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Full bispectra from primordial scalar and tensor perturbations in the most general single-field inflation model

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We compute the full bispectra, namely both auto and cross bispectra, of primordial curvature and tensor perturbations in the most general single-field inflation model whose scalar and gravitational equations of motion are of second order. The formulae in the limits of k-inflation and potential-driven inflation are also given. These expressions are useful for estimating the full bispectra of temperature and polarization anisotropies of the cosmic microwave background radiation.

Subject Index E80

1. Introduction

The non-Gaussianities of the temperature and polarization anisotropies of the cosmic microwave background (CMB) radiation are currently receiving increasing attention because they are important tools for discriminating models of inflation [1–3]. Ongoing and near-future projects such as the Planck satellite [4], the CMBpol mission [5], and the LiteBIRD satellite [6] should reveal the properties of the temperature and polarization anisotropies in detail. Such E-mode polarization anisotropies are sourced by both curvature and tensor perturbations [7–12], while only tensor (and vector) perturbations can generate B-mode polarization anisotropies [13,14].[1] Therefore, even when one estimates

[1] Though vector perturbations can also generate both E-mode and B-mode polarization anisotropies, they only have a decaying mode in linear theory and hence are suppressed in the standard inflationary cosmology based on scalar fields.

the "auto" bispectra of the temperature and E-mode polarization fluctuations, both the auto bispectra and the cross bispectra of the primordial curvature and tensor perturbations are indispensable.

For slow-roll inflation models with the canonical kinetic term [15–17], Maldacena evaluated the full bispectra, including the cross bispectra, of the primordial curvature and tensor perturbations [18]. Inflation models are now widely generalized into more varieties such as k-inflation [19,20], DBI inflation [21], ghost inflation [22], G-inflation [23,24], and so on. However, almost all the works on the non-Gaussianities in these inflation models concentrate only on the auto bispectrum of the curvature perturbations [25–30], which is insufficient for evaluating the bispectra of the temperature and E-mode polarization anisotropies of the CMB, as explained above. To our surprise, as far as we know, the full bispectra of the primordial curvature and tensor perturbations have not yet been obtained even for k-inflation [19,20], except for Ref. [31], where the primordial scalar–scalar–tensor cross bispectrum has been calculated for inflation models with an arbitrary kinetic term. There are several related works on the primordial cross bispectra. In Refs. [32,33], the authors show the primordial tensor–scalar cross bispectra induced from a holographic model and the scalar–scalar–tensor correlation has been discussed in the calculation of the trispectrum of the scalar fluctuations [34], the so-called "graviton exchange", and also in the context of the one-loop effects of the scalar power spectrum [35]. In Refs. [36,37], the authors calculate the correlation between primordial scalar and vector (magnetic fields) fluctuations in possible inflationary models of generating primordial magnetic fields.

Among the inflation zoo, the generalized G-inflation model [38] occupies a unique position in that it includes practically all the known well behaved single inflation models, since it is based on the most general single-field scalar–tensor Lagrangian with the second-order equation of motion, which was proposed by Horndeski more than thirty years ago [39] and was recently rediscovered in the context of the generalized Galileon [40,41]. Indeed, it includes standard canonical inflation [1,2,15–17], non-minimally coupled inflation [42–47] including Higgs inflation [48–53], extended inflation [54], k-inflation [19,20], DBI inflation [21], R^2 inflation [3,55], new Higgs inflation [56], G-inflation [23,24], and so on. Thus, once we analyze the properties of the primordial curvature and tensor perturbations in the generalized G-inflation, one can apply the result for any specific single-field inflation model.

So far, the power spectra of scalar and tensor fluctuations have been studied in Ref. [38] and the general formulae for them are given there. It has been pointed out that the sound velocity squared of the tensor perturbations as well as that of the curvature perturbations can deviate from unity. Then the auto bispectrum of the curvature perturbations was estimated in Refs. [57,58] (see also Refs. [59,60]) and found to be enhanced by the inverse sound velocity squared and so on. More recently, the auto bispectrum of the tensor perturbations was investigated in Ref. [61] and found to be composed of two parts. The first is the universal one, similar to that from Einstein gravity, and predicts a squeezed shape, while the other comes from the presence of the kinetic coupling to the Einstein tensor and predicts an equilateral shape. What remains to be studied are the bispectra of the primordial curvature and tensor perturbations in the generic theory.

In the case of the most general single-field model, not only the auto bispectrum of scalar perturbations but also that of tensor perturbations can be large enough to be detected by cosmological observations, e.g., the Planck satellite, as explained in Ref. [61], which suggests that cross bispectra can be large as well. For such a case, it is not necessarily justified to consider only the auto bispectrum of curvature perturbations, even when you evaluate the auto bispectrum of temperature (or E-mode) fluctuations, because cross ones can significantly contribute to it even if the tensor-to-scalar ratio is (relatively) small. Furthermore, when we try to evaluate the cross bispectra including B-mode fluctuations, the cross bispectra of tensor and scalar perturbations are indispensable because B-mode fluctuations are produced only from tensor perturbations. These facts are quite manifest even without any reference or estimation.

In such a situation, in this paper, we compute the cross bispectra of the primordial curvature and tensor perturbations in the generalized G-inflation model. The formulae in the limits of k-inflation and potential-driven inflation are also given as specific examples.

The organization of this paper is given as follows. In the next section, we briefly review the most general single-field scalar–tensor Lagrangian with the second-order equation of motion. In Sect. 3, quadratic and cubic actions for the primordial curvature and tensor perturbations are given. The full bispectra, including the cross ones, for them are discussed in Sect. 4. The special limits for them in the cases of k-inflation and potential-driven inflation are taken in Sect. 5. The final section is devoted to a conclusion and discussion.

2. Generalized G-inflation—The most general single-field inflation model

The Lagrangian for generalized G-inflation is the most general one that is composed of the metric $g_{\mu\nu}$ and a scalar field ϕ together with their arbitrary derivatives but still yields second-order field equations. The Lagrangian was first derived by Horndeski in 1974 in four dimensions [39], and very recently it was rediscovered in a modern form as the generalized Galileon [40], i.e., the most general extension of the Galileon [62,63], in arbitrary dimensions. Their equivalence in four dimensions has been shown in Ref. [38]. The four-dimensional generalized Galileon is described by the Lagrangian:

$$\mathcal{L} = K(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] \\ + G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3], \tag{1}$$

where K and G_i are arbitrary functions of ϕ and its canonical kinetic term $X := -(\partial\phi)^2/2$. We are using the notation G_{iX} for $\partial G_i/\partial X$. The generalized Galileon can be used as a framework to study the most general single-field inflation model. Generalized G-inflation contains novel models, as well as previously known models of single-field inflation such as standard canonical inflation, k-inflation, extended inflation, and new Higgs inflation, and even R^2 or $f(R)$ inflation (with an appropriate field redefinition). The above Lagrangian can also reproduce the non-minimal coupling to the Gauss–Bonnet term [38].

3. General quadratic and cubic actions for cosmological perturbations

In this section, we present the quadratic and cubic actions for scalar- and tensor-type cosmological perturbations based on the most general single-field inflation model. Employing the Arnowitt–Deser–Misner formalism, we write the metric as

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + g_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t), \tag{2}$$

where

$$N = 1 + \alpha, \quad N_i = \partial_i\beta, \quad g_{ij} = a^2(t)e^{2\zeta}(e^h)_{ij}, \tag{3}$$

and $(e^h)_{ij} = \delta_{ij} + h_{ij} + (1/2)h_{ik}h_{kj} + \cdots$. We work in the gauge in which the fluctuation of the scalar field vanishes, $\phi = \phi(t)$. Concerning the perturbations of the lapse function and shift vector, α and β, it is sufficient to consider the first-order quantities to compute the cubic actions, as pointed out in Ref. [18]. The first-order vector perturbations may be dropped. The curvature perturbation in generalized G-inflation is shown to be conserved on large scales at nonlinear order in Ref. [64].

Substituting the above metric into the action and expanding it to third order, we obtain the action for the cosmological perturbations, which will be written, with trivial notations, as

$$S = \int \mathrm{d}t\mathrm{d}^3x(\mathcal{L}_{hh} + \mathcal{L}_{ss} + \mathcal{L}_{hhh} + \mathcal{L}_{shh} + \mathcal{L}_{ssh} + \mathcal{L}_{sss}). \tag{4}$$

The first two Lagrangians are quadratic in the metric perturbations, which have already been obtained in Ref. [38]. To define some notations used in this paper, we will begin with summarizing the quadratic results in the next subsection. The third and last cubic Lagrangians have been derived in Refs. [61] and [57,58], respectively, but for completeness they are also reproduced in this section. The mixture of the scalar and tensor perturbations, $\mathcal{L}_{shh}$ and $\mathcal{L}_{ssh}$, is computed for the first time in this paper.

3.1. Quadratic Lagrangians and primordial power spectra

The quadratic terms are obtained as follows [38].

3.1.1. Tensor perturbations. The most general quadratic Lagrangian for tensor perturbations is given by

$$\mathcal{L}_{hh} = \frac{a^3}{8}\left[\mathcal{G}_T\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}h_{ij,k}h_{ij,k}\right], \tag{5}$$

where

$$\mathcal{F}_T := 2[G_4 - X(\ddot{\phi}G_{5X} + G_{5\phi})], \tag{6}$$

$$\mathcal{G}_T := 2[G_4 - 2XG_{4X} - X(H\dot{\phi}G_{5X} - G_{5\phi})]. \tag{7}$$

Here, a dot indicates a derivative with respect to t, $G_{i\phi} := \partial G_i/\partial\phi$, and the propagation speed of gravitational waves is defined as $c_h^2 := \mathcal{F}_T/\mathcal{G}_T$.[2] The linear equation of motion derived from the Lagrangian (5) is

$$E_{ij}^h := \partial_t(a^3\mathcal{G}_T\dot{h}_{ij}) - a\mathcal{F}_T\partial^2 h_{ij} = 0. \tag{8}$$

In deriving the above equations, we have not assumed that the background evolution is close to de Sitter. They can therefore be used for an arbitrary homogeneous and isotropic cosmological background.

We now move to the Fourier space to solve this equation:

$$h_{ij}(t, \mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}h_{ij}(t, \mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{9}$$

It is convenient to use the conformal time coordinate defined by $\mathrm{d}\eta = \mathrm{d}t/a$. We approximate the inflationary regime by de Sitter spacetime and take $\mathcal{F}_T$ and $\mathcal{G}_T$ to be constant.[3]

[2] In cases where the graviton propagation speed is smaller than light speed, nothing special happens: the light-cone alone determines the causality. In the opposite case, it has been argued that such a theory cannot be UV-completed as a Lorentz-invariant theory [65], though others reach the opposite conclusion [66]. We need further investigation in this case.

[3] As seen in Eqs. (27) and (28), $\mathcal{F}_S$ and $\mathcal{G}_S$ depend on $\mathcal{F}_T$ and $\mathcal{G}_T$. The time derivatives of $\mathcal{F}_S$ and $\mathcal{G}_S$ affect the spectral index of the power spectrum of the scalar curvature perturbations and they are required to be small from the current cosmological observations. Hence, the assumption that the time derivatives of $\mathcal{F}_T$ and $\mathcal{G}_T$ are small are natural from observational perspectives, although one cannot rule out the case where $\mathcal{F}_T$ and $\mathcal{G}_T$ have strong time-dependence without conflicting the current cosmological observations, strictly speaking. In this exceptional case, we must say that the assumption that the time derivatives of $\mathcal{F}_T$ and $\mathcal{G}_T$ are small is made just for simplicity.

The quantized tensor perturbation is written as

$$h_{ij}(\eta,\mathbf{k}) = \sum_s [h_{\mathbf{k}}(\eta)e^{(s)}_{ij}(\mathbf{k})a_s(\mathbf{k}) + h^*_{-\mathbf{k}}(\eta)e^{*(s)}_{ij}(-\mathbf{k})a^\dagger_s(-\mathbf{k})], \tag{10}$$

where under these approximations the normalized mode is given by

$$h_{\mathbf{k}}(\eta) = \frac{i\sqrt{2}H}{\sqrt{\mathcal{F}_T c_h k^3}}(1 + ic_h k\eta)e^{-ic_h k\eta}. \tag{11}$$

Here, $e^{(s)}_{ij}$ is the polarization tensor with the helicity states $s = \pm 2$, satisfying $e^{(s)}_{ii}(\mathbf{k}) = 0 = k_j e^{(s)}_{ij}(\mathbf{k})$. We adopt the normalization such that

$$e^{(s)}_{ij}(\mathbf{k})e^{*(s')}_{ij}(\mathbf{k}) = \delta_{ss'}, \tag{12}$$

and choose the phase so that the following relations hold:

$$e^{*(s)}_{ij}(\mathbf{k}) = e^{(-s)}_{ij}(\mathbf{k}) = e^{(s)}_{ij}(-\mathbf{k}). \tag{13}$$

The commutation relation for the creation and annihilation operators is

$$[a_s(\mathbf{k}), a^\dagger_{s'}(\mathbf{k}')] = (2\pi)^3\delta_{ss'}\delta(\mathbf{k} - \mathbf{k}'). \tag{14}$$

The two-point function can be written as

$$\langle h_{ij}(\mathbf{k})h_{kl}(\mathbf{k}')\rangle = (2\pi)^3\delta(\mathbf{k} + \mathbf{k}')\mathcal{P}_{ij,kl}(\mathbf{k}), \tag{15}$$

$$\mathcal{P}_{ij,kl}(\mathbf{k}) = |h_{\mathbf{k}}|^2\Pi_{ij,kl}(\mathbf{k}), \tag{16}$$

where

$$\Pi_{ij,kl}(\mathbf{k}) = \sum_s e^{(s)}_{ij}(\mathbf{k})e^{*(s)}_{kl}(\mathbf{k}). \tag{17}$$

The power spectrum, $\mathcal{P}_h = (k^3/2\pi^2)\mathcal{P}_{ij,ij}$, is thus computed as

$$\mathcal{P}_h = \frac{2}{\pi^2}\frac{H^2}{\mathcal{F}_T c_h}\bigg|_{c_h k\eta = -1}. \tag{18}$$

3.1.2. Scalar perturbations. The quadratic Lagrangian for the scalar perturbations is given by

$$\mathcal{L}_{ss} = a^3\left[-3\mathcal{G}_T\dot{\zeta}^2 + \frac{\mathcal{F}_T}{a^2}\zeta_{,i}\zeta_{,i} + \Sigma\alpha^2 - \frac{2}{a^2}\Theta\alpha\beta_{,ii} + \frac{2}{a^2}\mathcal{G}_T\dot{\zeta}\beta_{,ii} + 6\Theta\alpha\dot{\zeta} - \frac{2}{a^2}\mathcal{G}_T\alpha\zeta_{,ii}\right], \tag{19}$$

where

$$\begin{aligned}\Sigma := {} & XK_X + 2X^2K_{XX} + 12H\dot{\phi}XG_{3X} + 6H\dot{\phi}X^2G_{3XX} - 2XG_{3\phi} - 2X^2G_{3\phi X}\\ & - 6H^2G_4 + 6[H^2(7XG_{4X} + 16X^2G_{4XX} + 4X^3G_{4XXX})\\ & - H\dot{\phi}(G_{4\phi} + 5XG_{4\phi X} + 2X^2G_{4\phi XX})]\\ & + 30H^3\dot{\phi}XG_{5X} + 26H^3\dot{\phi}X^2G_{5XX} + 4H^3\dot{\phi}X^3G_{5XXX}\\ & - 6H^2X(6G_{5\phi} + 9XG_{5\phi X} + 2X^2G_{5\phi XX}),\end{aligned} \tag{20}$$

$$\begin{aligned}\Theta := {} & -\dot{\phi}XG_{3X} + 2HG_4 - 8HXG_{4X} - 8HX^2G_{4XX} + \dot{\phi}G_{4\phi} + 2X\dot{\phi}G_{4\phi X}\\ & - H^2\dot{\phi}(5XG_{5X} + 2X^2G_{5XX}) + 2HX(3G_{5\phi} + 2XG_{5\phi X}).\end{aligned} \tag{21}$$

Varying Eq. (19) with respect to α and β, we get the first-order constraint equations:

$$\Sigma\alpha - \frac{\Theta}{a^2}\partial^2\beta + 3\Theta\dot{\zeta} - \frac{\mathcal{G}_T}{a^2}\partial^2\zeta = 0, \tag{22}$$

$$\Theta\alpha - \mathcal{G}_T\dot{\zeta} = 0, \tag{23}$$

which are solved to yield

$$\alpha = \frac{\mathcal{G}_T}{\Theta}\dot{\zeta}, \tag{24}$$

$$\beta = \frac{1}{a\mathcal{G}_T}\left(a^3\mathcal{G}_S\psi - \frac{a\mathcal{G}_T^2}{\Theta}\zeta\right), \tag{25}$$

with $\psi := \partial^{-2}\dot{\zeta}$. Plugging Eqs. (24) and (25) into Eq. (19), we obtain

$$\mathcal{L}_{ss} = a^3\left[\mathcal{G}_S\dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}\zeta_{,i}\zeta_{,i}\right], \tag{26}$$

where we have defined

$$\mathcal{F}_S := \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a}{\Theta}\mathcal{G}_T^2\right) - \mathcal{F}_T, \tag{27}$$

$$\mathcal{G}_S := \frac{\Sigma}{\Theta^2}\mathcal{G}_T^2 + 3\mathcal{G}_T. \tag{28}$$

The sound speed is given by $c_s^2 := \mathcal{F}_S/\mathcal{G}_S$. The linear equation of motion derived from the Lagrangian (26) is

$$E^s := \partial_t(a^3\mathcal{G}_S\dot{\zeta}) - a\mathcal{F}_S\partial^2\zeta = 0. \tag{29}$$

The scalar two-point function can be calculated in a way similar to the case of the tensor perturbations. We move to the Fourier space:

$$\zeta(t,\mathbf{x}) = \int\frac{\mathrm{d}^3k}{(2\pi)^3}\zeta(t,\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{30}$$

and proceed in the de Sitter approximation, assuming that $\mathcal{F}_S$ and $\mathcal{G}_S$ are almost constant. The quantized curvature perturbation is written as

$$\zeta(\eta,\mathbf{k}) = \xi_{\mathbf{k}}(\eta)a(\mathbf{k}) + \xi^*_{-\mathbf{k}}(\eta)a^\dagger(-\mathbf{k}), \tag{31}$$

where the normalized mode is given by

$$\xi_{\mathbf{k}}(\eta) = \frac{iH}{2\sqrt{\mathcal{F}_S c_s k^3}}(1 + ic_s k\eta)e^{-c_s k\eta}. \tag{32}$$

The commutation relation for the creation and annihilation operators is

$$[a(\mathbf{k}), a^\dagger(\mathbf{k}')] = (2\pi)^3\delta(\mathbf{k}-\mathbf{k}'). \tag{33}$$

Thus, the power spectrum is calculated as

$$\langle\zeta(\mathbf{k})\zeta(\mathbf{k}')\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}\mathcal{P}_\zeta, \tag{34}$$

$$\mathcal{P}_\zeta = \frac{1}{8\pi^2}\frac{H^2}{\mathcal{F}_S c_s}\bigg|_{c_s k\eta=-1}. \tag{35}$$

From Eqs. (18) and (35), the tensor-to-scalar ratio r is given by

$$r := \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} = 16\frac{\mathcal{F}_S c_s}{\mathcal{F}_T c_h}, \tag{36}$$

where we have assumed that the relevant quantities remain practically constant between the horizon crossings of tensor and scalar perturbations that occur at different times in the case $c_h \neq c_s$ [67].

3.2. *Cubic Lagrangians*

We now present the most general cubic Lagrangians composed of the tensor and scalar perturbations. We would like to emphasize that in deriving the following Lagrangians the slow-roll approximation is *not* used, as discussed in the literature [68,69].

3.2.1. Three tensors. The Lagrangian involving three tensors was derived in Ref. [61]:

$$\mathcal{L}_{hhh} = a^3\left[\frac{\mu}{12}\dot{h}_{ij}\dot{h}_{jk}\dot{h}_{ki} + \frac{\mathcal{F}_T}{4a^2}\left(h_{ik}h_{jl} - \frac{1}{2}h_{ij}h_{kl}\right)h_{ij,kl}\right], \tag{37}$$

where we defined

$$\mu := \dot{\phi}XG_{5X}. \tag{38}$$

As discussed in Ref. [61], this cubic action for the tensor perturbation h_{ij} is composed only of two contributions. The former has one time derivative on each h_{ij} and newly appears in the presence of the kinetic coupling to the Einstein tensor, i.e., $G_{5X} \neq 0$. On the other hand, the latter has two spacial derivatives and is essentially identical to the cubic term that appears in Einstein gravity. Therefore, in what follows, we use the terminologies "new" and "GR" for the corresponding terms.

3.2.2. Two tensors and one scalar. The interactions involving two tensors and one scalar are given by

$$\begin{aligned}\mathcal{L}_{shh} = a^3&\left[\frac{3\mathcal{G}_T}{8}\zeta\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{8a^2}\zeta h_{ij,k}h_{ij,k} - \frac{\mu}{4}\dot{\zeta}\dot{h}_{ij}^2 - \frac{\Gamma}{8}\alpha\dot{h}_{ij}^2 - \frac{\mathcal{G}_T}{8a^2}\alpha h_{ij,k}h_{ij,k} - \frac{\mu}{2a^2}\alpha\dot{h}_{ij}h_{ij,kk}\right]\\ &- a\left[\frac{\mathcal{G}_T}{4}\beta_{,k}\dot{h}_{ij}h_{ij,k} + \frac{\mu}{2}\left(\dot{h}_{ik}\dot{h}_{jk}\beta_{,ij} - \frac{1}{2}\dot{h}_{ij}^2\beta_{,kk}\right)\right],\end{aligned} \tag{39}$$

where

$$\begin{aligned}\Gamma := {}&2G_4 - 8XG_{4X} - 8X^2G_{4XX}\\ &- 2H\dot{\phi}(5XG_{5X} + 2X^2G_{5XX}) + 2X(3G_{5\phi} + 2XG_{5\phi X}).\end{aligned} \tag{40}$$

This quantity can also be expressed in a compact form, $\Gamma = \partial\Theta/\partial H$.

Substituting the first-order constraint equations into Eq. (39), the Lagrangian reduces to

$$\begin{aligned}\mathcal{L}_{shh} = a^3&\left[b_1\zeta\dot{h}_{ij}^2 + \frac{b_2}{a^2}\zeta h_{ij,k}h_{ij,k} + b_3\psi_{,k}\dot{h}_{ij}h_{ij,k} + b_4\dot{\zeta}\dot{h}_{ij}^2 + \frac{b_5}{a^2}\partial^2\zeta\dot{h}_{ij}^2\right.\\ &\left.+ b_6\psi_{,ij}\dot{h}_{ik}\dot{h}_{jk} + \frac{b_7}{a^2}\zeta_{,ij}\dot{h}_{ik}\dot{h}_{jk}\right] + E_{shh},\end{aligned} \tag{41}$$

where

$$b_1 = \frac{3\mathcal{G}_T}{8}\left[1 - \frac{H\mathcal{G}_T^2}{\Theta\mathcal{F}_T} + \frac{\mathcal{G}_T}{3}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathcal{G}_T}{\Theta\mathcal{F}_T}\right)\right], \tag{42}$$

$$b_2 = \frac{\mathcal{F}_S}{8}, \tag{43}$$

$$b_3 = -\frac{\mathcal{G}_S}{4}, \tag{44}$$

$$b_4 = \frac{\mathcal{G}_T}{8\Theta\mathcal{F}_T}\left(\mathcal{G}_T^2 - \Gamma\mathcal{F}_T\right) + \frac{\mu}{4}\left[\frac{\mathcal{G}_S}{\mathcal{G}_T} - 1 - \frac{H\mathcal{G}_T^2}{\Theta\mathcal{F}_T}\left(6 + \frac{\dot{\mathcal{G}}_S}{H\mathcal{G}_S}\right)\right] + \frac{\mathcal{G}_T^2}{4}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mu}{\Theta\mathcal{F}_T}\right), \tag{45}$$

$$b_5 = \frac{\mu\mathcal{G}_T}{4\Theta}\left(\frac{\mathcal{F}_S\mathcal{G}_T}{\mathcal{F}_T\mathcal{G}_S} - 1\right), \tag{46}$$

$$b_6 = -\frac{\mu}{2}\frac{\mathcal{G}_S}{\mathcal{G}_T}, \tag{47}$$

$$b_7 = \frac{\mu}{2}\frac{\mathcal{G}_T}{\Theta}, \tag{48}$$

and

$$E_{shh} = \frac{\mu}{4\mathcal{G}_S}\frac{\mathcal{G}_T^2}{\Theta\mathcal{F}_T}\dot{h}_{ij}^2 E^s + \frac{\mathcal{G}_T^2}{2\Theta\mathcal{F}_T}\left(\frac{\zeta}{2} + \frac{\mu}{\mathcal{G}_T}\dot{\zeta}\right)\dot{h}_{ij}E_{ij}^h. \tag{49}$$

The last term, E_{shh}, can be removed by redefining the fields as

$$h_{ij} \to h_{ij} + \frac{\mathcal{G}_T^2}{\Theta\mathcal{F}_T}\left(\zeta + \frac{2\mu}{\mathcal{G}_T}\dot{\zeta}\right)\dot{h}_{ij}, \tag{50}$$

$$\zeta \to \zeta + \frac{\mu}{8\mathcal{G}_S}\frac{\mathcal{G}_T^2}{\Theta\mathcal{F}_T}\dot{h}_{ij}^2. \tag{51}$$

The contribution to the correlation function is, however, negligible, because the above field redefinitions involve at least one time derivative of the metric perturbation, which vanishes on super-horizon scales.

3.2.3. Two scalars and one tensor. The interactions involving one tensor and two scalars are given by

$$\mathcal{L}_{ssh} = a\Bigg[2\Theta\alpha\beta_{,ij}h_{ij} + \frac{\Gamma}{2}\alpha\beta_{,ij}\dot{h}_{ij} + \frac{\mu}{a^2}\alpha\beta_{,ij}h_{ij,kk} - \frac{3\mathcal{G}_T}{2}\zeta\beta_{,ij}\dot{h}_{ij} - 2\mathcal{G}_T\dot{\zeta}\beta_{,ij}h_{ij} + \mu\dot{\zeta}\beta_{,ij}\dot{h}_{ij}$$
$$- \mathcal{F}_T\zeta_{,i}\zeta_{,j}h_{ij} - 2\mathcal{G}_T\alpha_{,i}\zeta_{,j}h_{ij} + \mu\alpha_{,i}\zeta_{,j}\dot{h}_{ij} + \frac{\mathcal{G}_T}{2a^2}\beta_{,ij}\beta_{,k}h_{ij,k} + \frac{\mu}{a^2}\beta_{,ij}\beta_{,k}\dot{h}_{ij,k}\Bigg]. \tag{52}$$

Substituting the constraint equations, we obtain the reduced Lagrangian:

$$\mathcal{L}_{ssh} = a^3\Bigg[\frac{c_1}{a^2}h_{ij}\zeta_{,i}\zeta_{,j} + \frac{c_2}{a^2}\dot{h}_{ij}\zeta_{,i}\zeta_{,j} + c_3\dot{h}_{ij}\zeta_{,i}\psi_{,j} + \frac{c_4}{a^2}\partial^2 h_{ij}\zeta_{,i}\psi_{,j}$$
$$+ \frac{c_5}{a^4}\partial^2 h_{ij}\zeta_{,i}\zeta_{,j} + c_6\partial^2 h_{ij}\psi_{,i}\psi_{,j}\Bigg] + E_{ssh}, \tag{53}$$

where

$$c_1 = \mathcal{F}_S, \tag{54}$$

$$c_2 = \frac{\Gamma}{4\Theta}(\mathcal{F}_S - \mathcal{F}_T) + \frac{\mathcal{G}_T^2}{\Theta}\left[-\frac{1}{2} + \frac{H\Gamma}{4\Theta}\left(3 + \frac{\dot{\mathcal{G}}_T}{H\mathcal{G}_T}\right) - \frac{1}{4}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\Gamma}{\Theta}\right)\right] + \frac{\mu\mathcal{F}_S}{\mathcal{G}_T} + \frac{2H\mathcal{G}_T\mu}{\Theta} - \mathcal{G}_T\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mu}{\Theta}\right), \tag{55}$$

$$c_3 = \mathcal{G}_S\left[\frac{3}{2} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\Gamma}{2\Theta} + \frac{\mu}{\mathcal{G}_T}\right) - \left(3H + \frac{\dot{\mathcal{G}}_T}{\mathcal{G}_T}\right)\left(\frac{\Gamma}{2\Theta} + \frac{\mu}{\mathcal{G}_T}\right)\right], \tag{56}$$

$$c_4 = \mathcal{G}_S\left[-\frac{\mathcal{G}_T^2 - \Gamma\mathcal{F}_T}{2\Theta\mathcal{G}_T} - \frac{2H\mu}{\Theta} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mu}{\Theta}\right) + \frac{\mu}{\mathcal{G}_T^2}(\mathcal{F}_T - \mathcal{F}_S)\right], \tag{57}$$

$$c_5 = \frac{\mathcal{G}_T^2}{2\Theta}\left[\frac{\mathcal{G}_T^2 - \Gamma\mathcal{F}_T}{2\Theta\mathcal{G}_T} + \frac{2H\mu}{\Theta} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mu}{\Theta}\right) - \frac{\mu}{\mathcal{G}_T^2}(3\mathcal{F}_T - \mathcal{F}_S)\right], \tag{58}$$

$$c_6 = \frac{\mathcal{G}_S^2}{4\mathcal{G}_T}\left[1 + \frac{6H\mu}{\mathcal{G}_T} - 2\mathcal{G}_T\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mu}{\mathcal{G}_T^2}\right)\right], \tag{59}$$

and

$$E_{ssh} = \bar{f}_i\partial^{-2}\partial_i E^s + \bar{f}_{ij}E^h_{ij}, \tag{60}$$

with

$$\bar{f}_i := \frac{\Gamma}{2\Theta}\zeta_{,j}h_{ij} + \frac{\mu}{\mathcal{G}_T}\zeta_{,j}\dot{h}_{ij} + \frac{\mu}{a^2\Theta}\zeta_{,j}\partial^2 h_{ij} - \frac{\mu\mathcal{G}_S}{\mathcal{G}_T^2}\psi_{,j}\partial^2 h_{ij}, \tag{61}$$

$$\bar{f}_{ij} := \frac{\mathcal{G}_S}{\Theta\mathcal{G}_T}\left(\frac{\Gamma}{2} + \frac{\mu\Theta}{\mathcal{G}_T}\right)\zeta_{,i}\psi_{,j} - \frac{\mathcal{G}_T}{a^2\Theta^2}\left(\frac{\Gamma}{4} + \frac{\mu\Theta}{\mathcal{G}_T}\right)\zeta_{,i}\zeta_{,j}. \tag{62}$$

The field redefinition

$$h_{ij} \to h_{ij} + 4\bar{f}_{ij}, \tag{63}$$

$$\zeta \to \zeta - \frac{1}{2}\partial^{-2}\partial_i\bar{f}_i \tag{64}$$

removes the last term E_{ssh}. Since all the terms involve at least one derivative of the metric perturbation, the field redefinition does not contribute to the correlation function on super-horizon scales.

3.2.4. Three scalars. For completeness, here we give the cubic Lagrangian for the scalar perturbations derived in Refs. [57,58]. The cubic Lagrangian for the scalar perturbations is given by

$$\mathcal{L}_{sss} = -\frac{a^3}{3}(\Sigma + 2X\Sigma_X + H\Xi)\alpha^3 + a^3\left[3\Sigma\zeta + \Xi\dot{\zeta} + (\Gamma - \mathcal{G}_T)\frac{\zeta_{,ii}}{a^2} - \frac{\Xi}{3a^2}\beta_{,ii}\right]\alpha^2 - 2a\Theta\alpha\zeta_{,i}\beta_{,i} + 18a^3\Theta\alpha\zeta\dot{\zeta} + 4a\mu\alpha\dot{\zeta}\zeta_{,ii} - \frac{\Gamma}{2a}\alpha(\beta_{,ij}\beta_{,ij} - \beta_{,ii}\beta_{,jj}) + \frac{2\mu}{a}\alpha(\beta_{,ij}\zeta_{,ij} - \beta_{,ii}\zeta_{,jj}) - 2a\Theta\alpha\beta_{,ii}\zeta - 2a\Gamma\alpha\beta_{,ii}\dot{\zeta} - 2a\mathcal{G}_T\alpha\zeta\zeta_{,ii} - a\mathcal{G}_T\alpha\zeta_{,i}\zeta_{,i}$$

$$+3a^3\Gamma\alpha\dot{\zeta}^2+2a^3\mu\dot{\zeta}^3+a\mathcal{F}_T\zeta\zeta_{,i}\zeta_{,i}-9a^3\mathcal{G}_T\dot{\zeta}^2\zeta+2a\mathcal{G}_T\beta_{,i}\zeta_{,i}\dot{\zeta}-2a\mu\beta_{ii}\dot{\zeta}^2$$
$$+2a\mathcal{G}_T\beta_{,ii}\dot{\zeta}\zeta+\frac{1}{a}\left(\frac{3}{2}\mathcal{G}_T\zeta-\mu\dot{\zeta}\right)(\beta_{,ij}\beta_{,ij}-\beta_{,ii}\beta_{,jj})-2\frac{\mathcal{G}_T}{a}\beta_{,ii}\beta_{,j}\zeta_{,j}, \tag{65}$$

where

$$\begin{aligned}\Xi := {}& 12\dot{\phi}XG_{3X}+6\dot{\phi}X^2G_{3XX}-12HG_4\\ &+6[2H(7XG_{4X}+16X^2G_{4XX}+4X^3G_{4XXX})-\dot{\phi}(G_{4\phi}+5XG_{4\phi X}+2X^2G_{4\phi XX})]\\ &+90H^2\dot{\phi}XG_{5X}+78H^2\dot{\phi}X^2G_{5XX}+12H^2\dot{\phi}X^3G_{5XXX}\\ &-12HX(6G_{5\phi}+9XG_{5\phi X}+2X^2G_{5\phi XX}).\end{aligned} \tag{66}$$

Using the first-order constraint equations to remove α and β from the above Lagrangian, we obtain the following reduced expression:

$$\mathcal{L}_{sss}=\int \mathrm{d}t\mathrm{d}^3x a^3\mathcal{G}_S\left[\frac{\mathcal{C}_1}{6H}\dot{\zeta}^3+\mathcal{C}_2\dot{\zeta}^2\zeta+\mathcal{C}_3\frac{2c_s^2}{a^2}\zeta(\partial_i\zeta)^2+2\mathcal{C}_4\dot{\zeta}\partial_i\zeta\partial^i\psi+2\mathcal{C}_5\partial^2\zeta(\partial_i\psi)^2\right], \tag{67}$$

with $\psi=\partial^{-2}\dot{\zeta}$. There are five independent cubic terms with coefficients:

$$\begin{aligned}\mathcal{C}_1={}&-\frac{8\Xi\mathcal{G}_T^3}{3\Theta^3\mathcal{G}_S}+\frac{2H^2}{\Theta\mathcal{F}_S}\left[\frac{2\Xi\mathcal{G}_T^3}{\Theta^2}+\frac{3\mathcal{G}_T^3}{\Theta\mathcal{F}_S}(\mathcal{G}_S-2\mathcal{F}_S)+36\mu(\mathcal{G}_T-\mathcal{G}_S)+\frac{9\Gamma}{\Theta}\mathcal{G}_T(2\mathcal{G}_T-\mathcal{G}_S)\right]\\ &+2H\left[6\mu\left(\frac{1}{\mathcal{G}_S}-\frac{1}{\mathcal{G}_T}\right)+\frac{2(\Sigma-X\Sigma_X)\mathcal{G}_T^3}{\Theta^3\mathcal{G}_S}+\frac{\Xi\mathcal{G}_T}{\Theta^2}\left(\frac{3\mathcal{G}_T}{\mathcal{G}_S}-1\right)\right.\\ &\left.+\frac{3\mathcal{G}_T}{\Theta}\left(\frac{\mathcal{G}_S}{\mathcal{F}_S}+\frac{3\mathcal{G}_T}{\mathcal{G}_S}-1\right)+3\frac{\Gamma}{\Theta}\left(\frac{3\mathcal{G}_T}{\mathcal{G}_S}-2\right)\right]-\frac{6H^3\mathcal{G}_S\mathcal{G}_T^2}{\Theta^2\mathcal{F}_S^2}\left(6\mu+\frac{\Gamma\mathcal{G}_T}{\Theta}\right),\end{aligned} \tag{68}$$

$$\begin{aligned}\mathcal{C}_2={}&3+3H\mathcal{G}_S\left(\frac{\mu}{\mathcal{G}_T^2}+\frac{\Gamma}{2\Theta\mathcal{G}_T}-\frac{3\mathcal{G}_T}{2\Theta\mathcal{F}_S}\right)\\ &+3\frac{H^2\mathcal{G}_S}{\Theta\mathcal{F}_S}\left(8\mu+\frac{2\Gamma\mathcal{G}_T}{\Theta}-\frac{\mathcal{G}_T^3}{2\Theta\mathcal{F}_S}\right)+\frac{3H^3\mathcal{G}_S\mathcal{G}_T^2}{\Theta^2\mathcal{F}_S^2}\left(3\mu+\frac{\Gamma\mathcal{G}_T}{2\Theta}\right),\end{aligned} \tag{69}$$

$$\begin{aligned}\mathcal{C}_3={}&\frac{\mathcal{F}_T}{2\mathcal{F}_S}+H\left[\frac{(3\mathcal{G}_S-2\mathcal{G}_T)\mathcal{G}_T}{4\Theta\mathcal{F}_S}-\frac{\mu\mathcal{G}_S}{2\mathcal{G}_T^2}-\frac{\Gamma\mathcal{G}_S}{4\Theta\mathcal{G}_T}\right]\\ &+\frac{H^2\mathcal{G}_S}{\Theta\mathcal{F}_S}\left(\frac{\mathcal{G}_T^3}{4\Theta\mathcal{F}_S}-4\mu-\frac{\Gamma\mathcal{G}_T}{\Theta}\right)-\frac{H^3\mathcal{G}_S\mathcal{G}_T^2}{2\Theta^2\mathcal{F}_S^2}\left(3\mu+\frac{\Gamma\mathcal{G}_T}{2\Theta}\right),\end{aligned} \tag{70}$$

$$\mathcal{C}_4=-\frac{\mathcal{G}_S}{4\mathcal{G}_T}+3H\mathcal{G}_S\left(\frac{\mu}{2\mathcal{G}_T^2}+\frac{\Gamma}{4\Theta\mathcal{G}_T}-\frac{\mathcal{G}_T}{2\Theta\mathcal{F}_S}\right)+3\frac{H^2\mathcal{G}_S}{\Theta\mathcal{F}_S}\left(2\mu+\frac{\Gamma\mathcal{G}_T}{2\Theta}\right), \tag{71}$$

$$\mathcal{C}_5=\frac{3\mathcal{G}_S}{8\mathcal{G}_T}-\frac{3H\mathcal{G}_S}{4\mathcal{G}_T}\left(\frac{\mu}{\mathcal{G}_T}+\frac{\Gamma}{2\Theta}\right). \tag{72}$$

4. Primordial bispectra

Having obtained the general cubic Lagrangians composed of the scalar and tensor perturbations, we now compute the bispectra in this section. Here, we use the mode functions in exact de Sitter.

4.1. Three tensors

Let us consider the three-point function of the tensor perturbations:

$$\langle h_{i_1 j_1}(\mathbf{k}_1) h_{i_2 j_2}(\mathbf{k}_2) h_{i_3 j_3}(\mathbf{k}_3)\rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B^{(hhh)}_{i_1 j_1 i_2 j_2 i_3 j_3}, \tag{73}$$

$$B^{(hhh)}_{i_1 j_1 i_2 j_2 i_3 j_3} = \frac{(2\pi)^4 \mathcal{P}_h^2}{k_1^3 k_2^3 k_3^3} \left(\widetilde{\mathcal{A}}^{(\text{new})}_{i_1 j_1 i_2 j_2 i_3 j_3} + \widetilde{\mathcal{A}}^{(\text{GR})}_{i_1 j_1 i_2 j_2 i_3 j_3} \right), \tag{74}$$

where $\widetilde{\mathcal{A}}^{(\text{new})}_{i_1 j_1 i_2 j_2 i_3 j_3}$ and $\widetilde{\mathcal{A}}^{(\text{GR})}_{i_1 j_1 i_2 j_2 i_3 j_3}$ represent the contributions from the $\dot{h}^3$ and $h^2 \partial^2 h$ terms, respectively.

Each contribution is given by

$$\widetilde{\mathcal{A}}^{(\text{new})}_{i_1 j_1 i_2 j_2 i_3 j_3} = \frac{H\mu}{4\mathcal{G}_T} \frac{k_1^2 k_2^2 k_3^2}{K^3} \Pi_{i_1 j_1, lm}(\mathbf{k}_1) \Pi_{i_2 j_2, mn}(\mathbf{k}_2) \Pi_{i_3 j_3, nl}(\mathbf{k}_3), \tag{75}$$

$$\widetilde{\mathcal{A}}^{(\text{GR})}_{i_1 j_1 i_2 j_2 i_3 j_3} = \widetilde{\mathcal{A}} \left\{ \Pi_{i_1 j_1, ik}(\mathbf{k}_1) \Pi_{i_2 j_2, jl}(\mathbf{k}_2) \left[k_{3k} k_{3l} \Pi_{i_3 j_3, ij}(\mathbf{k}_3) - \frac{1}{2} k_{3i} k_{3k} \Pi_{i_3 j_3, jl}(\mathbf{k}_3) \right] + 5 \text{ perms of } 1, 2, 3 \right\}, \tag{76}$$

where $K = k_1 + k_2 + k_3$ and

$$\widetilde{\mathcal{A}}(k_1, k_2, k_3) := -\frac{K}{16} \left[1 - \frac{1}{K^3} \sum_{i \neq j} k_i^2 k_j - 4 \frac{k_1 k_2 k_3}{K^3} \right]. \tag{77}$$

The first term $\widetilde{\mathcal{A}}^{(\text{new})}_{i_1 j_1 i_2 j_2 i_3 j_3}$ is proportional to G_{5X} and hence vanishes in the case of Einstein gravity, while the second term $\widetilde{\mathcal{A}}^{(\text{GR})}_{i_1 j_1 i_2 j_2 i_3 j_3}$ is universal in the sense that it is independent of any model parameters and remains the same even in non-Einstein gravity.

In order to quantify the magnitude of the bispectrum, we define the two polarization modes as

$$\xi^{(s)}(\mathbf{k}) := h_{ij}(\mathbf{k}) e^{*(s)}_{ij}(\mathbf{k}), \tag{78}$$

and their relevant amplitudes of the bispectra as

$$\langle \xi^{(s_1)}(\mathbf{k}_1) \xi^{(s_2)}(\mathbf{k}_2) \xi^{(s_3)}(\mathbf{k}_3) \rangle = (2\pi)^7 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{\mathcal{P}_h^2}{k_1^3 k_2^3 k_3^3} \left(\widetilde{\mathcal{A}}^{s_1 s_2 s_3}_{(\text{new})} + \widetilde{\mathcal{A}}^{s_1 s_2 s_3}_{(\text{GR})} \right). \tag{79}$$

From Eqs. (75) and (76), the amplitudes $\widetilde{\mathcal{A}}^{s_1 s_2 s_3}_{(\text{new}),(\text{GR})}$ are easily calculated as [61]

$$\widetilde{\mathcal{A}}^{s_1 s_2 s_3}_{(\text{new})} = \frac{H\mu}{4\mathcal{G}_T} \frac{k_1^2 k_2^2 k_3^2}{K^3} F(s_1 k_1, s_2 k_2, s_3 k_3), \tag{80}$$

$$\widetilde{\mathcal{A}}^{s_1 s_2 s_3}_{(\text{GR})} = \frac{\widetilde{\mathcal{A}}}{2} (s_1 k_1 + s_2 k_2 + s_3 k_3)^2 F(s_1 k_1, s_2 k_2, s_3 k_3), \tag{81}$$

where

$$F(x, y, z) := \frac{1}{64} \frac{1}{x^2 y^2 z^2} (x + y + z)^3 (x - y + z)(x + y - z)(x - y - z). \tag{82}$$

As pointed out in Ref. [61], $\widetilde{\mathcal{A}}^{+++}_{(\text{new})}$ has a peak in the equilateral limit, while $\widetilde{\mathcal{A}}^{+++}_{(\text{GR})}$ has one in the squeezed limit.

It is convenient to introduce nonlinearity parameters, defined as

$$\widetilde{f}^{s_1 s_2 s_3}_{\mathrm{NL(new),(GR)}} = 30 \frac{\widetilde{\mathcal{A}}^{s_1 s_2 s_3}_{\mathrm{(new),(GR)}k_1=k_2=k_3}}{K^3}, \tag{83}$$

which are quantities analogous to the standard f_{NL} for the curvature perturbation. We find

$$\widetilde{f}^{s_1 s_2 s_3}_{\mathrm{NL(new)}} = -\frac{5}{10\,368}[3 + 2(s_1 s_2 + s_2 s_3 + s_3 s_1)] \frac{H\mu}{\mathcal{G}_T}, \tag{84}$$

or, more concretely,

$$\widetilde{f}^{+++}_{\mathrm{NL(new)}} = -\frac{5}{1152}\frac{H\mu}{\mathcal{G}_T}, \qquad \widetilde{f}^{++-}_{\mathrm{NL(new)}} = -\frac{5}{10\,368}\frac{H\mu}{\mathcal{G}_T}, \tag{85}$$

with $\widetilde{f}^{++-}_{\mathrm{NL(new)}} = \widetilde{f}^{+--}_{\mathrm{NL(new)}}$ and $\widetilde{f}^{---}_{\mathrm{NL(new)}} = \widetilde{f}^{+++}_{\mathrm{NL(new)}}$. (This symmetry arises because parity is not violated.) As for $\widetilde{f}^{s_1 s_2 s_3}_{\mathrm{NL(GR)}}$, we have

$$\widetilde{f}^{s_1 s_2 s_3}_{\mathrm{NL(GR)}} = \frac{85}{27\,648}[21 + 20(s_1 s_2 + s_2 s_3 + s_3 s_1)], \tag{86}$$

so that

$$\widetilde{f}^{+++}_{\mathrm{NL(GR)}} = \widetilde{f}^{---}_{\mathrm{NL(GR)}} = \frac{255}{1024}, \qquad \widetilde{f}^{++-}_{\mathrm{NL(GR)}} = \widetilde{f}^{+--}_{\mathrm{NL(GR)}} = \frac{85}{27\,648}. \tag{87}$$

As defined in Eq. (74), $B^{(hhh)}_{i_1 j_1 i_2 j_2 i_3 j_3}$ is normalized by $\mathcal{P}_h^2$. This normalization can be justified when one concentrates on the non-Gaussianity of the B-mode polarization. Because the B-mode polarization can be generated not by curvature perturbations but tensor perturbations (except for the lensing contribution), the size of the non-Gaussianity of the B-mode polarization can be directly characterized by $\widetilde{f}^{s_1 s_2 s_3}_{\mathrm{NL(new),(GR)}}$.

However, it should be noted that tensor perturbations can generate not only the B-mode polarization but also the temperature fluctuation and the E-mode polarization. The latter two are mainly generated by the curvature perturbations. Therefore, when one would like to quantify the auto and cross bispectra of the temperature fluctuation and the E-mode polarization, it would be better to normalize $B^{(hhh)}_{i_1 j_1 i_2 j_2 i_3 j_3}$ by $\mathcal{P}_\zeta^2$, namely,

$$B^{(hhh)}_{i_1 j_1 i_2 j_2 i_3 j_3} = \frac{(2\pi)^4 \mathcal{P}_\zeta^2}{k_1^3 k_2^3 k_3^3}\left(\mathcal{A}^{\mathrm{(new)}}_{i_1 j_1 i_2 j_2 i_3 j_3} + \mathcal{A}^{\mathrm{(GR)}}_{i_1 j_1 i_2 j_2 i_3 j_3}\right), \tag{88}$$

where $\mathcal{A}^{\mathrm{(new),(GR)}}_{i_1 j_1 i_2 j_2 i_3 j_3} = r^2 \widetilde{\mathcal{A}}^{\mathrm{(new),(GR)}}_{i_1 j_1 i_2 j_2 i_3 j_3}$, with r being the tensor-to-scalar ratio. In the same way, $\mathcal{A}^{s_1 s_2 s_3}_{\mathrm{(new),(GR)}} = r^2 \widetilde{\mathcal{A}}^{s_1 s_2 s_3}_{\mathrm{(new),(GR)}}$ and $f^{s_1 s_2 s_3}_{\mathrm{NL(new),(GR)}} = r^2 \widetilde{f}^{s_1 s_2 s_3}_{\mathrm{NL(new),(GR)}}$.

4.2. *Two tensors and one scalar*

The cross bispectrum of two tensors and one scalar is given by

$$\langle \zeta(\mathbf{k}_1) h_{ij}(\mathbf{k}_2) h_{kl}(\mathbf{k}_3)\rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B^{(\zeta hh)}_{ij,kl}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3), \tag{89}$$

where $B^{(\zeta hh)}_{ij,kl}$ is of the form

$$B^{(\zeta hh)}_{ij,kl} = \frac{2}{k_1^3 k_2^3 k_3^3} \frac{H^6}{\mathcal{F}_S \mathcal{F}_T^2 c_s c_h^2} \sum_{q=1}^{7} b_q \mathcal{V}^{(q)}_{ij,kl}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \mathcal{I}^{(q)}(k_1, k_2, k_3) + (\mathbf{k}_2, i, j \leftrightarrow \mathbf{k}_3, k, l). \tag{90}$$

Each contribution is given by

$$\mathcal{V}^{(1)}_{ij,kl} = \Pi_{ij,mn}(\mathbf{k}_2)\Pi_{kl,mn}(\mathbf{k}_3), \quad \mathcal{V}^{(2)}_{ij,kl} = \mathbf{k}_2\cdot\mathbf{k}_3\mathcal{V}^{(1)}_{ij,kl},$$
$$\mathcal{V}^{(3)}_{ij,kl} = \frac{\mathbf{k}_1\cdot\mathbf{k}_3}{k_1^2}\mathcal{V}^{(1)}_{ij,kl}, \mathcal{V}^{(4)}_{ij,kl} = \mathcal{V}^{(1)}_{ij,kl}, \quad \mathcal{V}^{(5)}_{ij,kl} = k_1^2\mathcal{V}^{(1)}_{ij,kl},$$
$$\mathcal{V}^{(6)}_{ij,kl} = \hat{k}_{1m}\hat{k}_{1n}\Pi_{ij,mm'}(\mathbf{k}_2)\Pi_{kl,nm'}(\mathbf{k}_3), \quad \mathcal{V}^{(7)}_{ij,kl} = k_1^2\mathcal{V}^{(6)}_{ij,kl}, \tag{91}$$

and

$$\mathcal{I}^{(1)} = \frac{1}{H^2}\frac{c_h^4k_2^2k_3^2(c_sk_1+K')}{K'^2},$$
$$\mathcal{I}^{(2)} = -\frac{1}{H^2}\frac{c_s^3k_1^3+2c_s^2c_hk_1^2(k_2+k_3)+2c_sc_h^2k_1(k_2^2+k_2k_3+k_3^2)+c_h^3(k_2+k_3)(k_2^2+k_2k_3+k_3^2)}{K'^2},$$
$$\mathcal{I}^{(3)} = \frac{1}{H^2}\frac{c_s^2c_h^2k_1^2k_2^2(K'+c_hk_3)}{K'^2}, \quad \mathcal{I}^{(4)} = \frac{2}{H}\frac{c_s^2c_h^4k_1^2k_2^2k_3^2}{K'^3}, \quad \mathcal{I}^{(5)} = \frac{2c_h^4k_2^2k_3^2(3c_sk_1+K')}{K'^4},$$
$$\mathcal{I}^{(6)} = \mathcal{I}^{(4)}, \quad \mathcal{I}^{(7)} = \mathcal{I}^{(5)}, \tag{92}$$

where $K' := c_sk_1 + c_h(k_2+k_3)$. Thus, it turns out that we only need to evaluate $\mathcal{V}^{(1)}_{ij,kl}$ and $\mathcal{V}^{(6)}_{ij,kl}$.

We would now like to define the amplitudes of the above cross bispectra in a similar way to the three-tensor case, for which we have adopted two different normalization conditions, (74) and (88), depending on whether we are interested in the B-mode polarization or the E-mode polarization and temperature fluctuations. The same ambiguity is present for the cases of these cross bispectra, too. Here we simply normalize them in terms of $\mathcal{P}_\zeta^2$, taking into account the fact that these bispectra generate the auto and cross bispectra of the temperature fluctuation and the E-mode polarization, too, which are mainly sourced by the curvature perturbation. Although this normalization may not be appropriate for those including the B-mode polarization, we do not examine this issue any further because the change of the normalization factor from $\mathcal{P}_\zeta^2$ to $\mathcal{P}_\zeta\mathcal{P}_h$ or $\mathcal{P}_h^2$ can readily be done by multiplying the appropriate powers of the tensor-to-scalar ratio r. Thus we adopt the following convention:

$$B^{(\zeta hh)}_{ij,kl} = \frac{(2\pi)^4\mathcal{P}_\zeta^2}{k_1^3k_2^3k_3^3}\mathcal{A}^{(\zeta hh)}_{ij,kl}, \tag{93}$$

where

$$\mathcal{A}^{(\zeta hh)}_{ij,kl} = 8H^2\frac{\mathcal{F}_Sc_s}{\mathcal{F}_T^2c_h^2}\sum_{q=1}^{7}b_q\mathcal{V}^{(q)}_{ij,kl}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\mathcal{I}^{(q)}(k_1,k_2,k_3) + (\mathbf{k}_2, i, j \leftrightarrow \mathbf{k}_3, k, l). \tag{94}$$

We also define the following cross bispectra:

$$\langle\zeta(\mathbf{k}_1)\xi^{(s_2)}(\mathbf{k}_2)\xi^{(s_3)}(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B^{(\zeta hh)}_{s2,s3}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3). \tag{95}$$

Here $B^{(\zeta hh)}_{s_2,s_3}$ and $\mathcal{A}^{(\zeta hh)}_{s_2,s_3}$ are given by

$$B^{(\zeta hh)}_{s2,s3} = \frac{(2\pi)^4\mathcal{P}_\zeta^2}{k_1^3k_2^3k_3^3}\mathcal{A}^{(\zeta hh)}_{s_2,s_3}$$
$$= \frac{2}{k_1^3k_2^3k_3^3}\frac{H^6}{\mathcal{F}_S\mathcal{F}_T^2c_sc_h^2}\sum_{q=1}^{7}b_q\mathcal{V}^{(q)}_{s2,s3}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\mathcal{I}^{(q)}(k_1,k_2,k_3) + (\mathbf{k}_2, s_2 \leftrightarrow \mathbf{k}_3, s_3), \tag{96}$$

where $\mathcal{V}^{(q)}_{s2,s3}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ is evaluated as

$$\begin{aligned}
\mathcal{V}^{(1)}_{s2,s3} &= \frac{1}{16k_2^2k_3^2}[k_1^2-(s_2k_2+s_3k_3)^2]^2, \quad \mathcal{V}^{(2)}_{s2,s3} = \mathbf{k}_2\cdot\mathbf{k}_3\mathcal{V}^{(1)}_{s2,s3} = \frac{k_1^2-k_2^2-k_3^2}{2}\mathcal{V}^{(1)}_{s2,s3},\\
\mathcal{V}^{(3)}_{s2,s3} &= \frac{\mathbf{k}_1\cdot\mathbf{k}_3}{k_1^2}\mathcal{V}^{(1)}_{s2,s3} = -\frac{k_1^2-k_2^2+k_3^2}{2k_1^2}\mathcal{V}^{(1)}_{s2,s3}, \quad \mathcal{V}^{(4)}_{s2,s3} = \mathcal{V}^{(1)}_{s2,s3}, \quad \mathcal{V}^{(5)}_{s2,s3} = k_1^2\mathcal{V}^{(1)}_{s2,s3},\\
\mathcal{V}^{(6)}_{s2,s3} &= \frac{K}{32k_1^2k_2^2k_3^2}(k_1-k_2-k_3)(k_1+k_2-k_3)(k_1-k_2+k_3)[k_1^2-(s_2k_2+s_3k_3)^2],\\
\mathcal{V}^{(7)}_{s2,s3} &= k_1^2\mathcal{V}^{(6)}_{s2,s3}.
\end{aligned} \tag{97}$$

4.3. *Two scalars and one tensor*

The cross bispectrum of two scalars and one tensor is given by

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)h_{ij}(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B^{(\zeta\zeta h)}_{ij}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3), \tag{98}$$

where $B^{(\zeta\zeta h)}_{ij}$ is of the form

$$B^{(\zeta\zeta h)}_{ij} = \frac{1}{4k_1^3k_2^3k_3^3}\frac{H^6}{\mathcal{F}_S^2\mathcal{F}_T c_s^2 c_h}\sum_{q=1}^{6}c_q\mathcal{V}^{(q)}_{ij}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\mathcal{J}^{(q)}(k_1,k_2,k_3) + (\mathbf{k}_1\leftrightarrow\mathbf{k}_2). \tag{99}$$

Each contribution is given by

$$\begin{aligned}
\mathcal{V}^{(1)}_{ij} &= k_{1k}k_{2l}\Pi_{ij,kl}(\mathbf{k}_3), \quad \mathcal{V}^{(2)}_{ij} = \mathcal{V}^{(1)}_{ij}, \quad \mathcal{V}^{(3)}_{ij} = \frac{1}{k_2^2}\mathcal{V}^{(1)}_{ij},\\
\mathcal{V}^{(4)}_{ij} &= \frac{k_3^2}{k_2^2}\mathcal{V}^{(1)}_{ij}, \quad \mathcal{V}^{(5)}_{ij} = k_3^2\mathcal{V}^{(1)}_{ij}, \quad \mathcal{V}^{(6)}_{ij} = \frac{k_3^2}{k_1^2k_2^2}\mathcal{V}^{(1)}_{ij},
\end{aligned} \tag{100}$$

and

$$\begin{aligned}
\mathcal{J}^{(1)} &= -\frac{1}{H^2}\frac{c_s^3(k_1+k_2)(k_1^2+k_1k_2+k_2^2)+2c_s^2c_h(k_1^2+k_1k_2+k_2^2)k_3+2c_sc_h^2(k_1+k_2)k_3^2+c_h^3k_3^3}{K''^2},\\
\mathcal{J}^{(2)} &= \frac{1}{H}\frac{c_h^2k_3^2[2c_s^2(k_1^2+3k_1k_2+k_2^2)+3c_sc_h(k_1+k_2)k_3+c_h^2k_3^2]}{K''^3},\\
\mathcal{J}^{(3)} &= \frac{1}{H^2}\frac{c_s^2c_h^2k_2^2k_3^2(c_sk_1+K'')}{K''^2},\\
\mathcal{J}^{(4)} &= \frac{1}{H}\frac{c_s^2k_2^2[c_s^2(k_1+k_2)(2k_1+k_2)+3c_sc_h(2k_1+k_2)k_3+2c_h^2k_3^2]}{K''^3},\\
\mathcal{J}^{(5)} &= \frac{2}{K''^4}[c_s^3(k_1+k_2)(k_1^2+3k_1k_2+k_2^2)+4c_s^2c_h(k_1^2+3k_1k_2+k_2^2)k_3\\
&\quad +4c_sc_h^2(k_1+k_2)k_3^2+c_h^3k_3^3],\\
\mathcal{J}^{(6)} &= \frac{1}{H^2}\frac{c_s^4k_1^2k_2^2(K''+c_hk_3)}{K''^2},
\end{aligned} \tag{101}$$

with $K'' := c_s(k_1+k_2)+c_hk_3$. Thus, it turns out that we only need to evaluate $\mathcal{V}^{(1)}_{ij}$.

As in the case of two tensors and one scalar, we normalize the bispectrum by $\mathcal{P}_\zeta^2$ as

$$B_{ij}^{(\zeta\zeta h)} = \frac{(2\pi)^4\mathcal{P}_\zeta^2}{k_1^3k_2^3k_3^3}\mathcal{A}_{ij}^{(\zeta\zeta h)}, \tag{102}$$

where

$$\mathcal{A}_{ij}^{(\zeta\zeta h)} = \frac{H^2}{\mathcal{F}_T c_h}\sum_{q=1}^{6} c_q \mathcal{V}_{ij}^{(q)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)\mathcal{J}^{(q)}(k_1, k_2, k_3) + (\mathbf{k}_1 \leftrightarrow \mathbf{k}_2). \tag{103}$$

We also define the following cross bispectra:

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\xi^{(s)}(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)B_s^{(\zeta\zeta h)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3). \tag{104}$$

Here $B_s^{(\zeta\zeta h)}$ and $A_s^{(\zeta\zeta h)}$ are given by

$$\begin{aligned} B_s^{(\zeta\zeta h)} &= \frac{(2\pi)^4\mathcal{P}_\zeta^2}{k_1^3k_2^3k_3^3}\mathcal{A}_s^{(\zeta\zeta h)} \\ &= \frac{1}{4k_1^3k_2^3k_3^3}\frac{H^6}{\mathcal{F}_S^2\mathcal{F}_T c_s^2 c_h}\sum_{q=1}^{6} c_q \mathcal{V}_s^{(q)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)\mathcal{J}^{(q)}(k_1, k_2, k_3) + (\mathbf{k}_1 \leftrightarrow \mathbf{k}_2), \end{aligned} \tag{105}$$

where $\mathcal{V}_s^{(q)}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is evaluated as

$$\mathcal{V}_s^{(1)} = \frac{K}{8k_3^2}(k_1 - k_2 - k_3)(k_1 + k_2 - k_3)(k_1 - k_2 + k_3), \tag{106}$$

and

$$\mathcal{V}_s^{(2)} = \mathcal{V}_s^{(1)}, \quad \mathcal{V}_s^{(3)} = \frac{1}{k_2^2}\mathcal{V}_s^{(1)}, \quad \mathcal{V}_s^{(4)} = \frac{k_3^2}{k_2^2}\mathcal{V}_s^{(1)}, \quad \mathcal{V}_s^{(5)} = k_3^2\mathcal{V}_s^{(1)}, \quad \mathcal{V}_s^{(6)} = \frac{k_3^2}{k_1^2k_2^2}\mathcal{V}_s^{(1)}. \tag{107}$$

Indeed, the above functions are independent of s due to the lack of parity violation.

4.4. *Three scalars*

Here we give the bispectrum defined by

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)B^{(\zeta\zeta\zeta)}(k_1, k_2, k_3). \tag{108}$$

The result is given in Refs. [57,58]:

$$\begin{aligned} B^{(\zeta\zeta\zeta)} = \frac{(2\pi)^4\mathcal{P}_\zeta^2}{4k_1^3k_2^3k_3^3}\Bigg[&\frac{(k_1k_2k_3)^2}{K^3}\mathcal{C}_1 + \frac{\mathcal{C}_2}{K}\left(2\sum_{i>j}k_i^2k_j^2 - \frac{1}{K}\sum_{i\neq j}k_i^2k_j^3\right) \\ &+ \mathcal{C}_3\left(\sum_i k_i^3 + \frac{4}{K}\sum_{i>j}k_i^2k_j^2 - \frac{2}{K^2}\sum_{i\neq j}k_i^2k_j^3\right) \\ &+ \mathcal{C}_4\left(\sum_i k_i^3 - \frac{1}{2}\sum_{i\neq j}k_ik_j^2 - \frac{2}{K^2}\sum_{i\neq j}k_i^2k_j^3\right) \\ &+ \frac{\mathcal{C}_5}{K^2}\left(2\sum_i k_i^5 + \sum_{i\neq j}k_ik_j^4 - 3\sum_{i\neq j}k_i^2k_j^3 - 2k_1k_2k_3\sum_{i>j}k_ik_j\right)\Bigg]. \end{aligned} \tag{109}$$

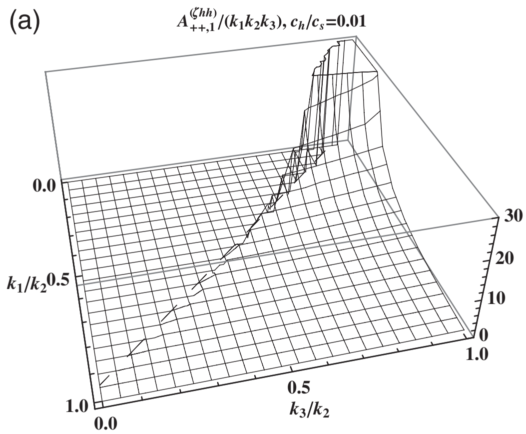

Fig. 1. $\mathcal{A}^{(\zeta hh)}_{++,1}(k_1k_2k_3)^{-1}$ as a function of k_1/k_2 and k_3/k_2 for $c_h/c_s = 0.01$ in (a) and $c_h/c_s = 10^2$ in (b), normalized to unity for $k_1 = k_2 = k_3$.

4.5. Shapes of the cross bispectra in momentum space

Let us discuss the shape of each cross bispectrum in momentum space. As shown in Ref. [61] and also mentioned in the previous subsection, for the bispectrum of the tensor mode, $\widetilde{\mathcal{A}}^{+++}_{\text{(new)}}$ and $\widetilde{\mathcal{A}}^{+++}_{\text{(GR)}}$ have, respectively, peaks in the equilateral and squeezed limits. The shape of the bispectrum of scalar perturbations was also discussed in Refs. [57,58] and the authors found that it is well approximated by the equilateral shape.

In a similar way to the auto bispectra of tensors and scalars, we can also discuss the shapes of the cross bispectra of tensors and scalars in momentum space. However, in contrast with the auto bispectra of tensors and scalars, the shapes of cross bispectra strongly depend on the sound speeds of the tensor and scalar perturbations, as can be seen in Eqs. (92) and (101). Here, we denote a term proportional to b_q in $\mathcal{A}^{(\zeta hh)}_{s_2,s_3}$ as $\mathcal{A}^{(\zeta hh)}_{s_2,s_3,q}$ and also a term proportional to c_q in $\mathcal{A}^{(\zeta\zeta h)}_{s}$ as $\mathcal{A}^{(\zeta\zeta h)}_{s,q}$. The shape of $\mathcal{A}^{(\zeta hh)}_{++,1}(k_1k_2k_3)^{-1}$ in k-space for two limiting cases is plotted in Fig. 1. The left panel (a) and the right one (b) are, respectively, for the cases with $c_h/c_s = 0.01$ and $c_h/c_s = 10^2$. This figure implies that $\mathcal{A}^{(\zeta hh)}_{++,1}(k_1k_2k_3)^{-1}$ has a peak in the squeezed limit ($k_1 \ll k_2 \sim k_3$) for both limiting cases. However, the sharpness of the peak seems to depend on the value of c_h/c_s. In Fig. 2 where $\mathcal{A}^{(\zeta hh)}_{++,2}(k_1k_2k_3)^{-1}$ is plotted, we find that $\mathcal{A}^{(\zeta hh)}_{++,2}(k_1k_2k_3)^{-1}$ for the case with $c_h/c_s = 0.01$ has a sharp peak in the squeezed limit together with a nontrivial shape in a wide region of the momentum space. For the case with $c_h/c_s = 10^2$ (shown in Fig. 2(b)), $\mathcal{A}^{(\zeta hh)}_{++,2}(k_1k_2k_3)^{-1}$ also has a peak in the squeezed limit.

In contrast to $\mathcal{A}^{(\zeta hh)}_{++,1}(k_1k_2k_3)^{-1}$ and $\mathcal{A}^{(\zeta hh)}_{++,2}(k_1k_2k_3)^{-1}$, both of which have a peak in the squeezed limit, $\mathcal{A}^{(\zeta hh)}_{++,3}(k_1k_2k_3)^{-1}$ does not have any sharp peak, but its shape strongly depends on the value of c_h/c_s, as shown in Fig. 3. In the case with $c_h/c_s \ll 1$, $\mathcal{A}^{(\zeta hh)}_{++,3}(k_1k_2k_3)^{-1}$ becomes large at $k_1 \ll k_2$, and then its shape looks to come close to so-called orthogonal type in the limit of $c_h/c_s \gg 1$. $\mathcal{A}^{(\zeta hh)}_{++,4}(k_1k_2k_3)^{-1}$ also strongly depends on the value of c_h/c_s. As can be seen in Fig. 4, the peak of $\mathcal{A}^{(\zeta hh)}_{++,4}(k_1k_2k_3)^{-1}$ shifts in the momentum space depending on c_h/c_s, and $\mathcal{A}^{(\zeta hh)}_{++,4}(k_1k_2k_3)^{-1}$ for small c_h has a finite value even in the squeezed limit. Although we do not show it here, we also found that $\mathcal{A}^{(\zeta hh)}_{++,5}$, $\mathcal{A}^{(\zeta hh)}_{++,6}$, and $\mathcal{A}^{(\zeta hh)}_{++,7}$ have almost the same shapes as $\mathcal{A}^{(\zeta hh)}_{++,4}$.

In Fig. 5, $\mathcal{A}^{(\zeta\zeta h)}_{+,q}(k_1k_2k_3)^{-1}$ for $q = 1, 2$ are plotted. This figure shows that $\mathcal{A}^{(\zeta\zeta h)}_{+,q}(k_1k_2k_3)^{-1}$ also has a strong dependence on c_h/c_s and there is no divergence feature in the whole region of the

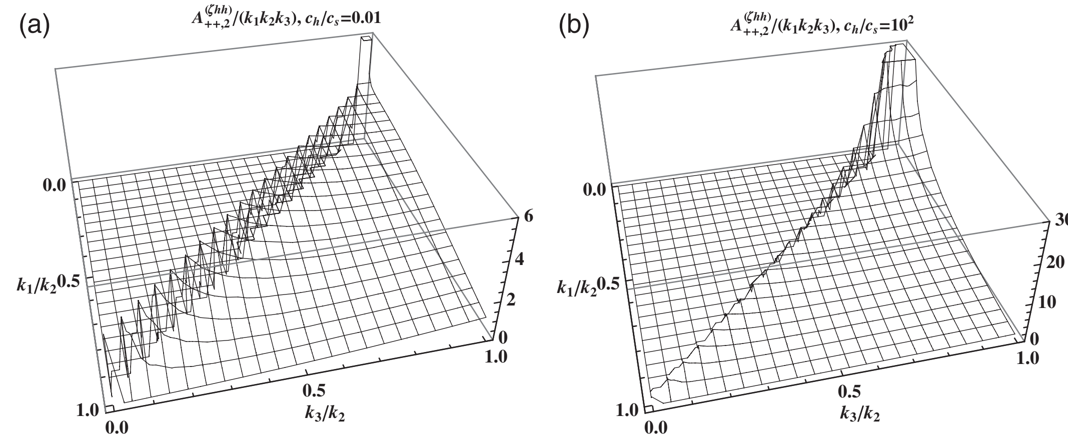

Fig. 2. $\mathcal{A}^{(\zeta hh)}_{++,2}(k_1k_2k_3)^{-1}$ as a function of k_1/k_2 and k_3/k_2 for $c_h/c_s = 0.01$ in (a) and $c_h/c_s = 10^2$ in (b), normalized to unity for $k_1 = k_2 = k_3$.

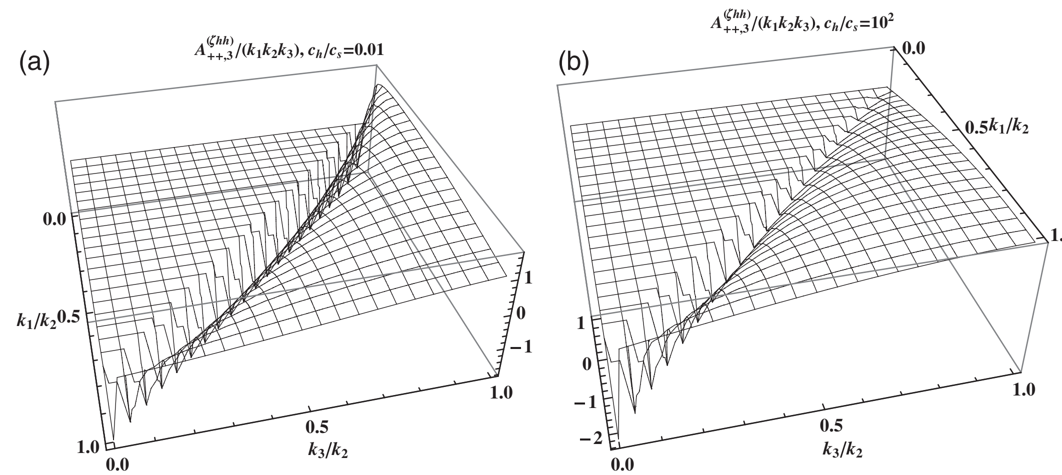

Fig. 3. $\mathcal{A}^{(\zeta hh)}_{++,3}(k_1k_2k_3)^{-1}$ as a function of k_1/k_2 and k_3/k_2 for $c_h/c_s = 0.01$ in (a) and $c_h/c_s = 10^2$ in (b), normalized to unity for $k_1 = k_2 = k_3$.

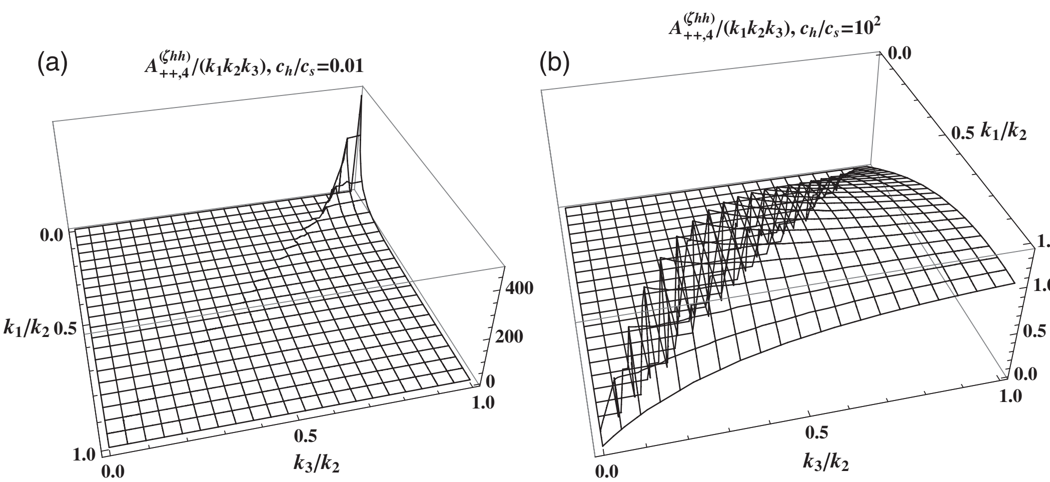

Fig. 4. $\mathcal{A}^{(\zeta hh)}_{++,4}(k_1k_2k_3)^{-1}$ as a function of k_1/k_2 and k_3/k_2 for $c_h/c_s = 0.01$ in (a) and $c_h/c_s = 10^2$ in (b), normalized to unity for $k_1 = k_2 = k_3$.

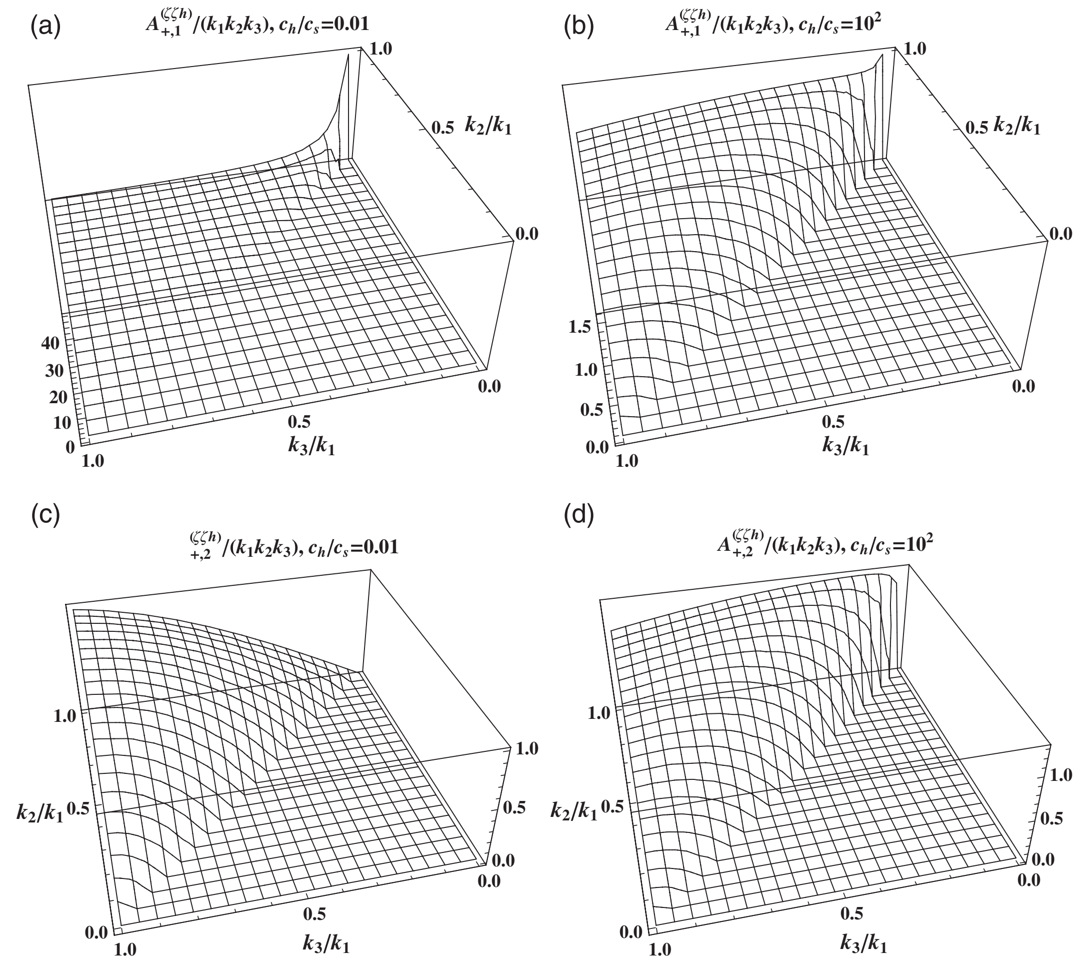

Fig. 5. $\mathcal{A}^{(\zeta\zeta h)}_{+,q}(k_1k_2k_3)^{-1}$ ((a) and (b) for $q = 1$, and (c) and (d) for $q = 2$) as a function of k_2/k_1 and k_3/k_1 for $c_h/c_s = 0.01$ in (a) and (c), and $c_h/c_s = 10^2$ in (b) and (d), normalized to unity for $k_1 = k_2 = k_3$.

momentum space, unlike the so-called local shape. Since we found that $\mathcal{A}^{(\zeta\zeta h)}_{+,q}$ for $q = 3, 4, 5, 6$ have almost the same shape as $\mathcal{A}^{(\zeta\zeta h)}_{+,2}$, we do not show the plots for these contributions.

A detailed analysis of the shapes of the cross bispectra, including a precise comparison with the standard local, equilateral, and orthogonal shapes, is currently in progress, along with a detailed analysis of CMB bispectra (X. Gao et al., manuscript in preparation).

5. Examples

In this section, we consider two representative examples of inflation to estimate the amount of non-Gaussianities from tensor and scalar perturbations. The first example is general potential-driven inflation, studied in Ref. [70]. This class of inflation models includes variants of Higgs inflation, enabled by enhancing the effect of Hubble friction. These potential-driven models have $c_s^2 = \mathcal{O}(1)$ and $c_h^2 \simeq 1$. Next, to see the impact of generic c_s^2 more clearly, we study k-inflation as another example.

5.1. *The case of potential-driven inflation models*

We wish to treat a wide class of potential-driven inflation models at one time. For this purpose, we introduce six ϕ-dependent functions to write

$$K = -V(\phi) + \mathcal{K}(\phi)X, \quad G_3 = h_3(\phi)X, \quad G_4 = g(\phi) + h_4(\phi)X, \quad G_5 = h_5(\phi)X. \tag{110}$$

In particular, the above form includes the different Higgs inflation models proposed so far [70]. These may also be regarded as the Taylor expansion of $K(\phi, X)$ and $G_i(\phi, X)$ with respect to X. Would-be leading terms in G_3 and G_5 have been removed without loss of generality.

The slow-roll dynamics of general potential-driven inflation models has been addressed in Ref. [70]. During inflation we assume that the following slow-roll conditions are satisfied:

$$\epsilon = -\frac{\dot{H}}{H^2} \ll 1, \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1, \quad \delta = \frac{\dot{g}}{Hg} \ll 1,$$
$$\alpha_2 = \frac{\dot{\mathcal{K}}}{H\mathcal{K}} \ll 1, \quad \alpha_i = \frac{\dot{h}_i}{Hh_i} \ll 1\ (i = 3, 4, 5). \tag{111}$$

It is convenient to define

$$u(\phi) := \mathcal{K} + \frac{h_4 V}{g}, \quad v(\phi) := h_3 + \frac{h_5 V}{6g}, \quad W(\phi) := \frac{1}{2}\left[u + \sqrt{u^2 - 4g^2 v \frac{\mathrm{d}}{\mathrm{d}\phi}\left(\frac{V}{g^2}\right)}\right]. \tag{112}$$

Under the slow-roll approximation the gravitational field equations reduce to

$$6gH^2 \simeq V, \quad 2\epsilon + \delta \simeq \frac{X}{gH^2}(u + 3H\dot{\phi}v). \tag{113}$$

Now it is easy to see that $\mathcal{F}_T \simeq \mathcal{G}_T \simeq 2g$ and

$$\mathcal{F}_S \simeq \frac{X}{H^2}(u + 4H\dot{\phi}v) \simeq \frac{g}{3}(2\epsilon + \delta)\left(4 - \frac{u}{W}\right),$$
$$\mathcal{G}_S \simeq \frac{X}{H^2}(u + 6H\dot{\phi}v) \simeq g(2\epsilon + \delta)\left(2 - \frac{u}{W}\right), \tag{114}$$

so that $\mathcal{F}_S$ and $\mathcal{G}_S$ are slow-roll suppressed. It can also be seen that $c_h^2 \simeq 1$ and $c_s^2 = \mathcal{O}(1)$.

The coefficients of the cubic terms are given by

$$b_1 \simeq \frac{Xu}{8H^2}, \quad b_2 \simeq \frac{g}{24}(2\epsilon + \delta)\left(4 - \frac{u}{W}\right), \quad b_3 \simeq -\frac{g}{4}(2\epsilon + \delta)\left(2 - \frac{u}{W}\right),$$
$$b_4 \simeq -\mu, \quad b_5 \simeq -\frac{\mu}{6H}\frac{1 - u/W}{2 - u/W}, \quad b_7 \simeq \frac{\mu}{2H}, \quad E_{shh} \simeq \frac{1}{4H}\zeta \dot{h}_{ij} E^h_{ij}, \tag{115}$$

and

$$c_1 \simeq \frac{g}{3}(2\epsilon + \delta)\left(4 - \frac{u}{W}\right), \quad c_2 \simeq \frac{g}{12H}(2\epsilon + \delta)\left(1 - \frac{u}{W}\right) + \frac{\dot{\phi}X}{4H^2}\left(3h_3 - \frac{h_5 V}{6g}\right),$$
$$c_5 \simeq \mu, \ \bar{f}_i \simeq \frac{1}{2H}\zeta_{,j}h_{ij} \quad \bar{f}_{ij} \simeq -\frac{1}{4a^2H^2}\zeta_{,i}\zeta_{,j}, \tag{116}$$

where $\mu = \dot{\phi}Xh_5$ as defined in (38). It turns out that the other coefficients are of higher order in the slow-roll parameter: $b_6 \sim c_3 \sim c_4 \sim c_6 = \mathcal{O}(\epsilon^2)$.

5.2. *The case of k-inflation*

To extract the effect of the nontrivial sound speed, let us consider k-inflation, which is the simplest model with a generic value of c_s^2. In the case of k-inflation, $K = K(\phi, X)$, $G_4 = M_{\rm Pl}^2/2$, $G_3 = 0 =$

G_5, we have

$$\mathcal{F}_T = \mathcal{G}_T = \Gamma = M_{\rm Pl}^2, \quad \mathcal{F}_S = M_{\rm Pl}^2\epsilon, \quad \mathcal{G}_S = \frac{M_{\rm Pl}^2\epsilon}{c_s^2}, \quad \Theta = M_{\rm Pl}^2 H, \quad \mu = 0, \tag{117}$$

with $c_h = 1$ and $r = 16\epsilon c_s$, which simplifies the coefficients in the cubic Lagrangians:

$$b_1 = b_2 = \frac{M_{\rm Pl}^2\epsilon}{8}, \quad b_3 = -\frac{M_{\rm Pl}^2\epsilon}{4c_s^2}, \quad b_4 = b_5 = b_6 = b_7 = 0,$$

$$E_{shh} = \frac{1}{4H}\zeta \dot{h}_{ij} E_{ij}^h, \tag{118}$$

$$c_1 = M_{\rm Pl}^2\epsilon, \quad c_2 = 0, \quad c_3 = \frac{M_{\rm Pl}^2\epsilon^2}{2c_s^2}, \quad c_4 = c_5 = 0, \quad c_6 = \frac{M_{\rm Pl}^2\epsilon^2}{4c_s^4}, \tag{119}$$

$$\bar{f}_i = \frac{1}{2H}\zeta_{,j}h_{ij}, \quad \bar{f}_{ij} = \frac{\epsilon}{2Hc_s^2}\zeta_{,i}\psi_{,j} - \frac{1}{4a^2H^2}\zeta_{,i}\zeta_{,j}. \tag{120}$$

Note that in deriving the above coefficients we have not invoked the slow-roll expansion.

6. Discussion

In this paper we have presented the full bispectra, including the cross bispectra of the primordial curvature and tensor perturbations, in the generalized G-inflation model, which is the most general single-field inflation model with second-order equations of motion.

In the event that full observations of these quantities could be made, we could extract many pieces of interesting information on the underlying theory. For example, by observing the three-point tensor correlation function, we can in principle determine the kinetic coupling to the Einstein tensor through μ. Another interesting quantity is the cross bispectrum of two tensors and one scalar. If we could observationally identify their coefficients b_2, b_3, and b_6, we could in principle determine $\mathcal{F}_S$, $\mathcal{G}_S$, $\mathcal{F}_T$, and $\mathcal{G}_T$ independently with the help of the three-tensor bispectrum, which would provide a consistency relation of the theory for the tensor-to-scalar ratio (36).

Let us next turn to the two-scalar and one-tensor bispectrum whose effective Lagrangian is given by (53). Its most interesting component is the first term proportional to $c_1 = \mathcal{F}_S$, which can be singled out by taking k_3 small. In the standard canonical inflation as well as in k-inflation, the coefficient simply takes

$$c_1 = \mathcal{F}_S = M_{\rm Pl}^2\epsilon = \frac{M_{\rm Pl}^2 r}{16c_s}$$

as derived in (119), where we have used the consistency relation in the last equality.

We can also show that this feature remains valid in the case where a sizable *local* non-Gaussianity is generated, as in the cases of the curvaton scenario [71,72] and the modulated reheating scenarios [73,74]. In such cases, the curvature perturbation ζ is sourced by another scalar field, which we denote by σ and its fluctuation by $\delta\sigma$. One can relate ζ and $\delta\sigma$ as

$$\zeta = N_\sigma(\sigma)\delta\sigma + \frac{1}{2}N_{\sigma\sigma}(\sigma)(\delta\sigma)^2, \tag{121}$$

using the δN-formalism [75–77]. Suppose that σ has the Lagrangian $\mathcal{L}_\sigma = \kappa(Y,\sigma)$ with $Y := -(\partial\sigma)^2/2$. Since the dynamics of σ is practically frozen during inflation and it practically behaves as a massless minimally coupled field, one can expand $\mathcal{L}_\sigma = \kappa(0,\sigma_0) + \kappa_\sigma(0,\sigma_0)Y$ in this regime where σ_0 is its expectation value in the domain including our horizon today. Then the mean-square fluctuation amplitude of σ is given by

$$\langle(\delta\sigma)^2\rangle = \frac{H^2}{4\pi^2\kappa_\sigma(0,\sigma_0)} = \frac{1}{N_\sigma^2(\sigma_0)}\mathcal{P}_\zeta, \tag{122}$$

the latter being an outcome of (121), and it determines the relation between $\delta\sigma$ and ζ, too. Then the effective Lagrangian representing tensor–scalar–scalar coupling is generated from the kinetic term of σ in this case and reads

$$\mathcal{L}_{ssh} = \frac{1}{2}\kappa_\sigma(0,\sigma_0)h^{\mu\nu}\sigma_{,\mu}\sigma_{,\nu} = \frac{1}{2}\left(\frac{H}{2\pi}\right)^2\mathcal{P}_\zeta h^{\mu\nu}\zeta_{,\mu}\zeta_{,\nu} = \frac{M_{\rm Pl}^2 r}{16}h_{ij}\zeta_{,i}\zeta_{,j}. \tag{123}$$

Note that in this case the sound speed is equal to unity. Thus we find that, if the sector responsible for the generation of curvature perturbations is minimally coupled to gravity with no extra Galileon-like terms, c_1 takes the same form whether they are generated by the inflaton or another scalar field. Thus this term can provide a test of the generalized Galileon as a source of the structure of the Universe.

The normalization of the cross bispectra is a nontrivial issue. In this paper, we have normalized them by the power spectrum of the curvature perturbation. This is mainly because these cross bispectra generate the auto and cross bispectra of the temperature fluctuation and the E-mode polarization, which are mainly sourced by the curvature perturbation. However, such a normalization may be inadequate for the cross bispectra including B-mode polarization. Therefore, we need to directly investigate the impacts on the CMB bispectra; it would be interesting to see the CMB cross bispectra between the temperature fluctuations and B-mode polarizations, which are sourced directly from the primordial cross bispectra of the scalar and tensor modes [31,78]. Constraining the model parameters by CMB bispectra is a work in progress (X. Gao et al., manuscript in preparation).

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