calculable the strong dynamics responsible for flavor hierarchies and in explaining the scale of electroweak symmetry breaking. This raises the attractive prospect that the hierarchy problem, Standard Model flavor hierarchy, and supersymmetric flavor problem may all share a common explanation and correlated features.

In this paper we wish to focus on specific models of flavor produced by coupling the Standard Model to a sector with strong conformal dynamics over a range of energies in which both sectors are supersymmetric. Among other features, such models have the virtue of considerable predictivity, as the anomalous dimensions (and hence fermion masses) of Standard Model fields are determined entirely by the gauge group and matter content of the superconformal sector. The primary challenge in building such models is to explicitly determine these anomalous dimensions, which at the time of the original work in [137] was difficult to achieve for simple models without a proliferation of superpotential couplings. In [141], considerable progress was made towards studying vector-like models of superconformal flavor using the a-maximization procedure [104] to determine the anomalous dimensions of both Standard Model and SCFT fields. Here our approach follows closely that of [141], using a-maximization to investigate the flavor spectrum arising from simple vector-like superconformal sectors.

In principle, this picture of flavor is related by a loose version of the AdS/CFT correspondence [19, 87, 132, 146, 155] to warped 5D models [143] in which each field has an exponential profile fixed by its bulk mass [77, 96]. Insofar as these 5D models possess the virtues of calculability and parametric freedom, one might naturally wonder whether 4D superconformal models have any real advantages over their 5D duals. Among other things, models based on 4D CFTs have fewer free parameters, as anomalous dimensions of all fields are fixed by the superconformal algebra and marginal interactions. Although various flavor textures may be realized in 5D by adjusting bulk masses, this leads to a proliferation of parameters, and it is not entirely clear whether a given 5D theory possesses a dual 4D CFT. Moreover, as the AdS/CFT correspondence is a strictly large-N duality, studying theories based on 4D

CFTs at small N may reveal features not readily accessible in 5D duals [49, 136, 147].

Our paper is organized as follows: In Section 2 we introduce the philosophy of superconformal flavor, relevant constraints, and previous results. In Section 3 we review the relation between R charges and scaling dimensions at superconformal fixed points, as well as the a-maximization procedure for determining the superconformal R-symmetry. Some subtlety arises when gauge-invariant operators saturate the unitarity bound, which influences both the a-maximization procedure and the contribution of SCFT states to the Standard Model β function. Having established the necessary tools, we turn in Section 4 to simple models of superconformal flavor based on $SU(N)$ gauge theories with an adjoint chiral superfield. In Section 5 we briefly treat related models based on $Sp(2N)$ gauge theories, which have the virtue of significantly smaller matter content charged under $SU(5)_{SM}$. In Section 6 we discuss the fixed points of these models and issues related to decoupling. We reserve for the Appendix the detailed constraints and numerical results of the a-maximization procedure as applied to the models in Sections 4 and 5.

5.2 Flavor hierarchy from flavor anarchy

The essential philosophy of superconformal flavor stems from the observation that fermion mass ratios and mixings may arise in the infrared from anarchy in the ultraviolet due strictly to quantum renormalization effects [76]. The size of renormalization effects required to explain the flavor hierarchy points to strongly coupled dynamics, which are in general incalculable. If, however, the dynamics are supersymmetric and approximately conformal, these strong renormalization effects may be estimated accurately [137]. In this section we will first review the means of generating Standard Model flavor hierarchies through large wavefunction renormalization, before turning to the approximately superconformal sectors that may be responsible. For simplicity, and to avoid potential conflicts with experimental constraints on baryon and lepton number-violating operators, we will restrict our attention to models of superconformal flavor operating at and above the GUT scale. This simplifying assumption leads us to consider Standard Model fermions strictly as components of GUT multiplets.

5.2.1 Standard Model flavor physics

The Standard Model Yukawa couplings are of the form (in unified notation)

$$W_{SM} \supset y_u^{ij} T_i T_j H_u + y_d^{ij} T_i \overline{F}_j H_d \tag{5.2.1}$$

where the $T_i \supset Q_i, U_i, E_i$ transform as a $\mathbf{10}$ of $SU(5)_{SM}$ and the $\overline{F}_i \supset L_i, D_i$ transform as a $\overline{\mathbf{5}}$, with $i = 1, 2, 3$. (We will not consider here the source of neutrino masses, but these may be included fairly easily.) The philosophy of superconformal flavor is simply that the Yukawa matrices $\mathbf{y}_u$ and $\mathbf{y}_d$ are not intrinsically hierarchical in the far ultraviolet, but rather contain $\mathcal{O}(1)$ factors consistent with effective field theory. The observed hierarchy in the Yukawas arises because the fields T_i and $\overline{F}_i$ inherit large wavefunction renormalization factors at some lower scale through coupling to an approximately conformal sector. When the infrared degrees of freedom are canonically normalized, the $\mathcal{O}(1)$ entries of the Yukawa matrices accumulate additional family-dependent suppression factors.

To see how this comes about, assume the fields $\Phi_i = T_i, \overline{F}_i$ of the Standard Model acquire large wavefunction renormalizations $Z_i(\mu)$ in the holomorphic basis at a scale $\mu \simeq M_{GUT}$:

$$\mathcal{L} = \int d^4\theta \sum_i Z_i(\mu) \Phi_i^\dagger \Phi_i \tag{5.2.2}$$

In the physical basis where fields are canonically normalized, this leads to suppression factors $\epsilon_i \equiv 1/\sqrt{Z_i}$ in the Yukawa couplings. In this notation, the Yukawa couplings are given by

$$W_{SM} \supset \epsilon_{T_i} \epsilon_{T_j} y_{u,0}^{ij} T_i T_j H_u + \epsilon_{T_i} \epsilon_{\overline{F}_j} y_{d,0}^{ij} T_i \overline{F}_j H_d \tag{5.2.3}$$

where $y_{u,0}^{ij}, y_{d,0}^{ij} \sim \mathcal{O}(1)$ are the anarchical Yukawa coefficients from the ultraviolet theory. To within these $\mathcal{O}(1)$ coefficients, the quark and lepton masses are therefore given by

$$
\begin{aligned}
(m_t, m_c, m_u) &\approx \frac{1}{\sqrt{2}} v \sin\beta \left(\epsilon_{T_3}\epsilon_{T_3}\epsilon_{H_u}, \epsilon_{T_2}\epsilon_{T_2}\epsilon_{H_u}, \epsilon_{T_1}\epsilon_{T_1}\epsilon_{H_u}\right) \\
(m_b, m_s, m_d) &\approx \frac{1}{\sqrt{2}} v \cos\beta \left(\epsilon_{T_3}\epsilon_{\bar{F}_3}\epsilon_{H_d}, \epsilon_{T_2}\epsilon_{\bar{F}_2}\epsilon_{H_d}, \epsilon_{T_1}\epsilon_{\bar{F}_1}\epsilon_{H_d}\right) \\
(m_\tau, m_\mu, m_e) &\approx \frac{1}{\sqrt{2}} v \cos\beta \left(\epsilon_{T_3}\epsilon_{\bar{F}_3}\epsilon_{H_d}, \epsilon_{T_2}\epsilon_{\bar{F}_2}\epsilon_{H_d}, \epsilon_{T_1}\epsilon_{\bar{F}_1}\epsilon_{H_d}\right)
\end{aligned}
\tag{5.2.4}
$$

where $v \approx 246$ GeV as usual. The resulting mixing angles in the CKM matrix are

$$
|V_{CKM}| \approx \begin{pmatrix} 1 & \epsilon_{T_1}/\epsilon_{T_2} & \epsilon_{T_1}/\epsilon_{T_3} \\ \epsilon_{T_1}/\epsilon_{T_2} & 1 & \epsilon_{T_2}/\epsilon_{T_3} \\ \epsilon_{T_1}/\epsilon_{T_3} & \epsilon_{T_2}/\epsilon_{T_3} & 1 \end{pmatrix}
\tag{5.2.5}
$$

which offers a fairly good parameterization of the observed values. It is natural, then, to consider what values of ϵ_i are required to match the observed masses of Standard Model fermions. This is not an exact science; the ϵ_i should be chosen to produce quark and lepton masses at the GUT scale, where they are subject to potentially sizable uncertainties due to supersymmetric threshold corrections. But a reasonable estimate gives (assuming $\epsilon_{H_u} \sim \epsilon_{H_d} \sim 1$) [13, 141]

$$
\begin{aligned}
\epsilon_{T_i} &\approx (0.001 \div 0.002, 0.03 \div 0.04, 0.7 \div 0.8) \\
\epsilon_{\overline{F}_i} &\approx \tan\beta \cdot (0.002 \div 0.01, 0.001 \div 0.007, 0.006 \div 0.02)
\end{aligned}
\tag{5.2.6}
$$

where the values of $\epsilon_{\overline{F}_i}$ come from considering the down-type quark masses; the result for lepton masses gives, encouragingly, similar results to within uncertainties due to threshold corrections.

There are clearly various ways to generate the hierarchy of (5.2.6). We will focus here on two types of flavor structures. The first is the so-called "ten-centered" structure, in which only the T_i of the Standard Model obtain significant ϵ factors from the conformal sector. The utility of these models stems from the observation that Standard Model flavor looks to be driven predominantly by a hierarchy among the different generations of T_i. For this to work also requires a large value of $\tan\beta$, which may cause problems with proton decay. Alternatively, we will also consider more "democratic" models with coupling to both T_i and $\overline{F}_i$. Here it is possible to accommodate much smaller values of $\tan\beta$, but one must be careful not to generate over-large hierarchies among the $\overline{F}_i$.

5.2.2 Superconformal flavor physics

Given that the flavor hierarchy may be explained by large wavefunction renormalization of Standard Model fields, it is now a matter of determining how such large renormalization might arise. Typically, the renormalization of Yukawa couplings and other dimensionless parameters in 4D scales logarithmically with energy, which is poorly suited to generating the required large factors (up to $Z_i \sim 10^6$) required to explain the range of quark masses. The key point, however, is that such significant effects may be realized in theories with approximately scale-invariant gauge couplings. If Standard Model fields Φ_i couple over some range of energies to fields charged under an approximately conformal gauge group G, these couplings may generate large anomalous dimensions γ_i. In this case the wave-function renormalization of the fields Φ_i is given in terms of the anomalous dimension γ_i by $\frac{d}{dt}\log Z_i \approx -\gamma_i$ (where $t = \log\mu$). If the group G becomes approximately conformal at a scale Λ and flows away from the conformal fixed point at a scale Λ_*, the resulting suppression factors take the form

$$\epsilon_{\Phi_i} = \exp\left(-\frac{1}{2}\int_{\log\Lambda_*}^{\log\Lambda}\gamma_i dt\right) \tag{5.2.7}$$

Of course, the running of Standard Model gauge and Yukawa couplings spoils conformal invariance. For an approximately superconformal fixed point, the anomalous dimensions are constant up to corrections of order $g_{SM}^2/16\pi^2$ and $y^2/16\pi^2$. In this approximation, the γ_i are constant, and so we find

$$\epsilon_{\Phi_i} = \left(\frac{\Lambda_*}{\Lambda}\right)^{\gamma_i/2} \tag{5.2.8}$$

When the anomalous dimensions are sufficiently large – typically $\gamma_i \sim \mathcal{O}(1)$ – and the range of energies $\Lambda_* < \mu < \Lambda$ sufficiently long, the suppression factors required by the flavor hierarchy will be readily generated.

The large anomalous dimensions γ_i for Standard Model fields may be generated by coupling to operators of the conformal sector via marginal interactions at the conformal fixed point. When the fixed point is superconformal, these anomalous dimensions become calculable. At such a fixed point, there is a simple relation between the scaling dimension of a chiral primary operator $\mathcal{O}$ and its superconformal R-charge, $\dim(\mathcal{O}) = \frac{3}{2}R_{\mathcal{O}}$. Correspondingly, the anomalous dimensions of operators at the superconformal fixed point are given by $\gamma_{\mathcal{O}} = 3R_{\mathcal{O}} - 2$. Since marginal superpotential terms at the fixed point must have R-charge 2, a superpotential coupling between SM and SCFT fields of the form $W = \Phi_i \mathcal{O}$ implies $R_{\Phi_i} = 2 - R_{\mathcal{O}}$ and hence $\gamma_i = 4 - 3R_{\mathcal{O}}$. Such couplings require $\mathcal{O}$ to transform nontrivially under Standard Model gauge symmetries. We may accomplish this by weakly gauging an $SU(5)_{SM}$ subgroup of the flavor symmetries in the SCFT, which amounts to a (small) explicit breaking of the global symmetry group. One caveat of this discussion is that the correct superconformal R charges may not always be readily determined, a challenge we will turn to in §3.

In this paper we will restrict our focus to vector-like superconformal sectors with fundamental matter and a rank-2 (adjoint or antisymmetric) tensor field. Such higher-rank fields introduce new gauge-invariant chiral operators whose canonical dimensions and R-charges may differ significantly from those of gauge-invariant operators formed by fundamental fields alone. As successful models of superconformal flavor require two or more operators with the same $SU(5)_{SM}$ charges but substantially different R-charges, rank-2 tensor fields therefore comprise an essential ingredient of superconformal flavor engineering. This situation is highly reminiscent of 4D models of composite flavor, in which various Standard Model families arise as mesons of identical SM charges but differing canonical dimensions [46, 130]. Likewise, we focus on vector-like conformal sectors due both to their ubiquity (as such gauge sectors often arise in string compactifications) and the simplicity with which their exotic states may be decoupled.

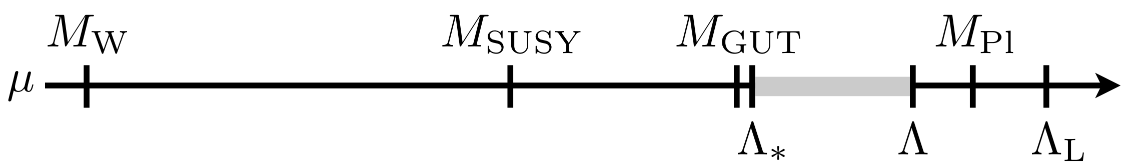

Figure 5.1: Cartoon of energy scales. We assume the gauge group G is approximately conformal in the energy range $\Lambda_* < \mu < \Lambda$; that the fields charged under G decouple from the Standard Model around $M_{GUT} \sim \Lambda_*$; and that any Standard Model Landau poles lie at a scale $\Lambda_L > \Lambda$ (and ideally $> M_{Pl}$). In principle, fields charged under G may be responsible for breaking supersymmetry at a scale M_{SUSY}.

5.2.3 Constraints on superconformal sectors

It is not quite enough to simply generate large wavefunction renormalization for Standard Model fermions. Weakly gauging a subgroup of the SCFT flavor symmetries leads to a plethora of extra states charged under both G and $SU(5)_{SM}$, which must be decoupled well before the weak scale, both to avoid spoiling Standard Model gauge coupling unification and violating observational bounds on SM-charged exotics. For simplicity, we assume the decoupling occurs at a scale $\sim \Lambda_*$, due to some small relevant deformations of the SCFT. It is not necessary to assume that all SCFT states acquire masses of $\mathcal{O}(\Lambda_*)$; it may be the case that some fields survive to lower energies, and perhaps are responsible for supersymmetry breaking at a lower scale.

If Standard Model fields decouple from the superconformal sector at Λ_*, we still need to worry about irrelevant operators induced at this scale. Foremost is the need to avoid operators violating baryon and lepton number. Although in general such operators will not be induced directly by the superpotential couplings of our theory, they are expected to appear as dimension-six operators in the Kähler potential with suppression of order $16\pi^2/\Lambda_*^2$. If $\Lambda_* \sim M_{GUT}$, this is fairly safe for all but the largest values of $\tan\beta$.

Whatever the scale of superconformal flavor, a principal constraint arises from the requirement that the Standard Model gauge couplings remain perturbative long enough for the observed flavor hierarchy to be generated. Consistency of our models requires that $SU(5)_{SM}$ remain a weakly-gauged subgroup of the SCFT flavor symmetries while at the approximately conformal fixed point, but the addition of so much extra matter charged under $SU(5)_{SM}$ tends to generate a Landau pole for g_5. Successful model-building amounts to ensuring that the Landau pole lie at a scale Λ_L above the window of energies in which the flavor hierarchy is produced.

This constraint may be enforced quite easily. The NSVZ beta function [139] for the $SU(5)_{SM}$ gauge coupling g_5 is given by

$$\beta_{g_5} = -\frac{g_5^3}{16\pi^2}\frac{[15 - \sum_i T(r_i)(1-\gamma_i)]}{1 - 5g_5^2/8\pi^2} = \frac{g_5^3}{16\pi^2}\frac{\mathfrak{b}}{1 - 5g_5^2/8\pi^2} \tag{5.2.9}$$

where $\mathfrak{b}$ is the "exact" β-function coefficient

$$\mathfrak{b} \equiv -15 + \sum_i (1-\gamma_i)T(r_i) = -3 \text{ Tr } \left[U(1)_R SU(5)_{SM}^2\right] \tag{5.2.10}$$

Given the R-charges of the SCFT fields charged under $SU(5)_{SM}$, we may compute $\mathfrak{b}$ for a given model and determine the scale Λ_L at which g_5 hits a Landau pole (subject to some subtleties arising when operators of the SCFT sector go free, as we will discuss in the next section). It is amusing to note that this β-function coefficient is equivalent to the $U(1)_R SU(5)_{SM}^2$ global anomaly coefficient at the conformal fixed point. This will turn out to play an important role in computing the contribution to β_{g_5} from gauge-invariant chiral operators that saturate the unitarity bound.

Using these results, we subject the models under consideration to a fairly simple criterion: that they generate an adequate flavor hierarchy over the range $\Lambda_* < \mu < \Lambda$ smaller than the hierarchy $\Lambda_* < \mu < \Lambda_L$ between decoupling and the Landau pole for g_5. For all models we compute Λ_L/Λ_* assuming Standard Model field content, an additional $SU(5)_{SM}$ adjoint Higgs Σ, and the field content of the superconformal sector. For ten-centered models, we compare this to the ratio Λ/Λ_* required to get within a factor of 3 of the observed hierarchy in up-type quark masses. For democratic models, we compare this to both the ratio Λ_T/Λ_* required to get within a factor of 3 of the up-type quark hierarchy, and the ratio Λ_F/Λ_* required to get within a factor of 3 of the lepton mass hierarchy for $\tan\beta = 10$.

5.3 R charges from a-maximization

Clearly, in order for the superconformal flavor mechanism to be effective it is necessary for the conformal sector to generate sufficiently large anomalous dimensions for Standard Model fields. Ideally, these anomalous dimensions should be calculable – often a challenging proposition for strongly coupled theories. A tremendous advantage is gained if the universe is supersymmetric over the energy range in which the flavor hierarchy is generated. In this case the superconformal algebra relates the scaling dimension of gauge-invariant chiral operators to their transformation properties under the superconformal $U(1)_R$ symmetry.

Recall that the superconformal algebra is the superalgebra $SU(2,2|1)$, the bosonic part of which consists of the familiar conformal $SO(4,2)$ and an additional non-anomalous $U(1)_R$. The charges of gauge-invariant chiral primary operators under this particular $U(1)_R$ give, in turn, their scaling dimension at the conformal fixed point. When this $U(1)_R$ may be readily identified, it provides a direct means of computing anomalous dimensions for fields coupled to the SCFT; such was the strategy employed in [137]. However, the utility of this approach is limited by the ease with which the superconformal $U(1)_R$ may be identified. In general, superconformal theories possess a variety of candidate $U(1)_R$ symmetries; the principle challenge lies in determining which $U(1)_R$ dictates the scaling dimensions of chiral primary operators at the fixed point.

Once the correct R-charges are known, unitarity imposes a bound relating the scaling dimension of a gauge-invariant operator $\mathcal{O}$ and its R-charge via the inequality [135]

$$\Delta(\mathcal{O}) \geq |\frac{3}{2}R(\mathcal{O})| \tag{5.3.1}$$

The inequality is saturated for chiral and antichiral primary operators; we will henceforth be interested solely in gauge-invariant chiral primaries for which the equality in (5.3.1) is exact. Once we know the scaling dimension, we can express the anomalous dimension γ of an operator in terms of its R-charges; for a chiral primary,

$$\Delta_{\mathcal{O}} \equiv 1 + \frac{1}{2}\gamma_{\mathcal{O}} = \frac{3}{2}R_{\mathcal{O}} \to \gamma_{\mathcal{O}} = 3R_{\mathcal{O}} - 2 \tag{5.3.2}$$

Clearly, if we can compute the R-charges of operators under the superconformal $U(1)_R$ symmetry, we may determine (up to the usual corrections of order $g_5^2/16\pi^2$ coming from the gauging of flavor symmetries) the anomalous dimensions of fields in a given model of flavor anarchy. But therein lies the rub; in general, a given theory will possess a variety of candidate $U(1)_R$ symmetries, none of which are obviously the superconformal $U(1)_R$. That is, if R_0 is some valid $U(1)_R$ symmetry, so too is

$$R_t = R_0 + \sum_i s_i F_i \tag{5.3.3}$$

where F_i are all the non-R flavor charges of the global symmetry group $\mathcal{F}$. The superconformal $U(1)_R$ corresponds to some specific choice of the s_i.

In truth, the situation is not quite so dire; the superconformal $U(1)_R$ is expected to commute with non-Abelian flavor symmetries, so we can restrict the linear combinations in (5.3.3) to only Abelian flavor generators. Moreover, if there is some sort of charge conjugation symmetry, then the $U(1)_R$ should commute with that as well, leaving only the F_i commuting with charge conjugation. For the simplest example of SQCD, these conditions are sufficient to imply that the superconformal $U(1)_R$ can't mix with any generators of the global symmetries $SU(F) \times SU(F) \times U(1)_B$, so that the superconformal $U(1)_R$ may be uniquely determined by the vanishing of the ABJ anomaly at the superconformal fixed point. For theories with additional matter content, however, one must somehow account for potential contributions from all possible abelian flavor symmetries.

The solution to this obstruction is a clever procedure called a-maximization [104], which amounts to the observation that the superconformal R charges are those that locally maximize the central charge a. Recall that a is the coefficient of the curvature term in the trace of the 4d energy-momentum tensor,

$$\langle T^\mu_\mu \rangle = -\frac{a}{16\pi^2}(\tilde{R}_{\mu\nu\rho\sigma})^2 + ... \tag{5.3.4}$$

Conveniently, supersymmetry allows us to compute a for a given theory in terms of traces of R-charges. The result, due to [11, 12], is

$$a = \frac{3}{32}\left[3\ \mathrm{Tr}\ R^3 - \ \mathrm{Tr}\ R\right], \tag{5.3.5}$$

where $\mathrm{Tr}\ R = \sum_i |r_i|(R_i - 1)$ is the sum over fermionic R-charges of the matter fields i in the theory, weighted by their dimensions r_i. The insight of a-maximization is that the correct values of the s_i corresponding to the superconformal $U(1)_R$ charge are given when the trial a function

$$a_t(s_i) = \frac{3}{32}\left[3\ \mathrm{Tr}\ R_t^3 - \ \mathrm{Tr}\ R_t\right] \tag{5.3.6}$$

has a local maximum as a function of the s_i. The R-charges given by the a-maximization procedure are precisely those appearing in the superconformal $U(1)_R$, and hence give the correct scaling dimension of gauge-invariant chiral primary operators in cases where it may not be determined by simpler means.[1]

Of course, as with so many clever things, this solution is contingent on being able to identify all the global $U(1)$ symmetries at the infrared fixed point. Clearly, if only a subset of the total $U(1)$ global symmetries have been identified, a-maximization over this incomplete subset will generally yield an incorrect result. Although it's sometimes sufficient to identify the $U(1)$ global symmetries in the ultraviolet, it is frequently the case that accidental global $U(1)$ symmetries emerge in the infrared. Most commonly these accidental $U(1)$'s are associated with gauge-invariant operators $\mathcal{O}$ saturating the unitarity bound in the IR. When such a field goes free, there arises a new $U(1)$ global symmetry associated with rotations of $\mathcal{O}$. These accidental $U(1)$s will spoil the a-maximization procedure unless appropriately accounted for.[2]

[1] Of course, the central charge a is of interest for more than simply a-maximization; for some time, this a was conjectured to obey a 4d analogue of the c-theorem, although counterexamples have subsequently been found [151].

[2] It is amusing to note that the counterexample to the conjectured a-theorem exploits precisely this loophole in the a-maximization procedure.

5.3.1 Accounting for accidental $U(1)$s

In the event that an operator $\mathcal{O}$ hits the unitarity bound, the a-maximization procedure will only yield correct R-charges provided that $a(R_t)$ is modified to account for $\mathcal{O}$ going free. In principle, this may be accomplished by replacing the putative contribution from $R_{\mathcal{O}}$ by the free-field value of $R_{\mathcal{O}} = 2/3$, via

$$a(R_t) \to a(R_t) + a(2/3) - a(\mathcal{O}) \tag{5.3.7}$$

This may be implemented physically using a procedure developed in [29]. If $\mathcal{O}$ transforms as some representation $r_{\mathcal{O}}$ of the global symmetry group, consider introducing an additional vector-like pair of gauge-invariant superfields L, M to the theory, where M transforms in the same flavor representation as $\mathcal{O}$ and L transforms in the appropriate conjugate representation. In addition to the new fields, include also a superpotential

$$W_{LM} = L(\mathcal{O} + hM). \tag{5.3.8}$$

Treating h as a perturbation, when $h = 0$ we see that M is a free field and $R(L) = 2 - R(\mathcal{O})$. Now turn on a small h; if $R(\mathcal{O}) > 2/3$ (i.e., when $\mathcal{O}$ is consistent with the unitarity bound), the term hLM is relevant, so L and M become massive and may be integrated out. In this case, the theory in the IR is identical to the original theory; the contributions of L and M to anomalies and a-maximization cancel entirely. However, when $\mathcal{O}$ violates the unitarity bound, the picture changes significantly. It's still the case that $R(L) = 2 - R(\mathcal{O})$, but now $R(\mathcal{O}) < 2/3$ implies that the coupling h is irrelevant and flows to zero in the IR. In that case, M is a free field with $R(M) = 2/3$, and the contributions of L and M to a no longer cancel. Indeed, adding L and M to the a-maximization procedure entails

$$a(R_t) \to a(R_t) + a(M) + a(L) \tag{5.3.9}$$

$$= a(R_t) + a(2/3) - a(\mathcal{O}) \tag{5.3.10}$$

$$= a(R_t) + \frac{\dim(r_{\mathcal{O}})}{96}(2 - 3R_{\mathcal{O}})^2(5 - 3R_{\mathcal{O}}) \tag{5.3.11}$$

Hence the addition of L and M to the theory precisely fixes a in the desired fashion when the field $\mathcal{O}$ goes free. Naturally, this prescription may be generalized to account for any number of operators hitting the unitarity bound.

Although the addition of L and M was introduced as a somewhat *ad hoc* procedure for fixing up the a-maximization procedure, such fields must additionally be accounted for in all anomaly calculations involving flavor symmetries of the SCFT [141]. In particular, the effects of L and M must be included in the running of the Standard Model gauge coupling g_5 when $SU(5)_{SM}$ is embedded in a weakly gauged subgroup of the superconformal global symmetries. Indeed, the inclusion of these contributions is crucial in correctly determining the effects of the SCFT sector on the running of g_5, particularly when determining the scale of potential Standard Model Landau poles.

At first glance, this may seem somewhat unusual; the fields L and M were introduced merely to account for gauge-invariant operators going free in the a-maximization procedure. The necessity of accounting for their contributions to other anomalies becomes most transparent when viewed from the perspective of the composite degrees of freedom in the IR. As noted in §2, the contribution of SCFT fields to the running of the gauge coupling g_5 is equivalent to their contribution to the $U(1)_R SU(5)^2_{SM}$ global anomaly of the SCFT. As such, anomaly-matching guarantees that these contributions must be the same whether computed in terms of the UV or IR degrees of freedom.

Consider then the contribution of a composite operator $\mathcal{O}$ to the NSVZ β-function for g_5:

$$\Delta\mathfrak{b} = (1-\gamma_{\mathcal{O}})T(r_{\mathcal{O}}) = 3(1-R_{\mathcal{O}})T(R_{\mathcal{O}}) \tag{5.3.12}$$

When $R_{\mathcal{O}}$ violates the unitarity bound, the naïve contribution from $\gamma_{\mathcal{O}} \neq 0$ computed via a-maximization is incorrect. But notice that when $\mathcal{O}$ goes free, the contributions from the corresponding L and M to the NSVZ β-function for g_5 are given by

$$\begin{aligned}\sum_{i=L,M}(1-\gamma_i)T(r_i) &= \sum_{i=L,M} 3(1-R_i)T(r_i) \\ &= T(r_{\mathcal{O}}) + 3(R_{\mathcal{O}}-1)T(r_{\mathcal{O}}) = \gamma_{\mathcal{O}}T(r_{\mathcal{O}})\end{aligned} \tag{5.3.13}$$

which precisely cancels the naïve contribution from $\mathcal{O}$ violating the unitarity bound and enforces $\gamma_{\mathcal{O}} = 0$. Thus incorporating the effects of L and M in the running of g_5 does not merely fix the a-maximization procedure; it also fixes the contribution of composite fields to all global anomalies of the SCFT (and hence also to β_{g_5}).

In general, these additional contributions have the effect of lowering the contribution of the SCFT to β_{g_5} (as one might expect, since the naïve $\gamma_{\mathcal{O}}$ are negative and increase $\mathfrak{b}$). As such, they play a key role in determining what candidate superconformal sectors may explain the flavor hierarchy before generating a Landau pole for the Standard Model gauge coupling.

5.4 Simple models with $SU(N)$

With these tools in hand, let us now turn to a series of simple vector-like models of superconformal flavor whose anomalous dimensions may be calculated using a-maximization. We will begin with models where the superconformal sector consists of an $SU(N)$ gauge group, adjoint superfield A, and some number of fundamental and antifundamental flavors. In [141] it was claimed that such models are incapable of generating a sufficient flavor hierarchy before hitting a Landau pole in g_5. We will find, to the contrary, that in many cases the Landau poles are sufficiently remote once the contributions from SCFT states to β_{g_5} are correctly accounted for.[3]

5.4.1 SQCD with an adjoint

Before focusing on specific models of superconformal flavor, it is worthwhile to review a few useful facts about $\mathcal{N} = 1$ supersymmetric $SU(N)$ QCD with F (anti)fundamental flavors Q ($\tilde{Q}$) and a single adjoint chiral superfield A. The theory with a polynomial superpotential for the adjoint was first studied extensively in [117–119], and later re-examined using a-maximization [120]. The dynamics of the theory are rendered

[3] Following correspondence with the authors of [141], their results have been revised to agree with those found here.

fairly simple by the addition of a simple superpotential for the adjoint of the form

$$W = \frac{s_0}{k+1}\ \mathrm{Tr}\ A^{k+1} \tag{5.4.1}$$

Such theories possess and $SU(F) \times SU(F) \times U(1)_B \times U(1)_R$ global symmetry; the transformation properties of $Q, \tilde{Q}$, and A under the gauge and global symmetries are shown in Table 5.1.

Table 5.1: Transformation properties of matter fields in SQCD with an adjoint

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)_B$	$U(1)_R$
Q	$\square$	$\square$	$\mathbf{1}$	1	$1-\frac{2}{k+1}\frac{N}{F}$
$\tilde{Q}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	-1	$1-\frac{2}{k+1}\frac{N}{F}$
A	**Adj**	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{2}{k+1}$

In general, it is often interesting to study the theory with a more general polynomial superpotential,

$$W = \sum_{i=0}^{k-1} \frac{s_i}{k+1-i}\ \mathrm{Tr}\ A^{k+1-i} \tag{5.4.2}$$

which breaks the remaining R-symmetry for nonzero s_i. The adjoint superpotential typically breaks the gauge group $SU(N) \to SU(r_1) \times ... \times SU(r_k) \times U(1)^{k-1}$ in the infrared.

Such theories possess stable vacua provided $F \geq N/k$. In the far infrared, they may be described in terms of a dual "magnetic" supersymmetric gauge theory consisting of a magnetic gauge group $SU(kF-N)$, F magnetic quarks and antiquarks $q, \tilde{q}$, a magnetic adjoint a, and gauge singlets $M_j \sim \tilde{Q}A^{j-1}Q$ representing mesons of the UV theory. Examination of the beta function for the magnetic gauge coupling reveals that the theory is interacting at its IR fixed point provided $N < \frac{2k-1}{2}F$.

5.4.2 A ten-centered model

Perhaps the simplest vector-like model of superconformal flavor with a rank-two tensor is an $SU(N)$ gauge theory with adjoint A and $F = 10$ fundamental and antifundamental flavors. For simplicity, we will also assume that the term $\mathrm{Tr}\, A^3$ is marginal at the conformal fixed point. We may embed $SU(5)_{SM}$ in the $SU(F) \times SU(F)$ global symmetry group as shown in Table 5.2.

Table 5.2: Embedding of a ten-centered model with $F = 10$

	$SU(5)_{SM}$	$SU(N)$
$Q_1 + \overline{Q}_2$	$\mathbf{5} + \overline{\mathbf{5}}$	$\square$
$\overline{Q}_1 + Q_2$	$\overline{\mathbf{5}} + \mathbf{5}$	$\overline{\square}$
A	$\mathbf{1}$	**Adj**.

The $(SU(N))$ gauge-invariant mesons transform under $SU(5)$ as

$$(Q_1 + \overline{Q}_2)(\overline{Q}_1 + Q_2) = 2 \times \mathbf{24} + 2 \times \mathbf{1} + \mathbf{10} + \mathbf{15} + \overline{\mathbf{10}} + \overline{\mathbf{15}} \tag{5.4.3}$$

With this field content, the most general couplings involving two SCFT fields and one Standard Model field are those incorporating the T_1 and T_2 fields. As such, our marginal superpotential terms at the conformal fixed point are

$$W = T_1 \overline{Q}_1 \overline{Q}_2 + T_2 \overline{Q}_1 A \overline{Q}_2 + A^3 \tag{5.4.4}$$

For the theory with $k = 2$ and $F = 10$, we require $N < 15$ for the theory to be interacting at the fixed point and $N < 20$ to have stable vacua. We will also find $R_{T_2} > 2/3$ only for $N > 10$, which gives us a window $10 < N < 15$ for this particular theory. By assumption, the β function for the $SU(N)$ gauge coupling vanishes at the fixed point, and the above operators are held to be marginal. The R charges, and hence the scaling dimensions, of the Standard Model fields T_1, T_2 may then be computed via a-maximization, the numerical results of which are reserved for Table C.1 in Appendix C. We find that several mesons are free fields at the fixed point: Q_1Q_2, $Q_1\overline{Q}_1$, $Q_2\overline{Q}_2$, and $\overline{Q}_1\overline{Q}_2$. Of these, only the $\overline{\mathbf{15}}$ of the last meson needs to be accounted for in the a-maximization procedure, since the $\overline{\mathbf{10}}$ component is set to zero in the chiral ring due to the superpotential couplings with T_1.

For $N = 11, 12$, the R-charges of SM fields are too small to generate the observed flavor hierarchy over any range of running. However, the theories with $N = 13, 14$ work beautifully. In both cases, a sufficient flavor hierarchy may be generated before g_5 hits a Landau pole. In order for this to work, it is crucial to correctly account for the effects of the mesons Q_1Q_2, $Q_1\overline{Q}_1$, $Q_2\overline{Q}_2$, and $\overline{Q}_1\overline{Q}_2$ going free when computing their contribution to $\beta(g_5)$. Thus we find that $SU(N)$ SQCD with an adjoint and $F = 10$ fundamental flavors provides a suitable model of superconformal flavor.

It is tempting to consider the same theory with marginal operator $\text{Tr } A^4$ at the conformal fixed point. For such a theory, we require $N < 25$ in order to be interacting and $N < 30$ for stability. We also find that T_2 violates the unitarity bound for $N < 12$, so we are interested in values $11 < N < 25$. The constraints on R-charges and results of a-maximization are reserved for Table C.2. For sufficiently large N – specifically, for $21 \leq N \leq 24$ – a sufficient flavor hierarchy may be generated before g_5 hits a Landau pole.

5.4.3 A more democratic model

Although ten-centered models capture much of the essential features of the flavor hierarchy, it is worth exploring whether a more complete hierarchy may be generated by coupling the SCFT to both T_i and $\bar{F}_i$ fields of the Standard Model. Extending our $SU(N)$ model to accommodate couplings to additional Standard Model representations is fairly simple; it requires only enlarging the flavor symmetry to $F > 10$. The simplest such model involves $F = 11$ fundamental and antifundamental flavors of the SCFT. The matter content and transformation properties under $SU(5)_{SM}$ are shown in Table 5.4.3.

Naturally, there is a significant increase in the number of $SU(N)$ gauge invariants transforming nontrivially under $SU(5)$:

$$(Q_1+\overline{Q}_2+Q_0)(\overline{Q}_1+Q_2+\overline{Q}_0) = 2\times\mathbf{24}+3\times\mathbf{1}+\mathbf{10}+\mathbf{15}+\overline{\mathbf{10}}+\overline{\mathbf{15}}+2\times\mathbf{5}+2\times\overline{\mathbf{5}} \quad (5.4.5)$$

Table 5.3: Matter content for $SU(N)$ theory with $F = 11$

	$SU(5)_{SM}$	$SU(N)$
$Q_1 + \overline{Q}_2 + Q_0$	$\mathbf{5} + \overline{\mathbf{5}} + \mathbf{1}$	$\square$
$\overline{Q}_1 + Q_2 + \overline{Q}_0$	$\overline{\mathbf{5}} + \mathbf{5} + \mathbf{1}$	$\overline{\square}$
A	$\mathbf{1}$	Adj.

For clarity, the transformation properties of the $SU(N)$ gauge-invariant mesons under $SU(5)$ is shown in detail in Table 5.4.

Table 5.4: Meson decomposition under $SU(5)$

Meson	$SU(5)$	Meson	$SU(5)$
Q_1Q_2	$\mathbf{10} + \mathbf{15}$	$Q_1\overline{Q}_0$	$\mathbf{5}$
$Q_1\overline{Q}_1$	$\mathbf{24} + \mathbf{1}$	Q_2Q_0	$\mathbf{5}$
$Q_2\overline{Q}_2$	$\mathbf{24} + \mathbf{1}$	$\overline{Q}_1Q_0$	$\overline{\mathbf{5}}$
$\overline{Q}_1\overline{Q}_2$	$\overline{\mathbf{10}} + \overline{\mathbf{15}}$	$\overline{Q}_2\overline{Q}_0$	$\overline{\mathbf{5}}$
		$Q_0\overline{Q}_0$	$\mathbf{1}$

Many of these mesons go free at the conformal fixed point: Q_1Q_2, $Q_1\overline{Q}_1$, $Q_2\overline{Q}_2$, $\overline{Q}_1\overline{Q}_2$, $\overline{Q}_1Q_0$, $\overline{Q}_2\overline{Q}_0$, and $Q_0\overline{Q}_0$. The $\overline{\mathbf{10}}$ component of $\overline{Q}_1\overline{Q}_2$ will be set to zero in the chiral ring, as will the $\mathbf{5}$s that couple to Standard Model matter. As for the vacua of the theory and the range of parameters, with our customary Tr A^3 deformation we require $N < 17$ for the fixed point to be interacting and $N \leq 22$ for stability of the vacuum.

Our marginal couplings at the fixed point now are[4]

$$W = T_1\overline{Q}_1\overline{Q}_2 + T_2\overline{Q}_1A\overline{Q}_2 + \overline{F}_1(Q_1\overline{Q}_0 + Q_2Q_0) + \overline{F}_2(Q_1A\overline{Q}_0 + Q_2AQ_0) + A^3 \tag{5.4.6}$$

The results of the a-maximization procedure are shown in Table C.3. We see that at $N = 11$ all the mesons of the SCFT are exactly free, and violate the unitarity bound

[4]Of course, it is also now possible to couple the SCFT fields to the Higgses H_u and H_d; it is technically natural to turn these couplings off, which we will do here for simplicity. For a discussion of the potential complications that arise from coupling SCFT fields to H_u and H_d, see [141].

for $N > 11$. So we can use our usual techniques to analyze the theory in the window $10 < N < 17$. The results are fairly encouraging; for $N > 13$ it is possible to generate a sufficient hierarchy for both the T_i and $\overline{F}_i$ before hitting a Landau pole of $SU(5)$.

5.4.4 Coupling to the adjoint Higgs

One way to address the potential Landau pole in the $SU(5)$ gauge coupling is to find other ways to reduce naive contributions from Standard Model GUT fields. Large anomalous dimensions do precisely that; since the contribution to β_{g_5} of a matter field in the representation r_i is proportional to $T(r_i)(1-\gamma_i)$, it's clear that large, positive anomalous dimensions γ_i can slow somewhat the progression of g_5 towards its Landau pole.

A simple way to implement this idea is to couple the superconformal sector to the $SU(5)_{SM}$ adjoint Higgs field Σ responsible for breaking $SU(5) \to SU(3) \times SU(2) \times U(1)$. Such couplings are, in general, allowed by the symmetries of models considered here, and are not unreasonable to include among the marginal interactions at the superconformal fixed point.

Consider, e.g., the model of § 5.4.2, incorporating now additional couplings to the adjoint Higgs Σ. The allowed interactions are now

$$W_{int} = T_1\overline{Q}_1\overline{Q}_2 + T_2\overline{Q}_1 A\overline{Q}_2 + (Q_1\overline{Q}_1 + Q_2\overline{Q}_2)\Sigma + A^3 \tag{5.4.7}$$

In principle this gives two extra constraints and one extra unknown, but in fact the two new terms are identical equations, so we still need a-maximization to solve for the R-charges. Doing a-maximization on the A^3 theory gives us only changes to the value of $\mathfrak{b}$; this dramatically improves the window of running for SU(5) couplings while preserving the nice predictions of the undeformed theory. There is a small trade-off in that the **24** component of the linear combination $Q_1\overline{Q}_1 + Q_2\overline{Q}_2$ is now set to zero in the chiral ring, but nonetheless the net effect is to lower $\mathfrak{b}$ significantly and thus render the Landau pole more remote.

5.5 Simple models with $Sp(2N)$

Although it is compelling that something as simple as $SU(N)$ SQCD with an adjoint leads to suitable models of superconformal flavor, it's useful to consider related models with different gauge groups. Symplectic groups, in particular, offer more "compact" theories of flavor, in the sense that Standard Model $SU(5)$ may be more efficiently embedded in their flavor symmetries.

In this section we will focus on $\mathcal{N} = 1$ supersymmetric $Sp(2N)$ gauge theory[5] with $2F$ fundamental flavors Q and an antisymmetric tensor A.[6] The IR behavior of $Sp(2N)$ theories with an antisymmetric tensor and polynomial superpotential $\mathrm{Tr}\ A^{k+1}$ was studied in detail in [100], while the theory without polynomial superpotential was analyzed using a-maximization in [140]. With some malice aforethought, we will focus here on the $k = 2$ superpotential with marginal operator $\mathrm{Tr}\ A^3$. This theory is interacting in the IR provided $N < \left(k - \frac{1}{2}\right) F - 2(k-1)$ and possesses stable vacua provided $N < kF$. The transformation properties of the matter fields under the relevant gauge and global symmetries is shown below in Table 5.5.

Table 5.5: Transformation properties of matter fields in $Sp(2N)$ with an antisymmetric tensor

	$Sp(2N)$	$SU(2F)$	$U(1)_R$
Q	$\square$	$\square$	$1 - \frac{2(N+k)}{(k+1)F}$
A	**Anti.**	**1**	$\frac{2}{k+1}$

5.5.1 A ten-centered model

As a warmup, let us begin with the simplest $Sp(2N)$ theory: $Sp(2N)$ with $2F = 10$ flavors of fundamental quark Q and antisymmetric tensor A. This theory was treated

[5] Here we are choosing notation such that $Sp(2) \sim SU(2)$

[6] The related $Sp(2N)$ theory with *symmetric* tensor A, studied extensively in [121], is less suitable for these simple models of flavor due to the different symmetry properties of the mesons QQ and QAQ.

in [141]; we review their results here before moving on to a more general model with larger flavor symmetry. The embedding is shown in Table 5.6.

Table 5.6: Embedding of a ten-centered $Sp(2N)$ model with $2F = 10$

	$SU(5)_{SM}$	$Sp(2N)$
$Q + \overline{Q}$	$\mathbf{5} + \overline{\mathbf{5}}$	$\square$
A	$\mathbf{1}$	**Anti.**

The mesons of the SCFT then transform under $SU(5)_{SM}$ as

$$(Q + \overline{Q})J(Q + \overline{Q}) = \mathbf{24} + \mathbf{1} + \mathbf{10} + \overline{\mathbf{10}} \tag{5.5.1}$$

As in the $SU(N)$ theory with $F = 10$, there are no $\mathbf{5}$ representations to combine with the $\overline{F}_i$ of the SM, making this a purely ten-centered model. In this case our desired couplings to SM fields are (including the customary cubic superpotential for the antisymmetric tensor)

$$W = T_1 \overline{Q}\overline{Q} + T_2 \overline{Q} A \overline{Q} + \text{ Tr } A^3 \tag{5.5.2}$$

For $k = 2$ and $F = 5$ we require $N \leq 5$ in order for the theory to possess an interacting fixed point. We find that the gauge invariant chiral operators QQ and $Q\overline{Q}$ go free in the range of interest, while $\overline{Q}\overline{Q}$ and $\overline{Q}A\overline{Q}$ are set to zero in the chiral ring. There are no baryons in the chiral ring of this theory, since putative baryons of an $Sp(2N)$ gauge theory may be expressed in terms of mesons.

The constraints and R-charges computed via a-maximization are reserved for Table C.5 of the the Appendix. Given the constraint on N, the possible theories are fairly proscribed. However, for $N = 5$ the theory generates a sufficient flavor hierarchy over a small range of energies. Equally attractive is the remoteness of Landau poles; the relative smallness of the additional Standard Model representations introduced by the SCFT ensures that g_5 remains perturbative many orders of magnitude above the GUT scale.

It is fairly straightforward to compute the R-charges for the simple extension to the $k = 3$ theory with $2F = 10$ flavors. The virtue of such theories is a larger window of N for which the IR fixed point is interacting – in this case, for $N \leq 8$. The lowered R-charge of A allows the mesons QAQ and $QA\overline{Q}$ to saturate the unitarity bound as well. For $8 \geq N \geq 5$ the outcome is encouraging: adequate flavor hierarchy with Landau poles far from the GUT scale.

5.5.2 A more democratic model

As before, we can consider extending the ten-centered model in §6.1 by enlarging the flavor symmetry of the superconformal sector. In this case, the simplest generalization is to increase the number of fundamental flavors to $2F = 12$ (recalling that we need an even number of flavors to cancel the global anomaly). As always, we may then weakly gauge an $SU(5)$ subgroup of the flavor symmetry. The corresponding transformation properties of the SCFT fields are shown in Table 5.7.

Table 5.7: $SU(5)$ embedding of a democratic $Sp(2N)$ model with $2F = 12$

	$SU(5)_{SM}$	$Sp(2N)$
$Q + \overline{Q} + Q_0 + \overline{Q}_0$	$\mathbf{5} + \overline{\mathbf{5}} + \mathbf{1} + \mathbf{1}$	$\square$
A	$\mathbf{1}$	**Anti.**

The enlarged flavor symmetry leads to a plethora of $Sp(2N)$ gauge-invariant chiral operators transforming under $SU(5)_{SM}$, which we list for convenience in Table 5.8.

Table 5.8: Meson decomposition of under $SU(5)$

Meson	$SU(5)$	Meson	$SU(5)$
QQ	$\mathbf{10}$	$\overline{QQ}$	$\overline{\mathbf{10}}$
$Q\overline{Q}$	$\mathbf{24} + \mathbf{1}$	$\overline{Q}Q_0$	$\overline{\mathbf{5}}$
QQ_0	$\mathbf{5}$	$\overline{QQ}_0$	$\overline{\mathbf{5}}$
$Q\overline{Q}_0$	$\mathbf{5}$	$Q_0\overline{Q}_0$	$\mathbf{1}$

Assuming our customary cubic superpotential term for the antisymmetric field, the theory possesses stable vacua provided $N < 12$ and is at an interacting IR fixed point provided $N < 7$.

The candidate marginal couplings at the conformal fixed point are thus

$$W = T_1\overline{QQ} + T_2\overline{Q}A\overline{Q} + F_1(QQ_0 + Q\overline{Q}_0) + F_2(QAQ_0 + QA\overline{Q}_0) + \text{ Tr } A^3 \quad (5.5.3)$$

As always, the gauge-invariant chiral operators with marginal couplings to Standard Model states are set to zero in the chiral ring ($\overline{QQ}, \overline{Q}A\overline{Q}$, and the linear combinations $QQ_0 + Q\overline{Q}_0, QAQ_0 + QA\overline{Q}_0$). Of the remaining chiral operators, $QQ, Q\overline{Q}, \overline{Q}Q_0, \overline{QQ}_0$, and $Q_0\overline{Q}_0$ saturate the unitarity bound and must be accounted for accordingly in the a-maximization procedure.

The superconformal R-charge assignments for this theory are shown in Table C.7. For $N = 6$ the theory produces a sufficient flavor hierarchy for both the T_i and $\overline{F}_i$ well below any potential Landau poles in g_5. The $k = 3$ theory with $2F = 12$ is essentially identical in features, albeit with a much larger window of colors (ranging up to $N < 11$ for an interacting fixed point).

5.6 Discussion

Thus far we have seen that a variety of models based on $SU(N)$ and $Sp(2N)$ superconformal gauge theories with rank-two tensor fields may give rise to the Standard Model flavor hierarchy above the GUT scale. However, our treatment has elided a few significant details that warrant some consideration – in particular, the effect of Standard Model field couplings on the superconformal fixed point, as well as the details of conformal symmetry breaking and decoupling – to which we now turn.

5.6.1 Saturating the unitarity bound

In the preceding sections, we have been interested in superpotential interactions coupling Standard Model and SCFT fields of the form $\delta W = \Phi_i \mathcal{O}$, where $\mathcal{O}$ is a gauge-invariant chiral operator comprised of matter fields of the SCFT. Thus far we have treated such interactions as a small deformation away from the original superconformal fixed point of the SCFT sector, but it is worth examining whether this approximation is completely justified. It is often the case in the undeformed SCFT that the scaling dimension of $\mathcal{O}$ saturates the unitarity bound, at which point an accidental $U(1)$ symmetry emerges to enforce $R_{\mathcal{O}} = 2/3$. When the SCFT has a dual description in which the magnetic dual of $\mathcal{O}$ is a free field, we generally interpret saturation of the unitarity bound as an indication that the field $\mathcal{O}$ has gone free.

The issue becomes somewhat more convoluted in the models considered here, where $\mathcal{O}$ is coupled additionally to Standard Model fields Φ_i by marginal superpotential interactions. In that case, when $\mathcal{O}$ hits the unitarity bound it is no longer the case that an accidental $U(1)$ emerges to enforce $R_{\mathcal{O}} = 2/3$, but rather $R_{\mathcal{O}} < 2/3$ is allowed. This R charge is not in conflict with the unitarity bound, as the F term for Φ_i sets $\mathcal{O}$ to zero in the chiral ring, so that the unitarity bound no longer pertains. One might become concerned about whether the interaction $\Phi_i \mathcal{O}$ in this case truly amounts to a small deformation of the superconformal fixed point, since it involves positing a marginal interaction between a Standard Model field and an otherwise-free operator.[7]

This question becomes fairly central in the models considered above, where generating an adequate flavor heirarchy before hitting a Standard Model Landau pole requires $R(\mathcal{O}) < 2/3$ (for at least one such $\mathcal{O}$) in every case.[8]

Thankfully, the new fixed points reached by coupling Standard Model fields to the SCFT are fairly well understood. Turning off the Standard Model gauge coupling and Yukawa interactions reduces these models to variations on $SU(N)$ SSQCD (analyzed via a-maximization in [29]) and its $Sp(2N)$ generalization. In this case, the role of the gauge singlets of SSQCD is played by Standard Model superfields. Although our models also differ from SSQCD by the inclusion of rank-two tensor fields, these do not significantly modify the relevant details. To understand the fixed point in detail,

[7]We thank Dan Green for an extensive discussion of this point. For further discussion, see also [31].

[8]Indeed, the only viable vector-like model of superconformal flavor that does not require $R(\mathcal{O}) < 2/3$ for some $\mathcal{O}$ is the $Sp(2N)$ theory with no polynomial superpotential for A, studied in [141]. However, this theory requires significantly more decades of running above the decoupling scale Λ_*, and may not fit between M_{GUT} and M_{Pl}.

let us review the results of [29]. Consider $SU(N)$ SQCD with F flavors $Q_i, \tilde{Q}_i$ and F' additional flavors $Q'_{i'}, \tilde{Q}'_{i'}$, as well as F'^2 singlets $S^{i'j'}$ with superpotential coupling

$$W = hS^{i'j'}Q'_{i'}\tilde{Q}'_{j'} \tag{5.6.1}$$

For $h = 0$, the IR fixed point is simply that of SQCD with $F + F'$ flavors, which we know to have an interacting fixed point for $3N/2 < F + F' < 3N$. Turning on $h \to h_* \neq 0$ amounts to a relevant deformation driving the theory to a new family of SCFTs in the IR, of which the usual fixed points of SQCD are special cases.

The fixed point may also be described by a dual $SU(F + F' - N)$ gauge theory. In the dual theory, the interaction (5.6.1) corresponds to a mass term for the singlets S and the mesons $M' \sim Q'\tilde{Q}'$, which may be integrated out. The remaining matter content at the fixed point consists of F flavors of magnetic quarks $q', \tilde{q}'$; F' flavors $q, \tilde{q}$; an $SU(F) \times SU(F)$ bifundamental meson M_{ij}, and $SU(F) \times SU(F')$ bifundamental mesons $P_{ij'}, P'_{ij'}$ with superpotential interaction

$$W = Mq'\tilde{q}' + Pq'\tilde{q} + P'\tilde{q}'q \tag{5.6.2}$$

The duality map for various gauge-invariant operators is

$$Q\tilde{Q} \to M, \quad S \to -q\tilde{q}, \quad Q\tilde{Q}' \to P, \quad Q'\tilde{Q} \to P', \quad Q^r Q'^{N-r} \to q'^{F-r} q^{F'-N+r} \tag{5.6.3}$$

Significantly, although M' and S have been integrated out of the dual theory, there remains a gauge-invariant chiral operator (identified with $-q\tilde{q}$) that has the same quantum numbers as the original singlets S.

Both the original theory and its dual share a $SU(F)_L \times SU(F)_R \times SU(F')_L \times SU(F')_R \times U(1)_B \times U(1)_{B'} \times U(1)_F \times U(1)_{R_0}$ flavor symmetry; the axial $SU(F + F')$ flavor symmetry is broken to $SU(F) \times SU(F') \times U(1)_F$ by $h \neq 0$. The R charges may be determined in the original theory by carrying out the a-maximization procedure subject to the constraints

$$N + F(R(Q) - 1) + F'(R(Q') - 1) = 0, \qquad R(S) + 2R(Q') = 2 \tag{5.6.4}$$

It's clear that the R charge of Q will differ from that of Q'; this is because the R-symmetry can mix with the $U(1)_F$ flavor symmetry, under which Q, Q' have opposite charges. Significantly, this implies in the original theory that $R(S) = 2 - 2R(Q')$, irrespective of whether $Q'\tilde{Q}'$ violates the unitarity bound. The duality map relates these R charges to those of the dual theory, such that

$$2R(Q) = R(M), \quad R(S) = 2R(q), \quad R(Q) + R(Q') = R(P) \tag{5.6.5}$$

In this case, the duality map implies $2R(q) = 2 - 2R(Q')$, so that the gauge invariant operator $q\tilde{q}$ inherits the R charge and scaling dimension of the singlets S.

Having established the duality map, it is fairly straightforward to understand the results of a-maximization in both the original theory and its dual. For fixed F'/F, as N/F is increased the theory goes successively through the phases: free electric fixed point; interacting fixed point with no mesons free; interacting fixed point with only $M = Q\tilde{Q}$ free; free magnetic fixed point. The meson M' does not appear in the phase diagram, as it has been set to zero in the chiral ring by F_S in the original theory, and equivalently has been integrated out in the dual theory. Nonetheless, in the original theory the field S gains a large anomalous dimension from its coupling to $M' = Q'\tilde{Q}'$, while in the dual theory the same anomalous dimension is developed by the dual gauge-invariant operator $-q\tilde{q}$. These results hold whether or not $M' = Q'\tilde{Q}'$ appears to violate the unitarity bound.

In terms of the models considered in Sections 4 and 5, the results are entirely analogous, although the Standard Model fields Φ_i transform under different representations of the flavor symmetry than the flavor bifundamental S of SSQCD. In terms of the original variables, an interaction $\Phi_i\mathcal{O}$ sets $\mathcal{O}$ to zero in the chiral ring and fixes $R(\Phi_i) = 2 - R(\mathcal{O})$ irrespective of whether $\mathcal{O}$ violates the unitarity bound. In terms of dual variables, Φ_i and $\mathcal{O}$ are integrated out, but there is a gauge invariant chiral operator $\tilde{\Phi}_i$ with the same quantum numbers and R charge as the original Φ_i. In this case, the low energy Standard Model degrees of freedom may be thought of as composites of the dual gauge group. Thus there appears to be nothing inconsistent about generating the flavor hierarchy by coupling Standard Model fields to SCFT operators that go free at the undeformed superconformal fixed point.

5.6.2 Decoupling

Thus far we have remained agnostic about what happens at and below the scale Λ_* at which conformality is broken and the theory flows away from its conformal fixed point. It certainly is necessary to decouple the SCFT fields carrying Standard Model charges, lest they come into conflict with observational limits on charged exotics. Thankfully, this may be accomplished easily in vector-like models simply by giving a vector mass to the fundamental quarks and antiquarks of the SCFT sector.

There are a variety of controlled ways of breaking conformal invariance. Perhaps the most typical way involves turning on vector masses for some or all flavors of fundamental matter at a scale $m_Q \sim \Lambda_*$, so that the theory no longer has enough flavors to remain conformal. Determining the correct IR degrees of freedom after conformal symmetry breaking is a fairly delicate matter; for a detailed discussion, see Appendix B of [141].

An alternative is to include a mass $m_A \sim \Lambda_*$ for the rank-two tensor A, along the lines of [148]. Below the scale m_A, A may be integrated out, leaving SQCD (or possibly a product group of SQCD theories, in the event that the vev of A breaks the original gauge group) with too few flavors to remain conformal. Thus the theory flows to a free fixed point that may be described in terms of the dual SQCD degrees of freedom. Breaking conformality in this fashion raises the possibility that the remaining degrees of freedom may break supersymmetry as in [103], although remaining matter charged under the Standard Model must still be decoupled in a controlled fashion.

5.7 Conclusion

The pattern of Standard Model flavor poses a considerable puzzle to theoretical physics; both the replication and hierarchy of fermion masses are without obvious explanation. It is exciting that a superconformal sector coupled to the Standard

Model may generate the observed fermion mass hierarchy from complete flavor anarchy over just a few decades in energy. Such a scenario, moreover, may not be entirely fantastic; many ultraviolet completions of the Standard Model give rise to additional gauge groups with bifundamental matter at high energies. If vector-like, these sectors may gracefully decouple at low energies and remain consistent with observational constraints.

When supersymmetric, these sectors have the virtue of calculability thanks to superconformal symmetry and the a-maximization procedure. As such, we may subject them to straightforward tests of consistency. Here we have found that simple theories of both $SU(N)$ and $Sp(2N)$ with fundamental matter and a rank-2 tensor field are capable of producing the observed flavor hierarchy before the unified Standard Model gauge coupling hits a Landau pole. Using these results, we have constructed both ten-centered and democratic models of superconformal flavor. It seems that a variety of potential models are viable, over a full range of $\tan\beta$. The challenge now rests in determining which, if any, such models may be realized in nature. Although models operating above the GUT scale are advantageous from the perspective of proton decay and other potentially dangerous baryon number violation, they are generally too remote to yield distinct experimental signatures beyond the observed Yukawa textures. It would be amusing to see if such models may be lowered to accessible energies without running afoul of observational bounds.

Much progress has been made in recent years towards understanding calculable supersymmetry breaking in vector-like gauge theories, beginning with [103]. Supersymmetry breaking vacua have been found in $SU(N)$ theories with fundamental and adjoint matter [9, 46], making it natural to consider whether both superconformal flavor and supersymmetry breaking may emerge from the same dynamics. The resulting correlations between the patterns of fermion and sfermion flavor may hold the key to explaining the small amount of observed flavor violation, as well as provide indications of the superconformal dynamics in the far ultraviolet.

Note added upon completion: After this work was completed, correspondence with the authors of [141] revealed that the discrepancy in results regarding the viability of $SU(N)$ theories with an adjoint arose from incorrect values of $\mathfrak{b}$ in the original version of [141]. Their values and conclusions have subsequently been revised and found to agree with those appearing in Section 4.

Appendix A

Various goldstini variations

A.1 Modulini masses from Supergravity

In order to calculate SUGRA effects on the ISS model we study a slightly more general case of the theory detailed in Section 3.4. We start with superfields X_i with superpotential

$$W = W_0 + f_a X_a \tag{A.1.1}$$

and Kähler potential

$$K = X_a X^\dagger_{a^\star} + \frac{1}{|\mu|^2} A_{ab^\star cd^\star} X_a X^\dagger_{b^\star} X_c X^\dagger_{d^\star} \quad , \tag{A.1.2}$$

from which we may define the modified Kähler potential

$$G = \frac{K}{M_P^2} + \log \frac{W}{M_P^3} + \log \frac{W^\star}{M_P^3} \tag{A.1.3}$$

and field derivatives as $G_a = \partial_a G$, $G_{ab^\star} = \partial_a \partial_{b^\star} G$ with $\partial_a = M_P \frac{\partial}{\partial X_a}$. For a modified Kähler potential of this form, then, once the goldstino direction has been rotated away, the fermion mass matrix is given as [27]:

$$m_{ab} = m_{3/2} \langle \nabla_a G_b + \frac{1}{3} G_a G_b \rangle \tag{A.1.4}$$

where $\nabla_a G_b = \partial_a G_b - \Gamma^c_{ab} G_c$. The Christoffel connection, Γ, is of crucial importance as it encodes the effects of $A_{ab^\star cd^\star}$. Now considering the leading terms in the fermion mass matrix under the assumption that $\sqrt{f} \sim \mu \ll M_P$ one finds:

$$m_{ab} = m_{3/2}\left(-\frac{2}{3}\frac{f_a f_b M_P^2}{W_0^2} - \frac{M_P^2}{W_0|\mu|^2}\delta^{cd^\star} A_{(ad^\star bl^\star)} f_c \langle X^\dagger_{l^\star}\rangle\right) \tag{A.1.5}$$

where $A_{(ad^\star bl^\star)}$ has been symmetrized over pairs of holomorphic indices.[1] At this stage it is appropriate to pause and consider the validity of this result. Throughout we have assumed that as $\sqrt{f} \sim \mu \ll M_P$ then $\langle X\rangle \ll M_P$ and $W_0 \ll M_P^3$. It may seem that if one takes the limit $A \to 0$ then $m_{ab} \propto f_a f_b$ which is a rank one matrix with only one non-zero eigenvalue. However taking this limit means that the scalar fields are no longer stabilised near the origin and the derivation of this result is no longer valid. Also it would appear from this result that the fermion mass matrix depends on the parameter μ; however we will see that $\langle X\rangle \sim |\mu|^2/M_P$ and this dependence drops out. Again, this independence only necessarily holds in the limit $\mu \ll M_P$.

Now considering the scalar potential $V = M_P^4 e^G(G_a G^a - 3)$ one finds that for vanishing cosmological constant

$$W_0 = M_P\sqrt{\frac{f_a f_a}{3}} = \frac{1}{\sqrt{3}} f_{eff} M_P \tag{A.1.6}$$

and at the minimum of the scalar potential

$$\langle X^\dagger_{l^\star}\rangle = -\frac{2|\mu|^2 W_0}{M_P^2} f_k (A_{(ab^\star kl^\star)} f_a f_{b^\star})^{-1} \tag{A.1.7}$$

where $(M_{kl^\star})^{-1}$ is understood as the standard matrix inverse. With these results in hand we can now write a general formula for the modulini mass matrix

$$m_{ab} = 2m_{3/2}\left(A_{(ad^\star bl^\star)}(A_{(ij^\star kl^\star)} f_i f_{j^\star})^{-1} f_{d^\star} f_k - \frac{f_a f_b}{f_{eff}^2}\right) \quad . \tag{A.1.8}$$

This equation is valid up to corrections of the order $\delta m \sim m_{3/2}|\mu|^2/M_P^2$. The extension

[1] By this we mean $A_{(ad^\star bl^\star)} = A_{ad^\star bl^\star} + A_{bd^\star al^\star} + A_{al^\star bd^\star} + A_{bl^\star ad^\star}$.

of this formula to one for matrix-valued fields can be found by replacing individual indices with pairs, i.e. $\{a\} \to \{ab\}$. At first Eq.(A.1.8) may appear rather opaque, however one important property can be observed by inspection. As described in [27] once the goldstino direction has been rotated away one expects that $G_a m_{ab} = 0$. This is clear from Eq.(A.1.4) when one enforces the condition of vanishing cosmological constant and that the fields are at the minimum of the potential. At the level of Eq.(A.1.8) one can see that this result also holds for any form of $A_{ab^\star cd^\star}$ as $f_a m_{ab} = f_b - f_b = 0$ by inspection.

A.2 Masses to all orders in f

As described in Section 3.4.1 the effective Kähler potential only includes corrections to second order in the SUSY breaking F-terms. To include higher order corrections at the level of the Kähler potential would require including higher order supercovariant derivatives. Therefore it is more straightforward to calculate the modulini masses to all orders in the F-terms by explicitly evaluating the loop diagram involving the exchange of scalar and fermionic partners of the heavy fields. In this manner the effects of SUGRA, and consequent R-symmetry breaking, are included by allowing for a non-zero vacuum expectation value for the fields which break SUSY. This vev can be calculated to all orders in the F-terms by including the tadpole term induced by SUGRA and calculating the scalar masses with the Coleman-Weinberg potential. Evaluating the one-loop contribution to the modulini masses using the masses and couplings in Eq.(3.4.5), and setting $h = 1$ for convenience, one finds

$$m_{ab,cd} = 2m_{3/2} \left(\frac{1}{2} \left(\frac{H(f_a)}{H(f_b)} + \frac{H(f_b)}{H(f_a)} \right) \delta_{ad}\delta_{bc} - \frac{f_a f_c}{f_{eff}^2} \delta_{ab}\delta_{cd} \right) \tag{A.2.1}$$

where

$$H(f) = \sum_{i=1}^{N_f - N_c} h(f, \tilde{\mu}_{0_i}^2) \tag{A.2.2}$$

and

$$h(f,\mu^2) = \frac{1}{f^2}\left(2f\mu^2 + 2f\mu^2\log\left(\frac{\mu^4}{\mu^4 - f^2}\right) + (\mu^4 + f^2)\log\left(\frac{\mu^2 - f}{\mu^2 + f}\right)\right) \tag{A.2.3}$$

Here $\tilde{\mu}_{0_i} \geq \sqrt{f}$ is the SUSY mass of the fields which have been integrated out. One can see that all dependence on the UV cut-off has cancelled and the masses are finite. The goldstini from the diagonal components of Φ still have mass $2m_{3/2}$ and the modulini from the off-diagonal components have mass $\geq 2m_{3/2}$, limiting to $2m_{3/2}$ when the F-terms are equal, as before. Therefore the results derived using the effective Kähler potential in Section 3.4.1 are qualitatively the same as those one finds when including the F-terms to all orders.

A.3 Decay widths

Starting with Eq.(3.5.1) we derive the decay width to multiple goldstini under the assumption that all but one messenger scales are the same. We take the first $N-1$ messenger scales equal to $\sqrt{x}\Lambda$ and the N^{th} messenger scale as Λ. Using this, the orthogonality of V_{ia}, the fact that $V_{Ni} = f_i/f_{eff}$ and that $\sum_i f_i V_{i,a\neq N} = 0$ we can simplify the sum over squares of the couplings:

$$\begin{aligned}\sum_{a=1}^{N-1}|C_a|^2 &= \sum_{a=1}^{N-1}\left|\sum_{i=1}^{N}\frac{f_i V_{ia}}{\Lambda_i^2}\right|^2 \\ &= \sum_{a=1}^{N-1}\frac{(x-1)^2 f_N^2}{x^2\Lambda^4}V_{Na}V_{aN}^T \\ &= \frac{(x-1)^2 f_N^2}{x^2\Lambda^4}\frac{f_{eff}^2 - f_N^2}{f_{eff}^2}\end{aligned} \tag{A.3.1}$$

Thus we have

$$\Gamma_{\phi^\dagger\to\zeta\psi} \simeq \frac{m_\phi}{16\pi}\left(\frac{(x-1)f_N}{x\Lambda^2}\right)^2\frac{f_{eff}^2 - f_N^2}{f_{eff}^2} \tag{A.3.2}$$

For decays to the gravitino similar steps lead to

$$|C_N|^2 = \left(\frac{f_{eff}^2 - (1-x)f_N^2}{x\Lambda^2 f_{eff}}\right)^2 \tag{A.3.3}$$

and

$$\Gamma_{\phi^\dagger \to G\psi} \simeq \frac{m_\phi}{16\pi}\left(\frac{f_{eff}^2 - (1-x)f_N^2}{x\Lambda^2 f_{eff}}\right)^2 \tag{A.3.4}$$

These results make no assumptions about the relative magnitudes of the various F-terms and therefore hold if there are multiple SUSY breaking sectors and all but one mediate SUSY breaking to the SSM in the same way. If all mediation sectors are the same this corresponds to the limit $x \to 1$.

Appendix B

Constraints on single-sector flavor

B.1 Constraints from Flavor Changing Neutral Currents

As is often the case with theories of supersymmetry breaking, the sfermion mass matrix is generally not diagonal in the same basis as the fermion mass matrix. The GIM mechanism does not operate for such general squark masses, leading to potential flavor-changing neutral currents in conflict with experimental bounds. In order to make meaningful contact with experimental limits, we will parametrize the contributions to flavor changing neutral currents (FCNCs) following [74].

In the single-sector models under consideration, the Yukawa matrices $\lambda_u, \lambda_d, \lambda_e$ are generated at the scale M_{flavor} with textures (4.1.2) dictated by the scaling dimensions of different composite states (in the case of the first two generations) or elementary states (in the case of the third generation) of the UV theory. When supersymmetry is broken, the squarks and sleptons of the first two generations may acquire SUSY-breaking soft masses directly, while all three generations acquire universal SUSY-breaking soft masses from gauge mediation. Barring additional superpotential terms mixing the mesons of the magnetic theory, these soft masses are all diagonal in the same basis as the non-diagonal Yukawa textures (4.1.2).

To reach the physical mass eigenbasis, the fermion mass matrices $M^u = v_u\lambda_u$,

$M^d = v_d\lambda_d$, and $M^e = v_d\lambda_e$ may be diagonalized by bi-unitary transformations

$$V_L^u M^u V_R^{u\dagger} = D^u V_L^d M^d V_R^{d\dagger} = D^d V_L^e M^e V_R^{e\dagger} = D^e \tag{B.1.1}$$

where, e.g., $D^u = \text{diag}\{m_u, m_c, m_t\}$. Likewise, we may write the 6×6 squark mass matrices $\tilde{M}^{u2}, \tilde{M}^{d2}, \tilde{M}^{e2}$ as

$$\tilde{M}^{x2} = \begin{pmatrix} \tilde{M}^{x2}_{LL} & \tilde{M}^{x2}_{LR} \\ \tilde{M}^{x2}_{RL} & \tilde{M}^{x2}_{RR} \end{pmatrix} \tag{B.1.2}$$

where $x = u, d, e$ and, for example, $\tilde{M}^{u2}_{LL}$ is the soft mass matrix for the squarks u_L coming from the doublets Q, while u_R are those coming from the singlets $\bar{u}$. Both $\tilde{M}^{x2}_{LL}$ and $\tilde{M}^{x2}_{RR}$ are Hermitian and come directly from soft masses, while $\tilde{M}^{x2}_{LR}$ and $\tilde{M}^{x2}_{RL}$ come from the trilinear A-terms. We will henceforth concentrate on the case where A terms are vanishingly small at the SUSY-breaking scale (they will be regenerated by RG flow, but still suppressed by a loop factor), so that $\tilde{M}^{x2}_{LL}$ and $\tilde{M}^{x2}_{RR}$ are the quantities of interest. For simplicity, we will also assume that $\tilde{M}^{x2}_{LL}$ and $\tilde{M}^{x2}_{RR}$ are identical.

Although the sfermion mass matrices $\tilde{M}^{x2}_{LL}, \tilde{M}^{x2}_{RR}$ are generated without off-diagonal elements, the transformation to the fermion mass eigenbasis (B.1.2) also rotates the sfermions and produces mass mixings between different generations of order

$$(\delta\tilde{M}^{x2}_{MN})_{ij} = \left(V_M^x \tilde{M}^{x2}_{MN} V_N^{x\dagger}\right)_{ij} \tag{B.1.3}$$

where the M, N refer to L and R. In the case where the off-diagonal terms in $\tilde{M}^{q2}_{LL}$ and $\tilde{M}^{q2}_{RR}$ are smaller than the diagonal ones (as they are in the models of interest) and the $V^x_{L,R}$ are close to the identity, it is conventional to parameterize FCNC constraints via bounds on the dimensionless quantities

$$(\delta^x_{MN})_{ij} = \frac{\left(V_M^x \tilde{M}^{x2}_{MN} V_N^{x\dagger}\right)_{ij}}{\sqrt{\left(V_M^x \tilde{M}^{x2}_{MN} V_N^{x\dagger}\right)_{ii}\left(V_M^x \tilde{M}^{x2}_{MN} V_N^{x\dagger}\right)_{jj}}}. \tag{B.1.4}$$

The δ_{ij} thus measure the relative size of the off-diagonal components in the sfermion mass matrices in a basis where the fermion mass matrices are diagonal. They can be constrained from measurements of e.g. $K^0 - \bar{K}^0$ or $D^0 - \bar{D}^0$ mixing and the rare decays $\mu \to e\gamma$ and $b \to s\gamma$.

B.1.1 Constraints on single-sector models

Relatively careful constraints on the sparticle spectrum may be placed on single-sector theories such as those considered here, owing to the fact that the Yukawa textures and soft masses are both specified by the dynamics. This allows the degree of alignment between fermion and sfermion masses to be quantified, thereby ameliorating more conservative bounds on arbitrary mass matrices. Here we will place bounds on first- and second-generation sfermion masses for flavor models involving a Yukawa texture of the form (4.1.2). These constraints are germane to the single-sector models developed above, but also pertain to other flavor models with similar textures.

Constraints for FCNCs are by far the strongest on the down quark sector, owing to relatively tight limits on the $K_L - K_S$ mass difference. As such, we will focus here on bounds arising from the down sector, under the assumption that the sfermion masses in all three sectors will be approximately similar; bounds on the up quark and lepton sector provide considerably weaker constraints on the soft spectrum.

For simplicity, we consider a Yukawa texture of the form

$$\lambda_d \simeq \begin{pmatrix} \epsilon^4 & 2\epsilon^3 & \frac{1}{4}\epsilon^2 \\ 2\epsilon^3 & 3\epsilon^2 & \epsilon \\ \frac{1}{4}\epsilon^2 & \epsilon & \frac{1}{4} \end{pmatrix}, \tag{B.1.5}$$

where we have chosen the numerical coefficients to give us nonzero eigenvalues approximately reproducing the down-sector quark masses when $\epsilon \sim 0.1$, $\tan\beta \sim 14$, and $v = 246$ GeV. This gives us down, strange, and bottom masses 3 MeV, 152 MeV, and 5 GeV, which are close to reality and give realistic FCNC bounds. Naturalness dictates that the stop mass lie around 1-2 TeV, which sets the scale of gauge-mediated contributions to all three generations (realistically, the longevity of the metastable

SUSY breaking vacuum requires the stop mass at the high scale to be not much less than ~ 3 TeV). When this is the only source of SUSY breaking, (B.1.4) is always diagonal and SUSY FCNCs are negligible. However, in addition to the gauge-mediated contribution, the first and second generation squarks and sleptons may obtain additional soft masses directly from SUSY-breaking, leading to an inverse hierarchy. The size of additional contributions to the soft masses $m_{\tilde{f}_1}, m_{\tilde{f}_2}$ of the first two generations is then constrained by FCNCs.

The FCNC constraints are strongest for the parameter $(\delta^d)_{12}$, which parameterizes mixing of the first and second generation down-type squarks and is constrained by $K^0 - \bar{K}^0$ mixing; the bound is approximately $(\delta^d)_{12} \leq 2.5 \times 10^{-3} \frac{\sqrt{m_{\tilde{f}_1} m_{\tilde{f}_2}}}{500}$ for $m_{\tilde{g}}^2 \simeq 0.3 m_{\tilde{f}_1} m_{\tilde{f}_2}$ (and weakens with increasing gluino mass). The constraints on first- and second-generation mixing in the up quark sector from $D^0 - \bar{D}^0$ are weaker by roughly a factor of 2, while the constraints on the lepton sector from $\mu \to e\gamma$ are weaker still. We may also constrain the matrix elements δ^d_{13} from $B^0 - \bar{B}^0$ mixing and δ^d_{23} from the rare process $b \to s\gamma$, though again these constraints prove far weaker than those arising from $K^0 - \bar{K}^0$ mixing.

B.1.2 Constraints from $K^0 - \bar{K}^0$

In order to constrain the possible values of $m_{\tilde{f}_1}$ and $m_{\tilde{f}_2}$ via the parameters $(\delta^d_{LL})_{12}$ and $(\delta^d_{RR})_{12}$, we can compute their contribution to the $K_L - K_S$ mass difference Δm_K. This difference has been measured within excellent precision to be very nearly $\Delta m_K = (3.483 \pm 0.006) \times 10^{-12}$ MeV [10]. There are Standard Model contributions to this quantity that parametrically fall within the measured value, but depend on hadronic uncertainties to an extent that the full contribution is unknown. Thus we can take as our constraint the requirement that our contribution to Δm_K does not exceed (in magnitude) the measured value. We can extract the contribution to Δm_K from squark mixing from [74]. These contributions depend on the gluino mass $m_{\tilde{g}}$ and the squark masses $m_{\tilde{f}_1}, m_{\tilde{f}_2}$ via the mixings $(\delta^d_{MN})_{12}$ for $M, N = L, R$. We will assume in our case that the LR and RL contributions are negligible and that $\delta_{LL} \simeq \delta_{RR}$, which is fairly accurate even when the Yukawa matrices are not entirely symmetric. This leads to by far the strongest constraints on the sfermion mass spectrum, as shown in Fig. B.1.

B.1.3 Constraints from other processes: $B^0 - \bar{B}^0$, $D^0 - \bar{D}^0$, $b \to s\gamma$, and $\mu \to e\gamma$

The mixings $(\delta^d_{MN})_{13}$ may similarly be constrained by $B^0 - \bar{B}^0$ mixing from their contribution to $\Delta m_B = (3.337 \pm 0.033) \times 10^{-10}$ MeV [10]. The calculation is essentially identical to that of the previous case, with the replacements $m_K \to m_B$, $m_s \to m_b$, $f_K \to f_B$, and $m_{\tilde{f}_2} \to m_{\tilde{f}_3}$. The resulting constraint is much weaker than that from $K^0 - \bar{K}^0$.

We may constrain mixing between the second and third generations via the rare decay $b \to s\gamma$, using the gluino-mediated contribution in [74]. In this case, we require that our contribution not exceed the measured branching ratio $BR(b \to s\gamma) = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4}$ [28]. The branching ratio is a strong function of squark mass, and is satisfied readily for squark masses above 1 TeV.

Although we have focused here on the down sector, similar constraints on $(\delta^u)_{12}$ and $(\delta^e)_{12}$ arise from $D^0 - \bar{D}^0$ mixing and the rare decay $\mu \to e\gamma$, respectively. Assuming the soft masses for all three sectors are parametrically similar, these constraints are generally weaker than those considered above, so we do not show them explicitly.

B.1.4 Constraints from tachyonic stop mass

Finally, we can take into account the upper bound placed on squark masses by the desire for a positive stop mass at the weak scale. As noted in [18], overly large masses for the first and second-generation squarks can drive the stop mass negative via their two-loop contribution to the stop mass RG. We can place a conservative bound on the masses of first- and second-generation squarks by just considering the interplay between one-loop gaugino contributions and two-loop squark contributions to the stop soft mass. In particular, we will ignore the contribution from the top Yukawa, which can drive the stop mass more negative. We will also ignore the running of the first and second generation squark masses, which is (verifiably) negligible. In this simplified case, we can solve the RGE for the stop mass analytically to find [18]

$$\begin{aligned} m_{\tilde{t}}^2(\mu) \simeq m_{\tilde{t}}^2(\Lambda) + \sum_i \frac{2}{b_i}(M_i^2(\Lambda) - M_i^2(\mu))C_i^{\tilde{t}} \\ -32\tilde{m}_{1,2}^2 \sum_i \frac{1}{2b_i}\left(\frac{g_i(\Lambda)^2}{16\pi^2} - \frac{g_i(\mu')^2}{16\pi^2}\right) C_i^{\tilde{t}} \end{aligned} \tag{B.1.6}$$

where $\tilde{t}$ can refer to $\tilde{t}_L$ or $\tilde{t}_R$ with appropriate choice of Casimirs (the stronger bound is on $\tilde{t}_L$), $i = 1, 2, 3$, b_i and C_i are the usual GUT-normalized β function parameters and Casimirs respectively, $\tilde{m}_{1,2}^2$ are the mean squark masses, μ is the low scale (taken to be 1 TeV), μ' is the scale where the heavy squarks decouple (taken to be 10 TeV), and Λ is the scale where SUSY is broken and RG flow commences (taken to be 100 TeV). We also take $M_i = g_i^2 M_0$, where $M_0 \sim \mu_\phi$ is the unified gaugino mass.

We may use the running of the stop mass to place two potential constraints on the masses of first- and second-generation sfermions. A weak constraint is the requirement that the stop retain a positive mass-squared at the weak scale; a stronger constraint is that the stop mass remain large enough at the weak scale to lift the Higgs mass above LEP limits. Aspects of both constraints are shown in Fig. B.1.

B.2 Unification

As is often the case for theories involving additional multiplets charged under the Standard Model, it is natural to consider whether the perturbative unification of Standard Model gauge couplings may be preserved and low-scale Landau poles avoided. Indeed, many models of metastable SUSY breaking suffer from the ubiquitous intermediate-scale Landau pole for the Standard Model gauge group. However, here it may be marginally possible to achieve unification at the GUT scale $\sim 10^{16}$ GeV.

Here we briefly recall the standard analysis of how extra $SU(5)$ multiplets affect the running of the gauge coupling. The relevant formula, found in e.g. §2 of [79], is that

$$\delta \alpha_{\mathrm{GUT}}{}^{-1} = -\frac{N}{2\pi} \log\left(\frac{M_{\mathrm{GUT}}}{M}\right) \tag{B.2.1}$$

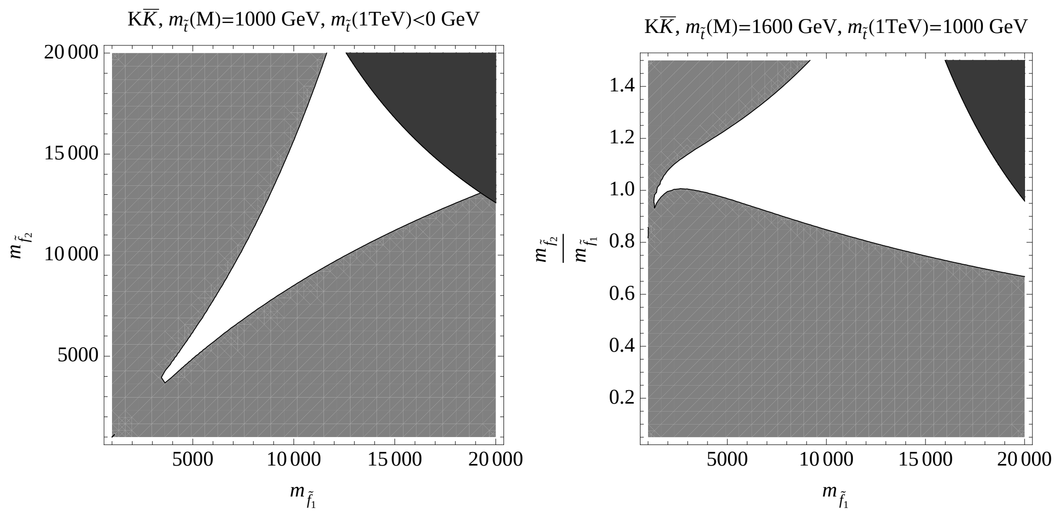

Figure B.1: Constraints on first and second generation sfermion masses. Light gray region ruled out by $K-\bar{K}$-mixing. (a) Dark gray region ruled out by tachyonic stops at the weak scale ($m_{\tilde{t}}(1\,\text{TeV}) < 0$). We assumed $m_{\tilde{g}} = 500$ GeV, $m_{\tilde{t}}(100\,\text{TeV}) = 1$ TeV. Note that the stop mass constraint disappears completely for the $m_{\tilde{f}_1}$ and $m_{\tilde{f}_2}$ mass range shown when $m_{\tilde{t}}(100\,\text{TeV}) \sim 3$ TeV, which is roughly the expected value in the models under consideration. (b) Dark gray region ruled out by stops being too light at the weak scale to give a Higgs mass above LEP limits ($m_{\tilde{t}}(1\,\text{TeV}) <$ 1000 GeV, assuming trilinear coupling is negligible). We assumed $m_{\tilde{g}} = 500$ GeV, $m_{\tilde{t}}(100\,\text{TeV}) = 1.6$ TeV.

where

$$N = \sum_{i=1}^{K} n_i \tag{B.2.2}$$

is the sum of the Dynkin indices n_i of the K extra $SU(5)$-charged matter multiplets. So each $\mathbf{5}$ or $\bar{\mathbf{5}}$ contributes 1 to the sum, each $\mathbf{10}$ contributes 3, each $\mathbf{15}$ contributes 7, and each $\mathbf{24}$ contributes 10.

The $4 \times (\mathbf{5} + \overline{\mathbf{5}})$ messengers we have at the ~ 100 TeV scale, in our "best" models, is a safe number to preserve perturbativity of α_{GUT}, in absence of additional SU(5) charges at higher scales below M_{GUT}. However, we have a large amount of additional matter at the scales Λ^2/Λ_0 and $\lambda\Lambda$. Even under the assumption that $\Lambda \sim M_{\text{GUT}}$ and we can ignore running due to the latter, the states at Λ^2/Λ_0 will contribute a total Dynkin index given by summing over the representations in brackets in (4.4.7), multiplied by 2 (to include the "spectators" they pair with). The total N just from (4.4.7) is 40, and makes it somewhat challenging to achieve unification before hitting a Landau pole, unless one pushes Λ_0 dangerously close to $M_{\rm GUT}$ or a larger Yukawa coupling is used.

It is important to remark that the non-spectator extra states are composites, which will in fact deconfine around the scale Λ. Such composites will clearly contribute differently to running at energies $> \Lambda$ (where we should use the electric description and count electric quark messengers), and it is conceivable that in some models this would vitiate the large threshold from encountering this plethora of states – this has played a crucial role in the ideas of [4,5]. However, in our concrete models even the electric "messenger index" would be quite large. In addition, the precise contribution in the energy regime around $\Lambda \sim M_{\rm GUT}$ does not seem easily calculable, and is naively quite significant. Thus, although in our construction we have suceeded in pushing the Landau pole to very high scales, comparable to $M_{\rm GUT}$, it would also be interesting to find models where this problem is completely solved – perhaps along the lines of [4,5].

B.3 Models with less extra matter

Generically, the class of models discussed in this paper exhibit a proliferation of charged matter coming from X and X_U. On one hand, this fact is an aesthetic nuisance since the corresponding masses, arising from cubic couplings in the electric theory, are naturally close to the high compositeness scale Λ. More importantly, as discussed in Appendix B, these states affect the RG running at very high energies, making perturbative unification challenging. In addition, the models contain a large number of messengers in the (ρ, Z_U) sector. These fields have masses $\sim$ 100 TeV, and thus affect the running of couplings more dramatically. In certain cases, like the one in §4.2.2 and the two composite generation example in [72], these states lead to Landau poles below the GUT scale.

Throughout the paper, we have adopted $SU(5)$ notation, mainly as a practical way of simplifying the group theory calculations, with the understanding that $SU(3)_C \times$

$SU(2)_L \times U(1)_Y$ quantum numbers could be easily re-introduced at any step. In the absence of Landau poles, a physical consequence of the entire field content (except the two light Higgs doublets) fitting into $SU(5)$ representations is unification. In this section we explore what happens if we build models dropping the $SU(5)$ condition. We will see that both the amount of extra matter in X and X_U and the number of messengers is substantially reduced. We illustrate our ideas with the adjoint model of §4.3. The minimal model corresponds to taking $N_c = 15$, $N_f = 8$ and embedding the $SU(3)_C \times SU(2)_L \times U(1)_Y$ into $SU(8)$ according to

$$\begin{array}{rcl} Q & \sim & [(\mathbf{3},\mathbf{1})_{x-1/3} + (\mathbf{1},\mathbf{2})_{x-1/2} + (\mathbf{1},\mathbf{1})_{x-1} + (\mathbf{1},\mathbf{1})_{x}] \quad + \quad (\mathbf{1},\mathbf{1})_0 \\ \tilde{Q} & \sim & [(\bar{\mathbf{3}},\mathbf{1})_{1/3-x} + (\mathbf{1},\mathbf{2})_{1/2-x} + (\mathbf{1},\mathbf{1})_{1-x} + (\mathbf{1},\mathbf{1})_{-x}] \quad + \quad (\mathbf{1},\mathbf{1})_0 \end{array} \tag{B.3.1}$$

There is a one parameter family of possible hypercharge assignments. X and X_U decompose as

$$\begin{array}{l} \big((\mathbf{3},\mathbf{2})_{1/6} + (\bar{\mathbf{3}},\mathbf{1})_{1/3} + (\mathbf{3},\mathbf{1})_{-2/3} + (\mathbf{1},\mathbf{2})_{-1/2} + (\mathbf{1},\mathbf{1})_1\big) \\ + \big[(\mathbf{8},\mathbf{1})_0 + (\bar{\mathbf{3}},\mathbf{2})_{-1/6} + (\mathbf{3},\mathbf{1})_{2/3} + (\mathbf{3},\mathbf{1})_{-1/3} + (\mathbf{1},\mathbf{3})_0 \\ \quad + 2\times(\mathbf{1},\mathbf{2})_{1/2} + (\mathbf{1},\mathbf{2})_{-1/2} + (\mathbf{1},\mathbf{1})_1 + 4\times(\mathbf{1},\mathbf{1})_0\big] \end{array} \tag{B.3.2}$$

namely, a full Standard Model generation plus additional matter, shown in square brackets. We see that the amount of extra matter in X and X_U has been reduced to less than a third of that in (4.4.7). x naturally drops out from (B.3.2), since it comes with opposite signs in the corresponding Q and $\tilde{Q}$ entries.

Let us now focus on the messengers coming from the (ρ, Z_U) sector. Their hypercharges do depend on the value of x. Interestingly, setting $x = 0$ we can form a $\mathbf{5}$ of $SU(5)$ by combining the $(\mathbf{3},\mathbf{1})_{1/3}$ from $\tilde{\rho}$ and the $(\mathbf{1},\mathbf{2})_{-1/2}$ from ρ (and similarly for $\bar{\mathbf{5}}$ and Z_U and $\tilde{Z}_U$). In this case, the messengers become

$$2\times[(\mathbf{5}+\bar{\mathbf{5}}) + ((\mathbf{1},\mathbf{1})_1 + (\mathbf{1},\mathbf{1})_{-1}) + 2\times(\mathbf{1},\mathbf{1})_0] \tag{B.3.3}$$

where we have used a hybrid $SU(5)$-Standard Model notation to emphasize that the entire low energy spectrum is in full $SU(5)$ representations modulo two $Y = \pm 1$ pairs. The number of charged messengers is also reduced, by approximately a 1/2 factor, with respect to the example in §4.3, pushing Landau poles to much higher energies than the ones discussed in Appendix B.2.

Appendix C

R-charges for Superconformal Flavor

Here we present the results of the a-maximization procedure applied to the various models considered earlier. The constraints relating various R charges arise from (a) the posited marginal interactions contained in the superpotential, and (b) the vanishing of the ABJ anomaly for the superconformal $U(1)_R$, which corresponds to the vanishing of β at the superconformal fixed point.

We subject the models under consideration to a fairly simple criterion: that they generate an adequate flavor hierarchy over the range $\Lambda_* < \mu < \Lambda$ smaller than the hierarchy $\Lambda_* < \mu < \Lambda_L$ between decoupling and the Landau pole for g_5. For all models we compute Λ_L/Λ_* assuming Standard Model field content, an additional $SU(5)_{SM}$ adjoint Higgs Σ, and the field content of the superconformal sector. For ten-centered models, we compare this to the ratio Λ/Λ_* required to get within a factor of 3 of the observed hierarchy in up-type quark masses. For democratic models, we compare this to both the ratio Λ_T/Λ_* required to get within a factor of 3 of the up-type quark hierarchy, and the ratio Λ_F/Λ_* required to get within a factor of 3 of the lepton mass hierarchy for $\tan\beta = 10$.

C.0.1 $SU(N)$ with $F = 10$ and A^3 superpotential

The R charges for this theory are constrained by the posited marginal operators and anomalies to obey

$$\begin{aligned} 2 &= R_{T_1} + R_{\overline{Q}_1} + R_{\overline{Q}_2} \\ 2 &= R_{T_2} + R_{\overline{Q}_1} + R_{\overline{Q}_2} + R_A \\ 2 &= 3R_A \\ 0 &= N + \frac{5}{2}(R_{Q_1} - 1) + \frac{5}{2}(R_{Q_2} - 1) + \frac{5}{2}(R_{\overline{Q}_1} - 1) + \frac{5}{2}(R_{\overline{Q}_2} - 1) + N(R_A - 1) \end{aligned} \tag{C.0.1}$$

In the window of interest several mesons go free: Q_1Q_2, $Q_1\overline{Q}_1$, $Q_2\overline{Q}_2$, and $\overline{Q}_1\overline{Q}_2$. All mesons involving A and all baryons are far from the unitarity bound. Of the free fields, only the $\overline{\mathbf{15}}$ of the $Q_2\overline{Q}_2$ needs to be accounted for, since the $\overline{\mathbf{10}}$ part is set to zero in the chiral ring. The a-maximization procedure gives us the following charges:

Table C.1: R charges for $SU(N)$ theory with $F = 10$ flavors and cubic adjoint superpotential

N	R_{T_1}	R_{T_2}	$R_{Q_{1,2}}$	$R_{\overline{Q}_{1,2}}$	R_A	$\mathfrak{b}$	$\frac{\Lambda_L}{\Lambda_*}$	$\frac{\Lambda}{\Lambda_*}$
11	1.448	0.781	0.257	0.276	0.667	33.884	$10^{1.78}$	-
12	1.572	0.905	0.186	0.214	0.667	34.528	$10^{1.75}$	-
13	1.705	1.039	0.119	0.147	0.667	35.925	$10^{1.68}$	$10^{1.65}$
14	1.849	1.182	0.058	0.076	0.667	38.080	$10^{1.58}$	$10^{1.19}$

C.0.2 $SU(N)$ with $F = 10$ and A^4 superpotential

This theory is a fairly trivial variation of the theory in §A.1; the R charges are constrained to obey

$$\begin{aligned} 2 &= R_{T_1} + R_{\overline{Q}_1} + R_{\overline{Q}_2} \\ 2 &= R_{T_2} + R_{\overline{Q}_1} + R_{\overline{Q}_2} + R_A \\ 2 &= 4R_A \\ 0 &= N + \frac{5}{2}(R_{Q_1} - 1) + \frac{5}{2}(R_{Q_2} - 1) + \frac{5}{2}(R_{\overline{Q}_1} - 1) + \frac{5}{2}(R_{\overline{Q}_2} - 1) + N(R_A - 1) \end{aligned} \tag{C.0.2}$$

In the window of interest several mesons go free: Q_1Q_2, $Q_1\overline{Q}_1$, $Q_2\overline{Q}_2$, and $\overline{Q}_1\overline{Q}_2$. For $N \geq 19$, we also add to the tally Q_1AQ_2, $Q_1A\overline{Q}_1$, $Q_2A\overline{Q}_2$, and $\overline{Q}_1A\overline{Q}_2$. At no point do any of the baryons go free. Of the free fields, only the $\overline{\mathbf{15}}$ of the $Q_2\overline{Q}_2$ and $Q_2A\overline{Q}_2$ need to be accounted for, since the $\overline{\mathbf{10}}$ parts are set to zero in the chiral ring. The a-maximization procedure gives us the following charges:

Table C.2: R charges for $SU(N)$ theory with $F = 10$ flavors and quartic adjoint superpotential

N	R_{T_1}	R_{T_2}	$R_{Q_{1,2}}$	$R_{\overline{Q}_{1,2}}$	R_A	$\mathfrak{b}$	$\frac{\Lambda_L}{\Lambda_*}$	$\frac{\Lambda}{\Lambda_*}$
14	1.383	0.883	0.292	0.308	0.5	47.826	$10^{1.26}$	-
15	1.482	0.982	0.241	0.259	0.5	50.080	$10^{1.20}$	$10^{1.95}$
16	1.585	1.085	0.192	0.208	0.5	52.920	$10^{1.13}$	$10^{1.49}$
17	1.690	1.190	0.145	0.155	0.5	56.345	$10^{1.07}$	$10^{1.33}$
18	1.797	1.297	0.099	0.102	0.5	60.363	$10^{1.00}$	$10^{1.21}$
19	1.900	1.400	0.050	0.050	0.5	61.300	$10^{0.98}$	$10^{1.11}$
20	2.000	1.500	0.000	0.000	0.5	61.000	$10^{0.99}$	$10^{1.02}$
21	2.100	1.600	-0.050	-0.050	0.5	61.300	$10^{0.98}$	$10^{0.95}$
22	2.200	1.700	-0.100	-0.100	0.5	62.200	$10^{0.97}$	$10^{0.89}$
23	2.300	1.800	-0.150	-0.150	0.5	63.700	$10^{0.95}$	$10^{0.84}$
24	2.400	1.900	-0.200	-0.200	0.5	65.800	$10^{0.92}$	$10^{0.79}$

C.0.3 $SU(N)$ with $F = 11$ and A^3 superpotential

In this case there are significantly more couplings in the superpotential. The marginal superpotential couplings and vanishing anomalies give us conditions

$$
\begin{aligned}
2 &= R_{T_1} + R_{\overline{Q}_1} + R_{\overline{Q}_2} \\
2 &= R_{T_2} + R_{\overline{Q}_1} + R_{\overline{Q}_2} + R_A \\
2 &= R_{F_1} + R_{Q_1} + R_{\overline{Q}_0} \\
2 &= R_{F_1} + R_{Q_2} + R_{Q_0} \\
2 &= R_{F_2} + R_{Q_1} + R_{\overline{Q}_0} + R_A \\
2 &= R_{F_2} + R_{Q_2} + R_{Q_0} + R_A \\
2 &= 3R_A \\
0 &= N + \frac{5}{2}(R_{Q_1} - 1) + \frac{5}{2}(R_{Q_2} - 1) + \frac{5}{2}(R_{\overline{Q}_1} - 1) + \frac{5}{2}(R_{\overline{Q}_2} - 1) \\
&\quad + \frac{1}{2}(R_{Q_0} - 1) + \frac{1}{2}(R_{\overline{Q}_0} - 1) + N(R_A - 1)
\end{aligned}
\tag{C.0.3}
$$

In the window of interest several mesons go free: Q_1Q_2, $Q_1\overline{Q}_1$, $Q_2\overline{Q}_2$, $\overline{Q}_1\overline{Q}_2$, $\overline{Q}_1 Q_0$, $\overline{Q}_2\overline{Q}_0$, and $Q_0\overline{Q}_0$. The $\overline{\mathbf{10}}$ component of $\overline{Q}_1\overline{Q}_2$ and the linear combination $Q_1\overline{Q}_0 + Q_2Q_0$ are set to zero in the chiral ring. The resulting R-charges are shown below:

Table C.3: R charges for $SU(N)$ theory with $F = 11$ flavors and cubic superpotential

N	R_{T_1}	R_{T_2}	R_{F_1}	R_{F_2}	$R_{Q_{1,2}}$	$R_{\overline{Q}_{1,2}}$	$R_{Q_0/\bar{Q}_0}$	R_A	$\mathfrak{b}$	$\frac{\Lambda_L}{\Lambda_*}$	$\frac{\Lambda_T}{\Lambda_*}$	$\frac{\Lambda_F}{\Lambda_*}$
11	1.333	0.833	1.333	0.667	0.333	0.333	0.333	0.667	36.665	$10^{1.65}$	-	-
12	1.452	0.952	1.433	0.766	0.266	0.274	0.301	0.667	36.098	$10^{1.67}$	$10^{2.14}$	-
13	1.578	1.078	1.529	0.862	0.202	0.211	0.269	0.667	36.674	$10^{1.64}$	$10^{1.49}$	$10^{1.78}$
14	1.711	1.211	1.623	0.956	0.142	0.145	0.236	0.667	37.976	$10^{1.59}$	$10^{1.31}$	$10^{1.20}$
15	1.851	1.351	1.715	1.048	0.085	0.075	0.200	0.667	40.002	$10^{1.51}$	$10^{1.15}$	$10^{0.91}$
16	1.995	1.495	1.806	1.140	0.032	0.002	0.162	0.667	42.758	$10^{1.41}$	$10^{1.02}$	$10^{0.74}$

C.0.4 $SU(N)$ with $F = 10$, A^3 superpotential, and marginal coupling to Σ

Very little changes from the simple case of §A.1 if we add couplings to the adjoint Higgs Σ of $SU(5)_{SM}$; only the contribution to $\mathfrak{b}$ is modified. The R-charges for additional coupling to the adjoint Higgs of $SU(5)$ are given below:

Table C.4: R charges for $SU(N)$ theory with $F = 10$ flavors, cubic superpotential, and coupling to $SU(5)$ adjoint

N	R_{T_1}	R_{T_2}	$R_{Q_{1,2}}$	$R_{\overline{Q}_{1,2}}$	R_A	R_Σ	$\mathfrak{b}$	$\frac{\Lambda_L}{\Lambda_*}$	$\frac{\Lambda}{\Lambda_*}$
11	1.448	0.781	0.257	0.276	0.667	1.467	23.884	$10^{2.52}$	-
12	1.572	0.905	0.186	0.214	0.667	1.600	24.528	$10^{2.46}$	-
13	1.705	1.039	0.119	0.147	0.667	1.733	25.925	$10^{2.32}$	$10^{1.649}$
14	1.849	1.182	0.058	0.076	0.667	1.867	28.080	$10^{2.15}$	$10^{1.190}$

C.0.5 $Sp(2N)$ with $2F = 10$ and A^3 superpotential

In this case the constraints from superpotential couplings and anomalies are

$$\begin{aligned} 2 &= R_{T_1} + 2R_{\bar{Q}} \\ 2 &= R_{T_2} + 2R_{\bar{Q}} + R_A \\ 0 &= 2(N+1) + 5(R_Q - 1) + 5(R_{\bar{Q}} - 1) + 2(N-1)(R_A - 1) \\ 2 &= 3R_A \end{aligned} \tag{C.0.4}$$

Now the operators that go free are the mesons QQ and $Q\overline{Q}$; the meson $\overline{QQ}$ is set entirely to zero in the chiral ring, and there are no baryons. The resulting R-charges are given below:

Table C.5: R charges for $Sp(2N)$ theory with $2F = 10$ flavors and cubic antisymmetric superpotential

N	R_{T_1}	R_{T_2}	R_Q	$R_{\overline{Q}}$	R_A	$\mathfrak{b}$	$\frac{\Lambda_L}{\Lambda_*}$	$\frac{\Lambda}{\Lambda_*}$
4	1.497	0.830	0.149	0.251	0.667	6.06	$10^{9.94}$	-
5	1.786	1.119	0.026	0.107	0.667	7.16	$10^{8.41}$	$10^{1.36}$

C.0.6 $Sp(2N)$ with $2F = 10$ and A^4 superpotential

The constraints in this case are a simple generalization of the previous case,

$$\begin{aligned} 2 &= R_{T_1} + 2R_{\bar{Q}} \\ 2 &= R_{T_2} + 2R_{\bar{Q}} + R_A \\ 0 &= 2(N+1) + 5(R_Q - 1) + 5(R_{\bar{Q}} - 1) + 2(N-1)(R_A - 1) \\ 2 &= 4R_A \end{aligned} \tag{C.0.5}$$

The operators that can go free are now QQ, $Q\overline{Q}$, and for sufficiently high N both QAQ and $QA\overline{Q}$. The resulting R-charges are shown below.

Table C.6: R charges for $Sp(2N)$ theory with $2F = 10$ flavors and quartic antisymmetric superpotential

N	R_{T_1}	R_{T_2}	R_Q	$R_{\overline{Q}}$	R_A	$\mathfrak{b}$	$\frac{\Lambda_L}{\Lambda_*}$	$\frac{\Lambda}{\Lambda_*}$
4	1.331	0.831	0.266	0.334	0.500	8.46	$10^{7.12}$	-
5	1.531	1.031	0.166	0.234	0.500	9.96	$10^{6.05}$	$10^{1.68}$
6	1.787	1.287	0.093	0.107	0.500	12.41	$10^{4.86}$	$10^{1.22}$
7	2.000	1.500	0.000	0.000	0.500	13.00	$10^{4.64}$	$10^{1.02}$
8	2.200	1.700	-0.100	-0.100	0.500	14.20	$10^{4.24}$	$10^{0.89}$

C.0.7 $Sp(2N)$ with $2F = 12$ and A^3 superpotential

The constraints from marginal superpotential terms and anomaly cancellation are

$$\begin{aligned}
2 &= R_{T_1} + 2R_{\bar{Q}} \qquad \text{(C.0.6)} \\
2 &= R_{T_2} + 2R_{\bar{Q}} + R_A \\
2 &= R_{F_1} + R_Q + R_{Q_0} \\
2 &= R_{F_1} + R_Q + R_{\bar{Q}_0} \\
2 &= R_{F_2} + R_Q + R_A + R_{Q_0} \\
2 &= R_{F_2} + R_Q + R_A + R_{\bar{Q}_0} \\
2 &= 3R_A \\
0 &= 2(N+1) + 5(R_Q - 1) + 5(R_{\bar{Q}} - 1) + (R_{Q_0} - 1) + (R_{\bar{Q}_0} - 1) + 2(N-1)(R_A - 1)
\end{aligned}$$

The gauge-invariant chiral operators set to zero in the chiral ring are $\overline{QQ}, \overline{Q}A\overline{Q}$, and the linear combinations $QQ_0 + Q\overline{Q}_0, QAQ_0 + QA\overline{Q}_0$. Of the remaining chiral operators, $QQ, Q\overline{Q}, \overline{Q}Q_0, \overline{QQ}_0$, and $Q_0\overline{Q}_0$ saturate the unitarity bound and must be accounted for accordingly in the a-maximization procedure. The resulting R-charges are shown below.

Table C.7: R charges for $Sp(2N)$ theory with $2F = 12$ flavors and cubic superpotential

N	R_{T_1}	R_{T_2}	R_{F_1}	R_{F_2}	R_Q	$R_{\overline{Q}}$	$R_{Q_0/\bar{Q}_0}$	R_A	$\mathfrak{b}$	$\frac{\Lambda_L}{\Lambda_*}$	$\frac{\Lambda_T}{\Lambda_*}$	$\frac{\Lambda_F}{\Lambda_*}$
4	1.333	0.667	1.333	0.667	0.333	0.333	0.333	0.667	9.000	$10^{6.70}$	-	-
5	1.549	0.882	1.523	0.856	0.195	0.225	0.282	0.667	8.483	$10^{7.10}$	-	-
6	1.847	1.181	1.674	1.007	0.100	0.076	0.227	0.667	9.120	$10^{6.61}$	$10^{1.19}$	$10^{1.02}$

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