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9 Conservation of Energy Prohibits Proton Decay

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Abstract. The proton can't decay, baryon number must be conserved. This is proven rigorously but it can be seen intuitively (using care not to be misled). The proton, the lightest strongly-interacting fermion, can be taken (roughly) to have a strong-interaction contribution to its mass. If it decays into fermions with no strong interaction then this contribution disappears, violating conservation of energy. This is made rigorous. Why pions are different, so can decay, is discussed.

9.1 Intuitive arguments

Experimentally the proton is stable; baryon number is conserved. Must it be so? Actually it is easy to understand why it must.

The proton is the lightest fermion with strong interactions. Suppose that the weak interaction caused it to decay to leptons (and perhaps photons) which are not affected by strong interactions. Then the weak interaction would turn off the strong one — a state with would go to a state without. But one interaction cannot turn off another. Anyone who doubts this can write a Hamiltonian with an interaction that turns off interactions.

This can also be seen with Feynman diagrams. Consider a diagram (of which there are an infinite number) in which a proton emits a virtual pion (kaon, or any other possibility) and then reabsorbs it. Going with that diagram there is another for which the proton emits the pion and then decays to leptons. The poor pion has to be reabsorbed. But it does not interact with leptons thus there is nothing that can absorb it.

Such diagrams violate conservation of energy (thus the entire set must). While there are various ways of considering this, protons cannot decay because of conservation of energy as we see formally below.

9.2 But pions can decay

While the weak interaction of baryons cannot turn the strong one off or on, mesons (like the pion) do decay to leptons. Why? The photon is analogous; its number is not conserved although charge is. However it is neutral, it couples to a neutral

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object, taken as a particle-antiparticle pair (the current is of this form). Electron-photon scattering can be regarded as creation of an electron-positron pair by the photon; the positron annihilates the electron and that of the pair replaces it. So while the electromagnetic interaction (of an electron) cannot be altered (it cannot be turned off — charge is conserved), photons can be taken as not directly interacting. Their creation or annihilation does not modify an interaction.

It is similar for mesons which can be regarded as having no strong interactions but rather couple to particle-antiparticle pairs. We can view meson-baryon scattering as annihilation of the baryon by its antiparticle with it replaced by the (identical) baryon of the pair. And decay can be seen as a baryon-antibaryon pair scattering into leptons or photons. So we take a Feynman diagram in which the meson goes into a virtual pair of baryons which then interact (annihilate) turning into non-strongly-interacting objects, leptons or photons. This does not violate conservation of energy. Because of what they are coupled to, baryon-antibaryon pairs, mesons can decay into objects not affected by strong interactions.

9.3 Mathematical analysis

With this intuitive understanding of why baryons cannot decay into only leptons but mesons can, we outline a more formal proof. It has been given previously with full details ([6], sec. IV-4, p. 212), but an outline clarifies the physical aspects and shows implications that can be revealing in other ways. To study the decay we consider the action of the Hamiltonian, H, on a proton. Acting on a state at $t = 0$, time-translation operator $\exp(iHt)$ gives the state at time t. Can this take a baryon to a state whose fermions are only those not having strong interactions?

What is a proton? Mathematically (the only way it can be treated) it is a function obeying Dirac's equation with mass m_P and the (here irrelevant and suppressed electromagnetic, gravitational), weak and strong interactions (whose forms are irrelevant). It is the presence of interactions that determines what a proton is. (Correctly an object is an eigenstate of the two Poincaré invariants ([2], sec. 6.3, p. 114). For a free particle, and one with an electromagnetic interaction, Dirac's equation is equivalent. Whether this is true with other interactions seems unknown so consequences of, perhaps important, differences if any are not clear. And putting interactions in invariants, which must be done whether Dirac's equation is used or not, might limit them. Particles are also eigenstates or sums of the momentum operators ([3], sec. 5.4, p. 93), of which the Hamiltonian is one. We ignore these, and refer to Dirac's equation but discussions should be of invariants which might be revealing. The statefunction (a better term than wavefunction) of the proton is a solution of coupled nonlinear equations. We need information about it but cannot solve so represent it in a way that allows analysis using an expansion. The arguments though are exact; we do not calculate so need not truncate.

The physical particle, labeled with a capital, that obeying Dirac's equation with all interactions, is a sum of states (schematically):

$$|P) = c(x,t)|p) + \sum c(x,t)_{p\pi}|p)|\pi) + \ldots + \sum c(x,t)_{K\Lambda}|K)|\Lambda) + \ldots, \tag{9.1}$$

summing over all states to which the proton is connected by interactions including any number of pions and so on. The summations represent ones over internal labels and integrals over momenta. All are suppressed; this argument is very general. State $|p)$ is the function satisfying Dirac's equation with the weak interaction, but not the strong. The effect of that is given by this sum which is thus an eigenstate with mass m_P, of the total Hamiltonian, including all interactions. Individual terms in the sum differ in energy and momenta; it is the sum that has the eigenvalues. Thus what this does is to write the strongly-interacting object as a sum of terms of objects (states satisfying equations) that have no strong interactions. Hence we are putting in the strong interaction explicitly by writing the physical object as this sum. We then can distinguish between objects with and without the particular interaction we wish to consider.

Coefficients are determined by the requirement that this be a solution of that set of complete equations for all objects to which the proton is coupled, directly or indirectly, and normalization $(P|P) = 1$. Also the initial state, a wavepacket, taken as a proton at rest, gives the coefficients at $t = 0$.

The effect of the Hamiltonian is seen from that of H which is a sum of the free particle Hamiltonians for the proton, pion (and so on) and leptons, plus terms for weak and strong interactions. The state of the system is a sum of terms, one the state of the proton, another (if decay were possible) the product of pion and lepton states and such, each summed over other labels and with integrals over momenta or space. The free part of H changes the phase. The weak interaction part, were decay possible, decreases the coefficient of the proton in the sum, while increasing that of the (say) pion plus lepton, initially zero — starting as a proton, the state becomes a sum of the proton, its contribution decreasing, plus the pion plus lepton state, with increasing contribution (and so on for other states). For the decaying pion the behavior is similar: starting as a pure pion it becomes a sum of that plus a state of leptons with the contribution of the first decreasing, of the second increasing.

What goes wrong? The weak interaction acts on $|p)$ supposedly causing it to decay, so the final state is

$$|fs) = \sum d_{\pi l}c|\pi)|l) + \sum e_{\pi\pi l}|l)|\pi)|\pi) + \dots \\ + \sum d_{lll}c|l)|l)|l) + \sum d_{\pi lll}\sum c_{p\pi}|l)|l)|l)|\pi) + \dots, \quad (9.2)$$

showing the transition to a pion plus a lepton, and to three leptons, and so on, with coefficients of non-occurring terms zero. The energy of $|fs)$ is m_P, not m_p, so needs contributions from all terms. But $|fs)$ is say a lepton plus a pion so other terms, to which this is orthogonal, cannot contribute.

Hence we can see why the proton cannot decay. State $|p)$ satisfies the equation with the weak interaction, thus is caused to decay by it. But its mass is not the physical mass since the equation it satisfies is not the equation satisfied by the physical object. However the physical object decays (were that possible) because of the decay of each of these terms. Each term however gives a state with energy less than the mass of the mass of the physical object. It is only the sum of masses that equals the physical one. But these (smaller) masses cannot be summed because the final states are different. The physical proton can decay to only one (for each decay) and the mass of that one is less than the mass of the initial state.

The decay of the proton cannot conserve energy, thus cannot occur. An intuitive way (which must be used very carefully as it can be misleading) of looking at this is that the proton has a contribution to its mass because of its strong interaction and this disappears when it decays.

Similarly decays of leptons (the τ) to baryons are ruled out.

9.4 Charge conservation is similar

The argument for electric-charge conservation is the same. Charge conservation is related to gauge invariance, a partial statement of Poincaré invariance ([3], sec. 3.4, p. 43) — this relates an allowed interaction to the Poincaré group. An interaction violating charge conservation would not transform under gauge transformations as other terms in the Hamiltonian, giving Poincaré transformations (on massive objects) that induce gauge transformations (on massless ones) resulting in physically identical observers who undergo the different gauge transformations — these cannot be fully specified — thus physically identical, but who see different Hamiltonians. The Hamiltonian would not be well-defined, implying inconsistent physics. It is fortunate that charge is conserved.

9.5 Implications

There are other implications requiring investigation; we mention a few in hope of stimulating such. All interactions known are of lowest order. Why? For electromagnetism linearity is enforced by gauge (Poincaré) transformations ([3], sec. 4.2, p. 57). For strong interactions, take a particle, a Δ or P, that emits a pion. Higher order terms would couple it not to a single pion, but to more. Intuitively we can guess why only lowest order occurs since it gives diagrams which we interpret (purely heuristically) as two or more pions emitted sequentially. Higher-order means that these are emitted together. However this is the limit of the lowest order in which the time between emissions goes to zero. A higher order interaction would be this limit which is included in the lowest order as one case; higher-order terms adding nothing would be irrelevant. Summing all diagrams, and integrating over time, would give contributions from terms that have the same effect as higher-order ones, thus changing only the value of the sum, so the value of the coupling constant — an experimental parameter (at present). Thus we could not distinguish contributions from terms of different order implying higher order would be undetectable. This regards particles as virtual. But consider a decay in two steps, each emitting a pion. If the intermediate object's life were sufficiently short this would be equivalent to pions being emitted simultaneously. If a nucleon had an interaction of the form $NNN'\pi$, the emission of an NN' pair could be thought of as due to the decay of a pion, and the interaction taken as the limit of the emission of a pion and then its decay, when its lifetime becomes zero, merely changing the sum. These are purely heuristic and must be investigated in greater depth.

9.6 Geometry is destiny(?)

This is part of a long investigation of how geometry through its transformation groups limits and determines physics [1,2,3,4,5,6]. Geometry is quite powerful, and quite limiting. And physics is quite limited — strangely enough to laws that allow life [7]. It is strange, and as analysis shows, inexplicable, incomprehensible.

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