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NAMBU STRING VERSUS DIRAC STRING IN EXTENDED QCD *

Y. M. CHO †
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

ABSTRACT

Motivated by a recent observation that the monopoles in grand unified theories tend to force QCD to become nontrivial, we analyze the global topology of the extended QCD as a nontrivial non-Abelian gauge theory. In the electric gauge not only the quarks but also the valence part of the potential develop the Nambu string, but the dual part of the potential remains free of any string singularity. In contrast, in the magnetic gauge the dual potential develops the Dirac string while the valence potential as well as the quarks become regular.

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†Permanent address: Dept. of Physics, Seoul National University, Seoul 151, Korea.

Recently the old problem of defining the color in QCD in the presence of the colored monopoles has been readdressed by some authors[1,2] within the context of grand unified theories. The observation is that selecting the color direction of the monopoles in grand unified theories[3] amounts to a topological obstruction to the color gauge symmetry in such a way that a choice of a global color SU(3) basis is not always possible. This observation raises two questions; first how to define the color,[2] but perhaps more importantly how to interprete QCD itself,[1] in the presence of such a topological obstruction. As for the definition of the color there is, of course, a much more deeply rooted problem which exists even without the topological obstruction: a theorem due to Schlieder[4] tells that the conserved color charge in classical non-Abelian gauge theories has such a severe gauge dependence that one can always choose a gauge in which the total charge vanishes completely, even when there is no topological obstruction. The impact of the topological obstruction to QCD itself, however, is indeed devastating because in the absence of a global SU(3) basis there can be no globally defined gauge potentials, and consequently no perturbative QCD. Obviously the conventional QCD is not suited to this kind of circumstance. Fortunately there exists in the literature an unconventional version of QCD called the extended QCD[5,6] which can accommodate this difficulty. The purpose of this letter is to clarify the global topology of the extended QCD and discuss how it works under the above circumstance.

Before we proceed, it is useful to clarify the problem that we face and put it in mathematical terms. Geometrically speaking the so–called topological obstruction[1,2] originates from the fact that[6,7] when one reduces a principal fibre bundle $P(M,G)$ (with the base manifold M and the structure group G) to a reduced bundle $P^*(M,H^*)$ where H^* is a closed subgroup of G, $P^*(M,H^*)$ could become nontrivial even when one starts from a trivial $P(M,G)$. Thus $P^*(M,H^*)$ does not always admit a global section even if $P(M,G)$ does. This means that even if one starts from a smooth potential of G which is reducible to H^*, the reduced potential will necessarily develop a string singularity when restricted to H^*, if the potential contains a long-range magnetic flux. In grand unified theories G is the unifying group and H^* is the unbroken little group of the Higgs fields which contains the color SU(3). So even if the Higgs fields do not break the color SU(3) they prevent us from having smooth color gauge potentials globally, in the presence of colored monopoles.[1,2] Under this circumstance we have no choice but to deal with a nontrivial QCD. The situation here is very similar to the case of Dirac's theory of monopole in which one has to deal with a nontrivial U(1) bundle.[8] Notice that the conventional QCD always assumes the existence of a global trivialization of the underlying principal fibre bundle.

So the problem is to construct a color gauge theory in the absence of a global section, and we will do so in several steps.[5,6] First start from the connection space (i.e., the space of the potentials) of a trivial $P(M,G)$ where from now on G is the color gauge group, and separate a subspace called the restricted connection[9] whose holonomy group becomes the Cartan's subgroup H^* of G. Notice that the restricted potential is globally defined since we start from a trivial bundle. More significantly the restricted potential has a dual structure: it can describe not only the color electric charges but also the color magnetic ones. Besides, it can be reduced to an Abelian potential on the reduced bundle $P^*(M,H^*)$. However, when the potential contains a color magnetic flux $P^*(M,H^*)$ becomes nontrivial so that the magnetic part of the potential develops a string singularity. A best way to separate the restricted connection is to impose the magnetic symmetry,[6,9] i.e., a left isometry H which commutes with G, on a generic connection on $P(M, G)$. For simplicity let us assume G = SU(2) and $H^* =$ U(1). In this case the magnetic symmetry may be written as

$$D_\mu \, \hat{m} = 0, \qquad (\hat{m}^2 = 1) \tag{1}$$

where $\hat{m}$ is a globally defined (except on isolated point singularities) scalar triplet which forms a left isometry Killing vector[6,10] in $P(M, G)$. Then the restricted potential $\hat{B}_\mu$ is obtained as the solution of the potential $\vec{B}_\mu$ of Eq. (1),

$$\hat{B}_\mu = A_\mu \, \hat{m} - \frac{1}{g} \, \hat{m} \times \partial_\mu \, \hat{m} \tag{2}$$

where $A_\mu = \hat{m} \cdot \vec{B}_\mu$ is the color electric potential which is globally defined with no string singularity. Now, when the second homotopy $\Pi_2(G/H)$ defined by $\hat{m}$ is nontrivial, $\hat{B}_\mu$ becomes dual so that its magnetic part $\hat{m} \times \partial_\mu \, \hat{m}$ describes point-like colored monopoles. The duality can be made more explicit in the magnetic gauge[6,9] where $\hat{m}$ becomes $\hat{\xi}_3$= (0, 0, 1),

$$\hat{B}_\mu \underset{\hat{m} \to \hat{\xi}_3}{\longrightarrow} (A_\mu + \tilde{C}_\mu) \, \hat{\xi}_3 \tag{3}$$

where $\tilde{C}_\mu$ is the singular magnetic potential. Notice that although $\tilde{C}_\mu$ can be absorbed into A_μ locally, this is not possible globally. Physically $\tilde{C}_\mu$ is the part of the potential which violates the Bianchi identity.

Due to the magnetic symmetry the field strength $\hat{G}_{\mu\nu}$ of $\hat{B}_\mu$ must be proportional to $\hat{m}$,

$$\hat{G}_{\mu\nu} = (F_{\mu\nu} + H_{\mu\nu}) \, \hat{m} \tag{4}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$H_{\mu\nu} = -\frac{1}{g} \hat{m} \cdot (\partial_\mu \hat{m} \times \partial_\nu \hat{m})$$

$$= \partial_\mu \tilde{C}_\nu - \partial_\nu \tilde{C}_\mu \quad .$$

This means that the field $\hat{m}$ not only specifies the global topology of the gauge symmetry, but also naturally selects the color electric direction, and it does so without breaking the gauge invariance since the magnetic symmetry (1) is a gauge covariant constraint which can be applied in any gauge. This observation allows us to circumvent the disturbing theorem by Schlieder[4] and the other difficulties[2] in defining the color in extended QCD.

In the magnetic gauge the most general potential $\vec{B}_\mu$ may be written as

$$\vec{B}_\mu = (A_\mu + \tilde{C}_\mu)\, \hat{\xi}_3 + X^1_\mu\, \hat{\xi}_1 + X^2_\mu\, \hat{\xi}_2 \tag{5}$$

where X^1_μ and X^2_μ are globally defined fields free of any string singularity. Now let us try to remove the string in $\vec{B}_\mu$ by going back to the original gauge which we will call the electric gauge from now on. In the electric gauge one finds

$$\vec{B}_\mu \underset{U}{\rightarrow} A_\mu\, \hat{m} - \frac{1}{g} \hat{m} \times \partial_\mu \hat{m} + X^1_\mu\, \hat{m}_1 + X^2_\mu\, \hat{m}_2 \tag{6}$$

where

$$\hat{m}_i = U\, \hat{\xi}_i \qquad (i = 1, 2, 3;\ \hat{m}_3 = \hat{m}) \quad .$$

Here come two points to be emphasized. First $\vec{X}_\mu = X^1_\mu\, \hat{\xi}_1 + X^2_\mu\, \hat{\xi}_2$ transforms covariantly under a gauge transformation. This is due to the Affine nature of the connection space: since $\vec{X}_\mu$ is the difference between the two points ($\vec{B}_\mu$ and $\hat{B}_\mu$) in the connection space it must transform covariantly under a gauge transformation. More importantly, by writing $\vec{B}_\mu = \hat{B}_\mu + \vec{X}_\mu$ we have made a gauge independent decomposition of the potential into the dual part and the covariant part. The second point is that in the electric gauge the covariant part $\vec{X}_\mu$ develops a string singularity while the dual part $\hat{B}_\mu$ becomes regular. This is so because when $\Pi_2(G/H)$ is nontrivial, no global basis for G is possible. This can best be shown in the one monopole sector which may be reached by the following U,

$$U = \begin{bmatrix} \cos\theta\cos^2\phi + \sin^2\phi\ , & -\sin^2\frac{\theta}{2}\sin 2\phi\ , & \sin\theta\cos\phi\ , \\ -\sin^2\frac{\theta}{2}\sin 2\phi\ , & \cos\theta\sin^2\phi + \cos^2\phi\ , & \sin\theta\sin\phi\ , \\ -\sin\theta\cos\phi\ , & -\sin\theta\cos\phi\ , & \cos\theta \end{bmatrix}$$

where θ and ϕ are the angular variables of the spherical coordinates of M. This shows that $\vec{X}_\mu$ through $\hat{m}_1$ and $\hat{m}_2$ develops a string along the negative z-axis. This demonstrates the fact that we are dealing with a gauge theory in which no global section for its potentials exists. For obvious reasons[5,6] we call $\hat{B}_\mu$ the binding gluon and $\vec{X}_\mu$ the valence gluon.

The Lagrangian for the extended QCD may be written in the magnetic gauge as follows,

$$\begin{aligned} \mathcal{L} = &-\frac{1}{4}\, G_{\mu\nu}^2 - \frac{1}{2}|(\partial_\mu + igB_\mu)X_\nu - (\partial_\nu + igB_\nu)X_\mu|^2 \\ &+ \mu^2 X_\mu^* X^\mu + \bar{r}\,\gamma^\mu(i\partial_\mu + \frac{g}{2}\,B_\mu)r + \bar{b}\,\gamma^\mu(i\partial_\mu - \frac{g}{2}\,B_\mu)b \\ &- m(\bar{r}\,r + \bar{b}\,b) + \frac{g}{\sqrt{2}}(\bar{b}\,\gamma^\mu r X_\mu^* + \bar{r}\,\gamma^\mu b X_\mu) \\ &- ig\,G_{\mu\nu}\,X_\mu X_\nu^* + \frac{g^2}{2}[X_\mu^2 X_\nu^{*2} - (X_\mu^* X^\mu)^2] \end{aligned} \qquad (7)$$

where $G_{\mu\nu} = F_{\mu\nu} + H_{\mu\nu}$, $B_\mu = A_\mu + \tilde{C}_\mu$, $X_\mu = (1/\sqrt{2})(X_\mu^1 + iX_\mu^2)$, and (r, b) is the quark doublet. Notice that, unlike in the conventional QCD, the valence gluons can have a mass term in the Lagrangian without breaking the gauge invariance. This is so since the valence gluons form a covariant multiplet. Now in the magnetic gauge the red and blue quarks are assumed to be globally defined with no string singularity. However, in the electric gauge they acquire a string for the same reason that the valence gluons do. Explicitly in the one monopole sector one has

$$\begin{pmatrix} r \\ b \end{pmatrix} \underset{U}{\rightarrow} \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2}\, e^{-i\phi} \\ \sin\frac{\theta}{2}\, e^{i\phi} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} r \\ b \end{pmatrix} \quad ,$$

so that the quarks develop a string along the negative z-axis. In conclusion, in the electric gauge <u>all</u> the colored objects, the quarks as well as the valence gluons, develop a string while in the magnetic gauge only the binding gluon does. Naturally the string in the electric gauge may be called the Nambu string and that in the magnetic gauge the Dirac string. Notice, however, that all these strings are for the moment purely topological and carry no energy.[11]

Now a few words about the confinement are in order. For this one has to study the vacuum structure of the theory. In the absence of any colored objects (the quarks as well as the valence gluons) the theory may be described (in the electric gauge) by the two regular fields; a charged scalar field ϕ which represents the point-like magnetic charge and its regular dual magnetic potential C_μ given by

$$\begin{aligned} H^*_{\mu\nu} &= \frac{1}{2}\,\epsilon_{\mu\nu\rho\sigma}\, H^{\rho\sigma} \\ &= \partial_\mu C_\nu - \partial_\nu C_\mu \quad . \end{aligned}$$

Then one may suppose that the theory could be approximated by the following Lagrangian,

$$\mathcal{L}_0 = |(\partial_\mu + i\,\frac{4\pi}{g}\, C_\mu)\,\phi|^2 - \frac{1}{4}\, H^{*2}_{\mu\nu} \quad . \tag{8}$$

This Lagrangian with a proper renormalization condition is known to generate a dynamical symmetry breaking,[12] which in our case could be interpreted as a magnetic condensation of the vacuum. This will guarantee the dual Meissner effect which confines any colored flux. Once the condensation takes place, the Nambu strings will of course become physical and carry energy. Finally as for the asymptotic freedom[13] we notice that inside the hadrons (after the confinement) the magnetic potential $\tilde{C}_\mu$ in the Lagrangian (7) may safely be neglected, since the magnetic condensation must occur only outside the hadrons. In this approximation the Lagrangian describes exactly the conventional (i.e., the trivial) QCD. This will guarantee the asymptotic freedom for the extended QCD inside the hadrons.

To sum up, we have shown how a nontrivial QCD which does not allow a global section for its potentials could describe the strong interaction within the context of the extended QCD. Although our argument here is based on SU(2) it can easily be generalized to SU(3). The crucial difference between the conventional QCD and the extended one is the presence of the scalar field $\hat{m}$, of which the magnetic potential becomes a composite field. The role of $\hat{m}$ is trifold. First it specifies the global topology $\Pi_2(G/H)$ of the gauge symmetry. Secondly it automatically selects the color electric direction and thus allows us to define the color charge in a gauge invariant way. Thirdly and perhaps most importantly, it makes the physical meaning of the theory more transparent by providing us with a gauge independent decomposition of the gluon fields into the dual part and the valence part. The dual part could be important to study the vacuum structure of the theory and the valence part could be useful to discuss the spectrum of the glueballs and their mixing with $q\,\bar{q}$ pairs.[5,14] The idea of the magnetic condensation of the QCD vacuum has been speculated by many authors.[15] In extended QCD one may find ways to test this idea.

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