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Strong coupling constant from moments of quarkonium correlators

Peter Petreczky*
Physics Department, Brookhaven National Laboratory, Upton (NY)
E-mail: petreczk@quark.phy.bnl.gov

I discuss recent progress and challenges in determining α_s from moments of quarkonium correlators.

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*Speaker.

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The strong coupling constant can be determined using the moments of quarkonium correlators. On the lattice the moments of pseudoscalar quarkonium correlators are the most practical ones, since these have the smallest statistical errors. The moments of the pseudoscalar quarkonium correlator, are defined as

$$G_n = \sum_t t^n G(t),\ G(t) = a^6 \sum_{\mathbf{x}} (am_{h0})^2 \langle j_5(\mathbf{x},t) j_5(0,0)\rangle. \tag{1}$$

Here $j_5 = \bar{\psi}\gamma_5\psi$ is the pseudoscalar current, a is the lattice spacing, and m_{h0} is the bare lattice heavy quark mass. The moments G_n are finite for $n \geq 4$ (n even) in the $a \to 0$ limit and do not need renormalization because the explicit factors of the quark mass. The moments can be calculated in perturbation theory in $\overline{\mathrm{MS}}$ scheme

$$G_n = \frac{g_n(\alpha_s(\mu), \mu/m_h)}{am_h^{n-4}(\mu_m)}. \tag{2}$$

Here μ is the $\overline{\mathrm{MS}}$ renormalization scale, and $m_h(\mu_m)$ is the renormalized heavy quark mass in the $\overline{\mathrm{MS}}$ scheme. The scale μ_m at which the $\overline{\mathrm{MS}}$ heavy quark mass is defined can be different from μ [1], though most studies assume $\mu_m = \mu$. The coefficient $g_n(\alpha_s(\mu), \mu/m_h)$ is calculated up to 4-loop, i.e. up to order α_s^3 [2]–[3]. For practical applications it is better to consider the reduced moments

$$R_n = \begin{cases} G_n/G_n^{(0)} & (n=4) \\ \left(G_n/G_n^{(0)}\right)^{1/(n-4)} & (n \geq 6) \end{cases}, \tag{3}$$

where $G_n^{(0)}$ is the moment calculated from the free lattice correlation function, since the leading order lattice artifacts cancel out in this ratio, and thus the cutoff effects in R_n are proportional to $\alpha_s^m a^{2n}$, $m \geq 1$, $n \geq 1$. It is straightforward to write down the perturbative expansion for R_n:

$$R_n = \begin{cases} r_4 & (n=4) \\ r_n \cdot (m_{h0}/m_h(\mu)) & (n \geq 6) \end{cases}, \tag{4}$$

$$r_n = 1 + \sum_{j=1}^{3} r_{nj}(\mu/m_h) \left(\frac{\alpha_s(\mu)}{\pi}\right)^j. \tag{5}$$

There is also a contribution to the moments of quarkonium correlators from the gluon condensate [4]. From the above equations it is clear that R_4 as well as the ratios R_6/R_8 and R_8/R_{10} are suitable for the extraction of the strong coupling constant $\alpha_s(\mu)$. The calculation of α_s in lattice QCD using the moments of quarkonium correlators was pioneered in Ref. [5] and now is pursued by several groups [5]–[10]. Here I will discussed this approach using the newest lattice results based on the calculations in 3-flavor QCD with Highly Improved Staggered Quark (HISQ) action and several heavy quark masses $m_h = m_c$, $1.5m_c$, $2m_c$ and $3m_c$ with m_c being the charm quark mass [10].

One of the challenges for accurate determination of the strong coupling constant from the moments of quarkonium correlators is a reliable continuum ($a \to 0$) extrapolation. There is also a window problem. We would like to work with the large value of m_h for perturbation theory to be reliable, at the same time to control the cutoff effects which grow with increasing m_h. So, one

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has to find a window, where $m_h/\Lambda_{QCD} \gg 1$ and $am_h \ll 1$. This problem is not specific to the moments method but is present in all lattice methods of α_s determination, except for the Schrödinger functional method (see discussions in the new FLAG report [11]).

To illustrate the challenge of continuum extrapolation of the moments in Fig. 1, I show the cutoff dependence of R_4 and R_6/R_8 together with continuum extrapolations. One can see that the cutoff effects is significant and simple a^2 extrapolations only work for the smallest three lattice spacings, for details see Ref. [10].

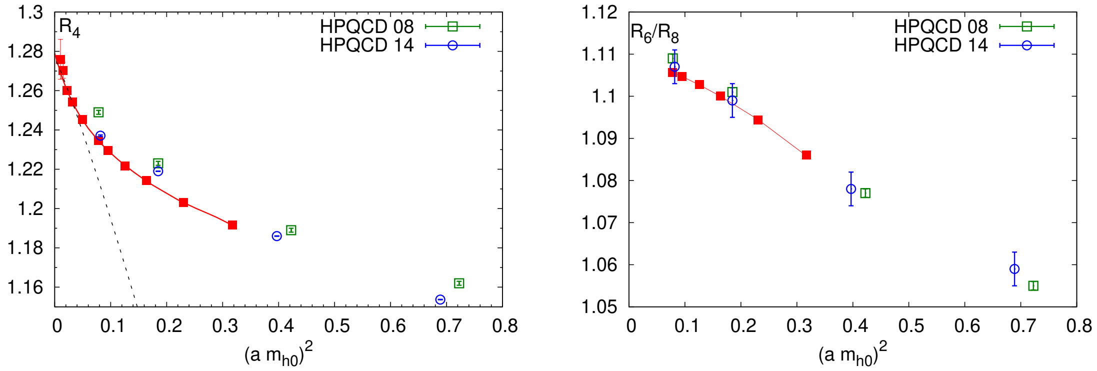

Figure 1: The lattice spacing dependence of R_4 and R_6/R_8 for $m_h = m_c$. The filled symbols correspond to the lattice results of Ref. [10], while the open symbols correspond to HPQCD results from Refs. [5, 7]. The solid line corresponds to polynomial fit, see text. The dashed line corresponds to simple a^2 fit. The errors for the HPQCD-14 result for R_6/R_8 have been obtained by propagating the errors on R_6 and R_8.

If one has data only at large lattice spacings, the continuum limit for R_4 can be easily underestimated, while the continuum limit for R_6/R_8 can be easily overestimated. One way to check for correctness of continuum extrapolations is to compare the results obtained for α_s using R_4 and R_6/R_8. The details of continuum extrapolations are discussed in Ref. [10]. Despite the difficulties of the continuum extrapolations of the moments, the final continuum results obtained in different lattice calculations seem to agree reasonably well, see discussions in Refs. [10]–[11].

From the continuum extrapolated value of R_4 or ratios R_6/R_8 and R_8/R_{10}, the value of $\alpha_s(\mu)$ can be obtained at scales comparable to the heavy quark mass (so that there are no large logarithms). The results for $\alpha_s(\mu = m_h)$ from Ref. [10] are shown in Fig. 2 and Table 1. In Fig. 2, I also compare the results from different lattice determinations. It is clear that performing lattice calculations at different values of the quark mass allows one to map out the running of the coupling constant at relatively low energy scales. It also helps to control the systematic errors of the weak coupling expansion. The running coupling constant extracted from moments of quarkonium correlators in Ref. [10] agrees with the result obtained from the static quark anti-quark energy [12] but is lower than the values of α_s obtained by HPQCD collaboration from the moments of quarkonium correlators. Since the continuum extrapolated lattice results on the moments and their ratios are in a reasonably good agreement with each other the source of this discrepancy must be related to the way comparison of the lattice and weak coupling results is performed. In Refs. [10] $\mu = m_h$, while in HPQCD studies $\mu = 3m_h$.

From the values of $\alpha_s(\mu = m_h)$ one can extract the 3-flavor Λ-parameter, $\Lambda_{\overline{MS}}^{n_f=3}$, which is given

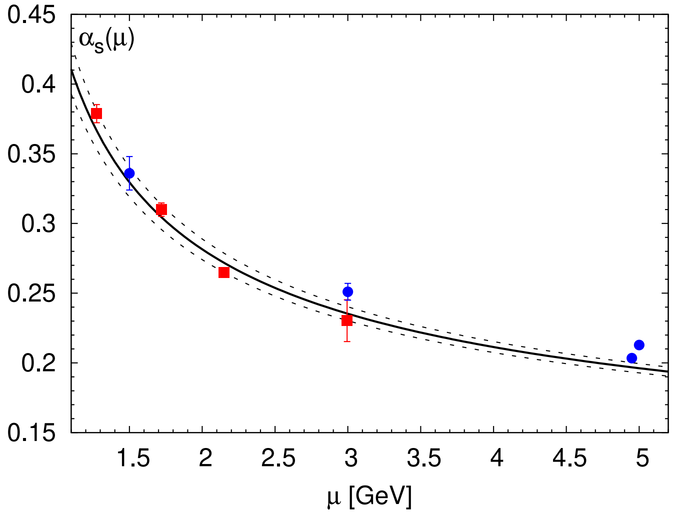

Figure 2: The running coupling in three-flavor QCD constant corresponding to $\Lambda_{\overline{\rm MS}}^{n_f=3} = 301(16)$ MeV. The solid line corresponds to the central value, while the dashed lines show the error band. The blue circles from left to right correspond to the determination of α_s for the static quark anti-quark energy [12] and from the moments of quarkonium correlators [5]–[7]. The result of Ref. [6] has been shifted horizontally for better visibility.

Table 1: The values of $\alpha_s(\mu = m_h)$ for different heavy quark masses, m_h, extracted from R_4, R_6/R_8, and R_8/R_{10}. The heavy quark mass is given in units of m_c. The first, second, and third errors correspond to the lattice, perturbative truncation, and the error due to the gluon condensate. The fifth column lists the averaged value of α_s. The last column gives the value of $\Lambda_{\overline{\rm MS}}^{n_f=3}$ in MeV.

m_h	R_4	R_6/R_8	R_8/R_{10}	average	$\Lambda_{\overline{\rm MS}}^{n_f=3}$
1.0	0.3815(55)(30)(22)	0.3837(25)(180)(40)	0.3550(63)(140)(88)	0.3788(65)	315(9)
1.5	0.3119(28)(4)(4)	0.3073(42)(63)(7)	0.2954(75)(60)(17)	0.3099(48)	311(10)
2.0	0.2651(28)(7)(1)	0.2689(26)(35)(2)	0.2587(37)(34)(6)	0.2649(29)	285(8)
3.0	0.2155(83)(3)(1)	0.2338(35)(19)(1)	0.2215(367)(17)(1)	0.2303(150)	284(48)

in the last column of Table 1. If the perturbative errors are under control, the value of $\Lambda_{\overline{\rm MS}}^{n_f=3}$ obtained from lattice results at different values of the heavy quark m_h should agree. Table 1, however, shows that there is a tension between $\Lambda_{\overline{\rm MS}}^{n_f=3}$ obtained for $m_h = 2m_c$ and the values obtained at smaller quark mass. Performing a weighted average of the $\Lambda_{\overline{\rm MS}}^{n_f=3}$ values in Table 1, I get $\Lambda_{\overline{\rm MS}}^{n_f=3} = 301 \pm 16$ MeV, where the assigned error reflects the spread of the results in Table 1. This value of the Λ-parameter corresponds to $\alpha_s(m_Z, n_f = 5) = 0.1161(12)$, which is about two sigma lower than the most recent result from HPQCD [7], but is in good agreement with the previous determination using the moments of charmonium correlator in 3-flavor QCD [8]. The analysis of Ref. [8] was criticized by the new FLAG report arguing that the perturbative uncertainties have been underestimated and for that reason was given a red symbol for the perturbative behavior [11]. The main argument of this criticism is the fact that $\mu = m_c$ is a low scale and that using higher renormalization scales $\mu =$

$sm_c,\ s > 1$ leads to larger values of α_s. While the raised point is certainly valid, the problems with perturbation theory is not specific to the analysis of Ref. [8] and should affect other determinations of α_s from the moments as well. In particular, if $\mu \neq m_h$ other choices of μ_m need to be considered and varying μ and μ_m independently will lead to much larger perturbative error [1].

In summary, the determination of α_s from the moments of quarkonium correlators, while promising also appears to be challenging. One of the challenge is the control of the continuum extrapolations, which requires many calculations at small lattice spacings. So far this requirement is only met in the 3-flavor calculations with HISQ action [10]. Despite this, there seems to be an agreement between the continuum extrapolated lattice results on the moments of the quarkonium correlators from different groups. This implies that differences in the quoted α_s values are not caused by problems in the lattice calculations, but rather the way lattice and perturbative calculations are combined to obtain α_s. It should be noted that the moments of the quarkonium correlators can be used to extract also the values of the heavy quark masses, and different lattice results agree quite well, see discussion in Ref. [10].

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