Towards clarifying the central engine of long gamma-ray bursts

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Abstract

Towards clarifying the formation processes of a system of black hole and massive accretion disk, we updated our full GR numerical code with microphysics (a finite temperature EOS, electron capture, local neutrino emission, and simplified treatment of neutrino cooling). Our new code shows an advanced performance that our results of spherical collapse agree quite well with those by the Boltzmann transport. We also calculate the gravitational-wave spectrum from convective activities occurred after the bounce.

1 Introduction

The observed spectroscopic connections between several SNe and long gamma-ray bursts (LGRBs) with SNe suggested that at least some LGRBs are associated with supernovae and the core collapse of massive stars. The collapsar model [1], in which the central engine of LGRBs is composed of a rotating black hole (BH) surrounded by a massive accretion disk formed after the collapse of rapidly rotating massive stellar core is currently one of the promising models of central engine of LGRBs.

To clarify the formation process of such BH-disk system, it is necessary to perform multidimensional simulations in full general relativity employing a finite temperature EOS and detailed microphysics. In this article, we briefly report our recent progress of the works.

2 Basic equations

The basic equations are the baryon conservation equation

$$\nabla_\alpha(\rho u^\alpha) = 0, \tag{1}$$

the lepton conservation equations,

$$\nabla_\alpha(\rho Y_e u^\alpha) = S_{Y_e}, \tag{2}$$

$$\nabla_\alpha(\rho Y_{\nu_i} u^\alpha) = S_{Y_{\nu_i}}, \tag{3}$$

where $\nu_i = \nu_e, \bar{\nu}_e, \nu_\mu$, and ν_τ, and the local conservation equation of the energy-momentum $\nabla_\alpha(T^{\rm Total})^\alpha_\beta = 0$. Here ρ and u^μ are the rest mass density and the 4-velocity.

The total stress-energy-momentum tensor is the sum of the fluid part $(T^{(\rm f)}_{\alpha\beta})$ and the neutrino part $(T^{(\nu)}_{\alpha\beta})$ as $(T^{\rm Tot})_{\alpha\beta} = (T^{\rm F})_{\alpha\beta} + (T^\nu)_{\alpha\beta}$, where $(T^{\rm F})_{\alpha\beta}$ is the stress-energy-momentum tensor of the fluid assumed to take the form of the perfect fluid $(T^{\rm F})_{\alpha\beta} = (\rho + \rho\varepsilon^{\rm F} + P^{\rm F})u_\alpha u_\beta + P^{\rm F} g_{\alpha\beta}$, and $\varepsilon^{\rm F}$ and $P^{\rm F}$ denote the specific internal energy density and the pressure of the fluid. The neutrino part $(T^\nu)_{\alpha\beta}$ is formally divided into 'trapped-neutrino' $((T^{\nu,\rm t})_{\alpha\beta})$ and 'streaming-neutrino' $((T^{\nu,\rm s})_{\alpha\beta})$ parts as

$$(T^\nu)_{\alpha\beta} = (T^{\nu,\rm t})_{\alpha\beta} + (T^{\nu,\rm s})_{\alpha\beta}.$$

We assume that the stress-energy-momentum tensor of the trapped-neutrino part takes the form of the perfect fluid and is combined with the fluid part to give

$$T_{\alpha\beta} \equiv (T^{\rm F})_{\alpha\beta} + (T^{\nu,\rm t})_{\alpha\beta} = (\rho + \rho\varepsilon + P)u_\alpha u_\beta + P g_{\alpha\beta}.$$

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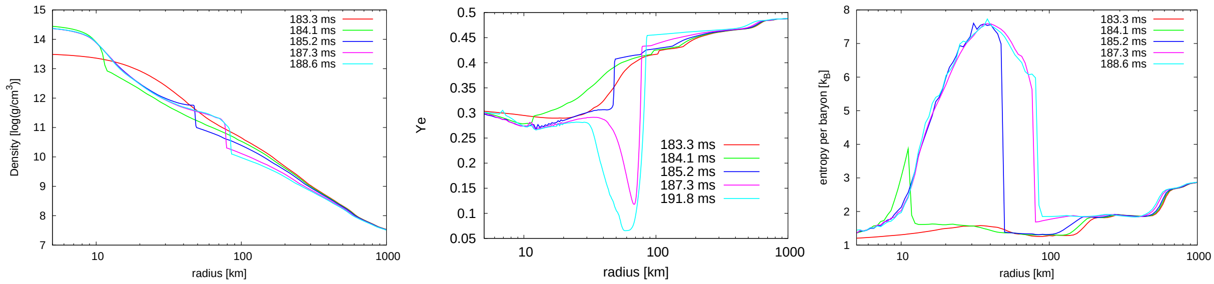

Figure 1: Radial profiles of density (the left panel), electron fraction (the middle panel), and entropy per baryon profile (the right panel) at selected time slices.

Thus, the specific internal energy density and the pressure are the sum of the contributions from the baryons (free protons, free neutrons, α-particles, and heavy nuclei), leptons (electrons, positrons, and *trapped neutrinos*), and radiation $P = P^{\rm F} + P_\nu = P_B + P_e + P_r + P_\nu$, $\varepsilon = \varepsilon^{\rm F} + \varepsilon_\nu = \varepsilon_B + \varepsilon_e + \varepsilon_r + \varepsilon_\nu$, where subscripts '$B$', '$e$', '$r$', and '$\nu$' denote the components of the baryons, electrons and positrons, radiation, and trapped neutrinos, respectively. We employ a finite temperature equation of state by Shen et al [2] for baryons. Fermions are treated as ideal Fermi gases. For radiation, we set $\varepsilon_r = a_r T^4/\rho$ $P_r = a_r T^4/3$, where a_r is the radiation constant.

On the other hand, the streaming-neutrino part is assumed to have the form

$$(T^{\nu,{\rm s}})_{\alpha\beta} = E_\nu n_\alpha n_\beta + F_\alpha n_\beta + F_\beta n_\beta + 1/3\gamma_{\alpha\beta}E_\nu,$$

where n_α and $\gamma_{\alpha\beta}$ are the unit normal and the metric of a three-dimensional hypersurface.

In terms of the above decomposition, we solve

$$\nabla_\beta T^\beta_{\ \alpha} = -Q_\alpha, \tag{4}$$

$$\nabla_\beta (T^{\nu,{\rm s}})^\beta_{\ \alpha} = Q_\alpha, \tag{5}$$

where the source term $Q_\alpha = Q u_\alpha$ is interpreted as the cooling rate due to the emission of (streaming) neutrinos. To summarize we solve Eq. (1), (2), (3), (4), and (5).

The source terms consists of the local and 'leakage' processes ($S_{Y_{\nu_i}} = S^{\rm local}_{Y_{\nu_i}} + S^{\rm leak}_{Y_{\nu_i}}$, $Q = Q^{\rm leak} = \sum Q^{\rm leak}_{Y_{\nu_i}}$). The local processes are the electron, positron, and neutrino captures [3] which is responsible to S_{Y_e} and S_{Y_i} electron-positron pair annihilation [4], plasmon decay [5], and Bremsstrahlung [6] which are to S_{Y_i}.

The leakage processes are the local emission of neutrinos according to the local diffusion timescale. In general the cross sections of neutrinos with matter can be written as $\sigma_i(\epsilon_\nu) = \tilde{\sigma}_i \epsilon_\nu^2$. The the opacity and the optical depth are similarly given by $\kappa(\epsilon_\nu) = \tilde{\kappa}\epsilon_\nu^2$ and $\tau(\epsilon_\nu) = \tilde{\tau}\epsilon_\nu^2$. We include neutrino absorption on free nucleons and heavy nuclei, neutrino scattering by electrons, free nucleons and heavy nuclei as opacity sources [6]. Then we define the diffusion timescale by

$$t^{\rm diff}_\nu(\epsilon_\nu) \equiv f^{\rm diff}\frac{\tau^2}{c\kappa} = f^{\rm diff}\frac{\tilde{\tau}^2}{c\tilde{\kappa}}\epsilon_\nu^2, \tag{6}$$

where $f^{\rm diff}$ is the constant of order unity. Then we define the local leakage rates by taking spectral average

$$S^{\rm leak}_{Y_{\nu_i}} \propto \int \frac{\hat{n}(\epsilon_\nu)}{t^{\rm diff}_\nu(\epsilon_\nu)} d\epsilon_\nu = \frac{1}{f^{\rm diff}}\frac{4\pi g_{\nu_i}}{h^3c^2}\frac{\tilde{\kappa}}{\tilde{\tau}^2}(k_BT)F_0(\eta_{\nu_i}), \tag{7}$$

$$Q_{Y_{\nu_i}} \propto \int \epsilon_\nu \frac{\hat{n}(\epsilon_\nu)}{t^{\rm diff}_\nu(\epsilon_\nu)} d\epsilon_\nu = \frac{1}{f^{\rm diff}}\frac{4\pi g_{\nu_i}}{h^3c^2}\frac{\tilde{\kappa}}{\tilde{\tau}^2}(k_BT)^2F_1(\eta_{\nu_i}), \tag{8}$$

where $n_\nu = \int \hat{n}(\epsilon_\nu)d\epsilon_\nu$, h and k_B are the Plank constant and Boltzmann constant, g_ν is the weight factor and $F_k(x)$ is the Fermi-Dirac function.

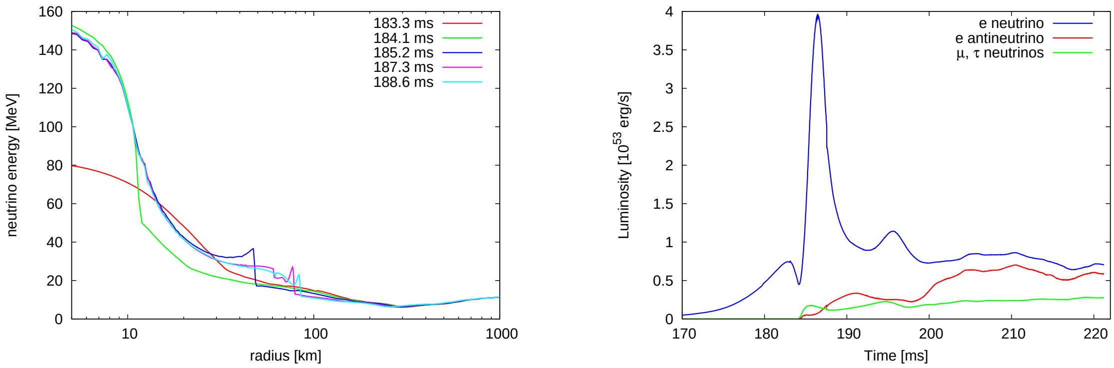

Figure 2: Radial profiles of electron-neutrino energy (the left panel) at selected time slices and time evolution of neutrino luminosities (right panel).

3 Results

First we compare the results of collapse of spherical core (model S15 [7]) with the results by Liebendöfer et al [8] in which the Boltzmann equations are solved for neutrinos. Figure 1 shows Radial profiles of density, electron fraction, and entropy per baryon profile at selected time slices after the bounce. Figure 2 shows radial profile of electron-neutrino energy and time evolution of neutrino luminosity. As the collapse proceeds, the central density exceeds the nuclear density and a bounce occurs ($t \approx 184$ ms) then a shock wave is formed and propagated outward. When the front of the shock wave pass through the neutrino sphere, a neutrino burst occur (see Fig. 2) and electron (lepton) fraction and entropy per baryon decreases rapidly ($t \approx 185$–187 ms). Note that negative gradients of electron fraction and entropy per baryon are formed in the core. Due to this neutrino burst, the shock wave eventually stalls at $r \approx 90$ km Our results shows good agreement with those by Liebendöfer et al.

It is well known that configuration with the negative gradients of lepton fraction and entropy per baryon are unstable against the Ledoux convection:

$$\left(\frac{\partial \rho}{\partial_{Y_l}}\right)_{P,s}\frac{dY_l}{dr} + \left(\frac{\partial \rho}{\partial_s}\right)_{P,Y_l}\frac{ds}{dr} > 0, \qquad \text{(unstable)}. \tag{9}$$

The left panel of Fig. 3 shows contour of entropy per baryon at a time slice after the neutrino burst. As this figure clearly show, the steep negative gradients of entropy and Y_l formed above the neutrino sphere lead to the convective overturn.

We approximately estimate the energy available by the convective overturn. The gravitational energy per unit mass released as a result of convective overturn Δh is given by

$$w = -g_{\rm eff}\left[\left(\frac{\partial \ln P}{\partial s}\right)_{\rho,Y_e}\left(\frac{\partial \ln P}{\partial \ln \rho}\right)^{-1}_{s,Y_e}(ds)_{\rm amb} + \left(\frac{\partial \ln P}{\partial Y_e}\right)_{\rho,s}\left(\frac{\partial \ln P}{\partial \ln \rho}\right)^{-1}_{s,Y_e}(dY_e)_{\rm amb}\right]\Delta h, \tag{10}$$

where 'amb' and 'blob' denote ambient and convective blob fluid element, and Δh is the overturn distance.

Thus, the energy available in the convective overturn of a region of mass ΔM and thickness Δh with a value of Δs at a distance r from the center of a proto-neutron star of mass M is approximately given by $W \sim 10^{51}\,\text{ergs}\left(\frac{\Delta M}{0.3M_\odot}\right)\left(\frac{\Delta h}{10\text{km}}\right)\left(\frac{|\Delta s|}{s}\right)\left(\frac{50\text{km}}{r}\right)^2\left(\frac{M}{M_\odot}\right)$, Similarly, with a value of ΔY_e, the available energy is given by $W \sim 10^{51}\,\text{ergs}\left(\frac{\Delta M}{0.3M_\odot}\right)\left(\frac{\Delta h}{10\text{km}}\right)\left(\frac{|\Delta Y_e|}{Y_e}\right)\left(\frac{50\text{km}}{r}\right)^2\left(\frac{M}{M_\odot}\right)$.

We see that if $\Delta s \sim s$ and $\Delta Y_e \sim Y_e$, $W \sim 10^{51}$ ergs. These numbers are achieved in our simulations for the regions below the neutrino sphere. Since the convection timescale in the proto-neutron star seems to be $\tau_{\rm ot} \lesssim R_{\rm NS} v_{\rm ot} \sim 10$ ms the energy gain by the overturn will be $L_{\rm ot} \gtrsim 10^{53}$ ergs/s. Since this free energy compensates the energy loss due to the photo-dissociation ($L_{\rm diss} \sim 10^{53}$ ergs/s) and the neutrino emission ($L_\nu \sim 10^{53}$ ergs/s at the time when the shock wave stalls), the shock wave are pushed outward.

Associated with the convective motions, gravitational waves are emitted. The gravitational waveforms are computed using a quadrupole formula. In the left panel of Fig. 3, we show the spectra of $h_{\rm char}$ due

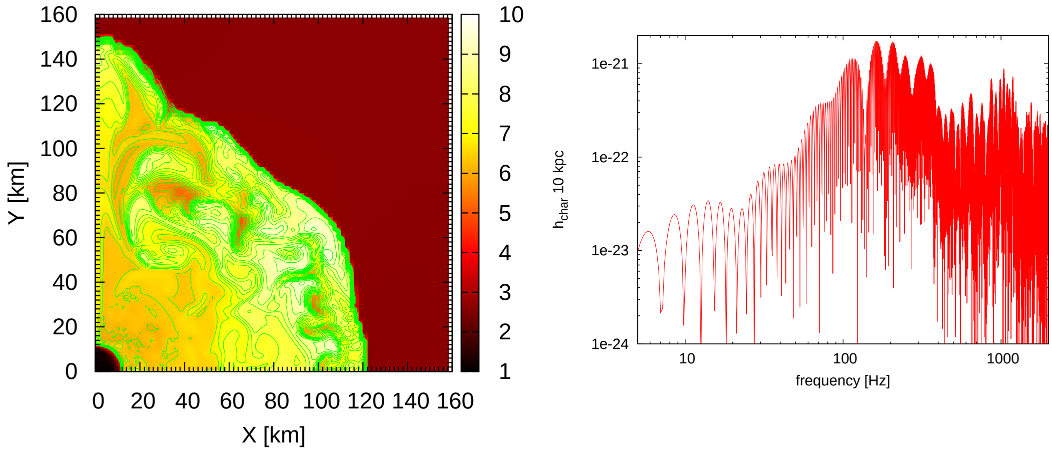

Figure 3: Contour of entropy per baryon (left panel) and gravitational-wave spectrum from convective motion (right panel).

to the convective motions. In contrast to spectra due to the core bounce, there is no dominant peak frequency in the power spectra. Instead, several maxima for the frequency range 100–500 Hz are present. This indicates that there exist a wide variety of scales of convective eddies with characteristic overturn timescales of 2–10 ms. The effective amplitude of the gravitational waves observed in the most optimistic direction is $h_{\rm char} \sim 10^{-21}$ for a event at a distance of 10 kpc, which is as large as that emitted at the bounce of rotating core collapse.

4 Summary

We perform a simulation of collapse of stellar core comparing results with those by spherical Boltzmann solver [8] and calculating spectrum of gravitational wave from convective activities. Our new code shows an advanced performance and can be used to clarify the formation process of LGRB engine. Fruitful scientific results will be reported in near future.

Numerical computations were performed on the NEC SX-9 at the data analysis center of NAOJ and on the NEC SX-8 at YITP.

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