Flip-flop Instability of Colliding Bunches with Round Cross Sections

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Abstract

Selfconsistent beam-beam interactions of colliding bunches changes their transverse sizes due to the flip-flop instability. Calculating the so-called dynamic β-functions at the interaction point, we study the flip-flop instability of colliding bunches with variations of the beam aspect ratios. Even for symmetrical unperturbed lattice conditions of colliding bunches the flip-flop instabilities conserving and changing the aspect ratios of colliding bunches occur within different and non-intersecting stopbands in the working plane of betatron tunes. In particular, this effect can limit the beam-beam performance of the colliding bunches with round cross sections.

1 INTRODUCTION

According to observations [1], under certain conditions transverse sizes of colliding bunches can spontaneously become very different, so that the size of one bunch may substantially exceed that of its partner thus, limiting the collider luminosity. This situation is not stable and after some time the bunches exchange their sizes. As was reported in Ref.[1], the variations in the transverse bunch sizes are not accompanied by the observations of the (dipole) bunch coherent signals. Such a phenomenon was called as the flip-flop effect. A comprehensive description of the flip-flop effect is embarrassed by a necessity to take into account the self-consistent nature of the beam-beam forces and by the fact that very frequently non-linear dependencies of the beam-beam force on the particle offsets are of the primary importance. The self-consistent behavior of the colliding bunches should essentially operate with simultaneous and correlative variations of sizes of both colliding bunches. For that reason, it cannot be studied using a single particle tracking only.

Generally, the difficulties in descriptions of the self-consistent behavior of colliding bunches make attractive the studies of simplified models predicting the flip-flop behavior. In Ref.[2] the flip-flop instability was found calculating self-consistent variations the bunch rms sizes at the interaction point. The equilibrium bunch sizes ware obtained as the period-one solutions to the envelope equations describing the transport of the bunch rms sizes in the phase space through the ring and taking into account nonlinear beam-beam kicks. The instability appears when the envelope equations for colliding bunches have multiple roots. It may occur since these equations are intrinsically nonlinear. To simplify calculations the beam-beam kick in Ref.[2] was calculated assuming the round cross sections of bunches and Gaussian distributions in bunches in transverse coordinates. The synchrotron radiation damping and the heating of bunches due to the fluctuations of particle synchrotron radiation was taken into account to provide stationary unperturbed bunch sizes. In Ref.[3] these calculations were generalized for the case of colliding bunches having flat cross sections at the interaction point (IP). As far as the calculations in Refs.[2, 3] are focused on the descriptions of nonlinear phenomena, their performances demand numerous artificial assumptions.

It was mentioned already in Ref.[2] and then in Ref.[4] that a reasonable though simplified description of the flip-flop instability can be obtained calculating in a self-consistent way the so-called dynamic β-functions of colliding bunches at IP. Within the framework of this model the instability is though a self-consistent, but essentially incoherent effect. The calculations of β-functions can be done using the linear approximation for the beam-beam kick in the particle offsets. If we assume short bunches ($\sigma_s \ll \beta$), the linear part of the beam-beam kick depends on the transverse sizes of colliding bunches only. The periodicity conditions for β-functions enable the calculations of their self-consistent values. Emittances of colliding bunches are not affected by the linear part of the beam-beam kick. For this reason, the calculated variations of the β–functions result in the self-consistent variations of transverse bunch sizes. Generally, spontaneous variations of β-functions at the interaction region change the lattice betatron functions along the ring. That may result in the variations of the diffusion coefficients and in relevant variations of the beam emittances. We neglect such variations assuming that these variations of the bunch emittances occur during the time intervals essentially longer than the instability growth rates. An obvious disadvantage of the model is that due to linear dependence of the beam-beam force on the particle offsets the model predicts the instabilities near the parametric resonances only.

In the papers [2, 3, 4] the calculations were done assuming that the beam-beam interaction does not change the aspect ratios of the colliding bunches. That is a very strong assumption. We report the main results of more general study (see, e.g. in Ref.[5]), when such an assumption is not used a priory.

2 ENVELOPE EQUATIONS

We assume head-on collisions, Gaussian distributions in colliding bunches, a short bunch length (σ_s) as compared to the value of β-function at the interaction point (IP) and zero dispersion function at IP. Then, for small betatron particle offsets from the closed orbit ($|x| \ll \sigma_x$ and $|z| \ll \sigma_z$)

the beam-beam kicks read

$$\delta z'_\pm = -k_{z,\pm} z_\pm, \quad k_{z,\pm} = \frac{2N_\mp r_e}{\gamma_\pm \sigma_{z\mp}(\sigma_{z\mp} + \sigma_{x\mp})}, \quad (1)$$

and

$$\delta x'_\pm = -k_{x,\pm} x_\pm, \quad k_{x,\pm} = \frac{2N_\mp r_e}{\gamma_\pm \sigma_{x\mp}(\sigma_{z\mp} + \sigma_{x\mp})}. \quad (2)$$

Here, suffixes $\pm$ mark the values referring to the electron (mark $-$) and the positron bunches, $\sigma_{x,z}$ are the rms bunch width and height, $N_\pm$ are the numbers of particles in the bunches, $\gamma_\pm = E_\pm/mc^2 \gg 1$ are the relativistic factors of particles, $r_e = e^2/mc^2$ is the classical radius of the electron; an independent variable is the path along the closed orbit (s), so that $x' = dx/ds$. For simplicity, we neglect the coupling of betatron oscillations in the ring arcs. Assuming also zero dispersion and the β-function slopes at IP as well as a linear map in the arcs without with equal betatron tunes ($\mu_0 = 2\pi\nu_0$) and betatron functions at IP for both colliding bunches we map the betatron oscillations though IP and through the arcs. Then, simple matrix multiplications result in the following expressions for the self-consistent tune shifts and β-functions:

$$\cos\mu_{z\pm} = \cos\mu_0 - \frac{k_{z\pm}\beta_{z0}}{2}\sin\mu_0, \quad (3)$$
$$\beta_{z\pm}\sin\mu_{z\pm} = \beta_{z0}\sin\mu_0, \quad (4)$$

$$\cos\mu_{x+} = \cos\mu_0 - \frac{k_{x,\pm}\beta_{x0}}{2}\sin\mu_0, \quad (5)$$
$$\beta_{x\pm}\sin\mu_{x\pm} = \beta_{x0}\sin\mu_0, \quad (6)$$

We assume that parameters for all bunches are chosen in the way that the beam-beam parameters are equal for the unperturbed bunch sizes:

$$B = 2\pi\xi_0 = \frac{(k_{z,\pm}\beta_z)_0}{2} = \frac{(k_{x,\pm}\beta_x)_0}{2}. \quad (7)$$

Defining $r = \sigma_x/\sigma_z$ and

$$z_\pm = \frac{\beta_{z0}}{\beta_{z\pm}}, \quad x_\pm = \frac{\beta_{x0}}{\beta_{x\pm}}, \quad q_\pm = \frac{1+r_0}{1+r_\pm},$$

we rewrite e.g. Eqs.(3) and (4) like follows

$$\cos\mu_{z\pm} = \cos\mu_0 - Bz_\mp q_\mp \sin\mu_0, \quad (8)$$

and

$$z_+^2 = 1 + 2B\cot\mu_0 z_- q_- - B^2 z_-^2 q_-^2. \quad (9)$$

Similarly, we find

$$x_\pm^2 = 1 + 2B\cot\mu_0 \frac{z_\mp q_\mp r_0}{r_\mp} - B^2\left(\frac{z_\mp q_\mp r_0}{r_\mp}\right)^2, \quad (10)$$

The oscillations are stable ($|\cos\mu| \le 1$) provided that

$$Bz_\pm q_\pm \le \cot\left(\frac{\mu_0}{2}\right), \quad Bz_\pm q_\pm \left(\frac{r_0}{r_+}\right) \le \cot\left(\frac{\mu_0}{2}\right). \quad (11)$$

For colliding bunches with round unperturbed cross sections at IP ($r_0 = 1$) and using $r = \sqrt{z/x}$, we rewrite Eqs.(9) and (10) in the following form

$$z_\pm^2 = 1 + 2B\cot\mu_0 w_\mp - B^2 w_\mp^2, \quad (12)$$

$$\left(\frac{z_\pm}{r_\pm^2}\right)^2 = 1 + 2B\cot\mu_0 \frac{w_\mp}{r_\mp} - B^2\left(\frac{w_\mp}{r_\mp}\right)^2, \quad (13)$$

where $w_\pm = 2z_\pm/(1+r_\pm)$. Among others, these envelope equations have the roots corresponding to $r_+ = r_- = 1$, when the beam-beam interaction does not change the bunch aspect ratios. In this case, the blowup in, say, vertical direction results in a proportional increase in the horizontal bunch size keeping the aspect ratio unaltered.

3 NON-SYMMETRICAL ASPECT RATIOS

More general cases were studied solving Eqs.(12) and (13) numerically. Typical results of these calculations are shown in Figs.1 and 2. These graphs contain only the points corresponding to stable betatron oscillations. As is seen from

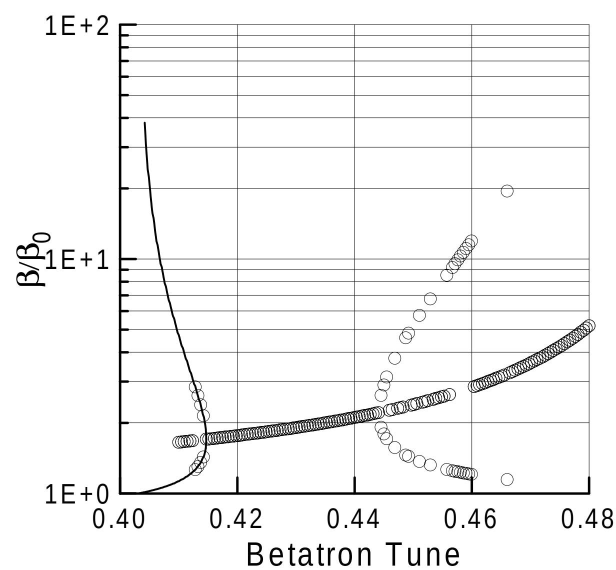

Figure 1: Dependence of the β–functions at the interaction point on the betatron tune. The cross sections of the unperturbed bunches are round ($r_0 = 1$); $\xi = 0.05$; open dots – roots from numerical solution, solid line – flip-flop β–functions assuming unaltered bunch aspect ratios.

these figures, with an increase in the unperturbed betatron tunes the flip-flop instability occurs in two regions. In the first, the instability does not change the aspect ratios of the bunches. So that after the blowup the initially round cross section of the bunch remains to be round. However, for higher betatron tunes appears a different flip-flop region, where the self-consistent beam-beam interaction apart from separation of β–functions of colliding bunches may result

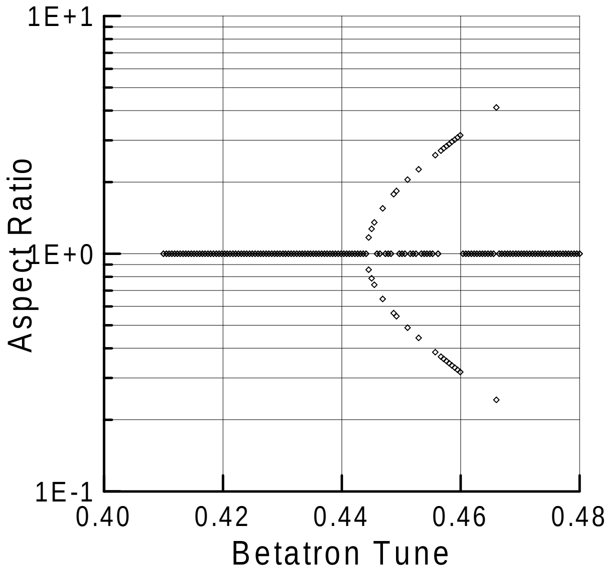

Figure 2: Dependence of the bunch aspect ratios on betatron tune. The cross sections of the unperturbed bunches are round ($r_0 = 1$); $\xi = 0.05$.

in the spontaneous separation of their aspect ratios. Because of this new instability, if one of the bunches becomes more flat in, say, the horizontal direction, then its partner becomes flat in the vertical direction and vice versa. In our model and for ordinary values of the beam-beam parameter ($\xi = 0.05$) such an instability occurs below the parametric resonances. We may expect that in more realistic cases, when the beam-beam force is a non-linear function of the particle offsets, similar splitting of beta-functions and aspect parameters will take place within the stopbands of non-linear beam-beam resonances. Note, that the region where the flip-flop instability changes the bunch aspect ratios has a different orientation in the tune diagram than that, where the bunch aspect ratios remain unaltered. In the case, when aspect ratios are altered, the differences in the bunch β–functions and of aspect ratios increase, when tune approaches the upper edge of the stopband. A decrease in the beam-beam parameter (ξ) decreases the widths of the instability stopbands, but does not eliminate these instabilities.

4 CONCLUSION

The self-consistent beam-beam interaction may result in a spontaneous breaking of the symmetry of the colliding bunches. Within the framework of used here simplified model we have found that self-consistent envelope equations may have multiple roots. These roots can describe both the symmetrical self-consistent variations in the betatron functions of the colliding bunches and the non-symmetrical ones. Symmetrical solutions exist for all tunes. The non-symmetrical solutions exist within definite stopbands of betatron tunes. For usual values of the beam-beam parameter and for our simplified model, these stopbands occur below parametric resonances. The non-symmetrical solutions may correspond to separation of the β–functions either with the conservation of the bunch aspect ratios, or the separation in the β–functions can be accompanied by the variations in the bunch aspect ratios of the colliding bunches. The stopbands for these instabilities are separated in the betatron tunes. Solutions with non-symmetrical aspect ratios occur closer to the upper edge of the parametric beam-beam resonance stopband. This circumstance can be used to control the self-consistent aspect ratios of the colliding bunches. In the case of the flip-flop instability with the variation of the bunch aspect ratios, one of the colliding bunches becomes flat in the horizontal direction, while its partner becomes flat in the vertical direction.

For more realistic cases, we may expect similar phenomena within the stopbands of the non-linear beam-beam resonances.

5 REFERENCES

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