PAIRING COHERENCE LENGTH IN NUCLEI*

V.V. BARAN[1,2], D.S. DELION[1,3,4]

[1]"Horia Hulubei" National Institute of Physics and Nuclear Engineering,
407 Atomiştilor, POB MG-6, RO-077125, Bucharest-Măgurele, România
[2]Department of Physics, University of Bucharest, 405 Atomiştilor, POB MG-11, RO-077125, Bucharest-Măgurele, România
[3]Academy of Romanian Scientists, 54 Splaiul Independenţei RO-050085, Bucharest, România
[4]Bioterra University, 81 Gârlei RO-013724, Bucharest, România
E-mail: virgil.baran@theory.nipne.ro, delion@theory.nipne.ro

Received February 23, 2015

We study pairing correlations by analyzing the coherence length in a HF+BCS formalism with various types of pairing potentials. We compare the density dependent delta (DDD) pairing interaction to effective Gaussian interactions with different width parameters. We then generalize the discussion to quartet correlations, and finally we evaluate the importance of proton-neutron correlations.

Key words: Coherence length, Density-dependent pairing interaction, Gaussian pairing interaction.

PACS: 21.30.Fe, 24.10.Cn, 25.70.Ef.

Coherence is an ubiquitous concept in the study of many-body systems, in particular nuclei. For example, collective excitations are microscopically described by a superposition of creation pair operators acting on the ground state, described by a coherent state within the Random Phase Approximation. Ground state properties of even-even nuclei are well reproduced by a pairing type interaction [2–4] and the wave function within the BCS approach is also of a coherent type [1].

A great amount of information about nuclear correlation can be obtained by looking at the coherence properties of the spatial distribution of the two-particle density [5, 6]. The main physical quantity that describes the coherence property is the coherence length, defined as the root mean square relative distance averaged over the density (the pairing density for superfluid nuclei). Previous studies [7–9] show that this quantity is relatively large, of the order of the nuclear size inside the nucleus and smaller around the nuclear surface.

In this work we first shall compare the coherence length given by the density dependent delta (DDD) pairing interaction to that of the Gaussian interaction with various width parameters. Then, we will generalize to quartet correlations, and analyze the corresponding quartet coherence length.

*Paper presented at the conference "Advanced many-body and statistical methods in mesoscopic systems II", September 1-5, 2014, Brasov, Romania.

1. THEORETICAL BACKGROUND

The most important two-body correlations beyond the mean field in even-even nuclei are given by the pairing interaction. We describe such systems within the standard HF+BCS approach, where the averaged particle number is conserved, separately for protons and neutrons. We consider in our basis bound sp states with negative energy, as well as relatively narrow sp resonances with positive energy up to $e_{max} =$ 10 MeV with a decay width $\Gamma \leq 1$ MeV (the background contribution is not relevant [13, 14]), given by a Woods Saxon central field with universal parameterization [16].

The information about the spatial properties of pairing correlations is contained in the anomalous density $\kappa(\mathbf{r_1},\mathbf{s_1};\mathbf{r_2},\mathbf{s_2}) = \langle BCS|\mathcal{A}\{\hat{\psi}(\mathbf{r_1},\mathbf{s_1})\hat{\psi}(\mathbf{r_2},\mathbf{s_2})\}|BCS\rangle$. After expanding in our basis of states and recoupling from the j-j to the L-S scheme, we retain only largest singlet component, $\kappa_1(\mathbf{r_1},\mathbf{r_2})$. By passing to the relative $\mathbf{r} = \mathbf{r_1} - \mathbf{r_2}$ and center of mass coordinate $\mathbf{R} = (\mathbf{r_1} + \mathbf{r_2})/2$ and averaging over their relative angle θ one obtains the averaged pairing tensor $\bar{\kappa}_1^2(r,R)$. We define the coherence length to be the relative distance averaged over this pairing tensor:

$$\xi(R) = \sqrt{\frac{\int \mathrm{d}r\ r^4\ \bar{\kappa}_1^2(r,R)}{\int \mathrm{d}r\ r^2\ \bar{\kappa}_1^2(r,R)}} \equiv \sqrt{\int \mathrm{d}r\ r^2\ w(r,R)} \tag{1}$$

2. NUMERICAL APPLICATION AND RESULTS

We analyzed all even-even nuclei with $20 < Z < 100$ and known experimental pairing gaps, determined by the binding energies of neighboring nuclei [15]. We solved the BCS equations by using two types of nucleon-nucleon pairing interactions. The use of the density dependent delta (DDD) interaction [12], $v(\mathbf{r},\mathbf{r}') = u_0\ \delta(\mathbf{r}-\mathbf{r}')\left(1-\rho_N(\mathbf{r})/\rho_N^{(0)}\right)$, given in terms of the nuclear density ρ_N and its central value $\rho_N^{(0)}$, is motivated by the fact that the strength of the effective pairing interaction depends upon the local density [11, 18]. The widely used Gaussian interaction is defined by $v(r_{12}) = -v_0 e^{-[r_{12}/r_0]^2}$ depending on the relative radius r_{12}. For the in-medium effective interaction there is no a priori reason to consider that its parameters are equal to those of the bare interaction. We can compare the cases r_0=2 fm, corresponding to the singlet "bare" value in the free space with the larger $r_0 = R_N = 1.2A^{1/3}$, corresponding to the nuclear geometrical radius. An extended radial dimensions of the pairing interaction also gives better results for the experimental α-decay widths [10]. The strength v_0 is chosen as for the Fermi level gap to reproduce the experimental gap.

In Fig. 1, we see large gap values for states below the Fermi level for the bare Gaussian. The values of the gaps given by the DDD interaction below the Fermi level

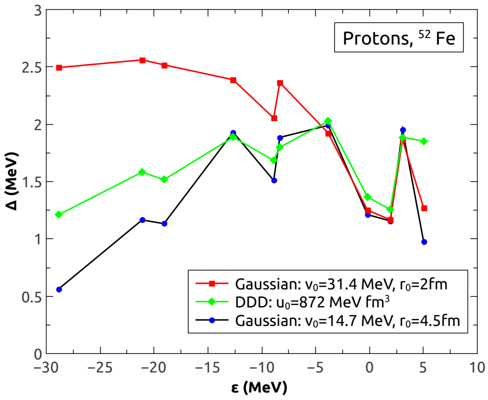

Fig. 1 – Paring gap *versus* sp energy ϵ in ^{52}Fe for DDD potential (diamonds) and Gaussian potentials with r_0= 2 fm (squares), $r_0 = R_N$ (circles).

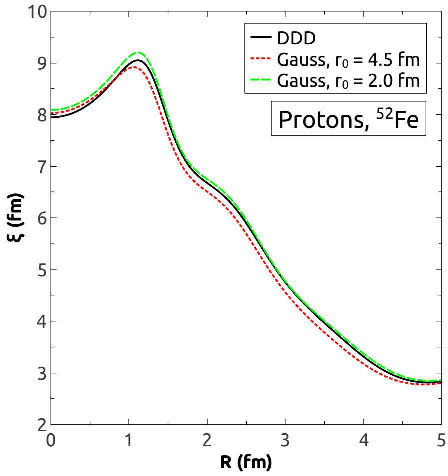

Fig. 2 – Proton coherence length divided by geometrical radius *versus* cm radius divided by geometrical radius in ^{52}Fe for DDD potential (solid line) and Gaussian potentials with r_0= 2 fm (long dashes), $r_0 = R_N$ (short dashes).

are significantly smaller than the Fermi gap and are similar with those given by the $r_0 = R_N$ Gaussian. We note that the corresponding coherence lengths are all very close in shape for all considered interactions, and in agreement with Ref. [7].

A systematic analysis for even-even nuclei with $20 < Z < 100$ revealed that the strength of the $r_0 = 2$fm Gaussian has almost the singlet "bare" value in the free space $v_0 \sim 35$ MeV for very light nuclei and as a result of the renormalization, the effective strength decreases down to $v_0 \sim 20$ MeV for heavy nuclei, except in the regions around magic numbers. For the bare Gaussian, the ratio of the coherence length to the nuclear radius decreases from 1.4 for light nuclei down to around unity for heavy nuclei. As a general trend, the coherence length is larger for neutrons, but the shell effects are stronger for protons. We then investigated the density dependent pairing interaction and we concluded that the strengths of the two analyzed interactions have similar properties for $Z > 40$.

The renormalization procedure for the realistic interaction like Bonn or Paris potentials [17] should determine which is the actual size of strength and width parameters of the effective pairing interaction.

Next we turn to quartet correlations. By considering the overlaps of the proton-proton and respectively neutron-neutron wave functions with the two-proton and respectively two-neutron part of the α-particle wave-function, we construct the quar-

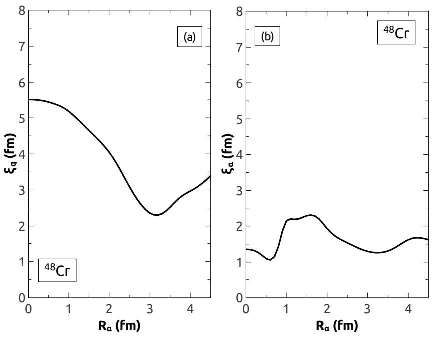

Fig. 3 – Quartet (a) and alpha (b) coherence lengths in ^{48}Cr.

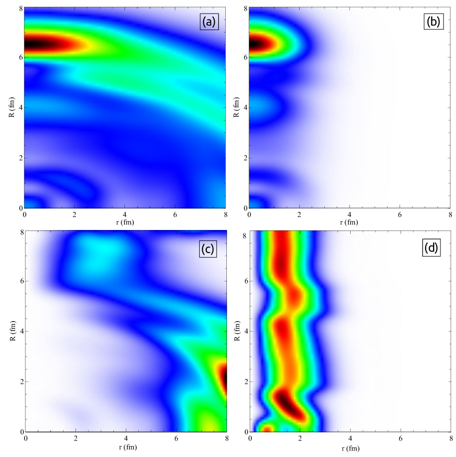

Fig. 4 – Quartet (a) and alpha (b) correlation tensors $\kappa_q(r\alpha, R\alpha)^2$ and $\kappa_\alpha(r\alpha, R\alpha)^2$. Quartet (c) and alpha (d) correlation densities $w_{q,\alpha}(r\alpha, R\alpha)$ defined as in Eq. (1), for ^{220}Ra.

tet tensor $\kappa_q(\mathbf{R}_\pi, \mathbf{R}_\nu) = \langle \kappa_\pi(\mathbf{r_1}, \mathbf{r_2}) | \phi_{00}^{(\beta_\alpha/2)}(r_\pi) \rangle \cdot \langle \kappa_\nu(\mathbf{r_3}, \mathbf{r_4}) | \phi_{00}^{(\beta_\alpha/2)}(r_\nu) \rangle$, where $\mathbf{r}_{\pi,\nu} = \mathbf{r}_{1,3} - \mathbf{r}_{2,4}$, $\mathbf{R}_{\pi,\nu} = (\mathbf{r}_{1,3} + \mathbf{r}_{2,4})/2$ an $\beta_\alpha \approx 0.5 fm^{-2}$. This plays the role of the pairing tensor in the quarteting case. By proceeding similarly with the pairing case (see Eq.1) we may now define the quartet coherence length ξ_q which is shown in Fig. 3 for ^{48}Cr as a function of the center of mass of the two pairs $\mathbf{R}_\alpha = (\mathbf{R}_\pi + \mathbf{R}_\nu)/2$.

By taking into account proton-neutron correlations, described by the part in the α-particle wave function dependent on the relative distance $\mathbf{r}_\alpha = \mathbf{R}_\pi - \mathbf{R}_\nu$, we may also define the so-called alpha coherence length. The corresponding correlation tensor is $\kappa_\alpha(r_\alpha, R_\alpha) = \kappa_q(r_\alpha, R_\alpha)\phi_{00}^{(\beta_\alpha)}(r_\alpha)$, and the coherence length is $\xi_\alpha(R_\alpha)$.

We see from Fig.3 that the quartet coherence length bears some resemblance to the pairing coherence length, being larger in the internal region. Taking into account proton-neutron correlations significantly alters its shape, as the α-coherence length oscillates about the geometrical dimension of the α-particle of 1.9 fm.

We also present the shapes of the quartet and α correlation tensors and densities (defined as in Eq.1) in Fig.4. We note that while in the quartet case the quantities are significant also at large distances, in the α case the pn correlations only allow nonzero values at a small r_α.

3. CONCLUSIONS

We have performed an analysis of the coherence length for various types of pairing interaction. We compared DDD potential to various parameterizations of the Gaussian interaction. We have shown that the strength of the bare width Gaussian for light nuclei is close to the singlet "bare" value in the free space $v_0 \sim 35$ MeV and decreases for heavy nuclei. Also, a larger width Gaussian has similar properties to the commonly used density dependent pairing potential, the closest being the case with $r_0 = R_N$.

It turns out that the coherence length has similar properties for all considered interactions. It is larger than the geometrical radius for light nuclei and approaches this value for heavy nuclei.

We also presented the generalization from pairing to quarteting correlations, and analyzed the quarteting correlation tensor and density that give the quarteting coherence length. It turns out that proton-neutron correlations are very important, as they give a completely different shape for the corresponding α-coherence length.

Acknowledgements. This work has been supported by the project PN-II-ID-PCE-2011-3-0092 and NuPNET-SARFEN of the Romanian Ministry of Education and Research.

REFERENCES

1. Ring P and Schuck P 1980 *The Nuclear Many Body Problem* (Springer-Verlag, New York-Berlin).
2. Dean D J and Hjorth-Jensen M 2003 *Rev. Mod. Phys.* **75** 607
3. Zelevinsky V and Volya A 2004 *Nucl. Phys.* A **731** 299
4. Yoshida S and Sagawa H 2008 *Phys. Rev.* C **77** 054308
5. Ferreira L, Liotta R J, Dasso C H, Broglia R A and Winther A 1984 *Nucl. Phys.* A **426** 276
6. Hagino K and Sagawa H 2005 *Phys. Rev.* C **72** 044321
7. Pillet N, Sandulescu N and Schuck P 2007 *Phys. Rev.* C **76** 024310
8. Pillet N, Sandulescu N, Schuck P and Berger J-F 2010 *Phys. Rev.* C **81** 034307
9. Viñas X, Schuck P and Pillet N 2010 *Phys. Rev.* C **82** 034314
10. Delion D S 2010 *Theory of Particle and Cluster Emission*, (Springer-Verlag, New York-Berlin)
11. Bertsch G and Esbensen H 1991, *Ann. Phys.* (N.Y.) **209** 327
12. Dobaczewski J, Nazarewicz W, Werner T R, Berger J F, Chinn C R and Decharge J 1996 *Phys. Rev.* C **53** 2809
13. Betan R Id, Dussel G G and Liotta R J 2008 *Phys. Rev.* C **78** 044325
14. Betan R Id 2012 *Nucl. Phys.* A **879** 14
15. Möller P and Nix J R 1995 *Nucl. Phys.* A **272** 502
16. Dudek J, Nazarewicz W and Werner T 1980 *Nucl. Phys.* A **341** 253
17. Delion D S, Baldo M and Lombardo U 1995 *Nucl. Phys.* A **593** 151
18. Borycki P J, Dobaczewski J, Nazarewicz W and Stoitsov M V 2006 *Phys. Rev.* C **73** 044319