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Three-body $B^0_{(s)}$ to $\phi\pi^+\pi^-$ decays

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We investigate the three-body decays of the B^0 meson to $\phi\pi^+\pi^-$ and the B^0_s meson to $\phi\pi^+\pi^-$. Hadronic three-body decays include both non-resonant and resonant contributions, on the basis of the factorization hypothesis. In this analysis, resonant structure is exhibited only in the $\pi^+\pi^-$ channel, whose resonant contribution can be described by S-wave, P-wave, and D-wave $\pi^+\pi^-$ contributions from $f_0(980)$, ρ, and f_2 mesons, and other possible resonance. Therefore, the theoretical values at the scale m_b are $(1.69 \pm 0.19) \times 10^{-7}$ and $(3.28 \pm 0.17) \times 10^{-6}$, while the corresponding experimental results are $(1.82 \pm 0.25) \times 10^{-7}$ and $(3.48 \pm 0.23) \times 10^{-6}$, respectively. Comparing the numerical analysis values with the experimental values shows that our results at the scale μ_b are in agreement.

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1. Introduction

The three-body decays of $B^0_s \longrightarrow \phi\pi^+\pi^+$ and $B^0 \longrightarrow \phi\pi^+\pi^-$ have not been observed before and were recorded by the LHCb experiment and tabulated by the Particle Data Group (PDG) [1,2].

The three-body decay of the D meson to $K\pi\pi$ was analyzed a long time ago [3]. In these sorts of three-body decay, the final-state mesons are supposed to be light. The momentum of the output mesons and the amplitude matrix elements are written by the variables $s = (p_B - p_3)^2$ and $t = (p_B - p_1)^2$ [4,5], and the Dalitz plot technique should be used integrated from $s_{\min}$, $t_{\min}$ to $s_{\max}$, $t_{\max}$ to compute the decay width. This double integral encompasses all angles between the momenta of the output mesons. The amplitudes of the three-body decays can be obtained, and the Feynman quark diagrams should be plotted. The direct three-body $B^0 \rightarrow \phi\pi^+\pi^-$ decay receives two separate parts: one from the point-like weak transition and the other from the pole diagrams that involve three-point or four-point strong vertices. First, we consider parameters that appear in the factorized term of the hadronic matrix element. In the case of $\langle B^0 \rightarrow \pi\pi\rangle \times \langle 0 \rightarrow \phi\rangle$, both π mesons are located in the form factor. In fact, two-meson matrix element transition of the B meson is described to the π mesons. The ϕ is placed in the decay constant. In addition, there is the emission-annihilation process $\langle B^0 \rightarrow 0\rangle \times \langle 0 \rightarrow \phi\pi\pi\rangle$. The total amplitude is computed as the sum of the amplitudes of the non-resonant and resonant contributions. The resonant contribution is evaluated by Dalitz plot analysis. The amplitude matrix element is related to multiplying the B meson by the pion pair transition in the different waves by the vacuum to ϕ meson transition. There are several resonances in the S-wave, P-wave, and D-wave $\pi^+\pi^-$ contributions with $\pi^+\pi^-$ invariant mass in the range $400 < m(\pi^+\pi^-) < 1600\,\mathrm{Mev}/c^2$. Analysis of the resonant contribution defined by the Breit–Wigner function is used to investigate the intermediate states $f_0\phi$, $\rho\phi$, and $f_2\phi$. Other three-body $B^0_s \rightarrow \phi\pi^+\pi^-$ decay is investigated. The parameters appearing in the factorized term are $\langle B^0_s \rightarrow \phi\rangle \times \langle 0 \rightarrow \pi\pi\rangle$ and $\langle B^0_s \rightarrow 0\rangle \times \langle 0 \rightarrow \phi\pi\pi\rangle$. The $0 \rightarrow \pi\pi$ matrix element is supposed to be proportional to the pion scalar, vector, and tensor form factors; then, the different resonances f_{0i}, ρ_i, and f_2 show in the $\pi^+\pi^-$ interaction.

2. Amplitude analysis

In this section, the amplitude and branching ratio of $B^0 \to \phi\pi^+\pi^-$ and $B_s^0 \to \phi\pi^+\pi^-$are obtained by using a factorization method. We have to contemplate the non-resonant and resonant contributions separately. Hadronic weak decays are evaluated by the effective weak Hamiltonian [6]

$$H_{\mathrm{eff}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} \sum_i [V_{\mathrm{CKM}} C_i(\mu) O_i(\mu)]. \tag{1}$$

Here, G_{F} defines the Fermi coupling constant, the coefficients V_{CKM} are elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix [7,8], $C_i(\mu)$ are the Wilson coefficients [9], and $O_{1,2}$ denote current–current operators, $O_{3\text{–}6}$ being penguin operators and $O_{7\text{–}10}$ the electroweak penguin operators [10].

2.1. Non-resonant contribution

In the factorization approach, the Feynman diagrams for three-body $B^0 \to \phi\pi^+\pi^-$ decay are depicted in Fig. 1, which includes the factorizable and non-factorizable diagrams. In the penguin level, the two π mesons are located in the form factor and the ϕ meson is located in the decay constant. The emission annihilation diagrams in which ϕ is emitted via gluon exchange are named "hairpin diagrams." The current analysis includes non-factorizable effects. The non-factorizable terms are dependent on vertex corrections and hard spectator interactions [11]. The non-factorizable diagrams depicted in Fig. 2 should be taken into account. For studying $B^0 \to \phi\pi^+\pi^-$ we use the QCD factorization framework, which includes theoretical properties of QCD like color transparency and hard scattering. Thus, the amplitude of this decay includes $\langle B^0 \longrightarrow \pi^+\pi^-\rangle \times \langle 0 \longrightarrow \phi\rangle$ and $\langle B^0 \longrightarrow 0\rangle \times \langle 0 \longrightarrow \phi\pi^+\pi^-\rangle$. Thus, the decay amplitude is given by

$$\begin{aligned}\langle \phi\pi^+\pi^-|H_{\mathrm{eff}}|B^0\rangle = \frac{iG_{\mathrm{F}}}{2\sqrt{2}}&((a_3+a_5+a_7)\lambda_p\langle\pi^+\pi^-|(\bar{b}d)_{V-A}|B^0\rangle\\ &\times\langle\phi|(\bar{s}s)_{V-A}|0\rangle + (a_2 V_{ub}V_{us}^* + (a_3+a_5+a_7)\lambda_p)\\ &\times\langle 0|(\bar{b}d)_{V-A}|B^0\rangle\langle\phi\pi^+\pi^-|(\bar{u}u)_{V-A}|0\rangle\\ &\quad + (a_3+a_5+a_7+a_4+a_10)\lambda_p\langle 0|(\bar{b}d)_{V-A}|B^0\rangle\\ &\times\langle\phi\pi^+\pi^-|(\bar{d}d)_{V-A}|0\rangle + (a_6+a_8)\lambda_p\langle 0|(\bar{b}d)|B^0\rangle\\ &\times\langle\phi\pi^+\pi^-|(\bar{d}d)|0\rangle,\end{aligned} \tag{2}$$

where $\lambda_p = \sum_{p=u,c} V_{pb}V_{pd}^*$. The three-body matrix elements $\langle\pi^+\pi^-|(\bar{b}d)_{V-A}|B^0\rangle$ have the following general form [12]:

$$\begin{aligned}\langle\pi^-(p_1)\pi^+(p_2)|(\bar{b}d)_{V-A}|B^0(p_B)\rangle &= ir(p_B - p_1 - p_2)_\mu + iw_+(p_1+p_2)_\mu\\ &\quad + iw_-(p_1-p_2)_\mu + h\varepsilon_{\mu\nu\alpha\beta}p_B^\nu(p_1+p_2)^\alpha(p_2-p_1)^\beta.\end{aligned} \tag{3}$$

We need to consider point-like and pole diagrams depicted in Fig. 3. We also require the strong coupling constant of $B^*B\pi$ and $BB\pi\pi$. The form factors $w_\pm$ and r for the non-resonant decay are

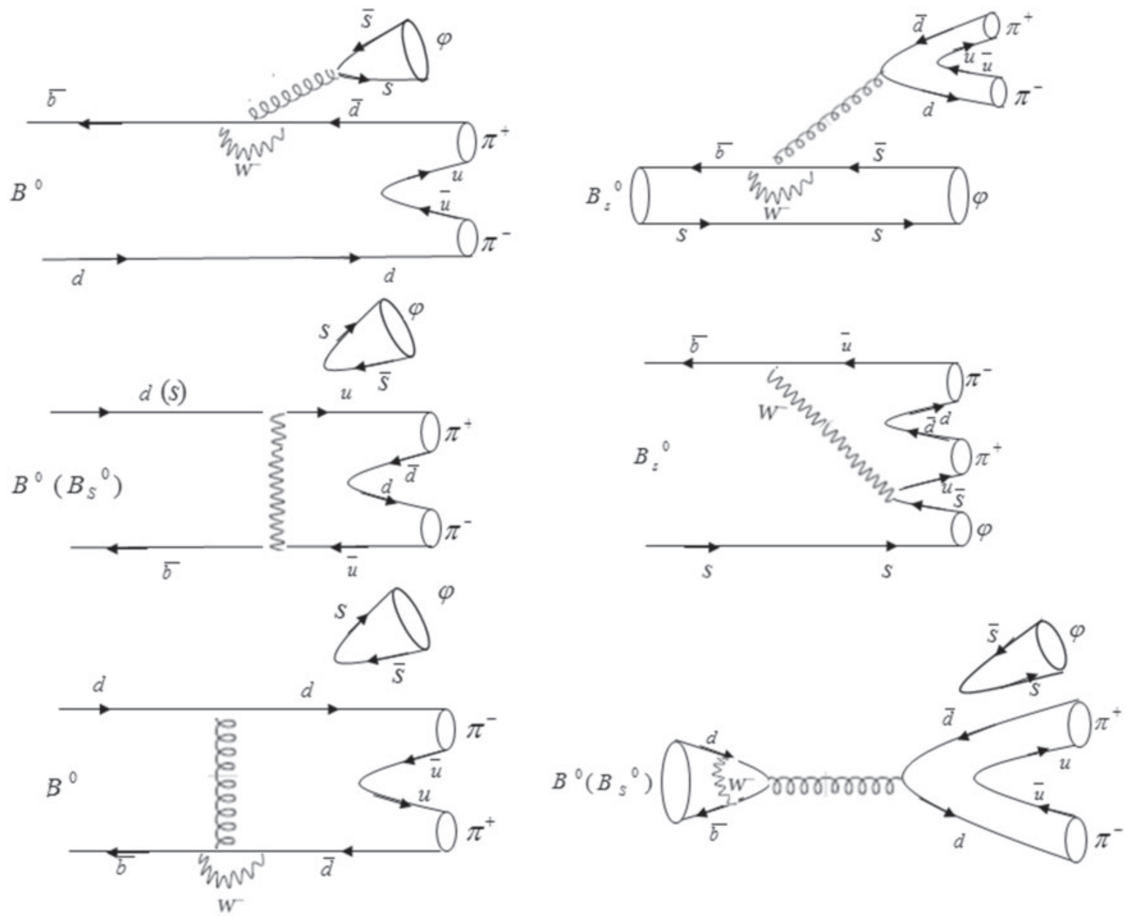

Fig. 1. $B^0(B_S^0) \longrightarrow \phi\pi^+\pi^-$ decay.

evaluated from these diagrams as [13]

$$
\begin{aligned}
r &= \frac{f_B}{2f_\pi^2} - \frac{f_B}{f_\pi^2}\frac{p_B.(p_2-p_1)}{(p_B-p_1-p_2)^2-m_B^2} + \frac{2gf_{B^*}}{f_\pi^2}\sqrt{\frac{m_B}{m_{B^*}}}\frac{(p_B-p_1).p_1}{(p_B-p_1)^2-m_{B^*}^2} \\
&\quad - \frac{4g^2 f_B}{f_\pi^2}\frac{m_B m_{B^*}}{(p_B-p_1-p_2)^2-m_B^2}\frac{p_1.p_2 - p_1.(p_B-p_1)p_2.(p_B-p_1)/m_{B^*}^2}{(p_B-p_1)^2-m_{B^*}^2}, \\
w_+ &= -\frac{g}{f_\pi^2}\frac{f_{B^*}m_{B^*}\sqrt{m_{B^*}m_B}}{(p_B-p_1)^2-m_{B^*}^2}\left(1-\frac{(p_B-p_1).p_1}{m_{B^*}^2}\right) + \frac{f_B}{2f_\pi^2}, \\
w_- &= \frac{g}{f_\pi^2}\frac{f_{B^*}m_{B^*}\sqrt{m_{B^*}m_B}}{(p_B-p_1)^2-m_{B^*}^2}\left(1+\frac{(p_B-p_1).p_1}{m_{B^*}^2}\right), \\
h &= 2g^2\frac{f_B}{f_\pi}\frac{m_B^2}{(m_B^2-m_\phi^2-s)(t+m_B^2-m_\pi^2)},
\end{aligned}
\tag{4}
$$

where g is a heavy-flavor independent strong coupling. The decay constants of the vector meson are defined as [14]

$$
\langle 0|(\bar{s}s)_{V-A}|\phi(p_3,\varepsilon)\rangle = f_\phi m_\phi \varepsilon_\mu^*. \tag{5}
$$

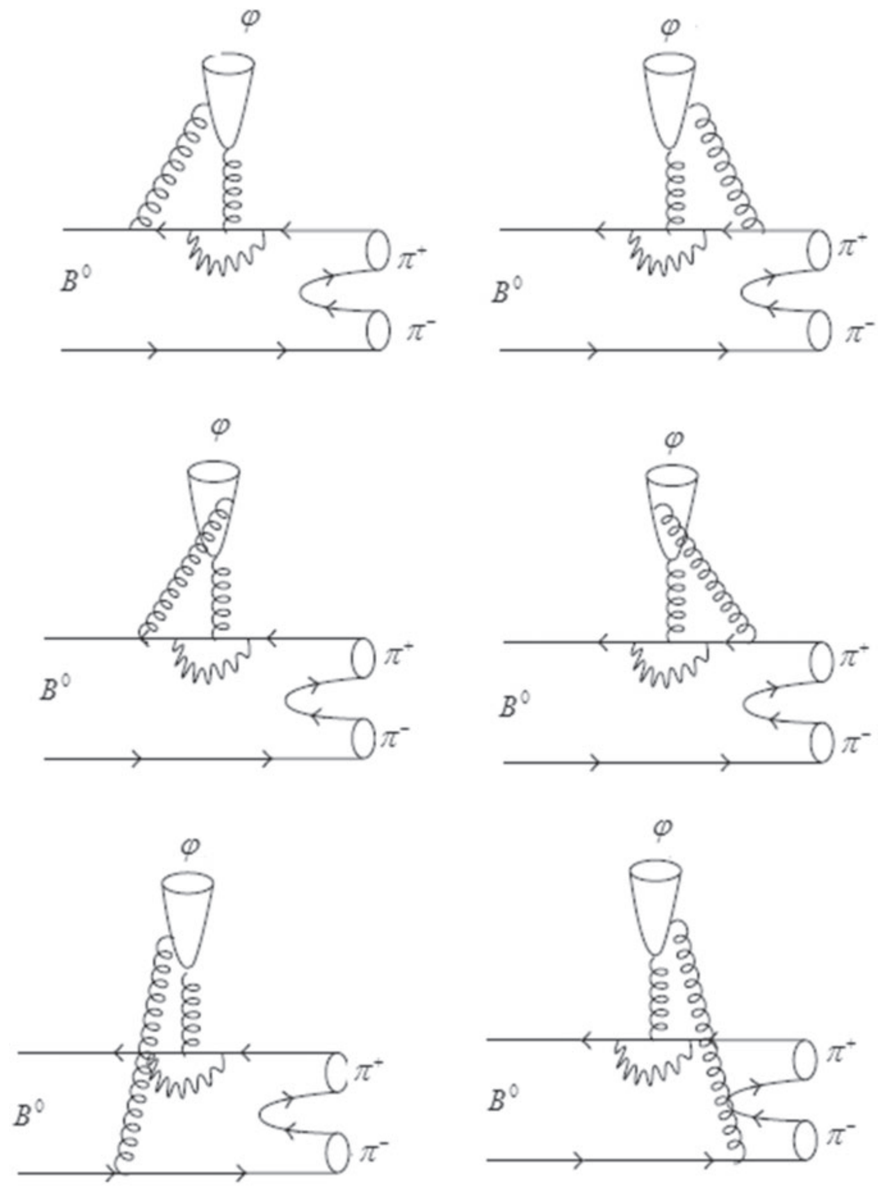

Fig. 2. Non-factorizable diagrams for $B^0 \longrightarrow \phi\pi^+\pi^-$ decay.

For the multiplication of the matrix element, we have

$$\langle\pi^-(p_1)\pi^+(p_2)|(\bar{b}d)_{V-A}|B^0(p_B)\rangle\langle 0|(\bar{s}s)_{V-A}|\phi(p_3,\varepsilon)\rangle$$
$$= if_\phi m_\phi(r\varepsilon.p_3 + w^+\varepsilon.(p_1+p_2) + w^-\varepsilon.(p_2-p_1)), \quad (6)$$

where, under the Lorentz condition, $\varepsilon.p_3 = 0$. The structure of the polarization vector can be described as

$$\varepsilon^{\lambda=0} = (|p_3|, 0, 0, p_3^0)/m_3,$$
$$\varepsilon^{\lambda=\pm1} = \frac{\mp(0, 1, \pm i, 0)}{\sqrt{2}}. \quad (7)$$

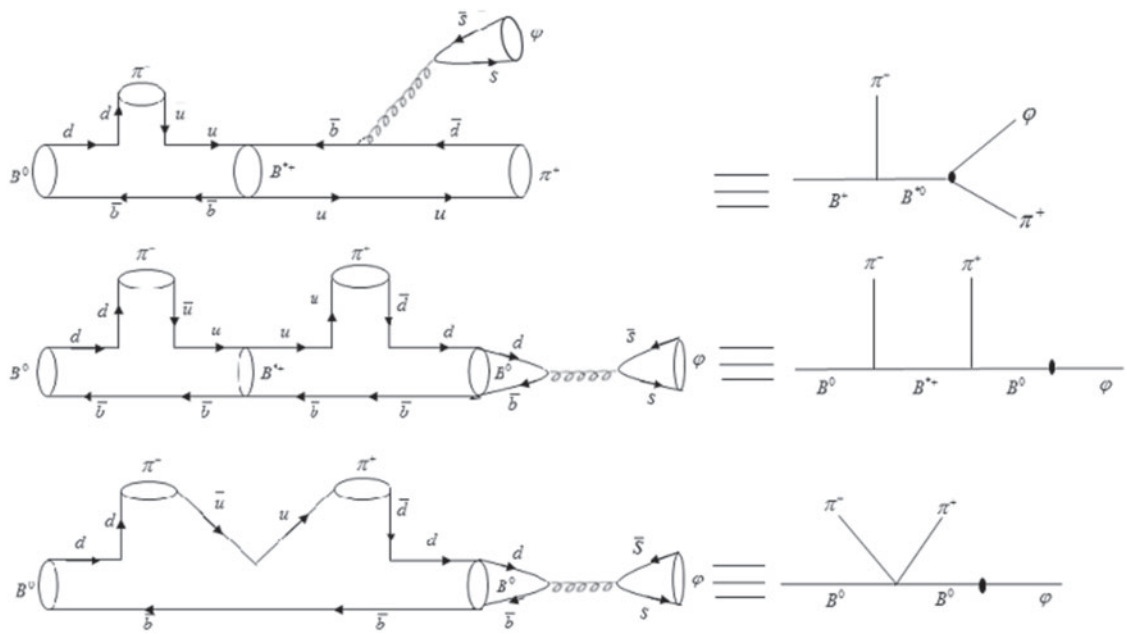

Fig. 3. Point-like and pole diagrams for $B^0 \longrightarrow \phi\pi^+\pi^-$ decay.

The energy–momentum conservation can be shown by:

$$p_B = p_1 + p_2 + p_3. \tag{8}$$

We define the following three invariants that are not independent as:

$$\begin{aligned} s_{12} &= (p_1 + p_2)^2 = (p_B - p_3)^2, \\ s_{13} &= (p_1 + p_3)^2 = (p_B - p_2)^2, \\ s_{23} &= (p_2 + p_3)^2 = (p_B - p_1)^2. \end{aligned} \tag{9}$$

According to the definition of four-momentum conservation, we have, from the invariants,

$$s_{12} + s_{13} + s_{23} = m_B^2 + m_1^2 + m_2^2 + m_3^2; \tag{10}$$

we let $s_{12} = s$ and $s_{23} = t$. In the center of mass of $\pi^-(p_1)$ and $\pi^+(p_2)$, we find

$$\begin{aligned} |p_1| = |p_2| &= \frac{1}{2}\sqrt{s - 4m_1^2}, \\ p_1^0 = p_2^0 &= \frac{1}{2}\sqrt{s}, \\ |p_3| &= \frac{1}{2\sqrt{s}}\sqrt{(m_B^2 - m_3^2 - s)^2 - 4sm_3^2}, \\ p_3^0 &= \frac{1}{2\sqrt{s}}(m_B^2 - m_3^2 - s), \end{aligned} \tag{11}$$

and the cosine of the helicity angle θ between the direction of p_2 and that of p_3 reads

$$\cos\theta = \frac{1}{4|p_2||p_3|}(m_B^2 + m_3^2 + 2m_2^2 - s - 2t). \tag{12}$$

With these definitions, we obtain

$$\varepsilon.(p_1+p_2)=2p_1^0\varepsilon^0,$$
$$\varepsilon.(p_2-p_1)=2|\varepsilon||p_1|\cos\theta. \tag{13}$$

The matrix elements of the annihilation process are written by

$$\langle\pi^-(p_1)\pi^+(p_2)\phi(p_3)|(\bar{d}d)_{V-A}|0\rangle=\frac{2i}{f_\pi}\left(p_{2\mu}-\frac{p_B.p_2}{p_B^2-p_3^2}p_{B\mu}\right)F^{\phi\pi\pi}(q^2),$$
$$\langle\pi^-(p_1)\pi^+(p_2)f_0(p_3)|(\bar{d}d)|0\rangle=v\frac{f_Bm_B^2}{f_\pi m_b}\left(1-\frac{s_{13}-m_1^2-m_3^2}{m_B^2-m_3^2}\right)F^{\phi\pi\pi}(q^2), \tag{14}$$

where

$$v=\frac{m_\pi^2}{m_u+m_d}. \tag{15}$$

The form factor $F(q^2)$ in Eq. (14) is described as

$$F^{M_1M_2M_3}(q^2)=\frac{1}{1-q^2/\Lambda_x^2}. \tag{16}$$

We intend to compute the branching ratios for $B\longrightarrow\phi\pi\pi$ decay in the improved QCD factorization approach. In order to do this, we consider the vertex corrections to this decay indicated by f_{I} andf_{II} in factorization. Their effects can be mixed with the Wilson coefficients [15]:

$$a_2=c_2+\frac{c_1}{3}+\frac{\alpha_s}{4\pi}\frac{C_F}{3}c_1\left(-18+12\ln\frac{m_c}{\mu}+f_{\mathrm{I}}+f_{\mathrm{II}}\right),$$
$$a_3=c_3+\frac{c_4}{3}+\frac{\alpha_s}{4\pi}\frac{C_F}{3}c_4\left(-18+12\ln\frac{m_c}{\mu}+f_{\mathrm{I}}+f_{\mathrm{II}}\right),$$
$$a_5=c_5+\frac{c_6}{3}+\frac{\alpha_s}{4\pi}\frac{C_F}{3}c_6\left(+6-12\ln\frac{m_c}{\mu}-f_{\mathrm{I}}-f_{\mathrm{II}}\right),$$
$$a_7=c_7+\frac{c_8}{3}+\frac{\alpha_s}{4\pi}\frac{C_F}{3}c_8\left(+6-12\ln\frac{m_c}{\mu}-f_{\mathrm{I}}-f_{\mathrm{II}}\right),$$
$$a_9=c_9+\frac{c_{10}}{3}+\frac{\alpha_s}{4\pi}\frac{C_F}{3}c_{10}\left(-18+12\ln\frac{m_c}{\mu}+f_{\mathrm{I}}+f_{\mathrm{II}}\right), \tag{17}$$

where the term f_{I}, the hard scattering function, results from the vertex corrections and f_{II} is expected with the hard gluon exchange involving the spectator quark in the D meson. The vertex corrections are given by [16]:

$$f_{\mathrm{I}}=\frac{2\sqrt{6}}{f_\phi}\int dx\phi_\phi^L(x)\left[\frac{3(1-2x)}{1-x}\ln(x)-3\pi i+3\ln(1-r^2)+\frac{2r^2(1-x)}{1-r^2x}\right],$$
$$f_{\mathrm{II}}=\frac{4\pi^2}{N}\frac{if_Rf_Bf_\phi}{X_s}\int_0^1 d\rho\frac{\phi_|^B(\rho)}{\rho}\int_0^1 du\frac{\phi_{||}^\phi(u)}{u}\int_0^1 d\eta\frac{\phi_{||}^R(\eta)}{\eta}. \tag{18}$$

The leading twist distribution amplitudes are given in terms of an expansion in Gegenbauer polynomials [17]:

$$\phi_i(u,\mu) = 6u\bar{u}\left[1 + \sum_{n=1}^{\infty} \alpha_{n,i}(\mu) C_n^{3/2}(2u-1)\right], \quad i = \perp, \parallel. \tag{19}$$

The other three-body decay that is investigated is $B_s^0 \longrightarrow \phi\pi^+\pi^+$ decay. Feynman diagrams for this decay are shown in Fig. 1. The amplitude of this decay includes $\langle B_s^0 \longrightarrow \phi\rangle \times \langle 0 \longrightarrow \pi^+\pi^-\rangle$ and $\langle B_s^0 \longrightarrow 0\rangle \times \langle 0 \longrightarrow \phi\pi^+\pi^-\rangle$. Thus, under the factorization hypothesis, the decay amplitude $B_s^0 \longrightarrow \phi\pi^+\pi^-$ is given by

$$\begin{aligned}
\langle \phi\pi^+\pi^-|H_{\text{eff}}|B_s^0\rangle = \frac{iG_{\text{F}}}{2\sqrt{2}}&((a_2 V_{ub}V_{us}^* + (a_3 + a_5 + a_7)\lambda_p)\langle\phi|(\bar{b}s)_{V-A}|B_s^0\rangle \\
&\times \langle\pi^+\pi^-|(\bar{u}u)_{V-A}|0\rangle + ((a_3 + a_5 - 1/2(a_7 + a_9))\lambda_p \\
&\times \langle\phi|(\bar{b}s)_{V-A}|B_s^0\rangle\langle\pi^+\pi^-|(\bar{d}d)_{V-A}|0\rangle + (a_2 V_{ub}V_{us}^* \\
&+ (a_3 + a_5 + a_7)\lambda_p)\langle 0|(\bar{b}s)_{V-A}|B_s^0\rangle\langle\phi\pi^+\pi^-|(\bar{u}u)_{V-A}|0\rangle \\
&+ (a_3 + a_5 + a_7 + a_9)\lambda_p\langle 0|(\bar{b}s)_{V-A}|B_s^0\rangle \\
&\times \langle\phi\pi^+\pi^-|(\bar{d}d)_{V-A}|0\rangle).
\end{aligned} \tag{20}$$

The hadronic matrix elements for $B_s^0 \longrightarrow \phi$ can be described as [18]

$$\begin{aligned}
\langle\phi(p_3,\varepsilon)|(\bar{b}s)_{V-A}|B_s^0(p_{B_s})\rangle = i((m_3 + m_{B_s})\varepsilon_\mu A_1^{B_s\phi}(q^2) \\
- \frac{\varepsilon.p_{B_s}}{m_3 + m_{B^*}}(p_{B_s} + p_3)_\mu A_2^{B_s\phi}(q^2) \\
- 2m_3\frac{\varepsilon.p_{B_s}}{q^2}q_\mu(A_3^{B_s\phi}(q^2) - A_0^{B_s\phi}(q^2)),
\end{aligned} \tag{21}$$

where

$$A_3^{B_s\phi}(q^2) = \frac{m_{B_s} + m_3}{2m_3}A_1^{B_s\phi}(q^2) - \frac{m_{B_s} - m_3}{2m_3}A_2^{B_s\phi}(q^2). \tag{22}$$

The two-pion-creation matrix element of the weak interaction current can be expressed by

$$\langle\pi^+(p_2)\pi^-(p_1)|(\bar{q}q)_{V-A}|0\rangle = (p_1 - p_2)F_1^{\pi\pi}(q^2). \tag{23}$$

The non-resonant weak and electromagnetic form factor $F^{\pi\pi}$ is parametrized as follows [19]:

$$\begin{aligned}
F_{\text{em}}^{\pi\pi}(q^2) &= \frac{1}{1 - q^2/M_*^2 + i\Gamma_*/M_*}, \\
F_{\text{weak}}^{\pi\pi}(q^2) &= \frac{F^{\pi\pi}(0)}{1 - q^2/\Lambda_\chi^2 + i\Gamma_*/\Lambda_\chi},
\end{aligned} \tag{24}$$

using $\Gamma_* = 200$ MeV and $M_* = 600$ MeV, and $\Lambda_\chi = 830$ MeV is the chiral-symmetry-breaking scale. By multiplying matrix elements, we have

$$\begin{aligned}
&\langle\phi(p_3,\varepsilon)|(\bar{b}s)_{V-A}|B_s^0(p_{B_s})\rangle\langle\pi^+(p_2)\pi^-(p_1)|(\bar{q}q)_{V-A}|0\rangle \\
&= iF_1^{\pi\pi}(q^2)((m_{B_s} + m_3)\varepsilon.(p_1 - p_2)A_1^{B_s\phi}(q^2) - 2\frac{\varepsilon.p_{B_s}}{m_3 + m_{B^*}}(p_1.p_3 - p_2.p_3)A_2^{B_s\phi}(q^2)).
\end{aligned} \tag{25}$$

2.2. Resonant contribution

As noted before, the Dalitz plot model can indicate the presence of intermediate resonance. The amplitude decay of of the B meson into $\pi\pi$ in the different wave is to be appropriate to the pion non-strange scalar or vector form factor, depending on the wave studied; the three-body matrix element $\langle\pi^+\pi^-|V_\mu|B^0\rangle$ can be described by S-, P-, and D-waves from the $\pi\pi$ channel. Resonant effects are explained in terms of the Breit–Wigner formalism. Thus, the resonant contribution of $B^0 \to \phi\pi^+\pi^-$ can be written as follows:

$$\begin{aligned}\langle\pi^+(p_2)\pi^-(p_1)|(\bar{b}d)_{V-A}|B^0(p_B)\rangle^R &= \sum_i \frac{g^{T_i\to\pi^+\pi^-}}{s-m_{T_i}^2+im_{T_i}\Gamma_{T_i}}\varepsilon_{\sigma\gamma}p_1^\sigma p_1^\gamma \\ \times\,\langle T|(\bar{b}d)_{V-A}|B^0\rangle &- \sum_i \frac{g^{V_i\to\pi^+\pi^-}}{s-m_{V_i}^2+im_{V_i}\Gamma_{V_i}}\varepsilon^*.(p_1-p_2) \\ \times\,\langle V|(\bar{b}d)_{V-A}|B^0\rangle &- \sum_i \frac{g^{S_i\to\pi^+\pi^-}}{s-m_{S_i}^2+im_{S_i}\Gamma_{S_i}}\langle S|(\bar{b}d)_{V-A}|B^0\rangle. \end{aligned} \tag{26}$$

The three-body matrix element can receive contributions from the f_2 tensor meson, the ρ vector meson, and the f_0 scalar resonances. The form factors for the $B \longrightarrow S$ [20], $B \longrightarrow V$ [18], and $B \longrightarrow T$ [21] transitions are described by:

$$\begin{aligned}\langle f_0(p_q)|(\bar{b}d)_{V-A}|B^0(p_B)\rangle &= -i\Bigg[\left((p_B+p_q)_\mu - \frac{m_B^2-m_{f_0}^2}{q^2}q_\mu\right)F_1^{Bf_0}(q^2) \\ &\qquad + \frac{m_B^2-m_{f_0}^2}{q^2}q_\mu F_0^{Bf_0}(q^2)\Bigg], \\ \langle\rho(p_q,\varepsilon)|(\bar{b}d)_{V-A}|B^0(p_B)\rangle &= i\Bigg[(m_B+m_\rho)\varepsilon_\mu^* A_1^{B\rho}(q^2) \\ &\qquad - \frac{\varepsilon^*.p_B}{m_B+m_\rho}(p_B+p_q)_\mu A_2^{B\rho}(q^2) \\ &\qquad -2m_\rho\frac{\varepsilon^*.p_B}{q^2}q_\mu(A_3^{B\rho}(q^2)-A_0^{B\rho}(q^2))\Bigg], \\ \langle f_2(p_q,\varepsilon)|(\bar{b}d)_{V-A}|B^+(p_B)\rangle &= ih\varepsilon_{\mu\nu\lambda\rho}\varepsilon^{*\nu\alpha}p_{B\alpha}(p_B+p_q)^\lambda(p_B-p_q)^\rho + k(q^2)\varepsilon^{*\mu\nu}p_{B\nu} \\ &\qquad + \varepsilon^*_{\alpha\beta}p_B^\alpha p_B^\beta(b_+(q^2)(p_B+p_q)^\mu + b_-(q^2)(p_B-p_q)^\mu). \end{aligned} \tag{27}$$

The polarization tensor $\varepsilon^{\mu\nu}(p_q,\lambda)$ with momentum p and helicity λ is given by [22]

$$\begin{aligned}\varepsilon^{\mu\nu}(\pm2) &= \varepsilon^\mu(\pm1)\varepsilon^\nu(\pm1), \\ \varepsilon^{\mu\nu}(\pm1) &= \frac{1}{\sqrt{2}}(\varepsilon^\mu(\pm1)\varepsilon^\nu(0)+\varepsilon^\mu(0)\varepsilon^\nu(\pm1)), \\ \varepsilon^{\mu\nu}(0) &= \sqrt{\frac{1}{6}}(\varepsilon^\mu(+1)\varepsilon^\nu(-1)+\varepsilon^\mu(-1)\varepsilon^\nu(+1)) + \sqrt{\frac{2}{3}}(\varepsilon^\mu(0)\varepsilon^\nu(0)), \end{aligned} \tag{28}$$

where $\varepsilon^\mu(0,\pm1)$ represent the polarization vector of the massive vector state moving along the Z-axis [see Eq. (7)]. We have used the partial widths for determining the coupling constants $f_0 \to \pi\pi$,

$\rho \to \pi\pi$, and $f_2 \to \pi\pi$ [2]:

$$\begin{aligned}
\Gamma(f_0(980) \to \pi^+\pi^-) &= 34.2^{+13.9}_{-11.8}\,\text{MeV},\\
\Gamma(f_0(1370) \to \pi^+\pi^-) &= 10.8 \pm 2\,\text{MeV},\\
\Gamma(f_0(1500) \to \pi^+\pi^-) &= 35.8 \pm 4\,\text{MeV},\\
\Gamma(\rho(770) \to \pi^+\pi^-) &\sim 149.1 \pm 0.8\,\text{MeV},\\
\Gamma(\rho(1450) \to \pi^+\pi^-) &= 400 \pm 60\,\text{MeV},\\
\Gamma(f_2(1270) \to \pi^+\pi^-) &= 165 \pm 9\,\text{MeV}.
\end{aligned} \tag{29}$$

Note also that $g^{S\to M_1M_2}$, $g^{V\to M_1M_2}$, and $g^{T\to M_1M_2}$ are the coupling constants for the scalar, vector, and tensor mesons:

$$\begin{aligned}
\Gamma_{S\to M_1M_2} &= \frac{p_c}{8\pi m_S^2} g^2_{S\to M_1M_2}, \quad \Gamma_{V\to M_1M_2} = \frac{p_c^3}{6\pi m_V^2} g^2_{V\to M_1M_2},\\
\Gamma_{T\to M_1M_2} &= \frac{p_c^5}{15\pi m_T^4} g^2_{T\to M_1M_2},
\end{aligned} \tag{30}$$

where p_c is the center-of-mass momentum. Thus, the resonant amplitude can be obtained as follows:

$$\begin{aligned}
&M_R(B^- \longrightarrow \pi^-(p_1)\pi^+(p_2)\phi(p_3,\varepsilon))\\
&= \frac{iG_F}{\sqrt{2}}\frac{1}{2}(a_3+a_5+a_7)\lambda_p m_\phi f_\phi \times \Bigg[\frac{g^{f_2\to\pi^+\pi^-}}{s-m_{f_2}^2+im_{f_2}\Gamma_{f_2}}\varepsilon_T^{\alpha\beta}\varepsilon_\mu(p_B+p_{f_2})_\rho\\
&\quad \times (k(m_\phi^2)\delta^\mu_\alpha\delta^\rho_\beta + b_+(m_\phi^2)p_{3\alpha}p_{3\beta}g^{\mu\rho}) - \sum_i \frac{ig^{\rho_i\to\pi^+\pi^-}}{s-m_{\rho_i}^2+im_{\rho_i}\Gamma_{\rho_i}}\Bigg(-(m_B+m_{\rho_i})\varepsilon.(p_1-p_2)\\
&\quad \times A_1^{B\rho_i}(m_\phi^2) - \frac{A_2^{B\rho_i}(m_\phi^2)}{m_B+m_{\rho_i}}\varepsilon.(p_B+p_{\rho_i})p_B.(p_1-p_2)\Bigg)\\
&\quad - \sum_i \frac{ig^{f_{0i}\to\pi^+\pi^-}}{s-m_{f_{0i}}^2+im_{f_{0i}}\Gamma_{f_{0i}}}\varepsilon.(p_B+p_{f_{0i}})F_1^{Bf_{0i}}(m_\phi^2)\Bigg],
\end{aligned}$$

where the polarization tensor $\varepsilon_T^{\alpha\beta}$ follows Eq. (28). In $B_s^0 \to \phi\pi\pi$, the two-body matrix element $\langle\pi^+\pi^-|V_\mu|0\rangle$ can also receive contributions from the f_2 tensor meson, ρ vector meson, and f_0 scalar resonances in the $\pi^-\pi^+$ channel. As noted above, the resonant contribution of $B_s^0 \to \phi\pi\pi$ is investigated by using the Breit–Wigner formalism:

$$\begin{aligned}
\langle\pi^-(p_1)\pi^+(p_2)|(\bar{q}q)_{V-A}|0\rangle^R &= \sum_i \frac{g^{T_i\to\pi^+\pi^-}}{s-m_{T_i}^2+im_{T_i}\Gamma_{T_i}}\varepsilon_{\sigma\gamma}p_1^\sigma p_1^\gamma\\
&\quad \times \langle T_i|(\bar{q}q)_{V-A}|0\rangle - \sum_i \frac{g^{V_i\to\pi^+\pi^-}}{s-m_{V_i}^2+im_{V_i}\Gamma_{V_i}}\varepsilon^*.(p_1-p_2)\\
&\quad \times \langle V_i|(\bar{q}q)_{V-A}|0\rangle - \sum_i \frac{g^{S_i\to\pi^+\pi^-}}{s-m_{S_i}^2+im_{S_i}\Gamma_{S_i}}\langle S_i|(\bar{q}q)_{V-A}|0\rangle.
\end{aligned} \tag{31}$$

The decay constants of the scalar and vector mesons are defined as [23,24]

$$\langle 0|(\bar{q}q)_{V-A}|f_0(980)(p_{f_0})\rangle = f_{f_0} p^{\mu}_{f_0},$$
$$\langle 0|(\bar{q}q)_{V-A}|\rho(p_\rho,\varepsilon)\rangle = f_\rho m_\rho \varepsilon^*_\mu. \tag{32}$$

The polarization of the tensor meson satisfies the following relations [25]:

$$\varepsilon^{\mu\nu} = \varepsilon^{\nu\mu}, \qquad \varepsilon^{\mu}_{\mu} = 0, \qquad p_\mu \varepsilon^{\mu\nu} = p_\nu \varepsilon^{\mu\nu} = 0. \tag{33}$$

Therefore, the decay constant of the tensor meson is defined as

$$\langle 0|(\bar{q}q)_{V-A}|T(p_q,\varepsilon)\rangle = a\varepsilon_{\mu\nu}p^{\nu} + b\varepsilon_{\mu\nu}p^{\mu} = 0. \tag{34}$$

Then we are led to

$$\begin{aligned}
&\langle \phi(p_3,\varepsilon)|(\bar{b}s)_{V-A}|B^0_s(p_{B_s})\rangle\langle \pi^+(p_2)\pi^-(p_1)|(\bar{q}q)_{V-A}|0\rangle^R \\
&-\sum_i i\frac{g^{f_{0i}\to\pi^+\pi^-}}{s-m^2_{f_{0i}}+im_{f_{0i}}\Gamma_{f_{0i}}} f_{f_{0i}} \Bigg((m_{B_s}+m_3)\varepsilon.p_{B_s} \\
&\times A_1^{B_s\phi}(q^2) - \frac{\varepsilon.p_{B_s}}{m_{B_s}+m_\phi}(p_{B_s}+p_3).p_{f_{0i}} A_2^{B_s\phi}(q^2) \\
&-2m_3\frac{\varepsilon.p_{B_s}}{q^2} q.p_{f_{0i}}(A_3^{B_s\phi}(q^2) - A_0^{B_s\phi}(q^2)\Bigg) \\
&-\sum_i \frac{g^{\rho_i\to\pi^+\pi^-}}{s-m^2_{\rho_i}+im_{\rho_i}\Gamma_{\rho_i}} \varepsilon_{\rho_i}.(p_1-p_2) f_{\rho_i} m_{\rho_i} \Bigg((m_{B_s}+m_3)\varepsilon.\varepsilon_{\rho_i} \\
&\times A_1^{B_s\phi}(q^2) - \frac{\varepsilon.p_{B_s}}{m_{B_s}+m_\phi}\varepsilon_{\rho_i}.(p_{B_s}+p_3) A_2^{B_s\phi}(q^2) \\
&-2m_3\frac{\varepsilon.p_{B_s}}{q^2}\varepsilon_{\rho_i}.q(A_3^{B_s\phi}(q^2) - A_0^{B_s\phi}(q^2)\Bigg),
\end{aligned} \tag{35}$$

where $q = p_1 + p_2 = p_{B_s} - p_3$ and $p_R = p_1 + p_2$. Therefore,

$$\varepsilon.p_{f_0} = \varepsilon.p_{B_s} = \frac{m_{B_s}}{m_3}|p_3|. \tag{36}$$

Finally, the decay amplitude through resonance reads

$$\begin{aligned}
M_R(B^0_s \to \phi\pi^+\pi^-) = &-\sum_i i\frac{g^{f_{0i}\to\pi^+\pi^-}}{s-m^2_{f_{0i}}+im_{f_{0i}}\Gamma_{f_{0i}}} f_{f_{0i}} m_{B_s}|p_3| \\
&\times 2A_0^{B_s\phi}(s_{12}) - \sum_i \frac{g^{\rho_i\to\pi^+\pi^-}}{s-m^2_{\rho}+im_{\rho}\Gamma_{\rho}} f_{\rho_i} m_{\rho_i} \\
&\times \Bigg[-(m_{B_s}+m_3)\varepsilon.(p_1-p_2)A_1^{B_s\phi}(s_{12}) \\
&\quad -\frac{\varepsilon.p_{B_s}}{m_{B_s}+m_3}(2p_3.p_2 - 2p_3.p_1)A_2^{B_s\phi}(s_{12})\Bigg].
\end{aligned} \tag{37}$$

Table 1. The values of the Wilson coefficients c_i at three renormalization scales μ.

μ	c_1	c_2	c_3	c_4	c_5	c_6	c_7/α	c_8/α	c_9/α	c_{10}/α
$m_b/2$	1.137	−0.295	0.021	−0.051	0.010	−0.065	−0.024	0.096	−1.325	0.331
m_b	1.081	−0.19	0.014	−0.036	0.009	−0.042	−0.011	0.060	−1.254	0.223
$2m_b$	1.045	−0.113	0.009	−0.025	0.007	−0.027	−0.011	0.039	−1.195	0.144

Table 2. The form factor for the $B \to \rho$ transition.

Decay		F(0)	a_F	b_F
$B \to \rho$	A_1	0.232	0.42	−0.25
	A_2	0.187	0.98	−0.03
	A_3	−0.221	1.16	0.09
	V	0.289	1.32	0.34

The three-body decay width is written as [26]

$$\Gamma(B \longrightarrow M_1M_2M_3) = \frac{1}{(2\pi)^3 32 M_B^2} \int_{s_{\min}}^{s_{\max}} \int_{t_{\min}}^{t_{\max}} |M_{NR}(B \longrightarrow M_1M_2M_3) + M_R(B \longrightarrow M_1M_2M_3)|^2 dtds, \tag{38}$$

where

$$\begin{aligned} s_{\min} &= (m_1 + m_2)^2, \\ s_{\max} &= (m_B - m_3)^2, \\ t_{\min,\max} &= m_2^2 + m_3^2 - \frac{1}{s}[(s - m_B^2 + m_1^2) \times (s + m_2^2 - m_3^2) \\ &\quad \pm \lambda^{1/2}(s, m_B^2, m_1^2)\lambda^{1/2}(s, m_2^2, m_3^2)], \end{aligned} \tag{39}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$.

3. Numerical results

We need to determine the values of several physical ingredients for the numerical analysis. The Fermi coupling constant, G_F, is taken to be 1.66×10^{-5} GeV. The values of the decay constants and meson masses in units of MeV are [2,23,27]:

$$\begin{aligned} &m_{\pi^\pm} = 139.57 \pm 0.00035,\ m_\phi = 1019 \pm 0.016,\ m_{B^0} = 5279.63 \pm 0.15, \\ &m_{f_0} = 990 \pm 20,\ m_{B_s} = 5366.89 \pm 0.19,\ m_\rho = 775.26 \pm 0.25, \\ &m_{f_2} = 1275.5 \pm 0.8,\ m_{B*} = 5324.65 \pm 0.25,\ f_{f_0} = 0.37,\ f_\phi = 221, \\ &f_\pi = 130,\ f_\rho = 218 \pm 2,\ f_B = 176 \pm 42,\ f_{B^*} = 194 \pm 6. \end{aligned} \tag{40}$$

The coupling constants are $g_{B^*B\pi} = 32 \pm 5$ [28] and $g_{B_s^*BK} = 10.6$ [29]. The values of c_i at three scales, $\mu = m_b/2$, m_b, and $2m_b$, are given in Table 1 [30]. The form factors $A_i^{B\rho}$ are shown in Table 2 [31]. The form factors of the transition $B_s \to \phi$ at $q^2 = 0$ applied in Eq. (20) are given in Table 3 [32]. The $B \to f_2$ transition form factor is depicted in Table 4 [33,34]. The parameter in the form

Table 3. The parameters for the $B_s \to \phi$ transition.

Decay	A_1	A_2	V
$B_s \to \phi$	0.29	0.25	0.24

Table 4. The parameters for the $B \to f_2$ transition.

Decay	k	b_+	b_-
$B \to f_2$	0.425	-0.014	0.014

Table 5. The decay branching ratios at scales $\mu = m_b/2$, m_b, and $2m_b$.

Mode	$B^0 \to \phi\pi^+\pi^-$ $(\times 10^{-7})$	$B_s^0 \to \phi\pi^+\pi^-$ $(\times 10^{-6})$
BR($2m_b$)	2.16 ± 0.21	5.5 ± 0.20
BR(m_b)	1.69 ± 0.19	3.28 ± 0.17
BR($m_b/2$)	0.98 ± 0.15	2.34 ± 0.13
$\mathrm{BR_{Exp}}$	1.82 ± 0.25	3.48 ± 0.23

factor of the $B \to f_0(d\bar{d})$ transition is 0.3 ± 0.05 [35]. The branching ratios of these decays at three scales are calculated as shown in Table 5.

4. Conclusion

In this research, we have computed the branching ratios of the $B^0 \to \phi\pi^+\pi^-$ and $B_s^0 \to \phi\pi^+\pi^-$ decays obtained from studies of three-body decays. The branching ratios of hadronic three-body decays are evaluated by applying the factorization approach. The Dalitz plot model for decays is determined by considering several resonant and non-resonant amplitudes. In order to examine the branching ratios of three-body decays, we investigated resonant and non-resonant contributions. There are factorizable and non-factorizable contributions such that the non-factorizable terms corresponding to the hard spectator interactions and vertex corrections are computed, which is why the improved QCD factorization approach was applied. Eventually, we computed the branching ratios at the three scales $m_b/2$, m_b, and $2m_b$.

In generalized factorization, the computed branching ratios of $B^0 \to \phi\pi^+\pi^-$ and $B_s^0 \to \phi\pi^+\pi^-$ had the values 1.69 ± 0.19 and 3.28 ± 0.17 at scale m_b, while the experimental results were 1.82 ± 0.25 and 3.48 ± 0.23 respectively. Comparison between our obtained values and experiment indicates relative agreement with experimental information: the values of the branching ratios on the scale m_b correspond to the experimental values. The resonant contribution considered in the computation of the branching ratio in the $B_s^0 \to \phi\pi^+\pi^-$ decay is dominant, and it approximately corresponds with the numerical values published by the PDG. Also, in the $B^0 \to \phi\pi^+\pi^-$ decay, the non-resonant contribution considered in our computation is dominant, and it corresponds approximately with the experimental results. In summary, the calculated branching ratios for the sum of non-resonant and resonant amplitudes are consistent with experimental results.

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