Particle Accelerators, 1993, Vol. 41, pp.29–39
Reprints available directly from the publisher
Photocopying permitted by license only

PAIR CREATION FROM BEAM-BEAM INTERACTION IN LINEAR COLLIDERS*

TOSHIAKI TAUCHI, KAORU YOKOYA[1] and PISIN CHEN[2]

[1]*National Laboratory for High Energy Physics, KEK, Oho, Tsukuba-shi, Ibaraki, 305, Japan*
[2]*Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309 USA*

(Received 6 July 1992; in final form 16 October 1992)

It has been recognized that e^+e^- pair creation during the collision of intense beams in linear colliders will cause potential background problems for high energy experiments. Detailed knowledge of the angular-momentum spectrum of these low energy pairs is essential to the design of the interaction region. In this paper, we derive the analytic formulae for the integrated cross-section of this process and we also modify the computer code ABEL(Analysis of Beam-beam Effects in Linear colliders) to include the pair creation processes, using the equivalent photon approximation. Special care has been taken on the non-local nature of the virtual photon exchanges. The simulation results are then compared with the analytic formulae, and applied to the next generation colliders such as JLC.

KEY WORDS: pair creation, linear colliders

1 INTRODUCTION

In future linear colliders, low energy e^+e^- pairs created during the beam crossing could cause background problems for the detectors. In the next generation of colliders, most such pairs will be made by *incoherent* processes, from the interaction of individual particles (e^+, e^- or beamstrahlung γ) in the two beams. This problem was first identified by Zolotarev *et al.*[1] At energies where the beamstrahlung parameter Υ is ≥ 1, the *coherent* production of a pair from a beamstrahlung photon interacting with the field of the oncoming beam becomes dominant, as first noted by Chen.[2] The seriousness of this problem lies in the transverse momenta that the pair particles carry when leaving the interaction point(IP) with large angles. One source of transverse momentum is from the kick by the field of the oncoming beam, which results in an outgoing angle $\theta \propto 1/\sqrt{x}$, where x is the fractional energy of the particle relative to the initial beam particle energy[3]. The second source comes from the inherent scattering angles of these pairs, which may already be large when they are created. This issue was first studied by Zolotarev *et al.*[1] and recently by Chen *et al.*[4]

In this paper we modify the ABEL[5] to include the incoherent pair creation processes using the equivalent photon approximation.[1,4] By this simulation we can correctly take

* Work supported by Department of Energy contract DE-AC03-76SF00515.

account of both the kick and the inherent angles of the pairs. The geometric reduction is also implemented in the ABEL. The simulation results are compared with the analytic calculations using the parameters of JLC as an example of the next generation linear colliders.

2 THE ANALYTIC FORMULAE

We consider three incoherent pair creation processes, the Breit-Wheeler (BW: $\gamma\gamma \to e^+e^-$) process, Bethe-Heitler (BH: $e^\pm\gamma \to e^\pm e^+e^-$) process and Landau-Lifshitz (LL: $e^+e^- \to e^+e^-e^+e^-$) process. In the calculations of these cross-sections the basic kernel is the same using the equivalent photon approximation. For the BW process both photons are real beamstrahlung photons; for the BH process one is real and the other is virtual; for the LL process both photons are virtual. The partial cross-sections with transverse momentum divided by γm_e, $x_t > x_t^\circ$, and outcoming angle, $\theta_\circ \leq \theta \leq \pi - \theta_\circ$, are calculated by the convolution of two photon energy spectra, $n_a(y_1), n_b(y_2)$, and the differential cross-section for $\gamma\gamma \to e^+e^-$, $\sigma_{\gamma\gamma}(y_1, y_2, c)$, as below.

$$\sigma(x_t^\circ, c_\circ) = g \int_{-c_\circ}^{c_\circ} \int_{y_-}^{1} \int_{y_b}^{1} dc dy_2 dy_1 n_a(y_1) n_b(y_2) \sigma_{\gamma\gamma}(y_1, y_2, c), \tag{1}$$

where y_i is the fractional photon energy and $c = \cos\theta$, and g=1/4 for the BW process and 1 for both BH and LL processes. The fractional energy x of the outcoming positron(or electron) to its angle θ is expressed by

$$x = \frac{2y_1y_2}{y_1(1-c) + y_2(1+c)} \tag{2}$$

As denoted in Eq. (1), the integration regions of two photon energies are

$$1 \geq y_1 \geq y_b = \frac{y_2 y_+}{y_2 - y_-}, \quad 1 \geq y_2 \geq y_-, \tag{3}$$

$$y_\pm = \frac{x_\circ}{2}(1 \pm c) = \frac{x_t^\circ}{2}\left(\frac{1 \pm c}{1 \mp c}\right)^{1/2}, \tag{4}$$

where $x_\circ$ and x_t° are the minimum energy and the minimum transverse energy. These lower bounds are very important for the calculations because the dominant contribution comes from them. The virtual and the beamstrahlung photon spectra are given by

$$n_v(y) = \frac{2\alpha}{\pi}\frac{1}{y}\ln(\frac{1}{y}) \qquad \text{and} \tag{5}$$

$$n_b(y) = \frac{1}{\pi}\Gamma(\frac{2}{3})(\frac{\alpha\sigma_z}{\gamma\lambda_e})(3\Upsilon)^{2/3}y^{-2/3} \equiv Ay^{-2/3}, \tag{6}$$

respectively, where σ_z, λ_e and γ are the beam bunch length, the electron Compton wavelength and a Lorentz factor for the beam, respectively. Beamstrahlung is characterized by a dimensionless and Lorentz invariant parameter, Υ, which is locally defined by $\Upsilon \equiv \gamma B/B_c$, where B is a magnetic field and B_c is the critical magnetic field, $B_c \equiv m_e^2/e$(4.4 GTesla). In our analytic equation we approximate Υ by its average value of Gaussian beams as follows,

$$\Upsilon = \frac{5}{6}\frac{Nr_e^2\gamma}{\alpha\sigma_z(\sigma_x + \sigma_y)} \tag{7}$$

where N and r_e is a beam intensity and the classical electron radius, respectively. Finally $\sigma_{\gamma\gamma}(y_1, y_2, c)$ is calculated by neglecting the obviously small terms $<O(\gamma^{-2})$.

$$\sigma_{\gamma\gamma}(y_1, y_2, c) = \frac{\pi r_e^2}{\gamma^2 y_1 y_2}\frac{1}{1-c^2}\left\{\frac{y_1^2(1-c)^2 + y_2^2(1+c)^2}{[y_1(1-c) + y_2(1+c)]^2}\right\} \tag{8}$$

$$\simeq \frac{\pi r_e^2}{\gamma^2 y_1 y_2}\frac{1}{1-c^2}, \tag{9}$$

The last approximation of Eq. (9) is only used in analytic calculations since the value in the parenthesis (Eq. (8)) varies slowly from 1/2 to 1 and it is well factorized outside the integral (Eq. (1)) , which is estimated to be 0.7 with an accuracy of a few % by comparing this approximation and the numerical integration with no such approximation. The resultant integrated cross-sections are expressed below as a function of x_t° and $\theta_\circ$, which are already multiplied by the factor of 0.7 in order to compensate for the last approximation,

$$\sigma_{\text{BW}}(x_t^\circ, \theta_\circ) = 1.69\frac{r_e^2}{\gamma^2}A^2\left(\frac{2}{x_t^\circ}\right)^{4/3}\ln\frac{1}{\tau_\circ} \tag{10}$$

$$\sigma_{\text{BH}}(x_t^\circ, \theta_\circ) = 3.55\frac{\alpha r_e^2}{\gamma^2}A\left(\frac{2}{x_t^\circ}\right)^{5/3}(\tau_\circ^{1/3} - \tau_\circ^{-1/3})(\ln\frac{x_t^\circ}{2} + 0.21) \tag{11}$$

$$\sigma_{\text{LL}}(x_t^\circ, \theta_\circ) = 0.83\frac{\alpha^2 r_e^2}{\gamma^2}\left(\frac{2}{x_t^\circ}\right)^2\ln\frac{1}{\tau_\circ}\left(\ln\frac{x_t^\circ}{2\tau_\circ}\ln\frac{x_t^\circ\tau_\circ}{2} + 3\ln\frac{x_t^\circ}{2} + 4.44\right) \tag{12}$$

where $\tau_\circ = \tan(\theta_\circ/2)$. However, they do not include the kick of created pairs by the opposing beam. The above expressions account for only one of the two particles (say positron) of the pair. To count the electron as well, we must multiply each expression by 2.

The above formulae are valid for large inherent angle scattering since they have a collinear singularity which is clearly seen in Eq. (9). The reason for this singularity is our neglect of the electron mass(m_e) in the derivation of the formulae. Remembering that the typical scattering angle is m_e/E_e, where E_e is the energy of e^- or e^+ of a pair, the valid angular region will be $\theta > m_e/E_e$. More detailed discussions on how to treat this singularity in event generation will be found in the subsequent sections.

3 THE ABEL SIMULATION

3.1 Event Generation

In ABEL, the beam bunches are described by ensembles of macro-particles. The number of macro-particles is typically 10^3 to 10^5. The whole process is divided into time slices. At each time step the bunches are further divided into longitudinal slices. The modification of ABEL is that the pairs are created in the collision between the macro-particles and the beamstrahlung photons in each longitudinal slice. There is no pair creation between the different slices; that is, the incoherent pair creation processes are "local" in the longitudinal direction. Then, the created particles(e^+ or e^-) are tracked in the Coulomb potential which is produced by the oncoming beam. As the transverse momenta of these particles are affected by the kick in the tracking, the integrated cross-sections for the processes in ABEL are given by $(x_\circ, \theta_\circ)$ instead of $(x_t^\circ, \theta_\circ)$, and there are no integrations over the beamstrahlung photon energy spectra for the BW and BH processes.

$$\sigma_{\text{BW}}(y_1, y_2, \theta_\circ) = 6.28\frac{r_e^2}{\gamma^2}\frac{1}{y_1 y_2}\ln\frac{1}{\tau_\circ} \tag{13}$$

$$\sigma_{\text{BH}}(y_1, \theta_\circ) = 8\frac{\alpha r_e^2}{\gamma^2}\frac{\ln\tau_\circ}{y_1}\left(\frac{\ln y_{min}+1}{y_{min}} - 1\right) \tag{14}$$

$$y_{min} = max\left(\frac{x_\circ}{2}(1-c_\circ),\ \frac{1}{\gamma^2}\right), \quad c_\circ = \cos\theta_\circ \tag{15}$$

$$\sigma_{\text{LL}}(x_\circ, \theta_\circ) = 0.64\frac{\alpha^2 r_e^2}{\gamma^2}\left(\frac{2}{x_\circ}\right)^2\left\{(\ln\tfrac{x_\circ}{2}+1)\{(\ln\tfrac{x_\circ}{2}+\tfrac{3}{2})(\tfrac{c_\circ}{1-c_\circ^2} - \ln\tau_\circ) + \tfrac{c_\circ\{1+\ln(1-c_\circ^2)\}}{1-c_\circ^2}\}\right\}, \tag{16}$$

where y_1, y_2 are the beamstrahlung photon energies. For the virtual photon energy, we use the distribution of Eq. (5). The energy of the outgoing particle is calculated by Eq. (2) with y_1, y_2 satisfying the boundary conditions of Eq. (3). In the above formulae we assume that the energy of macro-particles is the nominal one although ABEL takes account of their energy loss by beamstrahlung. This assumption is good enough for our purpose and conservative from a point of view of background consideration since most macro-particles lose only a few GeV and the total cross-sections depend on a logarithm of beam energy (see Eqs. (19)-(21)). The transverse momentum is calculated by the energy and the final scattering angle after the kicks.

For the generation of collinear pairs, a minimum cut-off angle θ_{cut} has to be introduced to regulate the singularity which is mentioned in the previous section,*i.e.* $\theta > \theta_{cut}$. The cut-off angle is a function of $m_e/E_e = 1/x\gamma$ and it depends on the processes of BW,BH,LL as follows,

$$\theta^i_{cut}(x) = f_i \frac{1}{x\gamma}, \qquad i = \mathrm{BW}, \mathrm{BH} \text{ or } \mathrm{LL} \tag{17}$$

where, f_{BW},f_{BH} and f_{LL} are patch factors to be determined by the normalization of their total cross-sections. In addition to these cut-off angles, the energies of two photons must fulfil the following threshold condition of an e^+e^- pair creation which is also ignored in the zero electron mass approximation.

$$y_1 y_2 > \frac{1}{\gamma^2} \ . \tag{18}$$

3.2 *Justification of Our Method*

The total cross-sections for the three processes are well known and are expressed below.

$$\sigma_{\mathrm{BW}} = \frac{3}{8}(\ln 4 + \frac{3}{2})\pi r_e^2 \gamma^{-2/3}(L - \ln 4)A^2 \tag{19}$$

$$\sigma_{\mathrm{BH}} = \frac{28}{3}\alpha r_e^2 (L - \frac{235}{42})A \tag{20}$$

$$\sigma_{\mathrm{LL}} = \frac{\alpha^2 r_e^2}{\pi}(\frac{28}{27}L^3 - 6.59L^2 - 11.8L + 104) \tag{21}$$

where $L = \ln 4\gamma^2$, and $\sigma_{\mathrm{BH}}, \sigma_{\mathrm{BW}}$ are already integrated over the beamstrahlung photon energy spectrum of Eq. (6). By comparing values calculated by the above equations and those of the numerical integration of Eq. (1) over $\gamma^{-1} < x < 1$ with the cut-off angles and the threshold conditions for the two photon energies, the patch factors are determined to be,

$$f_{\mathrm{BW}} = 0.5, \quad f_{\mathrm{BH}} = 0.92 \quad \text{and} \quad f_{\mathrm{LL}} = 0.7. \tag{22}$$

The total cross-sections thus calculated are summarized in Table1 for JLC[7] with $E_{beam} = 500$ GeV, for which A (the cross-section) is 1.11. In order to verify our method, the energy(x) distributions for BH and LL are also compared with the known formulae[6] and the results are shown in Fig.1 (a) and (b). As is clearly seen in these figures, the agreement between the two is good for the energy region of interest($E_e > 5$ MeV).

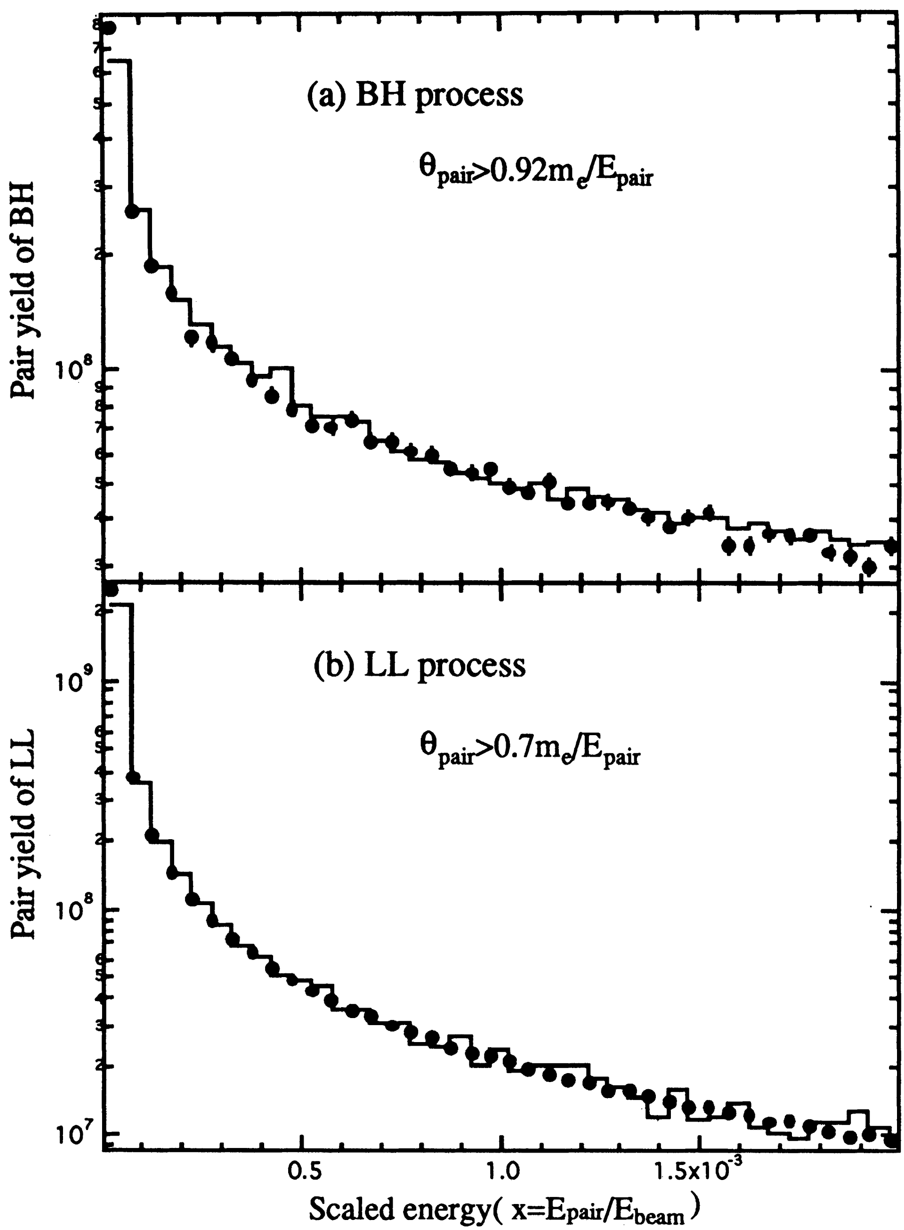

FIGURE 1: (a)Scaled energy (x) distributions of one of pair particles created in the BH process for JLC at E_{beam} = 500 GeV. The histogram is given by the known formula and plotted data are given by numerical integration with $f_{\mathrm{BH}} = 0.92$; (b) same as (a) for LL process, where $f_{\mathrm{LL}} = 0.7$.

TABLE 1: Total cross-sections for the three incoherent processes at E_{beam}=500 GeV in cm^2.

Process	Calculations by Eqs. (19)-(21)	Our method numerical integration of Eq. (1)
BW	1.9×10^{-27}	1.9×10^{-27}
BH	2.7×10^{-25}	2.8×10^{-25}
LL	5.6×10^{-26}	5.2×10^{-26}

3.3 Geometric Reduction

The finite impact parameter of the interactions in these processes comes from the transverse energy(q_t) of the virtual photon. The distribution of $y_t (\equiv q_t/\gamma m_e)$ is[6]

$$n_v(y_t) = \frac{y_t^2 dy_t^2}{(y_t^2 + y^2/\gamma^2)^2}. \tag{23}$$

The integral of the above equation corresponds to the logarithmic term of Eq. (5), and the range of its integration over y_t is from y/γ^2 to $1/\gamma$. For a given equivalent photon energy y, the dominant contribution to the integration comes from the region of small transverse momentum $y_t \sim y/\gamma$. In ABEL every virtual photon has finite transverse energy(y_t) according to Eq. (23). To account for this non-local nature of the virtual photon interaction, we first calculate the probability of pair creation which is proportional to the local intensities of the two beams (macro-particles or beamstrahlung photons) at a point. Defining the impact parameter(ρ) as $1/y_t \gamma m_e$, we get the non-local intensities of two beams separated by ρ from each other, where for the LL process ρ is actually a sum of two impact parameters. Then the reduction factor can be obtained by the ratio of "non-local" intensities divided by the "local"ones. If the separation is far beyond the beam (transverse) size, the pair creations will be suppressed strongly.†

ABEL creates the pairs at the position separated by ρ from the beam position and even outside the beam size. For analytic calculations, this geometric reduction can be taken into account by limiting the integration region of y_t $(1/\sigma_y \gamma m_e < y_t < 1/\gamma)$ in Eq. (23), where σ_y is a transverse beam size at a collision point.[8]

3.4 Effect of the Strong External Field

Since the $e^{\pm}$ pairs are created in the external field of the oncoming beam, which is a magnetic field of O(10^3) tesla at a TeV linear collider(JLC), cross-sections involving virtual photons, σ_{LL} and σ_{BH}, are affected and eventually suppressed by the magnetic field.[11] This effect is well explained by a radiation angle of a virtual photon in a magnetic field $\theta_H = 1/\gamma(\omega_c/\omega)^{1/3}$, which is compared with the angle in free space $\theta = 1/\gamma$, where ω and ω_c are the energy of the photon and a critical energy of a synchrotron radiation

† This *geometric reduction* effect was first observed at Novosibirsk,[9] and subsequently developed theoretically by several authors.[8,10]

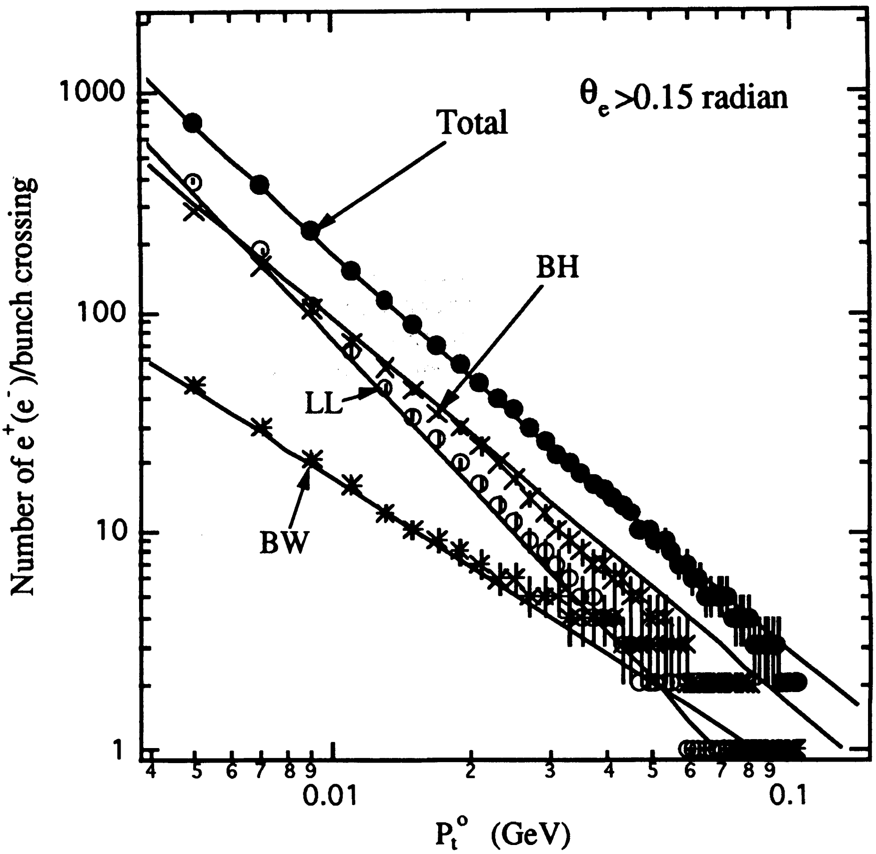

FIGURE 2: Comparison between the analytic calculation and the ABEL.

in the magnetic field, respectively. Requiring an inequality of $\theta > \theta_H$, the transverse energy of a virtual photon ω_t has a lower bound given by

$$\omega_t > \omega\theta_H = \frac{\omega}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3} \quad \text{or} \quad y_t > \frac{y}{\gamma}\left(\frac{\Upsilon}{y}\right)^{1/3}. \tag{24}$$

Thus the external magnetic field increases the minimal transverse energy transfer. This effect appears as a suppression of the pair creations with large impact parameters similar to geometric reduction. In ABEL the virtual photon participating in pair creation is required to fulfil the above condition (Eq. (24)).

4 NUMERICAL COMPARISON

For the numerical comparison between the analytic calculation and the ABEL, we estimate the yields from a 1 TeV linear collider, JLC,[7] where $\gamma = 10^6, \sigma_x/\sigma_y = 372/3.1$

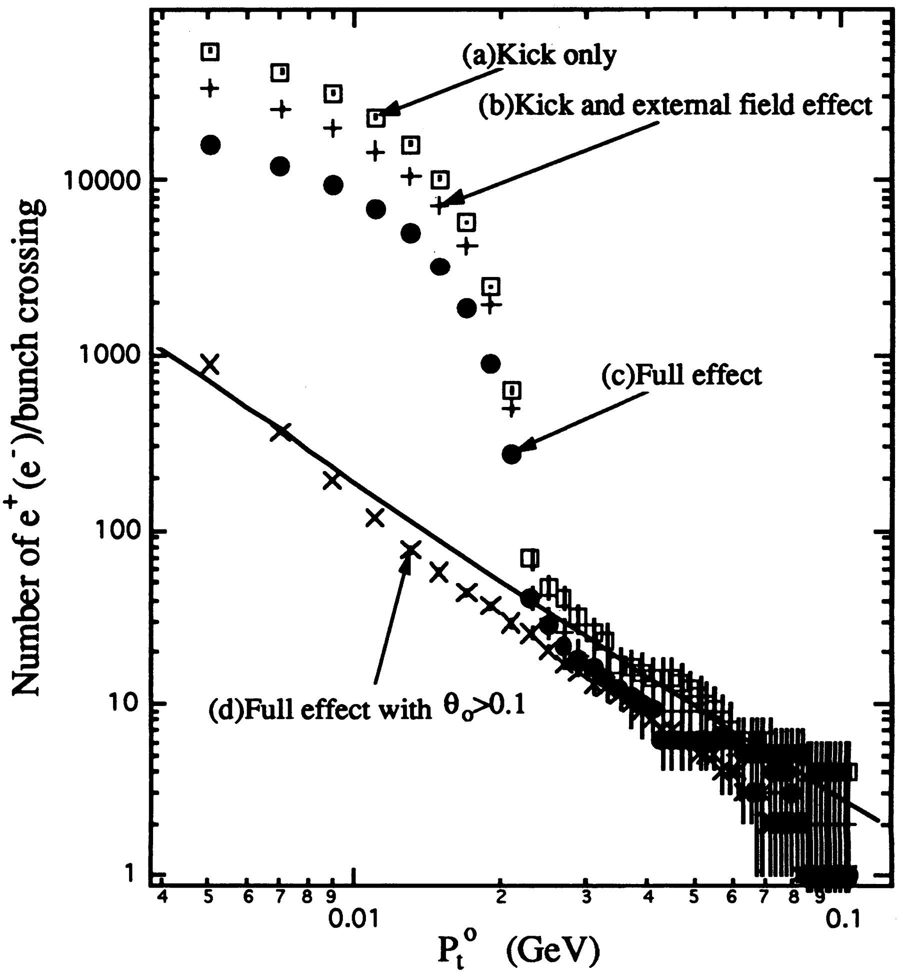

FIGURE 3: The ABEL simulation results with the various effects of (a)-(d) which are explained in the text.

nm, $\sigma_z = 112.8\ \mu$m, $< \Upsilon_> = 0.43$, luminosity $L = 8 \times 10^{31}$/cm^2/ bunch train (20 bunches per train) and 150 Hz RF pulse rate. Figure 2 shows the yields per bunch crossing calculated by the cross-sections of Eqs. (10),(11) and (12) and their sum as a function of $p_t^\circ = \gamma m_e x_t^\circ$ at $\theta > 0.15$ together with the results of the ABEL which are plotted with error bars. Major detectors for an experiment are required to be located in this angular region. The yields of the analytic calculations are already multiplied by 2 for e^+ and e^-. In this figure the effects of the kick, the non-local interaction and the external field in ABEL are switched off just for comparison. The agreement for all three processes is very good.

In addition to the large inherent angles of the pair creations, ABEL implements the kick, the geometric reduction and the external field effect as described in the previous section. To see these individual effects in detail, we simulated the pairs under the four kinds of conditions, that is (a) only the kick is taken into account; (b) the kick and the external field effect are considered; (c) and (d) include all effects, where (a), (b) and (c) have no angular constraints except for Eq. (17); and (d) requires the minimal angle $\theta_\circ = 0.1$ of the pair creation. The results are shown in Fig.3, where the total yields summed over the three processes are plotted in the same way as in Fig.2 , together with the total yield calculated by the analytic formulae which is drawn by a solid line. The geometric reduction rate is 70% at $p_t^\circ = 5$ MeV and it decreases gradually to 40% at $p_t^\circ > 25$ MeV. There is also 40-20% reduction due to the external field effect. However the latter reduction is mostly included in the former one, since these two reductions are due to essentially the same effect on the transverse energy transfer, and the geometric one is dominant in the present case of a very small beam spot size. By comparing (c) with (d) in Fig.3, a huge number of pairs for $p_t^\circ < 20$ MeV comes from the pairs created at a very forward angle ($\theta << 0.1$), a major part of whose transverse momenta are acquired by the kick. A sharp shoulder seen at about $p_t^\circ = 20$ MeV corresponds to the maximum kick angle of these pairs at $\theta > 0.15$. Beyond this shoulder ($p_t^\circ > 20$ MeV), the effect of the kick is small and the analytic calculation with the geometric reduction[8] is a good approximation of the ABEL result.

REFERENCES

1. M. S. Zolotarev, E. A. Kuraev and V. G. Serbo, Inst. Yadernoi Fiziki Preprint 81-63 (1981); SLAC TRANS-227 (1987)
2. P. Chen, SLAC-PUB-4822(1988); appeared in *Proc. DPF Summer Study, SNOWMASS '88,* World Scientific (1989).
3. P. Chen and V. I. Telnov, *Phys. Rev. Lett.* **63**, (1989) 1796. P. Chen, *Part. Acc.* **30**, (1990) 1013 .
4. P. Chen, T. Tauchi and D. V. Schroeder, 'Pair creation at Large Inherent Angles' Presented at the DPF Summer Study on High Energy Physics: Research Directions for the Decade, Snowmass, Colorado, June 25-July 13, 1990, SLAC-PUB-5652.
5. K. Yokoya, ABEL, A Computer Simulation Code for the Beam-Beam Interaction in Linear Colliders, KEK-Report-85-9, Oct. 1985, also *Nucl. Instr. Meth.* **B251**, (1986) 1.
6. V. B. Berestetskii, E. M. Lifshitz and L. P. Pitaevskii, *Relativistic Quantum Theory*, Part 1, Pergamon Press (1971).
7. K. Yokoya, 'Beam parameters for JLC' in the procceedings of LC91, protvino, Sep. 1991.
8. G. L. Kotkin, S. I. Polityko and V. G. Serbo, *Sov. J. Nucl. Phys.* **42**, (1985) 440.
9. A. E. Blinov, A. E. Bondar, Yu. I. Eidelman *et al.*, *Phys. Lett.* **13B**, (1982) 423. Yu. A. Tikhonov, *Candidates's Dissertation*, Inst. Nucl. Phys., Novosibirsk (1982).

10. V. N. Baier, V. M. Katkov and V. M. Strakhovenko, *Sov. J. Nucl. Phys.* **36**, (1982) 95. A. I. Burov and Ya. S. Derbenev, INP Preprint 82-07, Novosibirsk (1982).
11. V. N. Baier, V. M. Katkov, *Sov. Phys. Dokl.* **17**, (1973) 1068.