Crystals, Critical Fields, Collision Points, and a QED Analogue of Hawking Radiation

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1 Introduction

During the penetration of a crystal close to a crystallographic direction, the trajectory of the penetrating particle—due to the sequence of binary encounters—becomes indistinguishable from the trajectory obtained from 'smearing' (averaging) the charges along the string or plane, see Fig. 1.

From the resulting translational invariance in the longitudinal direction of the potential inside the crystal, follows a separation of the longitudinal and transverse motions, since the longitudinal momentum $p_\parallel$ is conserved. The result is a conserved 'transverse energy' and a transverse potential $U(r_\perp)$ in which the particle moves:

$$U(r_\perp) = \frac{1}{d}\int_{-\infty}^{\infty} V(r_\perp, z)\mathrm{d}z \tag{1}$$

where $V(r_\perp, z)$ is the potential of the atom at the location of the projectile.

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In the continuum model the transverse motion is given by:

$$\frac{d}{dt}\gamma m\dot{r}_\perp = -\frac{d}{dr_\perp}U(r_\perp(t)) \tag{2}$$

where the dot denotes differentiation with respect to time, t, and $r_\perp$ is the transverse coordinate. Using energy conservation and neglecting terms of order $1/\gamma^2$, the transverse energy reduces to

$$E_\perp = \frac{1}{2}pv\psi^2 + U(r_\perp) \tag{3}$$

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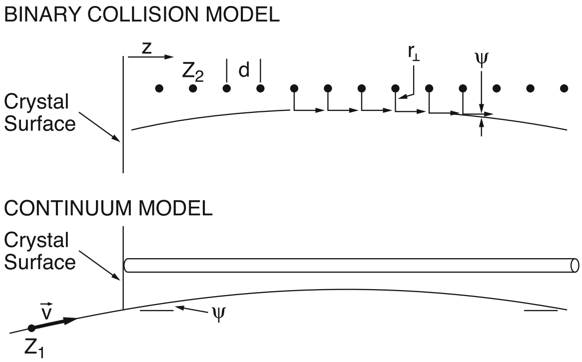

Fig. 1 A schematical drawing of the discrete nature of the scattering centers in a crystal and the resulting continuum approximation. The target atoms with atomic number Z_2 and distance d along the string, impose a curved trajectory on the penetrating particle with atomic number Z_1 through binary encounters over the transverse distance $r_\perp$. The resulting trajectory with entrance angle ψ can be accurately described as if being the result of interaction with a string of continuous charge distribution, i.e. the charges $Z_2 e$ being 'smeared' along the direction of motion z

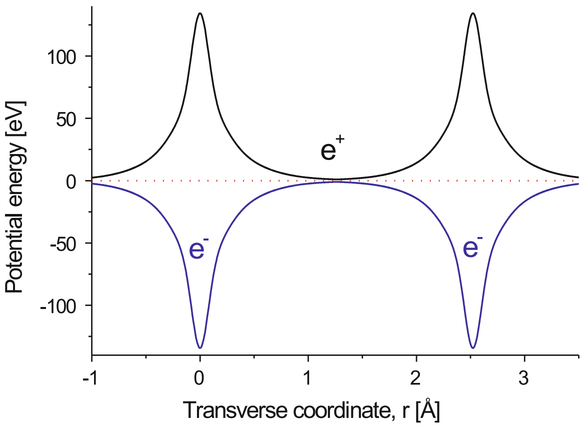

Fig. 2 The transverse potential energy for positrons and electrons, in the continuum approximation, for diamond along the $\langle 110 \rangle$ axis at room temperature. The main regions for channeled e^- and e^+ are indicated. The Doyle-Turner approximation for the atomic potential has been used [1]

where ψ is the particle angle to the axis [2, 3].

As can be seen from Fig. 2, the axial potential in diamond varies roughly 50 V over a transverse distance of 0.1 Å, corresponding to an electric field

$$\mathscr{E}_d \simeq 5 \cdot 10^{10}\ \mathrm{V/cm} \tag{4}$$

This extremely strong, and macroscopically continuous, electric field arises from the coherent interaction of the screened nuclear fields along the direction of motion.

It should be emphasized that although this field also appears in combination with channeled particles, it is not a necessary condition that the particle is channeled—the continuum approximation generally applies at angles one to two orders of magnitude higher than the critical angle for channeling.

For a thorough introduction to strong fields in crystals at high energies, see e.g. [4, 5].

1.1 The Critical Field

In the present connection, 'strong' means comparable to the quantum mechanical critical field in a Lorentz-invariant expression. In atomic physics, the scale for 'strong' fields is set by the electric field a 1s electron is exposed to in atomic hydrogen, the atomic unit $\mathcal{E}_a = e/a_0^2 = 5.14 \cdot 10^9$ V/cm, and the magnetic field that gives rise to the same force on the 1s electron $B_a = \mathcal{E}_a/c\alpha = 2.35 \cdot 10^5$ T. In quantum electrodynamics, on the other hand, the strength of the electric field is measured in units of the critical field $\mathcal{E}_0$ (and the corresponding magnetic field B_0), obtained from a combination of the electron charge and mass, the velocity of light and Planck's (reduced) constant (with values from [6])

$$\mathcal{E}_0 = \frac{m^2c^3}{e\hbar} = 1.323285 \cdot 10^{16}\,\mathrm{V/cm} \quad , \quad B_0 = 4.414005 \cdot 10^9\,\mathrm{T} \tag{5}$$

These scales of strength are thus related as $\mathcal{E}_a = \alpha^3\mathcal{E}_0$ and $B_a = \alpha^2 B_0$. As we shall see shortly, classical strong fields are even stronger $\mathcal{E}_c = \mathcal{E}_0/\alpha$ and $B_c = B_0/\alpha$.

The critical field $\mathcal{E}_0$ is frequently referred to as the Schwinger field [7], although it was treated as early as 1931 by Sauter [8, 9], following a supposition by Bohr on the magnitudes of fields relevant in the Klein paradox [10].

Rewriting the expression for the critical field to $\mathcal{E}_0 = mc^2/e\lambda\!\!\!^-$, where $\lambda\!\!\!^-_c = \hbar/mc$ is the reduced Compton wavelength, it appears that in a critical field a (virtual) electron may obtain an energy corresponding to the electron rest energy mc^2 while moving over a distance corresponding to the uncertainty of its location $\lambda\!\!\!^-_c$. Thus, the strong gradient of the potential combined with quantum uncertainty, as e.g. also seen in Zitterbewegung, may produce new particles—a QED phenomenon analogous to the Hawking radiation, discussed below.

In a classical analogue, a similar field strength is obtained from the field at a distance of a classical electron radius from the 'center' of the electron, $\mathcal{E}_c = e/r_e^2$. This field 'on the surface' of a classical electron is likewise where e transported over r_e yields mc^2, $\mathcal{E}_c = mc^2/er_e = \mathcal{E}_0/\alpha$, and, as must be required of a classical field, it does not contain $\hbar$. It is approximately equal to the Born-Infeld limiting field strength b_l. The latter was introduced 'dogmatically' by a Lagrangian $L = -b^2(1 - \sqrt{1 - (\mathcal{E}^2 - B^2)/b_l^2})$ (inspired by the relativistic $mc^2(1 - \sqrt{1 - v^2/c^2})$ where c is the limiting speed) and described a transition to non-linear, classical electrodynamics [11]. The classical strong field is thus 137 times larger than the quantum one, i.e. a tunneling process reduces the necessary field strength required to produce a pair in quantum theory [12].

1.2 Electrodynamical Invariants

We now consider the general case of a charged particle interacting with an electromagnetic field, following e.g. [13, 14]. Three dimensionless invariants can be constructed from the electromagnetic field strength tensor, $F_{\mu\nu}$, and the momentum four-vector p^ν (or, in the case of a photon, $\hbar k^\nu$):

$$\chi^2 = \frac{(F_{\mu\nu}p^\nu)^2}{m^2c^4\mathscr{E}_0^2} \tag{6}$$

$$\Xi = \frac{F_{\mu\nu}^2}{\mathscr{E}_0^2} = \frac{2(\mathbf{B}^2 - \boldsymbol{\mathscr{E}}^2)}{\mathscr{E}_0^2} \tag{7}$$

$$\Gamma = \frac{e_{\lambda\mu\nu\rho}F^{\lambda\mu}F^{\nu\rho}}{\mathscr{E}_0^2} = \frac{8\boldsymbol{\mathscr{E}}\cdot\mathbf{B}}{\mathscr{E}_0^2} \tag{8}$$

where $e_{\lambda\mu\nu\rho}$ is the antisymmetric unit tensor and contraction is indicated by repeated indices. For an ultra-relativistic particle moving across fields $\mathscr{E} \ll \mathscr{E}_0$, $B \ll B_0$ with an angle $\theta \gg 1/\gamma$ the invariants fulfill $\chi \gg \Xi, \Gamma$ and $\Xi, \Gamma \ll 1$. The relation of χ to the fields $\boldsymbol{\mathscr{E}}$ and $\mathbf{B}$ is given by [14]

$$\chi^2 = \frac{1}{\mathscr{E}_0^2m^2c^4}((\mathbf{p}c \times \mathbf{B} + E\cdot\boldsymbol{\mathscr{E}})^2 - (\mathbf{p}c\cdot\boldsymbol{\mathscr{E}})^2) \tag{9}$$

For an ultrarelativistic particle moving perpendicularly to a pure electric or pure magnetic field this reduces to

$$\chi = \frac{\gamma\mathscr{E}}{\mathscr{E}_0} \quad \text{or} \quad \chi = \frac{\gamma B}{B_0} \tag{10}$$

Due to $\mathscr{E}_0$ being proportional to m^2, we note that χ scales with $1/m^2$ such that e.g. the coherent production of muon pairs from electrons becomes appreciable only at energies $207^2 \approx 4\cdot10^4$ times larger than electron-positron pairs.

For the emission of radiation it is the trajectory that is decisive. Therefore, it is insignificant if the field responsible for the path is electric or magnetic and as a consequence they are frequently used indiscriminately in radiation emission. Since χ is invariant, γB (or $\gamma\mathscr{E}$) is the same in any reference system and thus it is reasonable to transform to the electron frame. In this reference system, by definition the Lorentz factor of the electron is 1 and the field present in the frame of the laboratory is boosted by $\gamma = E/mc^2$, where E is the energy of the electron in the laboratory. This means that the field *in the rest-frame of the electron* can become critical for achievable γ-values.

2 Quantum Synchrotron Radiation

Concerning the recoil in the emission process, a classical calculation of the synchrotron radiation emission in a magnetic field leads to a spectrum which extends to $\omega_c \simeq 3\gamma^3 eB/2p = 3\gamma^3\omega_B/2$ [15, 16], i.e.

$$\frac{\hbar\omega_c}{E} \simeq \frac{3\gamma B}{2B_0} = \frac{3\gamma\kappa_f}{2} = \frac{3\chi}{2} \tag{11}$$

which for sufficiently large γ exceeds 1. Here $\omega_B = eBv/pc$ is the cyclotron angular frequency and χ is the strong field parameter. Thus, for γ values beyond a certain point, the classically calculated radiation spectrum extends beyond the available energy [4, 17–19]. In this case a quantum treatment taking recoil into account becomes necessary:

> " …the condition for quantum effects to be unimportant is that the momenta of the radiated quanta be small compared with the electron momentum" [17].

As a result of the quantum correction, the total radiated intensity for the classical emission is according to Schwinger reduced by a factor

$$I/I_{\mathrm{cl}} = 1 - 55\sqrt{3}\lambda\!\!\!^{-}_c\omega_B\gamma^2/16c \tag{12}$$

due to first order quantum corrections when $\chi \ll 1$ [17]. Including the second order term the reductions for small values of χ are [14]

$$I/I_{\mathrm{cl}} = 1 - 55\sqrt{3}\chi/16 + 48\chi^2 \qquad \chi \ll 1 \tag{13}$$

and asymptotically for large values of χ

$$I/I_{\mathrm{cl}} \simeq 1.2\chi^{-4/3} \qquad \chi \gg 1 \tag{14}$$

Furthermore, an approximate expression ("accuracy better than 2 % for arbitrary χ" [20, eq. (4.57)])

$$I/I_{\mathrm{cl}} \simeq (1 + 4.8(1+\chi)\ln(1+1.7\chi) + 2.44\chi^2)^{-2/3} \tag{15}$$

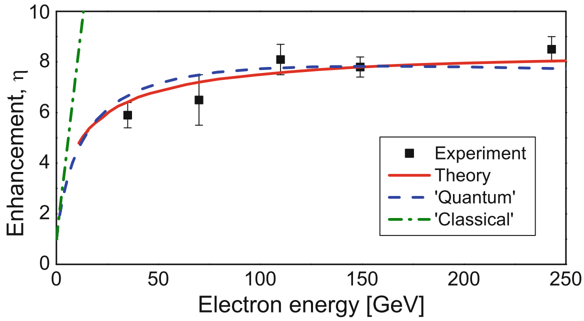

Fig. 3 Experiment [23] and theory [24] for radiation emission from electrons penetrating a tungsten crystal near the $\langle 111\rangle$ axis. For comparison, a *curve* based on Eq. (15) with a slightly arbitrary, but realistic $\overline{\chi} = 0.02 \cdot E$[GeV] (and vertical scale obtained as the best fit) is shown as the *dashed line* ('quantum'), and the corresponding classical expression as the *dash-dotted line* ('classical'). The enormous difference between the 'classical' and 'quantum' curves directly show the strong quantum suppression in the experimentally accessible regime

gives a compact analytical expression applicable e.g. in computer codes. From this, it is clear that the emission of synchrotron radiation is affected already at fairly small values of χ. A graphical representation compared to measured values is given in [21, 22].

In Fig. 3 is shown results for radiation emission from electrons impinging on a tungsten crystal close to the $\langle 111\rangle$ axis. As a consequence of the strong deflection upon the passage of the string of nuclei composing the axis, the electron is forced to emit radiation as in a constant field, as described above. This happens much like in normal synchrotron radiation emission, only in a much more intense field, $\lesssim 10^{11}$ V/cm, corresponding to 30.000 T. As a result of the high peak value of the χ parameter ($\chi_{\mathrm{W},\langle 111\rangle} \simeq 0.03 \cdot E$[GeV]), the radiation emission is subjected to strong quantum suppression. In the limit $\chi \ll 1$ the enhancement would be linear with increasing energy, as shown by the dash-dotted line. This is the case because synchrotron radiation emission is quadratic in energy and radiation from an amorphous foil is linear in energy, but due to the strong quantum suppression, the enhancement is reduced to the level shown by the dashed line, as also expected from equation (15). The good agreement between experimental values and theory shown in Fig. 3, combined with the equality of beamstrahlung and strong field theory shown in Fig. 4 provide a strong experimental indication that QED theory as applied to beamstrahlung—discussed in the following section—in the regime $1 \lesssim \Upsilon \lesssim 10$ is correct.

The accuracy of the experimental values is 5–10 %, enough to ascertain the validity of the theoretical approach. The quantum synchrotron behaviour of radiation emission in a strong field, is thus experimentally well confirmed at relatively high values of $1 \lesssim \chi \lesssim 10$.

3 Beam-Beam Interactions—Beamstrahlung

In the construction of linear colliders an important phenomenon is the emission of intense radiation due to the interaction of particles in one bunch with the electromagnetic field from the opposing bunch. This leads to the synchrotron radiation equivalent of particle deflection in the field of the bunch, instead of in a magnetic dipole: Beamstrahlung. As the emission of beamstrahlung has a direct and significant impact on the energy of the colliding particles, it is a decisive factor for e.g. the energy-weighted luminosity. Conversely, beamstrahlung emission may provide a method for luminosity measurement. It is therefore important to know if beamstrahlung theory is correct for the conceptual and technical design of the collision region—the center about which the rest of the machine is based.

The Lorentz factor γ in this case is understood as the Lorentz factor of each of the oppositely directed beams, measured in the laboratory system. Then relativistic velocity addition $v' = (v - V)/(1 - vV/c^2) = 2v/(1 + v^2/c^2)$ with $V = -v$ and $\gamma' = 1/\sqrt{1 - v'^2/c^2}$ yields the Lorentz factor γ' of one beam seen from a particle in the other beam of $\gamma' = 2\gamma^2 - 1$, usually shortened to $2\gamma^2$ in the ultrarelativistic limit. Thus, in the restframe of a particle in one bunch the field of the other bunch is boosted by a factor $\simeq 2\gamma^2$ and may approach or even exceed critical field values. The emission of beamstrahlung can be expressed as a function of χ (often called Υ in the accelerator physics community) which for the Stanford Linear Collider (SLC) is small $\simeq 10^{-3}$ but of the order unity for the next generation linear colliders [25]. For the planned Compact LInear Collider (CLIC) at CERN, the collision point is designed such that $\overline{\Upsilon} \simeq 4$. Quantum corrections to the emission of beamstrahlung therefore become crucial.

3.1 *Quantum Treatment of Beamstrahlung*

The discussion of quantum effects in radiation emission from energetic particles in collision with a counterpropagating bunch was started in the mid-80's [26–28]. In particular the suppression of the intensity stemming from the strong field deflection was of interest. It later continued with treatments of pair creation [29, 30]. Of particular relevance to the connection of beamstrahlung with emission from electrons penetrating crystals is the contribution by Baier, Katkov and Strakhovenko to the early development of the theory of beamstrahlung [31].

Shortly after the first publications on the relevance of quantum theory to beamstrahlung, Blankenbecler and Drell contributed a full quantum treatment of the problem, based on the eikonal approximation [32]. The scaling parameter in their approach is given by

$$C = \frac{m^2c^3RL}{4Ne^2\gamma^2\hbar} \tag{16}$$

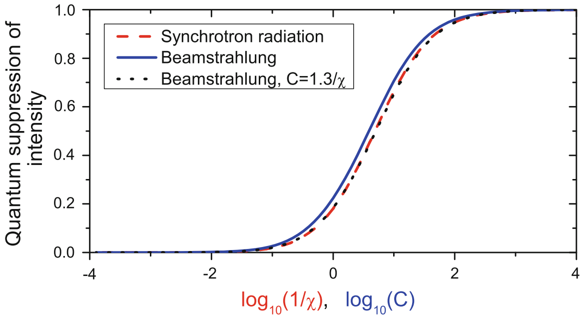

Fig. 4 The quantum suppression of radiation emission intensity, according to Eqs. (15) and (17)

representing the electric field from a homogeneously charged cylinder of length L and radius R holding N charges, in units of $\mathscr{E}_0 = m^2c^3/\hbar e$. The applicability of this scaling parameter was later elaborated upon by Solov'yov and Schäfer [33, 34]. From this model of a beam, the form factor $F = \delta/\delta_{\text{classical}}$, describing the quantal energy loss in units of the classical, was derived and approximated by:

$$F(C) = \left[1 + \frac{1}{b_1}[C^{-4/3} + 2C^{-2/3}(1 + 0.20C)^{-1/3}]\right]^{-1} \tag{17}$$

with $b_1 = 0.83$, see also [33]. Clearly, as stated by Blankenbecler and Drell, in the classical regime $\hbar \to 0$ in Eq. (16) such that C tends to infinity, and therefore the form factor tends to 1 according to Eq (17), as must be required.

As a result of the quantum correction, the total radiated intensity for the classical emission is reduced as given by equation (15).

In Fig. 4 is shown graphs based on Eqs. (17) and (15), where it has been assumed that $C = 1/\chi$. The curves are very similar, over the entire range of more than five orders of magnitude in χ. The expressions originating from the same phenomenon becomes even more evident by adjusting to $C = 1.3/\chi$ which results in the curves being indistinguishable on the plot. This is not a fortuitous coincidence: In Blankenbecler and Drell's theory, the bunch is treated as a homogeneously charged cylinder of length L and radius R holding N charges. At the distance r from the center of this cylinder the electric field is $2\gamma Ne/Lr$, which leads to an average (over r) field

$$\mathscr{E} = \frac{\gamma Ne}{LR} \tag{18}$$

which can be combined with Eq. (16) to give

$$C = \frac{m^2c^3}{\gamma \mathscr{E} e\hbar} = \frac{\mathscr{E}_0}{\gamma \mathscr{E}} = \frac{1}{\chi} \tag{19}$$

so it is legitimate to interchange C and $1/\chi$. The additional factor 1.3 that brings the curves into almost exact agreement, is due to the radiation intensity being non-linear in C, i.e. averaging over the field encountered and then calculating the intensity from this field may be different from calculating the intensity from the fields encountered and then averaging.

Early studies by Chen and Yokoya [25] showed that field gradient effects are small for a collider operating near $\Upsilon = \chi = 1/C = 1$, i.e. also for the planned CLIC at CERN where the expected value is as mentioned $\Upsilon \simeq 4$. It is therefore to a high degree of accuracy sufficient to use Eq. (17) derived for the homogeneously charged cylinder in calculations for beamstrahlung.

Nevertheless, due to the emission of quantum beamstrahlung, the 'useful' luminosity $\mathscr{L}_1$ (where $\mathscr{L}_1$ is defined as the luminosity for that part of the beam where the energy is still at least 99 % of the initial) becomes about 40 % of the nominal, due to the loss of energy in the beamstrahlung process. In a classical calculation the useful luminosity would have been at least an order of magnitude smaller. For such future colliders, $\gamma\gamma$-collisions, resulting e.g. also in hadronic interactions, may be generated from the beams themselves and the advantage of using leptonic beams ('clean' collisions) is to some extent lost. The beamstrahlung problem is unavoidable since single passage (as opposed to circular machines) forces small beam cross sections to give high luminosity. And since $\Upsilon \propto N\gamma/(\sigma_x + \sigma_y)\sigma_z$, with σ denoting the beam size, high energies and high luminosity means a high value for Υ. However, the problem may be partly alleviated by applying special bunch structures ('flat' beams, maximizing $\sigma_x + \sigma_y$) to avoid rapid beam deterioration from strong field effects [35].

Finally, it should be mentioned that effects of the spin of the particle become very important in the beamstrahlung emission. As polarimeters cannot be positioned at the intersection point of the crossing beams, reliable models for the degree of polarization after emission (immediately before the collision) must be developed for measurements with polarized beams to make sense.

4 Hawking Radiation and Unruh Effect

A fascinating analogy exists between the critical field and the Hawking radiation from a black hole: The gravitational acceleration at the Schwarzschild radius $R_S = 2GM/c^2$ equals $g(R_S) = c^4/4GM$ where G is Newton's constant and M the mass of the black hole. From the equivalence principle, locally the gravitational field is analogous to an accelerating frame of reference. The word *locally* is crucial in this context: A gravitational field and an accelerating frame are closely related, but they are not equal. It is impossible to 'transform away' a gravitational field by shifting to an accelerating frame due to the existence of tidal forces as expressed by the Riemann curvature tensor. But locally, e.g. for one test particle only, there is equivalence. Setting this gravitational acceleration equal to the acceleration of an electron in a critical field $g_0 = e\mathscr{E}_0/m = c^2/\lambda\!\!\!^{-}_c$ the condition $\lambda\!\!\!^{-}_c = 2R_S$ is obtained. In words: the black hole emits particles with (reduced) Compton wavelengths that are as large as or larger than the hole itself. This is approximately (within a factor 2 as for the calculation of the deflection of light by use of the equivalence principle) equal to the answer obtained in a full analysis of the Hawking radiation [36].

Likewise, the equivalence between the temperature of the Hawking radiation from a black hole and the temperature of the vacuum in a constantly accelerated frame [37, 38] has been widely discussed—the so-called Unruh effect. As channeled particles are subject to enormous fields and accelerations, outlines for possible detection schemes using strong crystalline fields have been put forward [39, 40]. In [39] it is estimated that a planar channeled positron with $\gamma \gtrsim 10^8$ will emit Unruh radiation as intense as the incoherent bremsstrahlung. These estimates, however, do not discuss the subtleties connected to the inherently non-constant acceleration for a channeled particle.

The Unruh effect gives rise to a Planckian photon spectral distribution at a temperature

$$T = \frac{\hbar a}{2\pi k_B c} \tag{20}$$

where a is the acceleration and k_B the Boltzmann constant.

Several other methods have been proposed to pursue the problem of measuring the Unruh temperature experimentally. According to Baier—who was an expert also in the field of radiative polarization [41]—the Unruh mechanism is a possible *interpretation* ("theoretical game") of the radiative depolarization in a storage ring [42], as originally suggested by Bell. For an overview of the suggested experimental methods and a review of the literature on the subject see e.g. [43, 44] (these chapters are mainly sophisticated theory chapters, but do contain references to experimental methods). In this connection, it may be mentioned that even the lightest charged composite, the positronium negative ion Ps^-, will have an essentially unaffected hyperfine structure when exposed to Unruh radiation during state-of-the-art acceleration in a high-gradient radio-frequency cavity [45].

The above mentioned analogy between the critical field and the Hawking radiation from a black hole becomes even more compelling by interpreting the field as a temperature as is done for the Unruh effect, Eq. (20): $T_0 = e\mathscr{E}_0\hbar/2\pi m k_B c$ [46] and inserting $g(R_S) = c^4/4GM$ instead of $g_0 = e\mathscr{E}_0/m$ in T_0 from which the correct Hawking temperature appears [47]:

$$T_0 = \frac{\hbar c^3}{8\pi G M k_B} \tag{21}$$

The Hawking radiation can thus be viewed as a critical field phenomenon, where the electromagnetic critical field is replaced by a gravitational field. Generally speaking, the uncertainty of the location of the particles is given by their (reduced) Compton wavelength, as evidenced e.g. by the Zitterbewegung. Thus, the interpretation that a quantum fluctuation—a virtual pair—can become real due to the presence of the critical field, where the rest mass energy is created over exactly this length, is valid in both cases. As the gravitational field at the Schwarzschild radius $g(R_S)$ is larger for small holes, short Compton wavelength—'hot'—radiation may be emitted, which is why light black holes possess a higher temperature than heavy ones—they possess a higher gradient.

It must be emphasized, though, that it is an analogy, not a one-to-one correspondence between electrodynamics and geometrodynamics. In the former case, for instance, the invariant Ξ, Eq. (7), is much smaller than one.

From the above qualitative considerations, it is clear that the QED analogy of Hawking radiation, critical field radiation, is of high importance to be investigated experimentally. This is perhaps even more the case as long as the gravitational version is not within observational reach in the foreseeable (perhaps even imaginable) future.

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