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Light Meson Spectroscopy Using AdS/QCD Soft Wall Model with Extra UV Cutoff

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In this work we discuss how to construct the mass spectra for the light mesons using a modified version of the AdS/QCD soft wall model with an extra (hard) wall at the UV region. In the case of the pseudoscalar and axial sectors, we introduce a modification into the bulk mass in order to introduce the parity information of these particles. With this new form of the bulk mass it is possible to model the f_0, ρ, η and a_1 trajectories with the same holographic parameters, proving that the model has shown the universality expected for light meson trajectories. The obtained radial trajectories proved to be in agreement with the experimental results with a RMS error near to 22% for 23 mesonic states fitted with 3 parameters.

KEYWORDS: AdS/QCD, Meson mass spectra, Regge Trajectories.

1. Introduction

Throughout the last twenty years, the AdS/CFT correspondence [1–3] has been used to study a wide range of non-perturbative phenomena with significant success. Some examples are given by low-energy QCD vacuum properties such as the Quark-Gluon Plasma state, confinement, meson spectra and chiral symmetry breaking. In order to study all of them, two approaches can be employed: the first one, called top-down, allows you to mimic properties of a given non-perturbative phenomenology to conformal (and also non-perturbative) super Yang Mills $\mathcal{N} = 4$ at large N which is equivalent (holographic partner) to a type IIB (weak) gravity in AdS space. On the other hand, the bottom-up approach, allows you to start from a gravity theory in AdS (which essentially could be different from SUGRA type IIB) and try to look out for a QFT description (which is not essentially SYM $\mathcal{N} = 4$ at large N) of a non-perturbative system. We will use the latter approach to describe the light meson spectra.

This work is entirely developed using the AdS/QCD ideas exposed in [4] at zero temperature. But it can be extended to the finite density [5] or finite temperature [6] holography just by recalling that, according to the holographic dictionary, thermal or density effects can be add just by considering a proper black hole (neutral or charged), see [7].

This work is divided in three parts. First, we develop the holographic frame to model light mesons. Then we present our numerical results and finally some discussions and conclusions.

2. Holographic Model

As we said above, our start point is the AdS/QCD soft wall model with an UV hard cutoff. First consider the usual AdS Poincarè patch with a geometrical UV cutoff:

$$dS^2 = g_{MN}\, dx^M\, dx^N = \frac{R^2}{z^2}\left[dz^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu\right]\Theta\,(z - z_0)\,, \tag{1}$$

where z_0 is some energy scale that will be related to the natureness of the strong interaction inside the meson, and $\Theta\,(z)$ is the Heaviside step function.

The action for the scalar an vector mesons will be given by the expression

$$I = I_{\text{Scalar}} + I_{\text{Vector}}, \tag{2}$$

with

$$I_{\text{Scalar}} = -\frac{1}{2\, g_S^2}\int d^5 x\, \sqrt{-g}\, e^{-\Phi(z)}\left[g^{MN}\,\partial_M S\,\partial_N S + M_5^2\, S^2\right], \tag{3}$$

$$I_{\text{Vector}} = -\frac{1}{2\, g_V^2}\int d^5 x\, \sqrt{-g}\, e^{-\Phi(z)}\left[\frac{1}{2}F_{MN}\, F^{MN} + M_5^2\, g^{MN}\, A_M\, A_N\right], \tag{4}$$

where $\Phi\,(z) = \kappa^2\, z^2$ is the static quadratic dilaton profile, $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strength related to the U(1) field $A_M\,(z, x^\mu)$, the coupling $g_{S\,(V)}$ is a constant that fixes units on the scalar (vector) sector, and M_5 is the bulk mass that fixes the hadronic identity for the mesonic states via the expression $M^2\, R^2 = (\Delta - p)(\Delta + p - 4)$, where Δ is the conformal dimension of of the hadronic operator and $p = 0, 1$ for scalar and vectors respectively.

Recall that mesons are created by $q\bar{q}$ operators, that have dimension three. This bulk mass definition is associated to mesonic states in s-wave ($L = 0$) only with isospin (I) fixed to zero (f_0 mesons [11]) and one (ρ mesons [12]). But, if we want to include other states with L different from zero, we can introduce some sort of *twist* operator to the conformal dimension in order raise the value of L. This idea was exposed in [13].

But it is also possible to explore other properties using this twist operator idea. In this work, we will associate the twist operator Δ_P to the parity of the mesonic state at hand, modifying the bulk mass in the following form

$$M_5^2\, R^2 = \left(\Delta_{\text{Phys}} + \Delta_P - p\right)\left(\Delta_{\text{Phys}} + \Delta_P + p - 4\right). \tag{5}$$

With this modification, we can change the parity of the meson studied by varying Δ_P. In the case of vector mesons, ρ and a_1 mesons, the mesonic states differ in parity by one. $\delta_P = 0$ fixes the bulk mass for the ρ family and $\Delta_P = -1$ does the proper in the a_1 family. In the scalar sector, f_0 and η mesons have a parity difference of one also, so it is possible to extend the Δ_P choice done in the vector case: f_0 is labeled with $\Delta_P = 0$ and η corresponds to $\Delta_P = 1$. Numerical results support this parameter choice.

A natural question arises at this stage: are this values of the bulk mass creating stable solutions? The answer come from the ideas behind the Breitenlohner–Freeman limit [8–10]. The possible values of Δ_P are constrained by the stability of the solutions in the bulk. This impose the rules exposed in table I.

For example, in the usual AdS/QCD approach ($\Delta_P = 0$), $M_5^2\, R^2 = -3$ for scalar mesons and $M_5^2\, R^2 = 0$ for vector mesons. The results for the mass spectrum of these mesonic states using the modified soft wall model with UV hard cutoff are exposed in [14].

3. Holographic calculation of the mesonic spectra in a nutshell

The main details of the calculations can be found in [14] and [15], but it can be summarized as follows: from the action principles given in 3 and 4 we construct the equations of motion.

Table I. This table summarizes the fixing of Δ_P and the value of $M_5^2 R^2$ on each case of interest.

Meson Identity	Δ_P	$M_5^2 R^2$
Scalar meson	0	−3
Vector meson	0	0
Pseudoscalar meson	−1	−4
Axial vector meson	−1	−1

In the case of the vector field we have to cases: the ρ trajectory with $M_5^2 R^2 = 0$, and the axial mesons $M_5^2 R^2 = -1$. In the former case, the equations of motion are gauge invariant and we can fix the gauge $A_z = 0$. In the case axial mesons, the vector field is massive, so gauge invariance is broken. But, following the ideas exposed in [16], the gauge $A_z = 0$ is still valid. In fact, consider the equation of motion for the A_z component, $\Box A_z - \partial_z \left(\partial_\mu A^\mu\right) - M_5^2 A_z = 0$. Fixing $A_z = 0$ still allows to impose a plane wave expansion in the solutions since $\partial_\mu A^\mu = 0$ is still valid.

With the solutions, use the holographic prescription [3] and compute the 2-point function, which is written as a pole expansion. Recall that the bulk fields act as sources for the operators with dimension Δ that create mesons. The poles of these functions are the masses for each mesonic state at hand, i.e., scalar, vector, pseudoscalar or axial mesons, that in general have the following form [14, 15]

$$M^2_{\text{meson},\,n} = 4\,\kappa^2 \chi_n\left(\kappa,\, \Delta_P,\, z_0\right), \tag{6}$$

where n is the radial excitation number, κ is the dilaton slope related to the quark content inside the mesons, z_0 is the hard cutoff locus related to the nature of the strong interactions inside the meson and Δ_P is the parameter that has the parity information and is fixed according to table I; $\chi_n(\kappa, z_0, \Delta_P)$ are the poles in the 2-point function expansion, that depend on the parameter choice.

Since all of the mesonic states fitted in this work are bounded states of light quarks, it is expected that the κ and z_0 have the same value. This comes from the particle phenomenology since radial Regge slope parameter should be universal for all the mesons [17, 18]. Numerical results are exposed in the table II, with the parameters choice given by $\kappa = 0.45$ GeV and $z_0 = 5.0$ GeV^{-1}.

4. Conclusions

In this work we have used the AdS/QCD soft wall model with an extra UV cutoff to model the masses of light mesons (with strangeness fixed to zero). The total RMS error related to fit 23 meson states with three parameters is 21.6%.

On the phenomenological side, is quite interesting to notice that the entire set of masses was fitted by three flavor independent parameters (κ, z_0 and Δ_P) giving some clues about the universality of the model. In fact, from QCD potentials, we learn that the string tension, that defines the mass spectrum in the Regge trajectories, is a flavor and spin independent quantity.

It is interesting to point out also that the model is no so good with the first states of each family. This is observed in other AdS/QCD models as well. Phenomenologically speaking, lower states in the trajectories are more sensitive to the Coulombian term in the potential. Thus, it is expected that higher excited states become more *linear*, implying that the confinement term is the dominant one. This fact is reproduced in the model developed here.

The next logical step is the extension of these ideas to other hadronic states such as strange mesons and baryons.

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Table II. Numerical results obtained for light meson states with $\kappa = 0.45$ GeV and $z_0 = 5.0$ GeV^{-1}, with a total RMS error equivalent to 21.6%. Experimental values are obtained from PDG [19]. For the $\eta(1760)$ and $\eta(2225)$ states, their masses are taken from [20, 21]. For the $a_1(1420)$ state, its mass is read from [22].

States given by I_{Scalar} (3)					States given by I_{Vector} (4)				
n	State	M_{Exp} (MeV)	M_{Th} (MeV)	$\%M$	n	State	M_{Exp} (MeV)	M_{Th} (MeV)	$\%M$
$\Delta_P = 0$									
1	$f_0(980)$	990 ± 20	1070	7.5	1	$\rho(770)$	775.26 ± 0.25	975	20.5
2	$f_0(1370)^*$	1325	1284	3.1	2	$\rho(1450)$	1465 ± 25	1455	1.0
3	$f_0(1500)$	1504 ± 6	1487	1.1	3	$\rho(1570)$	1570	1652	5.0
4	$f_0(1710)$	1723^{+6}_{-5}	1674	3.0	4	$\rho(1700)$	1720 ± 20	1829	6.0
5	$f_0(2020)$	2011^{+60}_{-80}	1846	8.2	5	$\rho(1900)$	1909	1992	4.0
6	$f_0(2100)$	2101	2153	2.4	6	$\rho(2150)$	2153	2142	0.5
7	$f_0(2200)$	2189	2292	4.5	7	Not seen	-	-	-
8	$f_0(2330)$	2314	2424	4.5	8	Not seen	-	-	-
$\Delta_P = -1$									
1	$\eta(550)$	547.86 ± 0.017	975.25	43.8	1	$a_1(1260)$	1230 ± 40	809.0	52.2
2	$\eta(1295)$	1294 ± 4	1233.6	50	2	$a_1(1420)$	$1414^{\pm 15}_{\pm 13}$	1114.7	26.9
3	$\eta(1405)$	1408.8 ± 1.8	1455.3	3.2	3	$a_1(1640)$	1654 ± 19	1351.3	22.4
4	$\eta(1475)$	1476 ± 4	1652.9	10.7	4	Not seen	-	-	-
5	$\eta(1760)$	1760 ± 11	1829.2	3.8	5	Not seen	-	-	-
6	$\eta(2225)$	2216 ± 21	1992.7	11.3	6	Not seen	-	-	-

References

[1] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. **323**, 183 (2000).
[2] J. M. Maldacena, Int. J. Theor. Phys. **38**, 1113 (1999) [Adv. Theor. Math. Phys. **2**, 231 (1998)].
[3] A. V. Ramallo, Springer Proc. Phys. **161**, 411 (2015)
[4] N. R. F. Braga, M. A. Martin Contreras and S. Diles, Phys. Lett. B **763**, 203 (2016).
[5] N. R. F. Braga and L. F. Ferreira, Phys. Lett. B **773**, 313 (2017)
[6] N. R. F. Braga, M. A. Martin Contreras and S. Diles, Eur. Phys. J. C **76**, no. 11, 598 (2016)
[7] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998) doi:10.4310/ATMP.1998.v2.n3.a3 [hep-th/9803131].
[8] P. Breitenlohner and D. Z. Freedman, Annals Phys. **144**, 249 (1982).
[9] P. Breitenlohner and D. Z. Freedman, Phys. Lett. **115B**, 197 (1982).
[10] S. Moroz, Phys. Rev. D **81**, 066002 (2010).
[11] P. Colangelo, F. De Fazio, F. Giannuzzi, F. Jugeau and S. Nicotri, Phys. Rev. D **78**, 055009 (2008)
[12] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D **74**, 015005 (2006)
[13] A. Vega and I. Schmidt, Phys. Rev. D **79**, 055003 (2009)
[14] S. Cortes, M. A. Martin Contreras and J. R. Roldan, Phys. Rev. D **96**, no. 10, 106002 (2017).
[15] M. A. Martin Contreras, A. Vega and S. Cortés, arXiv:1811.10731 [hep-ph].
[16] N. R. F. Braga, M. A. Martin Contreras and S. Diles, EPL **115**, no. 3, 31002 (2016)
[17] A. V. Anisovich, V. V. Anisovich and A. V. Sarantsev, Phys. Rev. D **62**, 051502 (2000).
[18] J. K. Chen, Phys. Lett. B **786**, 477 (2018) doi:10.1016/j.physletb.2018.10.022
[19] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98**, no. 3, 030001 (2018).
[20] D. M. Li and B. Ma, Phys. Rev. D **77**, 074004 (2008).
[21] L. M. Wang, S. Q. Luo, Z. F. Sun and X. Liu, Phys. Rev. D **96**, no. 3, 034013 (2017).
[22] C. Adolph *et al.* [COMPASS Collaboration], Phys. Rev. Lett. **115**, no. 8, 082001 (2015).