The single-term formula for ground band energies of A=20-100 mass region nuclei

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Introduction

There are serval empirical formulae to express the ground state band level energies of nuclei. The simplest well-known expression for rotational spectra is

$$E = \frac{\hbar^2}{2\Im} J(J+1), \qquad (1)$$

where $\Im$ and J are the moment of inertia and spin of the nuclei. The Bohr-Mottelson expression for deformed nuclei [1] is

$$E(J) = AX + BX^2 + CX^3 \qquad (2)$$

where $X = J(J+1)$. Holmberg and Lipas [2] noted that the moment of inertia ($\Im$) of deformed nuclei increase with level energy linearly, i.e

$$\Im(J) = a + bE. \qquad (3)$$

By substituting Eq.(3) in Eq.(1), they obtained the two-parameter *ab* formula

$$E(J) = a\left[\sqrt{1 + bJ(J+1)} - 1\right] \qquad (4)$$

and compared the level energies calculated from the two-parameter *ab* formula (4) with experimental energies. Later Zeng et al [3] illustrated the nonlinearity of the relation between $\Im$ and E in the rotational spectra for low and high spins. However by rewriting Eq.(4) and comparing it with Eq.(1), they derived a new relation between $\Im$ and E:

$$\Im = \frac{1}{2ab}\left(\sqrt{1 + \frac{2}{a}E} + 1\right). \qquad (5)$$

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Substituting Eq.(5) into Eq.(1), they formulated a new energy expression called the *pq* formula:

$$E(J) = a\big(\{\kappa^2 + [\kappa^4 + \kappa^3]^{1/2}\}^{1/3} + \{\kappa^2 - [\kappa^4 + \kappa^3]^{1/2}\}^{1/3}\big)$$

where $\kappa = \big(bJ(J+1)\big)/2$. Further, Brentano et.al [4] noted the dependence of MI on the spin (J) and energy (E):

$$\Im = \Im_0(1 + aJ + bE) \qquad (6)$$

By dropping the energy-dependent term in Eq.(6) and the putting this in Eq.(1), Brentano et al obtained the two-parameter formula, called the soft rotor formula (SRF)

$$E = \frac{1}{\Im_0(1+\alpha J)} J(J+1). \qquad (7)$$

Gupta et.al [5] suggested a single- term expression for ground band level energies of a soft-rotor. They replaced the concept of the arithmetic mean of the two terms used in the Bohr-Mottelson expression by the geometric mean and introduced a two-parameter formula called the power law

$$E = aJ^b. \qquad (8)$$

By using eq.(8) for any spin (J) index b can be determined from the ratio

$$R_J = E(J)/E(2) = (J/2)^b. \qquad (9)$$

Taking logs on both sides, we get

$$b_J = log(R_J)/log(J/2) \qquad (10)$$

By using Eq.(10), one can evaluate the value of index b for different J in any given nucleus.

1. Result and Discussion

In Figure 1 we compared ground band energy of ^{74}Se, ^{78}Se calculated by ab-formula, Ejiri, soft rotor formula and power law and compared with the experimental data. As we know that the $R_{4/2}$ ratio of ^{74}Se is 2.1 and ^{78}Se is 2.5 therefore ^{74}Se shows vibrational nature and ^{78}Se shows the γ-soft behavior.

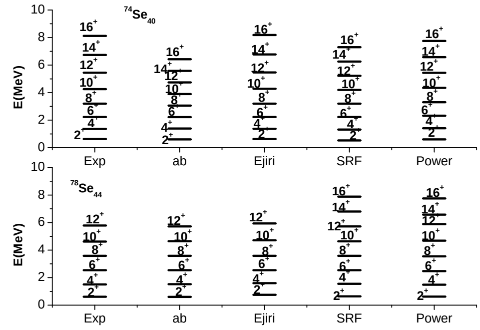

FIG. 1: Comparison between experimental energies with fits of *ab* formula, Ejiri, SRF and power law for ^{74}Se, ^{78}Se nuclei.

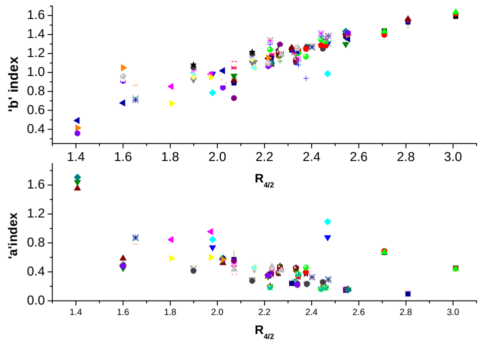

FIG. 2: Values of the fitted parameters a and b as a function of $R_{4/2}$ from the least squares fit.

The comparison shows that the power law is valid for both vibrational and soft nuclei. In ^{74}Se nuclei, Ejiri, SRF and power law show good agreement with the experimental values, but *ab* formula does not show good result for this nuclei. In case of ^{78}Se nuclei *ab*, ejiri, SRF and power law show excellent agreement with experimental values. Figure 2 shows the values of the fitted parameters b and a against the $R_{4/2}$ ratio. The value of b rises from 0.4 to 1.6 with increasing $R_{4/2}$. Similar is the case of a versus $R_{4/2}$ ratio. The value of a falls with increasing $R_{4/2}$. This reflects the smaller energy spacing in softer nuclei having ratio $R_{4/2} \leq 2.5$.

Conclusion

To summarize, we studied the power law which is applicable for both vibrational and soft nuclei. The point at which the value of b exhibits a sharp drop is an indication of the shape phase change in the nucleus. Thus the single-term expression in the form of a power formula is an alternative to the perturbation expansions.

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