Recent results in heavy quark physics from lattice QCD

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I discuss several new results from lattice heavy quark physics that have appeared in the last year.

1. INTRODUCTION

Lattice QCD calculations of charm and bottom physics have become one of the essential and fundamental tools of heavy quark physics. There exist many fully unquenched calculations (that is, including quark-antiquark pairs and with all uncertainties estimated), and new quantities are being added continually. Improved staggered fermions, which are for the most part what I report in this talk, have made progress the most rapidly. Advances in algorithms and methods have made possible unquenched calculations with several formulations of lattice fermions, and we can look forward to a comparison between fermion methods with many important quantities.

There are a variety of ways of putting heavy quarks on the lattice. All of them involve giving special treatment to the time dimension, through which a lot of four-momentum flows. They include the nonrelativistic operator expansions of NRQCD [1], HQET and the static approximation [2], the nonrelativistic normalization of the operators in the Wilson action in the Fermilab approach [3], and simply making the lattice spacing finer in the time direction using any action. A new action for moderately heavy quarks was introduced this year by the HPQCD collaboration, the "hisq" action. I will describe this more fully in a later section.

In the talk, I will describe a somewhat arbitrarily selected set of interesting new results that have appeared in the last year.

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2. $\overline{B} \to D^* l\nu$

The first unquenched determination of $|V_{cb}|$ using the decay $\overline{B} \to D^* l\nu$ appeared this year in joint work by the Fermilab lattice and MILC collaborations [4]. This calculation (as well as the other two Fermilab/MILC calculations discussed in this paper) used improved staggered ("Asqtad") fermions for the light quarks, and Wilson/clover fermions with the Fermilab normalizations for the heavy quarks. The original quenched determination of this quantity by the Fermilab lattice collaboration employed a complicated combination of amplitudes that had the virtue that most uncertainties vanished in the heavy quark symmetry limit [5]. This year's work used a much simpler double ratio:

$$\frac{< D^*|\bar{c}\gamma_j\gamma_5 b|\overline{B} >< \overline{B}|\bar{b}\gamma_j\gamma_5 c|D^* >}{< D^*|\bar{c}\gamma_4\gamma_5 c|D^* >< \overline{B}|\bar{b}\gamma_4\gamma_5 b|\overline{B} >} = |h_{A_1}(1)|^2 \quad (1)$$

Many errors still cancel in the new ratio (statistics, most operator normalization ...), and it is over an order of magnitude faster to compute. The chiral and continuum limit extrapolations are very mild. (For example, see Fig. 1 for the chiral extrapolation.)

The result is

$$h_{A_1}(1) = \mathcal{F}(1) = 0.921(23). \quad (2)$$

Using the recent PDG average for $|V_{cb}|\mathcal{F}(1)$, we find

$$|V_{cb}| = (38.7 \pm 0.9_{\rm exp} \pm 1.0_{\rm theo}) \times 10^{-3}. \quad (3)$$

This is about two sigma below the inclusive determination.

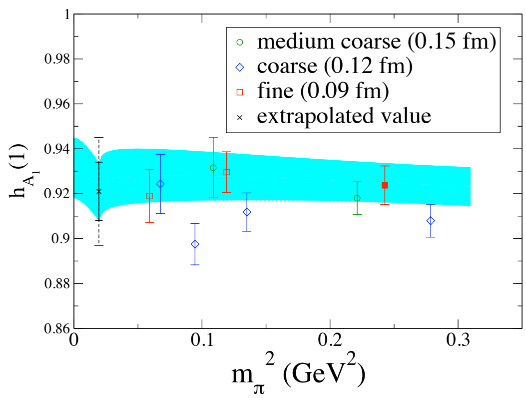

Figure 1. The chiral extrapolation of the quantity $h_{A_1}(1)$ describing $\overline{B} \to D^* l\nu$ decay.

3. $B \to \pi l \nu$

It is much more complicated to compare theory and experiment in the decay $B \to \pi l\nu$ than for most lattice results because the errors in both theory and experiment are highly dependent on the moment transfer t. Current lattice calculations have data for pion recoil momenta up to around 1 GeV, whereas the best experimental data is at higher recoil momentum. Comparison between theory and experiment is greatly simplified by mapping the momentum transfer t into a new variable z in the complex plane in such a way that the physical decay region is mapped into roughly $-0.3 \leq z \leq 0.3$. This was applied to lattice calculations this year in joint work by Fermilab and MILC [6].

The form factors may be written

$$f(t) = \frac{1}{P(t)\phi(t)} \sum_{k=0}^{\infty} a_k \; z(t)^k. \tag{4}$$

$P(t)$ and $\phi(t)$ are known functions. P incorporates the nonanalytic behavior in the B^* pole region, so that the rest of the function is analytic. ϕ is a perturbatively calculable function chosen such the the a_k have very simple behavior. (See [7] for a clear explanation.) With the proper choice of ϕ, analyticity and unitarity require that

$$\sum_{k=0}^{\infty} a_k^2 \leq 1. \tag{5}$$

Since $-0.3 \leq z \leq 0.3$, very general arguments imply that only five or six terms in the series are sufficient to describe the form factors to 1% accuracy. Becher and Hill have further argued that heavy quark theory implies that the bound on the a_k^2 in Eqn. 5 is $(\Lambda/m_b)^3$ [8], implying that only two or three terms in the series should be required to describe the form factors.

Last year, the BaBar experiment published results for $B \to \pi l\nu$ in twelve bins in $t = q^2$ [9], and with full correlation matrices for the uncertainties. (See Fig. 2.) This makes possible a clean extraction of V_{ub} with lattice QCD. Separate correlated fits to the lattice and experimental data can check that the shapes agree between theory and experiment. Then a combined correlated fit to the lattice and experimental data can be used to fix V_{ub} from the overall normalizations.

Fig. 3 shows the result of such a fit to the Fermilab/MILC lattice results presented at Lattice 2008. Fig. 4 shows the results of the same fit to the BaBar data. Fig. 5 shows the (fully correlated) fit to the combined data.

The results show that three terms in the z power series suffice to fit the data with good χ^2, in accordance with the calculations of Becher and Hill. The combined fit produces as an output $|V_{ub}|$, which is one of the fit parameters. The preliminary result is [6]

$$|V_{ub}| \times 10^3 = 2.94 \pm 0.35, \tag{6}$$

where the error is the combined theoretical and experimental error. This is the most accurate exclusive determination to date. It is about two sigma below the inclusive determinations. Final results will be out soon in a publication which is now being completed.

4. HISQ fermions

An important step forward in lattice fermion actions was made last year by the HPQCD Collaboration, with the "hisq" action ("highly improved staggered quarks")[10]. Ordinary naive

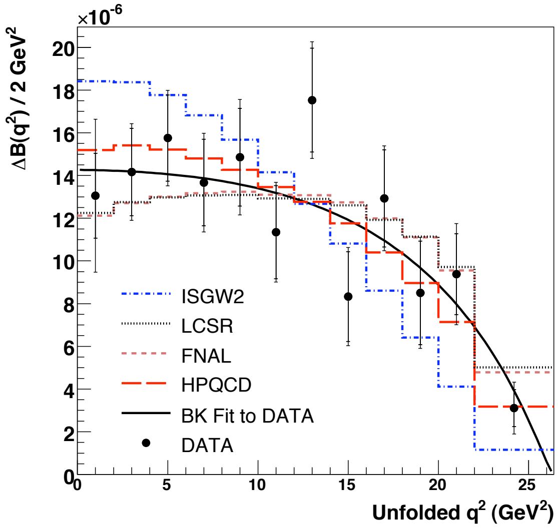

Figure 2. Results for the differential decay rate for $B \to \pi l \nu$ from BaBar. The data is given in twelve bins in q^2, with full correlation matrices for the uncertainties.

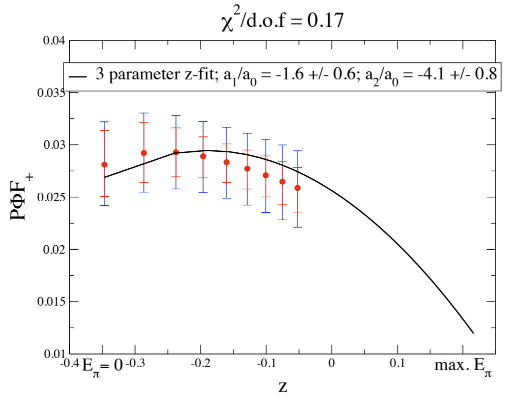

Figure 3. A z expansion fit to the Fermilab/MILC lattice results for $B \to \pi l \nu$.

and staggered fermions have poles in the propagator not only at zero momentum, but also at momentum π/a, as can be seen in the naive fermion propagator in Eqn. 7:

$$aG^{-1}(p) = \gamma \cdot \sin(ap) + am \tag{7}$$

Transitions between these poles (or "tastes" in staggered jargon) cause the leading discretization errors in staggered fermions. These errors can be reduced by suppressing the transitions that cause them in the following way. The link field in the action representing the gluons is replaced by a smeared link $U_\mu(x) \to \mathcal{F}_\mu U_\mu(x)$, defined as follows:

$$\mathcal{F}_\mu \equiv \prod_{\rho \neq \mu} \left(1 + \frac{a^2 \delta_\rho^{(2)}}{4} \right) \Bigg|_{\rm symm}, \tag{8}$$

where

$$\delta_\rho^{(2)} U_\mu(x) \equiv \frac{1}{a^2} (U_\rho(x) U_\mu(x + a\hat{\rho}) U_\rho^\dagger(x + a\hat{\mu}) - 2U_\mu(x) + U_\rho^\dagger(x - a\hat{\rho}) U_\mu(x - a\hat{\rho}) U_\rho(x - a\hat{\rho} + a\hat{\mu})). \tag{9}$$

This is illustrated in cartoon form in Fig. 6. The red link in the center is the original link, the blue staples represent the smearings. A single application of this smearing suppresses the leading one-gluon contributions to the unphysical taste-changing interactions, and is the basis of the so-called "Asqtad" action for improved staggered fermions (a^2, tadpole improved). The Asqtad action provides significantly reduced discretization error compared with the unimproved action.

The hisq action applies this smearing a second time, which suppresses two-gluon exchange contributions to taste-changing. It provides further dramatic suppression of taste changing. For charm quarks, the hisq action also resums all orders in ma in tree-level operator normalizations. This turns out to be significant only in the time-derivative operator (as could have been guessed). This procedure is very similar to the separate normalizations of time-like and space-like operators in the Fermilab treatment of Wilson fermions [3]. The HPQCD collaboration has applied this action to a variety of quantities in charmonium and charm heavy-light physics, with good agreement with data, and in particular with dramatically re-

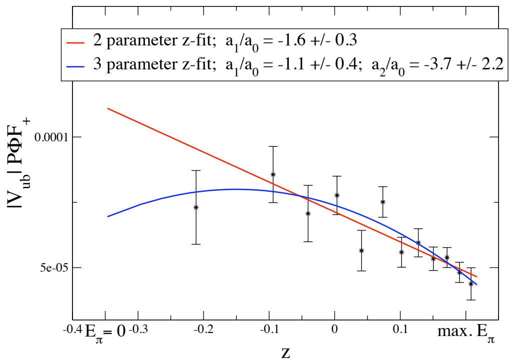

Figure 4. A z expansion fit to the BaBar experimental results for $B \to \pi l \nu$.

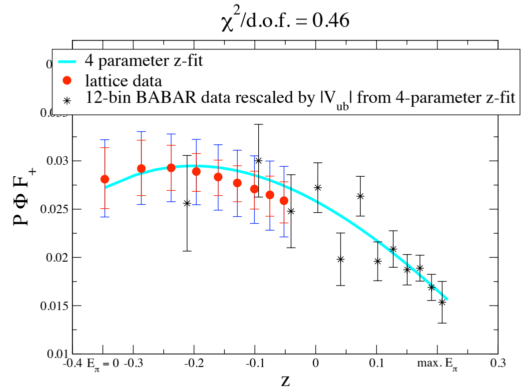

Figure 5. A z expansion combined fit to the lattice and experimental results for $B \to \pi l \nu$.

duced discretization errors.

5. The D and D_s decay constants

In 2005, the Fermilab lattice and MILC collaborations published determinations of the D and D_s meson decay constants to about 10% before experiments had reached that accuracy. When experiments subsequently reached the same accuracy and agreed, we claimed a success. However, as theory and experiment improve one expects eventually to see discrepancies between experiment and Standard Model predictions caused by Beyond-the-Standard-Model physics, so it is important to look at possible contradictions with an open mind. This year, we presented updated results [11]

$$f_D = 207(11)\text{ MeV} \quad (10)$$

$$f_{D_s} = 249(11)\text{ MeV}. \quad (11)$$

Also this year, the HPQCD collaboration published determinations of the π, K, D, and D_s decay constants using hisq fermions [12]. They obtained

$$f_\pi = 157(2)\text{ MeV} \quad (12)$$

$$f_K/f_\pi = 1.189(7) \quad (13)$$

$$f_D = 208(4)\text{ MeV} \quad (14)$$

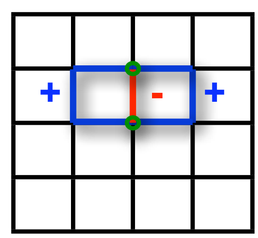

Figure 6. The paths in the smeared lattice gluon field given in Eqn. 10 that couple to momentum π/a gluons, but not to momentum zero gluons.

$$f_{D_s} = 241(3)\text{ MeV} \quad (15)$$

The first three HPQCD results agree very well with experiment, better than 2% and 4% for f_π and F_D, respectively, and better than a per cent for f_K/f_π. f_{D_s}, on the other hand, poses a puzzle. It is over three sigma away from the current experimental average of 270(8) MeV. (See Fig. 7.) It isn't unusual for three sigma discrepancy to appear and then disappear in new results. However, this discrepancy is dominated by the statistical error of the experiment, and three sigma statistical discrepancies are very rare. Since hisq

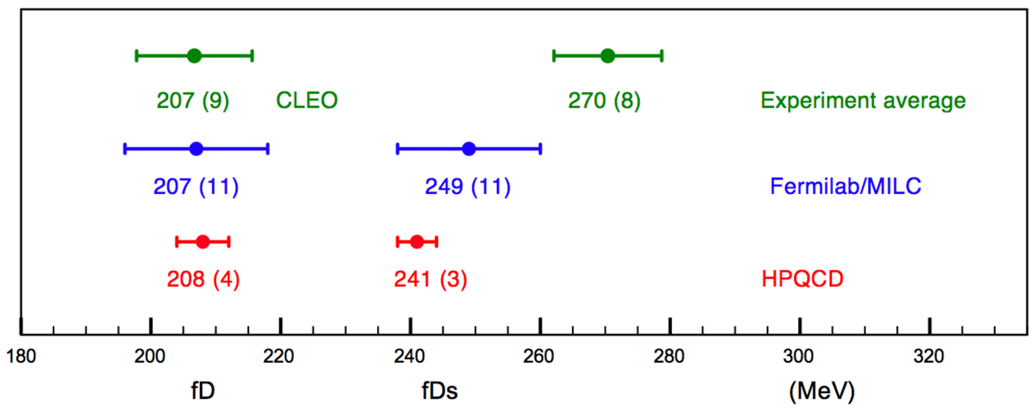

Figure 7. The values of the D and D_s meson decay constants obtained from experiment (top), from the Fermilab/MILC lattice calculation (middle), and from the HPQCD lattice calculations (bottom).

calculations are in their infancy, one may wonder if their uncertainties are perfect. However, it's possible to double and even triple the theory uncertainties and still have a three three sigma discrepancy. One would need to find a change in the analysis that moves the theory result by ten theory sigma to remove the discrepancy by changing the QCD. One further would need to do it in a way that doesn't change the results for F_D, which agrees well with experiment. All four decay constants were done with identical methods. The only difference between F_D and F_{D_s} is that a chiral extrapolation is required for the D meson and not for the D_s. They are identical calculations except that f_{D_s} should be more difficult in one respect. These results for f_{D_S} are the biggest puzzle in staggered fermion phenomenology at the moment, which has for the most part been a record of continuous phenomenological success.

At Lattice 2008, the European Twisted Mass collaboration presented nice results for f_D and f_{D_s} using twisted mass fermions [13]. They obtained $f_D = 197(7)(4)(0)(11)$ MeV and $f_{D_s} = 244(4)(3)(2)(10)$ MeV, in fine agreement with the staggered results. A quibble that I have with this analysis is that it uses two flavors of dynamical quarks, but does not include an estimate of the uncertainty that this causes. Based on the significant difference in decay constants between quenched (zero dynamical flavor) results and two and three flavor unquenched results, I would have guessed that leaving out the strange quarks could could move the results by several per cent. The strange quarks differ in this respect from the charm quark, whose loops are important only near the lattice spacing (since $m_c \sim 1/a$), and likely change physical results by a fraction of a per cent.

f_{D_s} is not a quantity for which model builders have been particularly clamoring for searches for new physics. Nevertheless, the situation is puzzling enough that Dobrescu and Kronfeld have written down models in which new physics might show up here, but not have been observed elsewhere yet [14].

6. m_c from correlation functions

The other new application of hisq fermions that I will to discuss is the determination of the charm quark mass from correlation functions. Some of the best determinations of m_c come from the moments of current-current correlation functions. These can be calculated in perturbation theory to high precision, to third order and in some cases to fourth order. By using dispersion relations, they can be determined experimentally from the annihilation cross section for $e^+e^- \to$ hadrons.

They are also easy to calculate with lattice QCD. They are the fundamental quantities from which hadron masses and other quantities are computed. For example, the mass of the ρ meson may be determine from the vector current correlation function of the light quarks:

$$G_\rho(t) = \sum_{\mathbf{x}} \langle 0|j_\mu(\mathbf{x},t) j_\mu(0,0)|0\rangle \quad (16)$$

$$= C_1 \exp(-M_\rho t) + \ldots \quad (17)$$

The ρ mass is determined from the large t behavior of this correlation function. The moments in t of this correlation function are the short distance quantities from which m_c has previously been obtained by comparing perturbative calculations with experiment. They were used by HPQCD to obtain m_c by comparing perturbative calculations with nonperturbative lattice calculations [15]. HPQCD used several techniques to increase the precision of their determination. The current-current correlation function is the only one that is accessible experimentally, but with the lattice all operator correlation functions are accessible. The pseudoscalar current correlation function can be calculated with the highest precision on the lattice, leaving the vector current correlator to serve as a cross check. Another trick for increasing the precision removes the leading discretization errors. These arise from the tree level quark propagators, and are the same in the nonperturbative correlators and in the tree-level correlators, which are known exactly. They can be removed by comparing the ratio of the full correlators and the tree-level correlators from the lattice and the continuum, rather than comparing the full correlators themselves.

This produces the result

$$m_c(m_c) = 1.266(14)\text{GeV}. \quad (18)$$

This is the most precise determination of m_c to date, an improvement by a factor of two over the precision of the determination from experimentally determined vector current correlation functions, which was $m_c(m_c) = 1.304(27)$GeV.

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