Periodic gravitational waves from a 3-body system

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Abstract

We make a proposal about a possible link between a 3-body system and periodic gravitational waves. None of N-body gravitating systems have been considered to emit periodic gravitational waves because of their chaotic orbits when N=3 (or more). However, by employing a figure-eight orbit as a toy model, we show that a 3-body system is capable of generating periodic waves, thereby suggesting that the true number of sources detectable by large-scale interferometers such as LIGO, VIRGO, GEO600 and TAMA300 may be larger than previously thought. A waveform generated by the special 3-body system is volcano-shaped and different from that of a binary system. Therefore, it will be possible to distinguish it in future observations.

1 Introduction

As you know very well, gravitational waves (GWs) are ripples in a curved spacetime generated by accelerated masses, predicted by General Relativity (GR). An evidence of GWs (though indirect) came in 1974 by discovery of binary pulser 'PSR1913+16' (J. Taylor, R. Hulse [Nobel Prize 1993]). The observed decrease in orbital period agrees with the theoretical value by GR. However, the amplitude is too small. No one has ever detected directly GWs.

The detection of GWs will open up new observation fields, especially concerning the physics of stellar central parts, black holes, the early universe, a test of GR and so on. The detection will lead to the Nobel Prize possibly in 20XX. There are several GWs detectors all over the world.

Most likely sources for the first detection are supposed to be (quasi-)periodic such as a single star in rotation and/or oscillation, and a binary star in inspiral and finally merging phases.

There are existing works on a binary plus the third body [1, 2, 3]. On the other hand, much less attention has been paid to N-body gravitating systems, because when $N = 3$(or more), orbits will be chaotic.

Our purpose is to show that a 3-body system can generate (quasi-)periodic GWs [4].

2 3-body Problem

3-body problem was investigated crucially by Jules-Henri Poincare (1854-1912): He gave a proof of being unintegralable. At present, there seems need of numerical computations. Furthermore, chaotic orbits seem to be unsuitable for periodic GWs. However, we know the existence of periodic orbits in 3-body system; Euler's a collinear solution (1765) and Lagrange's an equilateral triangle solution (1772).

We employ a figure-eight solution as a new model of a periodic orbit. It was found firstly by Cristopher Moore (1993) [5] via numerical computations, and secondly with a mathematical proof of existence by Alain Chenciner and Richard Montgomery (2000) [6].

One of its features are a stable orbit in the Newtonian gravity in spite of 3-body system [6, 7]. Each star chases each other on the same orbit with the total angular momentum = 0.

Our assumption:

- the orbital plane is taken as the x-y plane.
- 3-body system with three particles with equal mass.
- compact star like a neutron star or black hole.

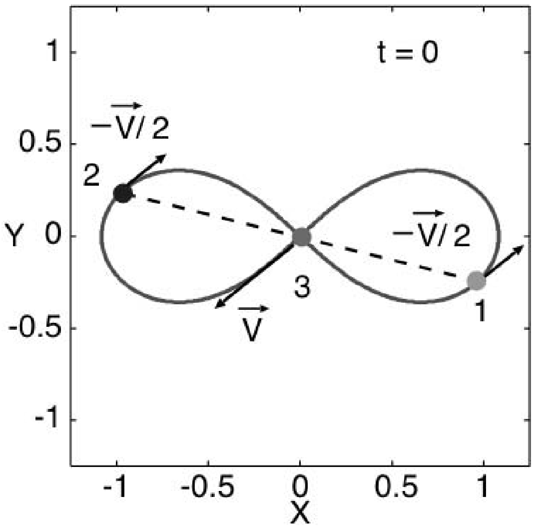

Figure 1: Three masses in a figure-eight at the initial time. Each mass is labeled by 1, 2 and 3. The initial velocity of each mass is denoted by an arrow. The distance between 1 and 3 is denoted by ℓ. In this plot, we take $\ell = 1$.

- when one mass arrives at the center of the 'figure-eight', ℓ (length parameter) is defined as a half of the separation between the remaining two masses.

The equation of motion of 3-body system is

$$m_i' \frac{d^2 \vec{r_i}'}{dt'^2} = -\frac{m_i' m_j' (\vec{r_i}' - \vec{r_j}')}{|\vec{r_i}' - \vec{r_j}'|^3} - \frac{m_i' m_k' (\vec{r_i}' - \vec{r_k}')}{|\vec{r_i}' - \vec{r_k}'|^3}, \tag{1}$$

where $i, j, k = 1 \sim 3$, $i \neq j \neq k$, and we define

$$m_i' \equiv \frac{m_i}{M_\odot}, \tag{2}$$

$$\vec{r_i}' \equiv \frac{\vec{r_i}}{\ell}, \tag{3}$$

$$t' \equiv \frac{t}{\sqrt{\ell^3/GM_\odot}}. \tag{4}$$

The orbital period becomes

$$T = 6.32591398 \sqrt{\frac{\ell^3}{GM_\odot}}. \tag{5}$$

3 Gravitational Waves

The quadrupole formula in the wave zone is

$$h_{ij}^{TT} = \frac{2G\ddot{\not{I}}_{ij}}{rc^4} + O(\frac{1}{r^2}), \tag{6}$$

where we define

$$\not{I}_{ij} = I_{ij} - \delta_{ij}\frac{I_{kk}}{3}, \tag{7}$$

$$I_{ij} = \sum_{A=1}^{N} m_A x_A^i x_A^j. \tag{8}$$

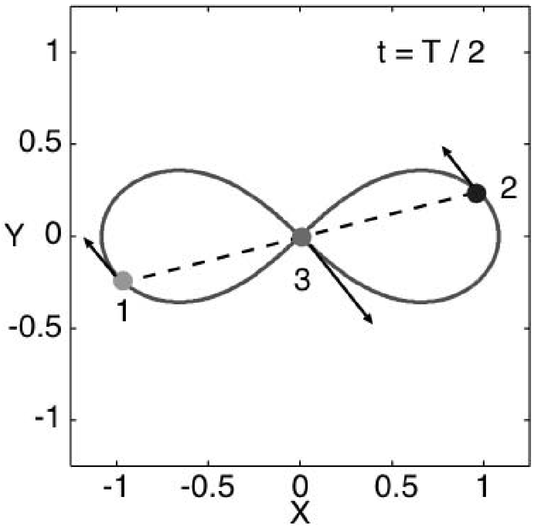

Figure 2: A figure-eight at $t = T/2$. The velocity of each mass is denoted by an arrow.

Here, we denote the Newtonian constant by G, the light speed by c, the space distance from an observer to a source by r, and Transverse and Traceless by TT.

The waveform is shown by Fig. 3. The amplitude of the GWs ($\ell = R_\odot$) is

$$h_{ij}^{TT} \sim 10^{-23} \left(\frac{m}{M_\odot}\right) \left(\frac{R_\odot}{\ell}\right) \left(\frac{10kpc}{r}\right), \tag{9}$$

which is of the same order as that of a binary system.

Next, we consider the angular momentum loss. $L^x = L^y = 0$, because a figure-eight orbit is on the x-y plane. The angular momentum loss rate dL^z/dt vanishes. This is because the total orbital angular momentum $= 0$.

Let us compare a figure-eight with the head-on collision of two non-spinning black holes [8, 9]. For this case, the angular momentum is not carried away. A crucial difference is that for two black holes in head-on collision, each black hole moves without any orbital angular momentum with respect to the common center of mass. On the other hand, in a figure-eight, each star has the angular momentum.

The energy loss rate is expressed as

$$\begin{aligned} \frac{dE}{dt} &= \frac{G}{5c^5} < I\!\!\!-^{(3)}_{ij} I\!\!\!-^{(3)}_{ij} > \\ &= 1.2 \times 10^{19} \left(\frac{m}{M_\odot}\right)^5 \left(\frac{R_\odot}{\ell}\right)^5 \text{erg/s} \quad \text{(figure-eight)} \\ &= 2.0 \times 10^{18} \left(\frac{m}{M_\odot}\right)^5 \left(\frac{R_\odot}{\ell}\right)^5 \text{erg/s} \quad \text{(binary)} \end{aligned} \tag{10}$$

where $\langle \cdots \rangle$ denotes the time average.

The radiation reaction time scale becomes

$$\begin{aligned} t_{GW} &\equiv \frac{E}{dE/dt} \\ &= 0.13 \left(\frac{M_\odot}{m}\right)^3 \left(\frac{\ell}{R_\odot}\right)^4 \text{Gyr} \quad \text{(figure-eight)} \\ &= 0.15 \left(\frac{M_\odot}{m}\right)^3 \left(\frac{\ell}{R_\odot}\right)^4 \text{Gyr} \quad \text{(binary)} \end{aligned} \tag{11}$$

Finally, we make comments on "three black holes just before merging" as a speculation. We take each mass as $10 \times M_\odot$, and ℓ as $10 \times$ Schwarzschild radius denoted by $M_{sch} = 2GM_\odot/c^2$. The frequency of

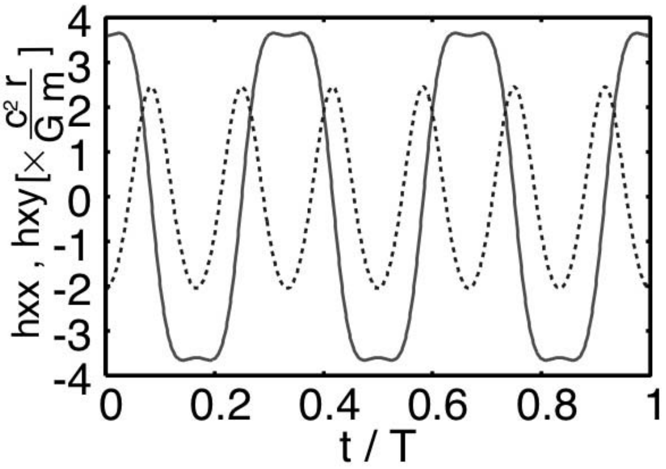

Figure 3: Gravitational waves from three masses in a figure-eight orbit. The dashed curve and the solid one denote h_{xx}^{TT} and h_{xy}^{TT}, respectively.

GWs is about a few kHz, around which the large-scale interferometric detectors are most sensitive. The amplitude of GWs becomes

$$h_{ij}^{TT} \sim 10^{-17} \left(\frac{m}{10M_\odot}\right)\left(\frac{10M_{sch}}{\ell}\right)\left(\frac{10kpc}{r}\right). \tag{12}$$

4 Conclusion

It is possible for 3-body system to generate periodic GWs. One example is a figure-eight orbit. A difference from a binary system is a waveform which is volcano-shaped. The emitted GWs carry away no total angular momentum. Our result may be encouraging; the true number of sources may be larger than previously thought, though would be very few such systems in the universe.

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