

Nuclear Structure for Reactions and Decays

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Abstract. Shell Model techniques are powerful tools to describe the nuclear structure effects observed in reactions and decays. This contribution describes several relevant shell model applications, including double beta decays, compound nuclear reactions, and direct nuclear reactions. In particular it discusses the role of spectroscopic factor extracted from transfer reactions in revealing proton-neutron correlations effects in nuclei. It also identifies the elements of the effective shell model Hamiltonian that are responsible for these strong correlations.

1. Introduction

Reactions and decays are generally viewed as processes with many particles in continuum. For structureless particles, the scattering theory is a very reliable tool. Many decays and reaction of structureless particles can be describe using the penetrability factor know since the early days of quantum mechanics and alpha decay theories. However, most of the nuclei have complicated structure, and the nuclear structure information is essential to realistically describe their reactions and decays. Shell Model (ShM) techniques are powerful tools to describe the nuclear structure effects observed in reactions and decays.

In some reactions, such as compound nuclear reactions that goes through excitation energy region involving a high density of states, mere knowledge of the density of states for given spin and parity would suffice. Indeed, those cases, a statistical theory due to Hauser and Feshbach [1] turned out to be very useful. In recent years, we proposed and advanced approach to the ShM spin- and parity-dependent nuclear level densities (NLD) [2, 3, 4, 5, 6, 7], and we started to use this approach to describe reaction cross-sections and reaction rates for nuclear astrophysics [8]. Due to the limited space, this approach will not be further developed here, but we direct the interested reader to the our papers listed in this paragraph.

Direct reactions, such as break-up and transfer reactions, are also of large interest in nuclear structure studies, nuclear astrophysics, nuclear medicine, and nuclear energy applications. The structureless cross section are usually calculated using a distorted wave Born approximation. The comparison with the experimental data requires a multiplicative spectroscopic factor that contains information about the nuclear structure of the transferred particle relative to the core. This information can be further used to tune the nuclear structure models, such as the ShM Hamiltonian. In this contribution we describe the spectroscopic factors of the N=28 isotopes, and we show that they contain essential information about the proton-neutron (pn) correlations in nuclei and about the pn part of the ShM Hamiltonian.

Neutrinoless double beta ($0\nu\beta\beta$) decay, which can only occur by violating the conservation of the total lepton number, if observed it will unravel physics beyond Standard Model (SM), and it will represent a major milestone in the study of the fundamental properties of neutrinos

[10]-[16]. Indeed, its discovery would decide if neutrinos are their own antiparticles [17], and would provide a hint on the scale of their absolute masses. That is why there are intensive investigations of this process, both theoretical and experimental. Recent results from neutrino oscillation experiments have demonstrated that neutrinos have mass and they can mix [18]-[20]. However, the neutrino oscillations experiments cannot be used to determine the lowest neutrino mass and the neutrino mass hierarchy. Neutrinoless double beta decay is viewed as one of the best routes to decide these unknowns. A key ingredient for extracting the absolute neutrino masses from $0\nu\beta\beta$ decay experiments is a precise knowledge of the nuclear matrix elements (NME) for this process.

There are potentially many mechanisms that could contribute to the neutrinoless double beta decay process, which will be briefly reviewed below. Several of these mechanisms do not provide contributions to the decay rate that explicitly depend on the neutrino masses, but their contributions will vanish if the neutrinos are not massive Majorana particles [17]. In this review we will concentrate to those mechanisms for which the decay rate explicitly depends on the neutron masses. In all cases the half-lives depend on the nuclear matrix elements that need to be accurately calculated using low-energy nuclear structure models. The two neutrino double beta ($2\nu\beta\beta$) decay is an associate process that is allowed by the Standard Model, and it was observed in about ten isotopes. Therefore, a good but not sufficient test of nuclear structure models would be a reliable description of the $2\nu\beta\beta$ half-lives.

Since most of the $\beta\beta$ decay emitters are open shell nuclei, many calculations of the NME have been performed within the pnQRPA approach and its extensions [21]-[32]. However, the pnQRPA calculations of the more common two-neutrino double beta decay half-lives, which were measured for about 10 cases [33], are very sensitive to the variation of the so called g_{pp} parameter (the strength of the particle-particle interactions in the 1^+ channel) [21]-[23], and this drawback still persists in spite of various improvements brought by its extensions [24]-[29], including higher-order QRPA approaches [30]-[32]. The outcome of these attempts was that the calculations became more stable against g_{pp} variation, but at present there are still large differences between the values of the NME calculated with different QRPA-based methods, which do not yet provide a reliable determination of the two-neutrino double beta decay half-life. Therefore, although the QRPA methods do not seem to be suited to predict the $2\nu\beta\beta$ decay half-lives, one can use the measured $2\nu\beta\beta$ decay half-lives to calibrate the g_{pp} parameters that are further used to calculate the $0\nu\beta\beta$ decay NME [34]. Another method that recently provided NME for most $0\nu\beta\beta$ decay cases of interest is the Interactive Boson Model (IBA-2) [35, 36].

On the other hand, the progress in computer power, numerical algorithms, and improved nucleon-nucleon effective interactions, made possible large scale shell model calculations of the $2\nu\beta\beta$ and $0\nu\beta\beta$ decay NME [37]-[39]. The main advantage of the large scale shell model (LSSM) calculations is that they seem to be less dependent on the effective interaction used, as far as these interactions are consistent with the general spectroscopy of the nuclei involved in the decay. Their main drawback is the limitation imposed by the exploding shell model dimensions on the size of the valence spaces that can be used. The most important success of the large scale shell model calculations was the correct prediction of the $2\nu\beta\beta$ decay half-life for ^{48}Ca [37, 40]. In addition, these calculations did not have to adjust any additional parameters, i.e. given the effective interaction and the Gamow-Teller (GT) quenching factor extracted from the overall spectroscopy in the mass-region (including beta decay probabilities and charge-exchange form factors), one can reliably predict the $2\nu\beta\beta$ decay half-life of ^{48}Ca .

Clearly, there is a need to further check and refine these calculations, and to provide more details on the analysis of the NME that could be validated by experiments. We have recently revisited [42] the $2\nu\beta\beta$ decay of ^{48}Ca using two recently proposed effective interactions for this mass region, GXPF1 and GXPF1A, and we explicitly analyzed the dependence of the double Gamow-Teller sum entering the NME on the excitation energy of the 1^+ states in

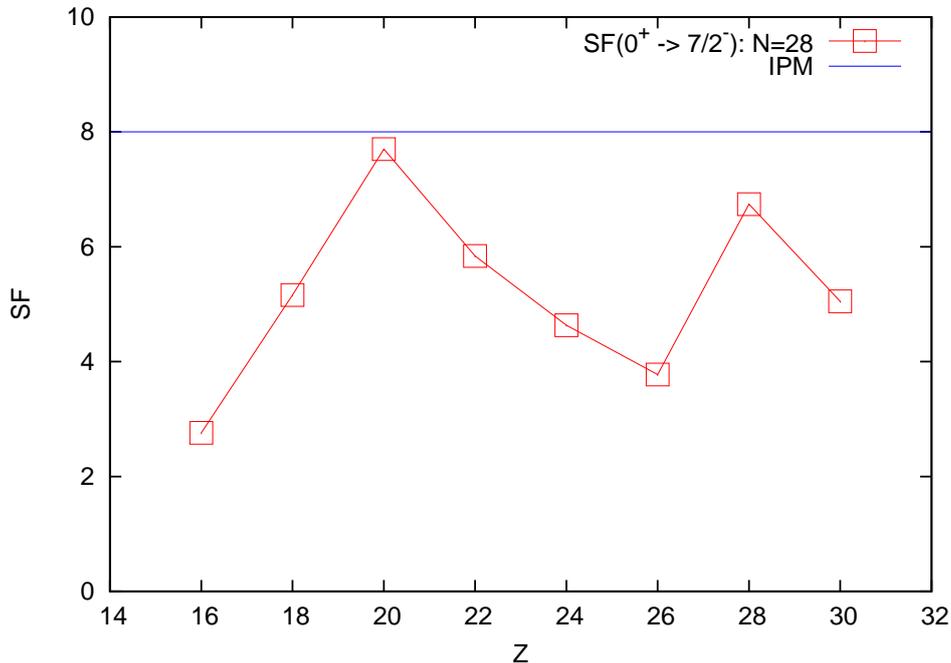


Figure 1. (Color online) The ShM prediction for the SF of the N=28 isotones.

the intermediate nucleus ^{48}Sc . This sum was recently investigated experimentally [43], and it was shown that the incoherent summation of the absolute values of the Gamow-Teller matrix elements would provide an incorrect NME, thus validating our prediction. We have also corrected by several orders of magnitude the probability of transition of the g.s. of ^{48}Ca to the first excited 2^+ state of ^{48}Ti . Future experiments on double beta decay of ^{48}Ca (CANDLES [44] and CARVEL [45]) may reach the required sensitivity of measuring such transitions, and our results could be useful for planning these experiments.

2. Signal of pn correlations in the spectroscopic factor of the N=28 isotones

Direct nuclear reactions, such as break-up and transfer reactions, are also largely dependent on the nuclear structure information, such as the spectroscopic factors. In reverse, information from direct nuclear reaction can be used to better understand the nuclear structure of exotic nuclei, such as the pn correlations and their effect on changes in the single particle energies and magic numbers. As an example we show here the case of the N=28 isotones.

The precise definition of the spectroscopic factor (SF) and its relation to DWBA cross-sections can be found in many references, e.g. [47, 48]. The SF is not a genuine observable, and it is useful to gauge it to some simple model, such as the independent particle model (IPM), which assumes that one removes/add nucleons to a spherical single particle shell, say $f_{7/2}$, and only pairing interaction act to remove degeneracy. If an even number n of nucleons are filling the shell, the SF for removing one nucleon is n . For example, if one removes one neutron from any of the N=28 isotones, for which the $f_{7/2}$ shell is fully filled with 8 neutrons, then the IPM SF is 8. Indeed, experimental results and full ShM calculation in pf shell for Ca isotopes shows that the IMP predictions are observed [47]. However, if one includes or subtracts proton pairs, full ShM calculation in pf and $sd - pf$ model spaces indicate large deviations from the simple IPM predictions as shown in Fig. 1. In particular, one can see a significant drop in the SF for ^{54}Fe and ^{46}Ar , while the SF prediction for ^{56}Ni is almost back to the IPM value. Clearly, this

behavior is related to correlations between the proton pairs filling the $d_{3/2}$ and $f_{7/2}$ shell and the N=28 neutron system. In fact, the ^{54}Fe SF was measured long time ago [51], and the ^{46}Ar and ^{56}Ni SFs were recently measured, e.g. Refs. [49] and [50], respectively. The experimental data clearly backs-up the theoretical predictions.

We further investigate the pn mechanisms responsible for the behavior of the SF in Fig. 1. We found out that the mechanism responsible for the 40pn correlations between the protons filling up the $f_{7/2}$ shell and the neutrons in the $f_{7/2}$ shell. Indeed, reducing to zero the $f_{7/2}$ pn matrix elements coupled to $J=0, 1$, and 7 , one gets the SF of ^{54}Fe close to its IPM value. These changes clearly break the isospin symmetry. Therefore, one could also say that isospin symmetry is responsible for these strong pn correlations effects [51]. What is new here is that we identified the specific pn pieces of the effective Hamiltonian responsible for this dramatic effect: a 50% reduction of the cross section for the neutron break-up of ^{54}Fe .

The mechanism behind the pn effects in the SF of ^{46}Ar is different. Here, there are clear correlations between the proton pairs filling the $d_{3/2}$ and the neutrons in the $f_{7/2}$ shell. We found out that by reducing by 0.6 MeV the pn part of the $T=0$ monopole matrix elements between the $d_{3/2}$ and $f_{7/2}$ shells one pushes down the position of the $d_{3/2}$ level, while keeping the pf single particle energies unchanged. Therefore, this change is reducing the interaction of the $d_{3/2}$ proton pairs with the neutrons in the $f_{7/2}$ shell. As a consequence, the ^{46}Ar SF is going up towards the IPM value. Details will be presented elsewhere [52].

3. Two-neutrino double beta decay

LSSM calculations of $2\nu\beta\beta$ decay NME can now be carried out rather accurately for many nuclei [55]. In the case of ^{48}Ca , Ref. [37] reported for the first time a full pf -shell calculation of the NME for the $2\nu\beta\beta$ decay mode, for both transitions to the g.s. and to the 2_1^+ excited state of ^{48}Ti . As an effective interaction it was used the Kuo-Brown G-matrix [56] with minimal monopole modifications, KB3 [57]. In Ref. [42] we use the recently proposed GXPF1A two-body effective interaction, which has been successfully tested for the pf shell [58]-[60], to perform $2\nu\beta\beta$ decay calculations for ^{48}Ca . Our goal was to obtain the values of the NME for this decay mode, both for transitions to the g.s. and to the 2_1^+ state of ^{48}Ti , with increased degree of confidence, which will allow us in the next future to address similar calculations for the $0\nu\beta\beta$ decay mode of this nucleus [39]. The $2\nu\beta\beta$ transitions to excited states have longer half-lives, as compared with the transitions to the g.s., due to the reduced values of the corresponding phase space factors, but they are measurable in some cases, such as ^{100}Mo [61].

For the $2\nu\beta\beta$ decay mode the relevant NME are of Gamow-Teller type, which has the following expressions for the decay to states in the grand-daughter that has the angular momentum $J = 0, 2$ [10]-[15],

$$M_{GT}^{2\nu}(J^+) = \frac{1}{\sqrt{J+1}} \sum_k \frac{\langle J_f^+ || \sigma\tau^- || 1_k^+ \rangle \langle 1_k^+ || \sigma\tau^- || 0_i^+ \rangle}{(E_k + E_J)^{J+1}} \quad (1)$$

Here E_k is the excitation energy of the 1_k^+ state of intermediate odd-odd nucleus, and $E_J = \frac{1}{2}Q_{\beta\beta}(J^+) + \Delta M$. $Q_{\beta\beta}(J^+)$ is the Q-value corresponding to the $\beta\beta$ decay to the final J_f^+ state of the grand-daughter nucleus, and ΔM is the mass difference between the parent and the grand-daughter. The most common case is to look to decay to the 0^+ g.s. of the grand-daughter, but decays to the first excited 0^+ and 2^+ states are also investigated.

The $2\nu\beta\beta$ decay half-life expression is given by

$$\left[T_{1/2}^{2\nu, J} \right]^{-1} = G_J^{2\nu} |M_{GT}^{2\nu}(J)|^2 \quad (2)$$

where $G_J^{2\nu}$ are $2\nu\beta\beta$ phase space factors. Specific values of $G_J^{2\nu}$ for different $2\nu\beta\beta$ decay cases can

be found in different reviews, such as Ref. [11]. For a recent analysis of $G_J^{2\nu}$ see Ref. [36]. In Ref. [42] we explicitly analyzed the dependence of the double-Gamow-Teller sum entering the NME Eq. (1) on the excitation energy of the 1^+ states in the intermediate nucleus ^{48}Sc . This sum was recently investigated experimentally [43], and it was shown that indeed, the incoherent sum (using only absolute values of the Gamow-Teller matrix elements) would provide an incorrect NME, validating our prediction. We have also corrected by several orders of magnitude the probability of transition of the g.s. of ^{48}Ca to the first excited 2^+ state of ^{48}Ti .

In Ref. [42] we fully diagonalized 250 1^+ states in the intermediate nucleus to calculate the $2\nu\beta\beta$ decay NME for ^{48}Ca . This procedure can be used for somewhat heavier nuclei using the J-scheme shell model code NuShellX [41], but for large dimension cases one needs an alternative method. The pioneering work on ^{48}Ca [37] used a strength-function approach that converges after a small number of Lanczos iterations, but it requires large scale shell model diagonalizations when one wants to check the convergence. Ref. [46] proposed an alternative method, which converges very quickly, but it did not provide full recipes for all its ingredients, and it was never used in practical calculations. Recently [62], we proposed a simple numerical scheme to calculate all coefficients of the expansion proposed in Ref. [46]. Following Ref. [46], we choose as a starting Lanczos vector, L_1^\pm , either the initial or final state in the decay (only 0^+ to 0^+ transitions are considered), on which we apply the Gamow-Teller operator. This approach is very efficient for large model spaces, as for example the $jj55$ space (consisting of the $0g_{7/2}$, $1d$, $2s$, and $h_{11/2}$ orbits), which for ^{128}Te leads to m-scheme dimensions of the order of 10 billions. In the calculation of ^{48}Ca decay we use the standard quenching factor, 0.77, for the Gamow-Teller operator $\sigma\tau$. We checked the result reported in Ref. [42] using this alternative method and we found the same result. The novel result report here for the first time is for the transition to the first excited 0^+ state in ^{48}Ti at 2.997 MeV. The matrix element we found is 0.05, very close to that for the transition to the g.s. Using the phase space factor from Ref. [11] we found the the half-life for this transition is 1.6×10^{24} y. We recall here that our results reported in [42] for the half-lives of the transitions to g.s. and to the first 2^+ excited state are 3.3×10^{19} y and 8.5×10^{23} y, respectively.

We also calculated using the same techniques the $2\nu\beta\beta$ decay NME for ^{76}Ge and ^{82}Se in $jj44$ valence space ($0f_{5/2}$, $1p$, $0g_{9/2}$) using the JJ44PN effective interaction briefly described in Ref. [53], and with JUN45 interaction [63]. Our results are about 25-40% larger than the corresponding NMEs extracted from experimental data [33]. The experimental NMEs could be reproduced if a small quenching factor, 0.6, is used. Such a reduction of the GT quenching factor would require some deeper understanding. An alternative explanation could be related to the missing spin orbit partners of the $0f_{5/2}$ and $0g_{9/2}$ orbits, which lead to a significant violation ($\sim 50\%$) of the Ikeda sum-rule. We could show that by adding the $0f_{7/2}$ and $0g_{7/2}$ orbits to the valence space for the intermediate nucleus one can fully recover the sum-rule. In addition, preliminary investigations in this larger valence space indicate a decrease of the NME towards the experimental value. We also found a similar behavior for the $2\nu\beta\beta$ matrix elements of ^{136}Xe , ^{130}Te , and ^{128}Te using the MC interaction of Ref. [64] in the $jj55$ model space. An even more drastic decrease of the effective quenching factors for these nuclei are reported by Caurier et al. (see e.g. Table 2 of Ref. [65]). The half-life of ^{136}Xe was recently measured by different groups [66, 67, 68]. Using the the value of 2.23×10^{21} y [67] and the newer phase space factors [36] one gets 0.022 MeV^{-1} for $M_{GT}^{2\nu}(0_{gs}^+)$. Typical extra-quenching, as for the ^{76}Ge and ^{82}Se cases, is necessary to describe this small matrix element. The Ikeda sum-rule can be satisfied adding the $0g_{9/2}$ and $0h_{9/2}$ orbits to the $jj55$ model space for the intermediate nucleus ^{136}Cs . However, the issues related to the choice of the effective interaction and the removal of the center-of-mass spurious contributions have to be resolved.

4. Neutrinoless double beta decay

The $0\nu\beta\beta$ decay $(Z, A) \rightarrow (Z + 2, A) + 2e^-$ requires the neutrino to be a massive Majorana fermion, i.e. it is identical to the antineutrino [17]. We already know from the neutrino oscillation experiments that in the weak interaction some of the neutrinos have mass and the mass eigenstates are mixed by the PNMS matrix U_{lk} , where l is the lepton flavor and k is the mass eigenstate number (see e.g. [69]). However, the neutrino oscillations experiments cannot decide the mass hierarchy, the mass of the lightest neutrino, and some of the CP non-conserving phases of the PNMS matrix (assuming that neutrinos are Majorana particles).

Considering only contributions from the exchange of light, left-handed(chirality), Majorana neutrinos [16], the $0\nu\beta\beta$ decay half-live is given by

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G^{0\nu} |M^{0\nu}|^2 \left(\frac{|\langle m_{\beta\beta}\rangle|}{m_e}\right)^2, \quad (3)$$

Here, $G^{0\nu}$ is the phase space factor, which depends on the $0\nu\beta\beta$ decay energy, $Q_{\beta\beta}$ and the nuclear radius [11, 70]. The effective neutrino mass, $\langle m_{\beta\beta}\rangle$, is related to the neutrino mass eigenstates, m_k , via the lepton mixing matrix, U_{ek} ,

$$\langle m_{\beta\beta}\rangle / m_e \equiv \eta_{\nu L} = \sum_k m_k U_{ek}^2 / m_e. \quad (4)$$

m_e is the electron mass. The NME, $M^{0\nu}$, is given by

$$M^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} - M_T^{0\nu}, \quad (5)$$

where $M_{GT}^{0\nu}$, $M_F^{0\nu}$ and $M_T^{0\nu}$ are the Gamow-Teller (GT), Fermi (F) and tensor (T) matrix elements, respectively. Using closure approximation these matrix elements are defined as follows:

$$\begin{aligned} M_\alpha^{0\nu} &= \left\langle 0_f^+ \left| \sum_{m,n} \tau_{-m} \tau_{-n} O_{mn}^\alpha \right| 0_i^+ \right\rangle \\ &= \sum_{j_p j_{p'} j_n j_{n'} J_\pi} TBTD(j_p j_{p'}, j_n j_{n'}; J^\pi) \langle j_p j_{p'}; J^\pi T | \tau_{-1} \tau_{-2} O_{12}^\alpha | j_n j_{n'}; J^\pi T \rangle_a, \end{aligned} \quad (6)$$

where O_{mn}^α are $0\nu\beta\beta$ transition operators, $\alpha = (GT, F, T)$, $|0_i^+\rangle$ is the g.s. of the parent nucleus, and $|0_f^+\rangle$ is the final 0^+ state of the grand daughter nucleus. The two-body transition densities (TBTD) can be obtained from ShM calculations [54]. Expressions for the anti-symmetrized two-body matrix elements (TBME) $\langle j_p j_{p'}; J^\pi T | \tau_{-1} \tau_{-2} O_{12}^\alpha | j_n j_{n'}; J^\pi T \rangle_a$ can be found elsewhere, e.g. Refs. [71, 54]. One should also notice that due to rotational invariance of the TBME, transitions to final 2^+ of grand daughter are forbidden in closure approximation and strongly suppressed in general. Assuming that one unambiguously measures a $0\nu\beta\beta$ half-life, and one can reliably calculate the NME for that nucleus, one could use Eqs. (3) and (4) to extract information about the lightest neutrino mass and the neutrino mass hierarchy [69]. In addition, one could consider the contribution from the right handed currents to the effective Hamiltonian, which can mix light and heavy neutrons of both chiralities (L/R)

$$\begin{aligned} \nu_{eL} &= \sum_{k=light} U_{ek} \nu_{kL} + \sum_{k=heavy} U_{ek} N_{kL} \\ \nu_{eR} &= \sum_{k=light} V_{ek} \nu_{kR} + \sum_{k=heavy} V_{ek} N_{kR}, \end{aligned} \quad (7)$$

where N_k are the heavy neutrinos that are predicted by several see-saw mechanisms for neutrino masses [69]. U_{lk} and V_{lk} are the left and right-handed components of the unitary matrix that diagonalizes the neutrino mass matrix. One should also mention that there are several other mechanisms that could contribute to the $0\nu\beta\beta$ decay, such as the exchange of supersymmetric (SUSY) particles (e.g. gluino and squark exchange [76]), etc, whose contributions are not directly related to the neutrino masses, but indirectly via the Schechter-Valle theorem [17]. Assuming that the masses of the light neutrinos are smaller than 1 MeV and the masses of the heavy neutrinos, M_k , are larger than 1 GeV, the particle physics and nuclear structure parts get separated, and the inverse half-life can be written as

$$\begin{aligned} \left(T_{1/2}^{0\nu}\right)^{-1} &= G^{0\nu} \left| \eta_{\nu L} M^{0\nu} + \langle \lambda \rangle \tilde{X}_\lambda + \langle \eta \rangle \tilde{X}_\eta + (\eta_{NL} + \eta_{NR}) M^{0N} \right. \\ &\quad \left. + \eta_{\lambda'} M_{\lambda'}^{0\nu} + \eta_{\tilde{q}} M_{\tilde{q}}^{0\nu} + \eta_{KK} M_{KK}^{0\nu} \right|^2, \end{aligned} \quad (8)$$

where $\eta_{\nu L}$ was defined in Eq. (4), and

$$\begin{aligned} \eta_{NL} &= \sum_{k=heavy} U_{ek}^2 \frac{m_p}{M_k}, \quad \eta_{NR} \approx \left(\frac{M_{WL}}{M_{WR}}\right)^4 \sum_{k=heavy} V_{ek}^2 \frac{m_p}{M_k} \\ \langle \lambda \rangle &= \epsilon \sum_{k=light} U_{ek} V_{ek}, \quad \langle \eta \rangle = \left(\frac{M_{WL}}{M_{WR}}\right)^2 \sum_{k=light} U_{ek} V_{ek}. \end{aligned} \quad (9)$$

Here ϵ is the mixing parameter for the right heavy boson W_R and the standard left-handed heavy boson W_L , $W_R \approx \epsilon W_1 + W_2$, M_{WR} and M_{WL} are their respective masses, and m_p is the proton mass. The $\eta_{\lambda'}$ and $\eta_{\tilde{q}}$ are the R-parity violation contributions in supersymmetric (SUSY) Grand Unified Theories (GUT) related to the long range gluino exchange and squark-neutrino mechanism, respectively [69]. Finally, the η_{KK} term is due to possible Kaluza-Klein (KK) neutrino exchange in extra-dimensional model [72]. The set of nuclear matrix elements $M^{0\nu}$, \tilde{X}_λ , \tilde{X}_η , M^{0N} , $M_{\lambda'}^{0\nu}$, and $M_{\tilde{q}}^{0\nu}$ are discussed in many reviews, e.g. Ref. [69]. The $M_{KK}^{0\nu}$ analysis can be found in Ref. [72]. In particular, using the factorization ansatz [72] one gets

$$\eta_{KK} M_{KK}^{0\nu} = \frac{\langle m \rangle_{SA}}{m_e} M^{0\nu} + m_p \langle m^{-1} \rangle M^{0N} \quad (10)$$

where $\langle m \rangle_{SA}$ and $\langle m^{-1} \rangle$ KK masses depend on the brane shift and bulk radius parameters, and are given in Table II of [72]. One can see that the mass parameters $\langle m \rangle_{SA} / m_e$ and $m_p \langle m^{-1} \rangle$ has the effect of modifying $\eta_{\nu L}$ and η_{NR} respectively. $|m_p \langle m^{-1} \rangle| < 10^{-8}$ and it could in principle compete with η_{NR} . $|\langle m \rangle_{SA} / m_e|$ varies significantly with the model parameters and it could also compete with $\eta_{\nu L}$. One needs to go beyond the factorization ansatz, and use information from several nuclei [73] to discern any significant contribution from the KK mechanism.

Constraints from colliders experiments suggest that terms proportional with the mixing angles, ϵ , $U_{ek(heavy)}$, and $V_{ek(light)}$ are very small [77]. Information from colliders also puts some limits on $(M_{WR}, M_N) \sim (2.5 GeV, 1.4 GeV)$. Based on this information and the present limit on the $0\nu\beta\beta$ decay of ${}^{76}\text{Ge}$ one can estimate that $|\eta_{\nu L}| < 10^{-6}$, and $|\eta_{NR}| < 10^{-8}$. They also suggest that $|\langle \lambda \rangle| < 10^{-8}$ and $|\langle \eta \rangle| < 10^{-9}$. In addition, these contributions should be signaled by the different angular and energy distribution of the outgoing electron, signals which are under investigation at SuperNEMO [74]. Here we assume that these contributions are small and can be neglected. In addition, if $\langle \lambda \rangle$ is small, Eq. (9) suggests that η_{NL} is small. Then, the half-life can be written as

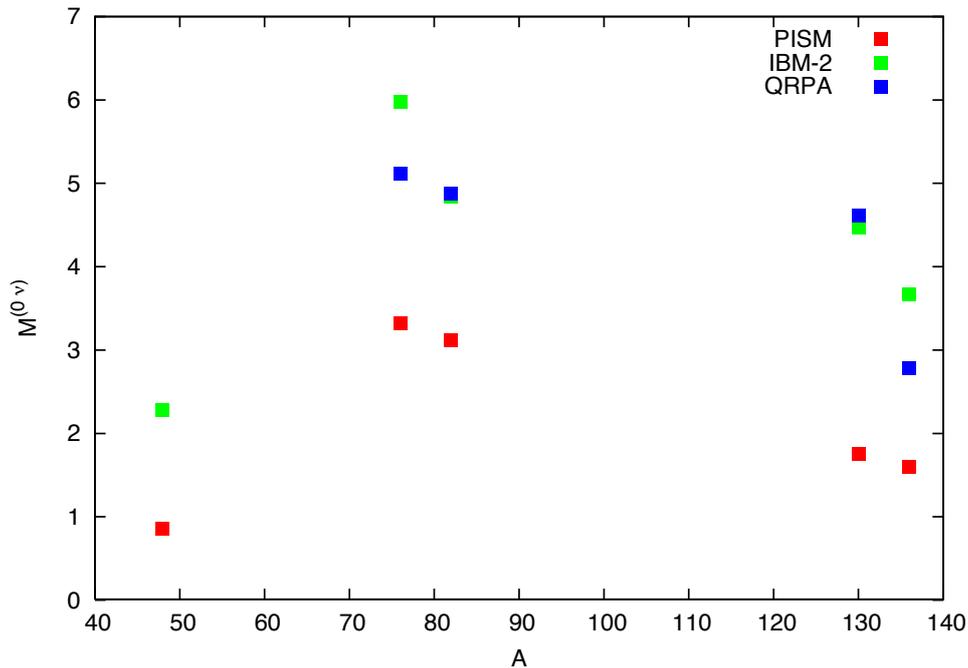


Figure 2. (Color online) The light neutrino-exchange NME for a set of relevant double-beta decay emitters. PISM are the results of present work. IBM-2 results are taken from [36], and QRPA results are taken from [76].

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G^{0\nu} \left(|M^{0\nu}|^2 |\eta_{\nu L}|^2 + |M^{0N}|^2 |\eta_{NR}|^2 \right), \quad (11)$$

where we used the fact that the interference between the left-handed terms and the right-handed terms is negligible [69]. We also neglect the SUSY and KK contribution until a hint of their existence is provided by colliders experiments, or future results of $0\nu\beta\beta$ decay experiments show that these contributions are necessary [73].

The structure of the M^{0N} is as described in Eqs. (5)-(8), with slightly different form factors H_α (see e.g. page 68 of Ref. [69]). A similar treatment can be observed for the contributions from R-parity breaking SUSY mechanisms [76]. A detailed description of the matrix elements of O_{12}^α for the jj -coupling scheme consistent with the conventions used by modern shell model effective interactions is given in Ref. [54]. One should also mention that our method [54] of calculating the TBTD, Eq. (6), is different from that used in other shell model calculations [39]. We included in the calculations the recently proposed higher order terms of the nucleon currents, three old and recent parametrization of the short-range correlations (SRC) effects, finite size (FS) effects, intermediate states energy effects, and we treated careful few other parameters entering the into the calculations. We found very small variation of the NME with the average energy of the intermediate states, and FS cutoff parameters, and moderate variation vs the effective interaction and SRC parametrization. We could also show that if the ground state wave functions of the initial and final nucleus can be accurately described using only the valence space orbitals, the contribution from the core orbitals can be neglected. This situation is different from that of the nuclear parity-nonconservation matrix elements [78], for which the ‘‘mean-field’’ type contribution from the core orbitals could be significant [79].

Comparisons of light neutrino-exchange matrix elements, $M^{0\nu}$, can be found in several recent publications (see, e.g. [36]). Our results are similar to those of other calculations based on the

shell model. We include here in Fig. 2 our results for ^{48}Ca , ^{76}Ge , ^{82}Se , ^{130}Te , and ^{136}Xe of the light neutrino-exchange matrix elements, M^{0N} , in comparison with the QRPA and IBA-2 results. Our corresponding ShM results for heavy neutrino-exchange, M^{0N} , were presented in a recent publication [75]. To our knowledge, no other results of shell model calculations for these matrix elements were reported so far. Based on these calculations one can extract “single-mechanism” limits for $\eta_{\nu L}$, η_{NR} from the experimental results or, assuming that both mechanisms contributing to the half-life in Eq. (11) compete, one can use the experimental data from two isotopes to assess the contribution of each mechanism [76]. This analysis will be reported elsewhere [80]. If the exchange of light neutrino would be determined as the dominant mechanism, then our results could possibly decide the light neutrino mass hierarchy and the lowest neutrino mass [69].

5. Conclusions and outlook

In conclusion, we emphasized the role of nuclear structure for reactions and decays. For compound nucleus reactions the spin- and parity-dependent NLD are essential ingredients, besides the traditional transmission probabilities used in reaction theories for point-like particles. We showed that we developed advanced tools based on the statistical spectroscopy to efficiently and accurately calculate ShM NLD.

Direct nuclear reactions are also largely dependent on the nuclear structure information, such as the spectroscopic factors. In reverse, information from direct nuclear reaction can be used to better understand the nuclear structure of exotic nuclei, such as the pn correlations and their effect on changes in the single particle energies and magic numbers. In the case of $N=28$ isotones we found strong pn correlations between the number of proton pairs filling the $f_{7/2}$ and $d_{3/2}$ shells and the SF for breaking up a neutron, which are reflected in the unusually small SF for the cases of ^{54}Fe and ^{46}Ar . We found that in both cases pieces of the pn part of the effective ShM Hamiltonian are responsible for these effects. In principle one should not be worried, because these pieces are present in any realistic ShM Hamiltonian, but they might be missing in most, if not all, mean-field approaches.

We also analyzed the $2\nu\beta\beta$ and $0\nu\beta\beta$ decays of nuclei with masses $A = 48 - 136$ using shell model techniques. We described very efficient techniques to accurately calculate the $2\nu\beta\beta$ NME in cases that involve large shell model dimension, which were tested for the case of ^{48}Ca , and we used them to make some predictions for ^{76}Ge , ^{82}Se , ^{128}Te , ^{130}Te , and ^{136}Xe . We conclude the $A > 48$ analysis by emphasizing the potential role of including all spin-orbit partners and satisfying the Ikeda sum-rule.

We reviewed the main contributing mechanisms to the $0\nu\beta\beta$ decay, and we showed that based on the present constraints from colliders one could reduce the contribution to the $0\nu\beta\beta$ half-life to the relevant terms described in Eq. (11). A reliable analysis of the $0\nu\beta\beta$ decay experimental data requires an accurate calculations of the associated NME for light and heavy neutrino-exchange contributions. We extended our recent analysis [54] of the $0\nu\beta\beta$ NME for ^{48}Sc to the experimentally relevant cases of ^{76}Ge , ^{82}Se , ^{130}Te , and ^{136}Xe . We also presented for the first time shell model results for the heavy neutrino-exchange nuclear matrix elements necessary to the analysis. However, more effort has to be done to include all spin-orbit partners in the valence space, eliminate the center-of-mass spurious contribution, and better understand the changes in the effective $0\nu\beta\beta$ transition operators [81]. In addition, the closure approximation used in the shell model calculations and by other methods (e.g. IBA-2 [35], PHFB [82], and GSM [83]) needs to be further checked for accuracy, especially for the heavy neutrino exchange NME.

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References

- [1] Hauser W and Feshbach H 1952 *Phys. Rev.* **87** 366
- [2] Horoi M, Kaiser J and Zelevinsky V 2003 *Phys. Rev. C* **67** 054309
- [3] Horoi M, Ghita M and V. Zelevinsky V *Phys. Rev. C* **69** 041307(R)
- [4] Horoi M, Ghita M and Zelevinsky V 2005 *Nucl. Phys.* **A785** 142c
- [5] Horoi M and Zelevinsky V 2007 *Phys. Rev. Lett.* **98** 262503
- [6] Scott M and Horoi M 2010 *EPL* **91** 52001
- [7] Sen'kov R A and Horoi M 2010 *Phys. Rev. C* **82** 024304
- [8] Horoi M and Sen'kov R A 2010 *PoS(NIC-X)222* <http://pos.sissa.it/> .
- [9] Sen'kov M, Horoi M and Zelevinsky V 2013 *Comp. Phys. Comm.* **184** 215 DOI: 10.1016/j.cpc.2012.09.006
- [10] Haxton W C and Stephenson G J 1984 *Prog. Part. Nucl. Phys.* **12** 409
- [11] Suhonen J and Civitarese O 1998 *Phys. Rep.* **300** 123
- [12] Faessler A and Simkovic F 1998 *J. Phys. G: Nucl. Part. Phys.* **24** 2139
- [13] Klapdor-Kleingrothaus H V 2001 "60 Years of Double-Beta Decay - From Nuclear Physics to Beyond Standard Model Particle Physics" World Scientific Singapore
- [14] Elliot S R and Engel J 2004 *J. Phys. G* **30** R183
- [15] Elliot S R and Vogel P 2002 *Annu. Rev. Nucl. Part. Sci.* **52** 115
- [16] Avignone F T , Elliott S R and Engel J 2008 *Rev. Mod. Phys.* **80** 481
- [17] Schechter J and Valle J W F 1982 *Phys. Rev. D* **25** 2951
- [18] Aharmim B *et al* 2005 *Phys. Rev. C* **72** p 055502.
- [19] Arsepella C *et al* 2008 *Phys. Lett. B* **658** 101; Araki T *et al* 2005 *Phys. Rev. Lett.* **94** 081801
- [20] Schwetz T 2008 *Nucl. Phys. B (proc. Suppl)* **188** 158
- [21] Vogel P and Zirnbauer M R 1986 *Phys. Rev. Lett.* **57** 3148
- [22] Grotz K and Klapdor H V 1986 *Nucl. Phys.* **A 460** 395
- [23] Suhonen J, Taigel T and Faessler A 1988 *Nucl. Phys.* **A 486** 91
- [24] Staudt A, Muto K and Klapdor-Kleingrothaus H V 1990 *EPL* **13** 31
- [25] Civitarese O, Faessler A, Suhonen J and Wu X R 1990 *Phys. Lett. B* **251** 333
- [26] Stoica S and Kaminski W A 1993 *Phys. Rev. C* **47** 867; Stoica S 1994 *Phys. Rev. C* **49** 2240
- [27] Raduta A A, Delion D S and Faessler A 1995 *Phys. Rev. C* **51** 3008
- [28] Pantis G, Simkovic F, Vergados J D and Faessler A 1996 *Phys. Rev. C* **53** 695
- [29] Bobyk A, Kaminski W A and Zareba P 1999 *Eur. Phys. J. A* **5** 385
- [30] Raduta A A, Faessler A, Stoica S and Kaminski W A 1991 *Phys. Lett. B* **254** 7
- [31] Toivanen J and Suhonen J 1995 *Phys. Rev. Lett.* **75** 410; 1997 *Phys. Rev. C* **55** 2314
- [32] Simkovic F, Schwieger J, Veselsky M, Pantis G and Faessler A 1997 *Phys. Lett. B* **393** 267
- [33] Barabash A S 2010 *Phys. Rev. C* **81** 035501
- [34] Rodin V A , Faessler A, Simkovic F and Vogel P 2006 *Nucl. Phys.* **A766** 107, erratum ibidem
- [35] Barea J and Iachello F 2009 *Phys. Rev. C* **79** 044301
- [36] Barea J, Kotila J and Iachello F 2012 *Phys. Rev. Lett.* **109** 042501
- [37] Caurier E, Poves A and Zuker A P 1990 *Phys. Lett.* **B252** 13
- [38] Caurier E, Nowacki F, Poves A and Retamosa J 1996 *Phys. Rev. Lett.* **77** 1954
- [39] Retamosa J, Caurier E and Nowacki F 1995 *Phys. Rev. C* **51** 371
- [40] Balys A *et al* 1996 *Phys. Rev. Lett.* **77** 5186
- [41] <http://www.garsington.eclipse.co.uk>
- [42] Horoi M, Stoica S and Brown B A 2007 *Phys. Rev. C* **75** 034303
- [43] Yako K *et al* 2009 *Phys. Rev. Lett.* **103** 012503
- [44] Umehara S *et al* 2006 *J. Phys.: Conference Series* **39** 356
- [45] Zdesenko Yu G *et al* 2005 *Astropart. Phys.* **23** 249
- [46] Engel J, Haxton W and Vogel P 1992 *Phys. Rev. C* **46** 2153R
- [47] Tsang M B *et al* 2005 *Phys. Rev. Lett.* **95** 222501
- [48] Tsang M B *et al* 2009 *Phys. Rev. Lett.* **102** 062501
- [49] Lee J *et al* 2010 *Phys. Rev. Lett.* **104** 112701
- [50] Tsang M B *et al* to be submitted
- [51] Suehiro T, Finck J E and Nolen J A 1979 *Nucl. Phys. A* **313** 141
- [52] Horoi M *et al* in preparation
- [53] Lee J, Tsang M B, Lynch W G, Horoi M and Su S C 2009 *Phys. Rev. C* **79** 054611
- [54] Horoi M and Stoica S 2010 *Phys. Rev. C* **81** 024321
- [55] Poves A 2009 *invited talk at the ECT* workshop* Trento, Italy, July 27-32, 2009

- [56] Kuo T T S and Brown G E 1968 *Nucl. Phys.* **A114** 235
- [57] Poves A and Zuker A P 1981 *Phys. Rep.* **71** 141
- [58] Honma M, Otsuka T, Brown B A and Mizusaki T 2004 *Phys. Rev. C* **69** 034335
- [59] Honma M, Otsuka T, Brown B A and Mizusaki T 2005 *Eur. Phys. J. A* **25** Suppl. 1 499
- [60] Horoi M, Brown B A, Otsuka T, Honma M and Mizusaki T 2006 *Phys. Rev. C* **73** 061305(R)
- [61] Hornish M J, De Braekeleer L, Barabash A S and Umatov V I 2005 *arXiv:nucl-ex/0512030*
- [62] Horoi M 2010 *AIP Procs.* **1304** 106
- [63] M. Honma M, T. Otsuka T, T. Mizusaki T and Hjorth-Jensen M 2009 *Phys. Rev. C* **80** 064323
- [64] Qi C and Xu Z X 2012 *arXiv:1207.5056 [nucl-th]*
- [65] Caurier E, Nowacki F and Poves A 2012 *arXiv:1112.5039v2 [nucl-th]*
- [66] Gavriljuk Ju M *et al* 2011 *arXiv:1112.0859v2 [nucl-ex]*
- [67] Auger M *et al* 2012 *Phys. Rev. Lett.* **109** 032505
- [68] Gondo A *et al* 2012 *Phys. Rev. C* **85** 045504
- [69] Vergados J D, Ejiri H and Simkovic F 2012 *arXiv:1205.0649v2 [hep-ph]*
- [70] Kotila J and Iachello F 2012 *Phys. Rev. C* **85** 034316
- [71] Neacsu A, Stoica S and Horoi M 2012 *arXiv:1208.5728 [nucl-th]*
- [72] Bhattacharyya G, Kalpdor-Kleingrothaus H V and Kovalenko S G 2003 *Phys. Rev. D* **67** 113001
- [73] Deppisch F and Päs H 2007 *Phys. Rev. Lett.* **98** 232501
- [74] Arnold R *et al* 2010 *arXiv:1005.1241 [hep-ex]*
- [75] Horoi M 2012 to be published in the Proceedings of the Carpathian Summer School of Physics 2012, Sinaia, Romania, June 24 - July 7, 2012, *AIP Proceedings*
- [76] Faessler A, Meroni A, Petcov S T, Simkovic F and J. Vergados J D 2011 *Phys. Rev. D* **83** 113003
- [77] Das S P, Deppisch F F, Kittel O and Valle J W F 2012 *Phys. Rev. D* **86** 055006
- [78] Horoi M and Brown B A 1995 *Phys. Rev. Lett.* **74** 231
- [79] Adelberger E G and Haxton W C 1985 *Annu. Rev. Nucl. Part. Sci.* **35** 501
- [80] Horoi M 2012 in preparation
- [81] Engel J and Hagen G 2009 *Phys. Rev. C* **79** 064317
- [82] Rath P K, Chandra R, Chaturvedi K, Raina P K and Hirsch J G 2010 *Phys. Rev. C* **82** 064310
- [83] Rodriguez T R and Martinez-Pinedo G 2010 *Phys. Rev. Lett.* **105** 252503