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PARTICLE PRODUCTION RATIOS IN THE DIFFRACTIVE EXCITATION MODEL*

Rudolph C. Hwa and David C. Robertson
Institute of Theoretical Science
University of Oregon, Eugene, Oregon 97403

and

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

ABSTRACT

Ratios of the invariant distributions of produced particles, $R(\pi^+/\pi^-)$, $R(\bar{p}/\pi^-)$, $R(K^-/\pi^-)$, and $R(K^+/p)$, have been calculated in the diffractive excitation model. Two free parameters are used to adjust the relative normalization. The shapes of the ratios as functions of x are essentially independent of any arbitrary parameters. The results agree with the data very well.

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One of the attractive features of the diffractive excitation model is that it can predict the shapes of the single particle spectra with essentially no adjustable parameters and yet with remarkable accuracy in a variety of inclusive reactions.[1-6] The model should therefore lend itself readily to the prediction of particle production ratios as functions of the longitudinal momenta. In this paper we discuss the production of K^+K^- and $\bar{p}p$ pairs as well as the associated production of $K^+\Sigma^0$ and the like. With two adjustable parameters, we calculate the particle ratios $R(\pi^+/\pi^-)$, $R(\bar{p}/\pi^-)$, $R(K^-/\pi^-)$, and $R(K^+/p)$ as functions of x. The agreement with the experimental data currently available is good.

The diffractive excitation model assumes that at very high energies the production processes are dominated by two-cluster formations. Under the supposition that the mean momentum of a particle in the rest frame of the associated cluster is limited, typically 350 MeV,[7] the overlap between the decay distributions of the right-going and left-going clusters in the center-of-mass system becomes insignificant as the total energy increases indefinitely. We may thus calculate the asymptotic shape of the single-particle spectrum for $a + b \rightarrow c$ + anything by considering only one cluster explicitly, the phase space for the other cluster being completely integrated over. Defining the invariant distribution for the detection of particle c to be

$$f_c(x, p_\perp) = x_0 \frac{d\sigma}{dx\,dp_\perp^2} = p_0 \frac{d\sigma}{dp_\parallel dp_\perp^2} \tag{1}$$

where $x = 2p_\parallel/\sqrt{s}$ and $x_0 = 2p_0/\sqrt{s}$, we have[1,2]

$$f_c(x, p_\perp) = x_0 \int \frac{d\sigma}{dM_1} g_c(M_1, x)\, n_c(M_1) dM_1 , \quad x > 0 . \tag{2}$$

The expression for $x < 0$ is similar (with M_1 replaced by M_2), so it will not be considered separately. In (2), $d\sigma/dM_1$ is the cross section for the production of a cluster of mass M_1 whatever the other cluster may be, i.e. M_2 being integrated over both the discrete and the continuum states. $g_c(M_1, x)$ is the probability that among the particles emanating from a cluster of mass M_1 a particle of type c, if created, is observed at x. We normalize the integral of $g_c(M_1, x)$ over all x to unity. The probable number of such particles is given by n_c inside the integral in (2). Because we attach a probabilistic interpretation to the number n_c, it is a continuous variable which can be less than one.

Since the data on particle production ratios at very high energies are available only for pp collisions,[8-10] let us for definiteness consider here only the proton cluster. When a proton is exicted to a massive state, it decays mainly by emitting pions. This is necessarily true when the excited state is not too massive. But if M_1 is large, finite probability exists for the creation of kaons and nucleon-antinucleon pairs. In order to facilitate the development of a concept for particle production, let us consider first the extreme case of a very massive cluster, $M_1 \to \infty$. We emphasize that this limit is taken under the condition $M_1^2 << s$, so s is even greater. By considering very large M_1 we remove the kinematical limitations on producing any type of particle. Thus if we assume that any particle i that is produced has a finite average energy E_i in the cluster rest frame, we have

$$M_1 = \sum_i n_i E_i + n_p' E_N + n_{KY}' E_{KY} \tag{3}$$

The first term on the right-hand side represents the summation over the three types of pions and all particles produced in pairs, the totality of which has zero baryon and hypercharge quantum numbers. n_i is the number of particles of type i produced. We exclude resonances in the counting, including only the detectable particles that are stable against strong decay. The last two terms of (3) represent the two modes that have the quantum numbers of the proton: proton itself and the kaon-hyperon state (K^+Y^o or K^oY^+). Other more massive possibilities such as $K^+K^o\Xi^o$ are ignored. It should be noted that n'_p does not include the protons produced in conjunction with an antinucleon, which should be accounted for in the summation term.[11] Evidently, baryon number conservation implies

$$n'_p + n'_{KY} = 1 \tag{4}$$

where n'_p and n'_{KY} are the probabilities for finding a leading proton or a kaon-hyperon pair, respectively, per collision after averaging over many events with the formation of clusters of the same mass M_1. In obvious notation E_{KY} signifies the average energy (always in the cluster rest frame) of the kaon-hyperon pair.

We now make the fundamental assumption that asymptotically (as $M_1 \to \infty$) the fraction, $\xi(m_i)$, of the total number of produced particles that correspond to a particular type i of mass m_i is constant. If n_1 designates the number of produced particles in a cluster of mass M_1, then we have

$$n_i = \xi(m_i)n_1 , \quad n_1 \to \infty . \tag{5}$$

The label i indicates the type of particle with mass m_i; the density of states describing the number of different types within a mass interval (m, dm) is an extra factor not included in $\xi(m_i)$. Using (3) and (5) we obtain

$$n_1 = (M_1 - E_0)/W \approx M_1/W, \quad M_1 \to \infty \tag{6}$$

where

$$E_0 \equiv n'_p E_N + n'_{KY} E_{KY} \tag{7}$$

$$W \equiv \sum_i \xi(m_i) E_i \quad . \tag{8}$$

From (6) we see that W has the meaning of grand average energy per particle in a cluster regardless of the particle type. We have learned from the study of the spectrum and average multiplicity of the pions[1,2] that n_π must be proportional to M_1 if an agreement with the data is to be achieved. The introduction of pair production of heavy particles into our consideration should not alter this conclusion. Hence, W must be a finite quantity. This means that the summation in (8) must converge; in fact, it should converge very quickly if W is to be roughly equal to E_π, a condition which ensures that the consideration of kaon- and antiproton-production does not alter the general features about the pion production already obtained by ignoring K and $\overline{p}$. It is not necessary for us to specify at this point what $\xi(m_i)$ is, except to mention that it should be a rapidly damping function of m_i.

We turn our attention next to the dependence of n_c on M_1 for low and intermediate values of M_1 where we are near the thresholds for the production of kaon pairs and nucleon-antinucleon pairs. Clearly the asymptotic formula given by (5) and (6) must be modified in order to account for the threshold effects. If we assume that π^+, π^- and π^o are produced with equal probability except for the necessity of preserving charge conservation, and that the production of p and n is also equally probable, then the probable numbers of π^+ and π^- are

$$n_{\pi^+} = \frac{1}{3}(n_\pi + \frac{1}{2}) \tag{9}$$

$$n_{\pi^-} = \frac{1}{3}(n_\pi - 1) \tag{10}$$

where n_π is the total number of pions produced in any given event. For low cluster masses only pions are produced, so we shall calculate n_π according to

$$n_\pi = (M_1 - E_N)/E_\pi \tag{11}$$

Corresponding to $< p_\perp > \sim 350$ MeV, we set $E_N = 1$ GeV and $E_\pi = 450$ MeV. The minimum value of M_1 is taken to be 1.4 GeV. However, since E_π and E_p may vary when M_1 is near threshold, we shall take the values of n_{π^+} and n_{π^-} to be given by (11), but not less than 1/2 and 1/3 respectively.

For $\bar{p}$ production the mass of the minimal cluster is approximately $M_1 = 3\,E_N$. At that mass although the state $pp\bar{p}$ with each particle having an average energy E_N is kinematically allowed, the probability for the occurrence of such a state is vanishingly small as compared to a state with one proton and many pions. That is, the value of $n_{\bar{p}}$ should become zero at the "threshold" $M_1 = 3\,E_N$. We therefore use the following formula which incorporates both the threshold effect and the asymptotic behavior:

$$n_{\bar{p}} = \xi(m_N)(M_1 - 3\,E_N)W^{-1} \tag{12}$$

Identical reasoning applies for the production of K^-, yielding

$$n_{K^-} = \xi(m_K)(M_1 - E_N - 2\,E_K)W^{-1} \; . \tag{13}$$

The characters of the proton and K^+ spectra are similar; they both have two components. For the proton, one component is the leading proton, the ground state into which the excited proton decays. The other component arises from the creation of nucleon-antinucleon pairs. For K^+, instead of the leading proton, we have the associated production of K^+ and a hyperon. The asymptotic behavior of n'_{KY} for large M_1 can conceivably be

$$\frac{n'_{KY}}{1-n'_{KY}} = \frac{n'_{KY}}{n'_p} = \frac{\xi\left[\frac{1}{2}(m_K + m_Y)\right]}{\xi(m_N)}, \qquad M_1 \to \infty . \tag{14}$$

However, near the KY threshold we expect n'_{KY} to vanish, so a reasonable formula for all M_1 is

$$n'_{KY}(M_1) = n'_{KY}(\infty)\left[1 - E_{KY}/M_1\right] \tag{15}$$

The leading K^+ spectrum thus calculated should then be added to the K^- spectrum to give the total K^+ distribution.

Having discussed $n_c(M_1)$ for all interesting types of particles c that can be and have been observed, we are now ready to apply (2) in the computation of the invariant distributions for the various particles and then to obtain their ratios. The quantity $d\sigma/dM_1$ in (2) is known in the diffractive excitation model[1,2] to be

$$\frac{d\sigma}{dM_1} = \frac{A}{M_1^2} \tag{16}$$

for large M_1, although by duality its validity may hold even for moderate values of M_1. The constant A need not be specified here since it is cancelled upon taking the particle ratio. The function $g_c(M_1, x)$ in (2) is taken to be the Lorentz transform to the c. m. system of a Gaussian distribution in the cluster rest frame

$$g_c(M_1, x) = \sqrt{\frac{3}{2\pi}} \exp\left(-\frac{3k_c^2}{2E_c^2}\right)\frac{dk_c}{dx} \tag{17}$$

$$k_c = \frac{1}{4}\left[(x_0 - x)\frac{s}{M_1} - (x_0 + x)M_1\right]. \qquad (18)$$

The dependence on the mass and transverse momentum of the particle c is contained in the relation between x_0 and x. We can either integrate over the transverse momentum or set it at the root-mean-square value; the result is essentially the same. We present the results in terms of the particle ratios which are defined to be

$$R\left(\frac{c}{c'}\right) = \frac{f_c(x, p_\perp)}{f_{c'}(x, p_\perp)} \qquad (19)$$

1. $\underline{R(\pi^+/\pi^-)}$. Since the average transverse momentum of the pions is about 350 MeV, we choose for the masses of the minimal cluster to be $(M_1)_{min}$ = 1.4 GeV. Incorporating (5) and (6) with (9) through (11) so as to arrive at a set of formulas for all M_1, we rewrite

$$n_{\pi^+} = \xi(m_\pi)\left[\frac{M_1 - E_0}{W} + \frac{1}{2}\right] \qquad (20)$$

$$n_{\pi^-} = \xi(m_\pi)\left[\frac{M - E_0}{W} - 1\right] \qquad (21)$$

where we set E_0 = 1 GeV, and W = 450 MeV. The factor $\xi(m_\pi)$, which is irrelevant to $R(\pi^+/\pi^-)$, is expressed explicitly here for later use in the calculation of other ratios. We integrate (2) over $p_\perp$, setting E_c in (17) equal to W. The result for $R(\pi^+/\pi^-)$ is shown in Figure 1. The data are given in references 9 and 10. The agreement is evidently good.

2. $\underline{R(\bar{p}/\pi^-)}$. We set E_N = 1 GeV in (12) and (17), and integrate (2) to get $f_{\bar{p}}(x)$. The shape of $R(\bar{p}/\pi^-)$ as a function of x is independent of $\xi(m_N)/\xi(m_\pi)$, which we choose to fit the normalization of the data. In fact, the shape is

essentially independent of any adjustable parameters; that is, all (energy) parameters are given reasonable values on the basis of the argument stated in footnote 7 and are not free to vary except in a limited range. The result is shown also in Figure 1. Note that the model predicts a flattening of the ratio for $x < 0.15$. At $x = 0$, $R(\bar{p}/\pi^-) = \xi(m_N)/\xi(m_\pi) \simeq 0.04$.

3. $\underline{R(K^-/\pi^-)}$. The calculation is identical to the above case except that E_K is set at 700 MeV corresponding to an average kaon $p_\perp$ of 350 MeV. The predicted shape is as shown in Figure 1. The only question is about the normalization which is proportional to $\xi(m_K)/\xi(m_\pi)$. In an attempt to speculate on a reasonable form for $\xi(m_i)$ we conjecture that the highly excited state (fireball) is a thermodynamic system of its constituents and that the creation of particles follows a distribution of the form[12]

$$\eta(m_i) = \eta_0 \exp(-m_i/T) \tag{22}$$

where T is some energy parameter. Here m_i represents the masses of all the particles that can be created, not only the "stable" particles π, K, and p, but also the resonances ρ, ω, etc. The function $\xi(m_i)$ differs from $\eta(m_i)$, however, in that it describes the proportion of various stable particles only, since they are the ones detected in the inclusive reactions. Since the resonances decay eventually into pions, plus a K or an N as the case may be, we expect

$$\frac{\xi(m_N)}{\xi(m_\pi)} << \frac{\eta(m_N)}{\eta(m_\pi)} \tag{23}$$

$$\frac{\xi(m_N)}{\xi(m_K)} \approx \frac{\eta(m_N)}{\eta(m_K)} \tag{24}$$

Hence, we have

$$\frac{\xi(m_K)}{\xi(m_\pi)} \approx \frac{\xi(m_N)}{\xi(m_\pi)} e^{(m_N - m_K)/T} \tag{25}$$

The value of T is to be determined by fitting $R(K^+/p)$ and is found to be ~ 760 MeV as is discussed below. Since $\xi(m_N)/\xi(m_\pi)$ is found from $R(\bar{p}/\pi^-)$ to be about 0.04, we have

$$\xi(m_K)/\xi(m_\pi) \approx 0.07 \tag{26}$$

Thus, the normalization and shape of $R(K^-/\pi^-)$ as shown in Figure 1 are predictions. The only experimental result on this is contained in the statement in reference 10 that $R(K^-/\pi^-)$ remains constant at 0.08 to within a factor of two for $0.1 < x < 0.4$, $0.3 < p_\perp < 0.5$ GeV/c.

4. $\underline{R(K^+/p)}$. If we substitute (15) for n_c in (2), we see that f_{K^+} is directly proportional to $n'_{KY}(\infty)$ which from (14), (22) and (24) is given by

$$n'_{KY}(\infty) = \frac{z}{1+z} \tag{27}$$

where

$$z = \exp\left[\left(m_N - \frac{m_K + m_Y}{2}\right)/T\right] \tag{28}$$

The total distribution f_{K^+} is obtained by adding the hyperon production component to the K^+K^- production component which is just f_{K^-}. We set $E_{KY} = 1.8$ GeV in the calculation. The parameter T is varied to fit the data of reference 10 on $R(K^+/p)$ as shown in Figure 2. For the choice of T = 760 MeV the fit is evidently good.

Conclusion. The shapes of the particle production ratios are essentially independent of adjustable parameters. All energy parameters (apart from T) follow from the same $< p_\perp >$ of about 350 MeV for all particles. The normalizations of $R(\bar{p}/\pi^-)$ and $R(K^+/p)$ are not predicted but obtained by adjusting two parameters to fit the data. However, $R(\pi^+/\pi^-)$ and $R(K^-/\pi^-)$ are predicted. They all agree well with data. It is interesting to note how flat some of these ratios are for small values of x.

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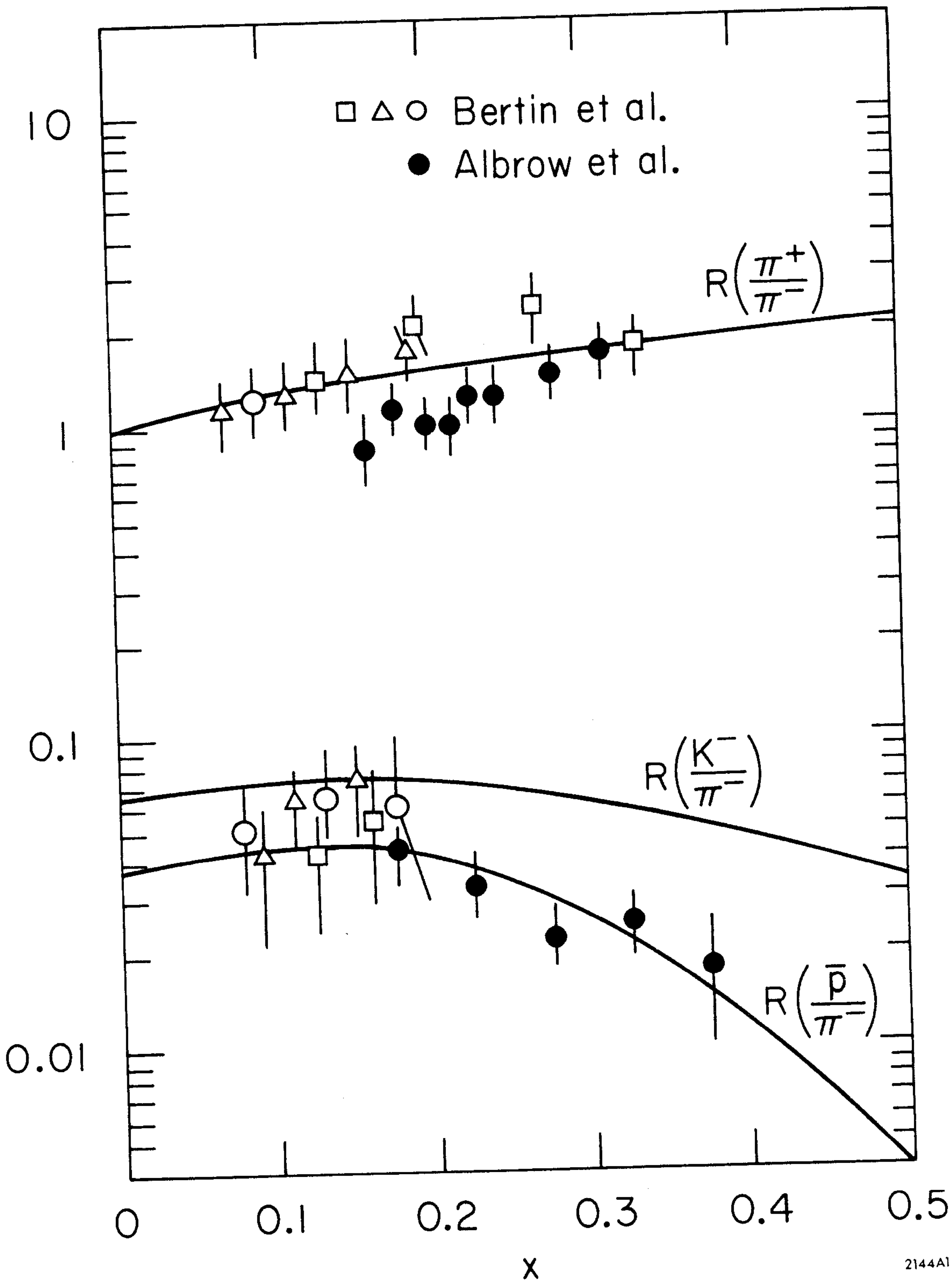
7. This estimate is made by inference from the average transverse momenta of the particles assuming that the distributions in the cluster rest frame are more or less isotropic.
8. J. V. Allaby et al. (to be published).
9. A. Bertin et al., Phys. Letters 38B, 260 (1972).
10. M. G. Albrow et al., Phys. Letters 40B, 136 (1972).
11. This separation is made only for the sake of bookkeeping. There is, of course, no way to determine in an actual event with an additional proton produced which one of the protons accompanies the antiproton. Such distinction is actually unnecessary.
12. Although the form of the distribution is similar to that of the thermodynamical model [see, for example, R. Hagedorn, Suppl. Nuovo Cimento 3, 147 (1965)], the concepts involved in the two models are different.

Figure Captions

Figure 1 Solid lines are theoretical curves of $R(\pi^+/\pi^-)$, $R(K^-/\pi^-)$ and $R(\bar{p}/\pi^-)$. The data points are for $R(\pi^+/\pi^-)$ and $R(\bar{p}/\pi^-)$ from references 9 and 10. Data for $R(K^-/\pi^-)$ are essentially constant at around 0.08 for $0.1 < x < 0.4$.

Figure 2 Theoretical result for $R(K^+/p)$ is shown in solid line. The data are from reference 10.



□ △ ○ Bertin et al.
● Albrow et al.
10
1
0.1
0.01
R(π⁺/π⁻)
R(K⁻/π⁻)
R(p̄/π⁻)
0
0.1
0.2
0.3
0.4
0.5
x
2144A1

Fig. 1

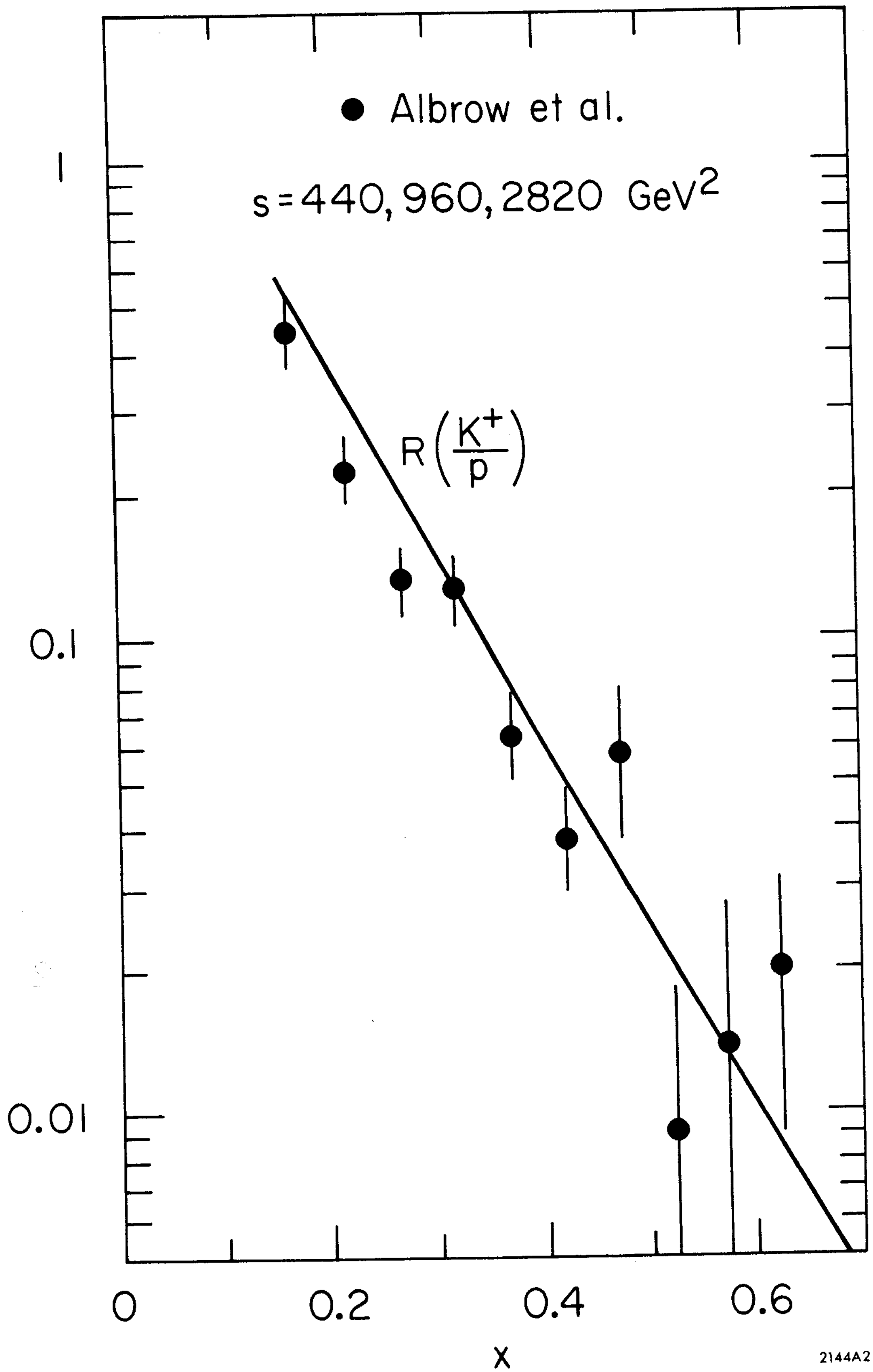


Fig. 2