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Ph.D. Thesis:

## A SEARCH FOR GRAVITATIONAL WAVES FROM PULSARS IN BINARY SYSTEMS USING AURIGA AND RADIOTELESCOPE OBSERVATIONS

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# Introduction

Several phenomena happening in the universe show that there's a lot of variability in the universe itself, and that this evolution happens not only in the global scale, but also locally. Examples of this feature are X-ray transients and GRBs [1], [2]. These phenomena can be, at present, all explained by the General Relativity theory, and the important fact for our purpose in this work is that General Relativity predicts the propagation of gravitational waves (G.W.). Now we want to focus on the problem of gravitational wave emission from some selected objects, and in particular we want to find a possible class of promising, well known, gravitational wave emitters, and to characterize them. In order to do this, it is important to remember that General Relativity prevents emission from evolved and isolated objects, such old neutron stars (N.S.) which are not for example in a binary system. This is a result of the Birkoff theorem [3], which we will talk about in more detail in chapter 1, which in fact can be seen as the fundamental difference between the case of gravitational waves and the one of electromagnetic waves. So, with the aim of focusing on promising objects, we have to move our interest onto systems in which the space-time is not stationary. So we will take as promising sources systems composed by two bodies, i. e. binary systems in which at least one of the objects is a compact object, namely neutron star. Moreover, it is well known from Taylor's measures about the binary pulsar PSR 1913+16 [4] that the "quadrupole formula" for the power radiated in gravitational waves is valid and really describes the dynamic of such a system, in the weak-gravity limit, namely when the perturbations in the elements of the metric tensor are small with respect to the background. This means that if we are not close to the extreme case of a black hole, we can successfully use the linearized equations of relativity to perform our estimations about radiated energy, waveforms and expected signal amplitudes. Our choice of focusing on this kind of objects is also "practical", because, at present, the exact description of the dynamics occurring in strong-relativity situations requires some further efforts, and we would have to deal with numerical relativity, which is not yet a widespread and easy methodology. So, we

will take into considerations also, and especially, isolated pulsars, but only binary pulsars. The problem we are going to deal with is to give an exact description of how the signal phase evolves as time goes by. In fact, if a neutron star emits some intrinsically periodic gravitational waves for some reasons (that we are going to discuss), the phase doesn't evolve linearly, especially due to effects of the relative motion of the two objects in the binary system, due to the Doppler effect [5]. Fortunately some of these objects show also an electromagnetic emission, for example in the radio field, and so they have been well studied by radioastronomers, which phase measurements characterized by a very good precision. Between the catalogs of binary radio pulsars, we have found 5 objects which are promising as gravitational wave emitters, and, for 4 of these 5 objects, radioastronomers can provide all the orbital parameters which are needed to accurately predict the time evolution of the gravitational we phase, as seen by an Earth-based detector. We will focus on these 4 objects, describe the astrophysical mechanisms that can lead to gravitational wave emission, calculate the expected signal amplitudes and compare them with the sensitivity of the AURIGA detector, in operation at the INFN National Laboratories of Legnaro, and other detectors, by means of their present or predicted sensitivity. We will find interesting upper limits on the amount of the radiated gravitational energy, and discuss how these upper limits can be used to upgrade a little bit the astrophysical knowledge of these systems. What follows is just a frame of how this work is organized.

In chapter one, we will discuss gravitational wave sources, introduce the quadrupole formula that we will use to calculate pulsar luminosity in gravitational waves, in terms of the geometrical parameters of the source. We will see why accretion is important, and discuss how accretion can occour and behave. Finally, we will discuss how, from the measured electromagnetical luminosity, one can infer the gravitational one.

In chapter two there is a complete description of the kinematic of the binary, namely the NS orbits and their effects on the gravitational signal phase. We will discuss the Solar System Barycenter frame of reference (SSB), and show that the so far achieved precisions on orbital parameters measurements is good enough to successfully perform our study. We will illustrate the objects of interest, and our selection criteria through the databases. Finally, we will discuss the detector motion in the SSB and how we can compose the relative motion of the neutron star respect to the barycenter of the binary systems, with the one of the detector, finally finding the phase evolution of the binary at the detector input.

In chapter three, we will focus on data analysis, giving the basic properties of the AURIGA data, together with a brief description of the AURIGA detector. In particular, we will see what the h-reconstructed data are, its transfer function and its noise features which are relevant to our study: the detector will be characterized as the product of a sequence of filters, that finally lead to the transfer function, as the instrument to use in order to deconvolve the output. As experimental data of detectors are obtained by sampling their analog outputs we will present our analysis in the discrete time domain. We will give an example of how a signal would be seen, if present, as added to the detector's noise. We will discuss the problem of the instabilities which can eventually arise in phase evolution of data, and see how these instabilities can be monitored if they really occur. We will apply to the data the etherodyne detection technique, that leads to the extraction of amplitude and phase of the signal. Then, the statistical interpretation of the results will be given.

In chapter four we will give our final results.

In chapter five we draw our conclusions. Here we can anticipate that we are not be able to reject the null hypothesis, i. e. the hypothesis that a signal is not present in the noise, and so we must give our upper limit on signal amplitudes. However, we will discuss in detail which are the astrophysical implications of these upper limits, giving the interesting astrophysical statements that can be inferred from our measure. Finally, we will discuss future perspectives in this field. In fact, it's worth noticing that the same method we implement and use in this work (analysis approach and numerical codes) can be applied to the case of interferometric Earth-based detectors, still existing (LIGO, VIRGO, GEO, TAMA) or upcoming in the future (Advanced LIGO, Advanced VIRGO). In this case, our calculations about expected amplitudes, compared with sensitivities that will be achieved in the future, lead to the conclusion that a detection of a real gravitational signal is possible. and that further efforts in this field do make sense. In particular, a good idea would be to apply the same method to the very recent measurements about X-transients in pulsars, which can be performed with no additional effort, searching for unexpected and interesting results; another possible application, which doesn't require a too large additional effort, is to apply this method to another class of stellar systems, known as Low-Mass X-ray Binaries.

### Chapter 1

# Pulsars as gravitational wave sources

#### 1.1 Radiation from an isolated source: the Birchoff theorem

The limit at which the linear theory results can be applied and can be considered reliable is not well known. The non-linearity of the gravitational field is one of its fundamental properties, and so, in general, the non-linearities in the field equations are important. For our purposes, it is important to study the possibility of the emission of gravitational waves from space isolated and limited sources, and so to introduce the concept of energy transfer, carried out by a gravitational wave. To make the calculations feasible, one needs to impose some geometrical conditions about the shape of the source. So, the simpler hypothesis is that the source has a spherical symmetry, which is in fact, with a good approximation, the case of old, isolated pulsars, that are not accreting, and there are no perturbations of any kind coming from the outer space outside the source. In this case, we can demostrate the validity of a general result, by means of the Birchoff theorem, which in practice avoids emission of gravitational wave from such a star. So, we have to look at different kind of sources, i.e. accreting pulsars in binary systems. The Birchoff theorem states that, if a source is limited, isolated and spherically symmetric, the gravitational field, in the vacuum space, is necessarily static, and so there is no emission of gravitational waves. To demonstrate the Birchoff theorem, we need to remember the result, coming from General Relativity, that, for a isolated and spherically symmetric distribution of mass, it is always possible to find a coordinate frame in which the general solution of the Einstein's equations in the vacuum space outside the source is the Schwarzschild

solution:

$$ds^{2} = \left(1 - \frac{2m}{r}\right)dt^{2} - \left(1 - \frac{2m}{r}\right)dr^{2} - r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}) .$$
 (1.1)

The first consequence of equation 1.1 is that  $g_{ab,0} = 0$ , and so the solution is stationary. Moreover [3], the coordinates adapt to the Killing vectorial field are

$$X^a = \delta^a_0 \ . \tag{1.2}$$

Given the fact that

$$X_a = g_{ab}X^b = g_{ab}\delta_0^b = g_{0a}\delta_a^0 = \left(1 - \frac{2m}{r}, \ 0, \ 0, \ 0\right) , \qquad (1.3)$$

we immediately see that the field  $X^a$  admits the orthogonality hypersurfaces expressed by the relation

$$X_a = \lambda f_{,a} , \qquad (1.4)$$

where

$$\lambda = X^2 = g_{00} \tag{1.5}$$

and

$$f_{,a} = t = \text{constant} . \tag{1.6}$$

So the (time- type) Killing vectorial field admits as orthogonality hypersurfaces all the hypersurfaces of the family given by the condition t = constant, and so the solution must be static. This result can also be thought in another way: the solution of 1.1 is symmetric respect to time, because it is invariant respect to the time reflexion  $t \to -t$ , and invariant respect to a generic time shift, because it is invariant respect to the transformation  $t \rightarrow t +$  constant. So we have demonstrated that, in the vacuum, a spherically symmetric solution is necessarily static (Birkhoff theorem). This is maybe an unexpected result, because in newtonian theory the spherical symmetry is not related to the time independence, and this result shows the particular character of the nonlinear differential equations at partial derivatives of General Relativity and how their solutions behave. In particular, the Birkhoff theorem implies that, if a spherically symmetric source changes its shape, but still remaining in a sperically symmetric shape, it can not diffuse any perturbations in the outer space. This means that a pulsating but spherically simmetric star, even if it is rotating, cannot emit gravitational waves. If such a source is confined

in a limited region of the space r < a for some value a, so the solution of the Einstein equation in the outer space must be the external Schwarzschild solution. However, the opposite is not in general true: a source that leads to an external Schwarzschild solution is not necessarily spherically symmetric, so, in general, a source doesn't inherit the symmetry of its own external field. If we consider the limit of the metric 1.1 for  $r \to \infty$ , we obtain the flat metric of special relativity written in spherical cohordinates, i. e.

$$ds^{2} = dt^{2} - dr^{2} - r^{2}(d\theta^{2} + \sin^{2}\theta d\phi^{2}) , \qquad (1.7)$$

and so a spherically symmetric solution in the vacuum is, necessarily, asymptotically flat. So, in order to have emission of gravitational waves, one needs to break the symmetry that is the hypothesis of the Birchoff theorem, namely the spherical symmetry. But the only break of the spherical symmetry is not enough to have a non-sationary field: for example, in the case of the Kerr metric, the field is still stationary. We need to have a time-varying quadrupole moment. One way to do this is for the body to have a non-zero quadrupole moment and rotation. Under these conditions, gravitational waves can be generated, as we show in section 1.2.

#### 1.2 Quadrupole formula and radiated energy

We are going to see, given the Einstein's equations of general relativity, the relation between the emission of gravitational waves and a rotating star, namely, how to get, with some approximations which can be applied in our case, the quadrupole formula. The starting point are the Einstein equations of General Relativity:

$$R_{ik} - \frac{1}{2}g_{ik}R = \frac{8\pi G}{c^2}T_{ik} , \qquad (1.8)$$

where  $R_{ik}$  is the Ricci tensor, which carries informations about the metric coefficients  $g_{ik}$ , holding them and their second derivatives in a non-linear way, R is the curvature scalar, build from the Ricci tensor by contracting indexes, and  $T_{ik}$  is the tensor holding the mass-energy distribution. If we are not in the case of strong gravity, which is our case, because our problem is not the one of a black hole, the metric can be written as the Minkovski's one  $g_{ik}^{(0)}$ , plus little terms:

$$g_{ik} = g_{ik}^{(0)} - h_{ik} , \quad h_{ik} \ll g_{ik}^{(0)} .$$
 (1.9)

In the vacuum space  $(T_{ik} = 0)$ , and in the linear approximation, it's possible to demonstrate that equations 1.8 can be all reduced to [6]

$$h_{ik;l}^{;l} + 2R_{limk}h^{lm} = 0 , (1.10)$$

where ; is the co-variant derivative [7] and  $R_{limk}$  is the background Riemann tensor; here equations 1.10 are written chosing (using the gauge freedom) the transverse-traceless gauge given by

$$\delta^{mn}h_{mn} = 0 \quad \text{and} \quad \delta^{mn}\partial_m h_{ni} = 0 \ . \tag{1.11}$$

In order to relate the propagation of the wave to the source which generated the wave itself, it's useful to introduce the energy-impulse pseudotensor as done in [7]. In fact, if we introduce for synthesis the quantities

$$\psi_{ik} = h_{ik} - \frac{1}{2} \delta_{ik} h , \qquad (1.12)$$

equations 1.10 become

$$\frac{1}{2}\Box\psi_{ik} = \frac{8\pi G}{c^4}\tau_{ik} , \qquad (1.13)$$

where  $\Box$  is the D'Alembert operator  $\Box = -g_i^{ik}0)\frac{\partial^2}{\partial x_i\partial x_k}$  or also  $\Box = \bigtriangleup -\frac{1}{c^2}\frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ if we want to write it using the Laplace symbol  $\bigtriangleup$  and  $\tau_{ik}$  is the energyimpulse pseudo-tensor introduced by [7]. It is important to notice that this object is not a tensor, meaning that its components don't transform as the ones of a tensor, and it contains in practice only the terms of  $T_{ik}$  that only refer to the field and not to the matter. From equation 1.13 one can write each component of the wave tensor as an integral over a volume of the space. In fact, using the concept of retarded time  $t' = t - \frac{R}{c}$  where R is the distance from the source, and using the well known theorems of the differential calculus, one can write

$$\psi_i^k = -\frac{4k}{c^4 R_0} \int (\tau_i^k)_{t-R/c} dV . \qquad (1.14)$$

Now we want to calculate these integrals. We must use the relation

$$\frac{\partial \tau_i^k}{\partial x^k} = 0 , \qquad (1.15)$$

which is true because the same condition is true for the tensor of the wave. Lowering the index, one finds

$$\frac{\partial \tau_{\alpha\gamma}}{\partial x^{\gamma}} - \frac{\partial \tau_{\alpha0}}{\partial x^{0}} = 0 = \frac{\partial \tau_{0\gamma}}{\partial x^{\gamma}} - \frac{\partial \tau_{00}}{\partial x^{0}} .$$
(1.16)

If we multiply equation 1.16 by  $x^{\beta}$  and integrate over all the volume, we find

$$\frac{\partial}{\partial x^0} \int \tau_{\alpha 0} x^\beta dV = \int \frac{\partial \tau_{\alpha \gamma}}{\partial x^\gamma} x^\beta dV = \int \frac{\partial (\tau_{\alpha \gamma} x^\beta)}{\partial x^\gamma} dV - \int \tau_{\alpha \beta} dV . \quad (1.17)$$

Given the fact that, at the infinity, it is  $\tau_{ik} = 0$ , the first integral in the second member of the previous equation is 0 because of the Gauss theorem. Then we have

$$\int \tau_{\alpha\beta} dV = -\frac{1}{2} \frac{\partial}{\partial x^0} \int (\tau_{\alpha0} x^\beta + \tau_{\beta0} x^\alpha) dV . \qquad (1.18)$$

Then, let's multiply the second part of 1.17 by  $x^{\alpha}x^{\beta}$  and integrate over the space: we find

$$\frac{\partial}{\partial x^0} \int \tau_{00} x^{\alpha} x^{\beta} dV = -\int (\tau_{\alpha 0} x^{\beta} + \tau_{\beta 0} x^{\alpha}) dV . \qquad (1.19)$$

If we compare 1.18 with 1.19, we have that

$$\int \tau_{\alpha\beta} dV = \frac{1}{2} \left(\frac{\partial}{\partial x^0}\right)^2 \int \tau_{00} x^\alpha x^\beta dV . \qquad (1.20)$$

Now, we have to recall that, when there is no electric charge and angular momentum, it is always true that  $\tau_{00} = \mu c^2$ , where  $\mu$  is the mass density. In fact, if  $u^i$  is the 4-velocity, it is  $T_i^k = \mu c^2 u_i u^k$ , i. e.  $T_0^0 = \mu c^2$ , and the energy-impulse pseudo-tensor has, by its definition, the property that  $\tau_{00} = T_{00}$ . So, equation 1.14 can be re-written as

$$\psi_{\alpha\beta} = -\frac{2k}{c^4 r} \frac{\partial^2}{\partial t^2} \int \mu x^{\alpha} x^{\beta} dV . \qquad (1.21)$$

If the distance r from the observer to the emitting body is large respect to the dimensions of the star emitting the wave, we can approximate the wave to a plane wave. So, if we introduce the tensor quadrupole moment of the emitting body as

$$D_{\alpha\beta} = \int \mu (3x^{\alpha}x^{\beta} - r^2\delta_{\alpha\beta})dV , \qquad (1.22)$$

the density of the energy flux which is radiated in the direction, for example, of the  $x^1$  axis is

$$\frac{k}{36\pi c^5 r^2} \begin{bmatrix} \cdots & \cdots & \cdots \\ (D_{22} - D_{33} \ 2)^2 + D_{23}^2 \end{bmatrix} .$$
(1.23)

It's worth noticing that  $\mu x_{\alpha} x_{\beta}$  is the inertia matrix of the source, and that eq. 1.23 is directly the quadrupole formula 1.24.

#### 1.3 G. W. emission from a rotating pulsar

In this section, to indicate derivatives we use the notation introduced in Weinberg [8], and the same signature (-1, 1, 1, 1) of the metric in the spacetime. Let's consider a non-axisymmetric neutron star, which is rotating with an angular velocity  $\omega$ , and an observator placed at a fixed distance rfrom the rotating star. The amplitude of the gravitational wave emitted by the rotating star, as observed at the time t', assuming that the quadrupole approximation described in [7] holds, is given by

$$h_{jk}(t,r) = \frac{2G}{c^4 r} I_{jk,00}(t'), \qquad (1.24)$$

where  $I_{jk}$  is the matrix of inertia of the pulsar, t' is the delayed time t' = t - r/c, G is the gravitational constant and c is the speed of light. It is very important to notice that the formula (1.24) has been experimentally verified with a very good precision by Taylor's measurements about the binary pulsar PSR 1913+16 [4]. In what follows we indicate the tensor I in a reference frame having its origin in the center of mass of the star and fixed respect to the star itself, and with its  $\hat{e}_z$  axis lying along the direction of the angular momentum of the star. In this frame, we can write the matrix I as

$$I = \begin{pmatrix} I_1 + I_2 + (I_1 - I_2)\cos(2\omega t) & -(I_1 - I_2)\sin(2\omega t) & 0\\ -(I_1 - I_2)\sin(2\omega t) & I_1 + I_2 - (I_1 - I_2)\cos(2\omega t) & 0\\ 0 & 0 & I_3 \end{pmatrix}$$
(1.25)

At this point, it's useful to define the ellipticity of the star, as  $\varepsilon \equiv \frac{I_1 - I_2}{I_3}$ . With this definition, and substituting the equation (1.25) in (1.24), one finds

$$h(t, r \stackrel{\wedge}{e_z}) = \frac{4G}{c^4 r} \varepsilon I_3 \omega^2 \begin{pmatrix} -\cos\left[2\omega\left(t - \frac{r}{c}\right)\right] & \sin\left[2\omega\left(t - \frac{r}{c}\right)\right] & 0\\ \sin\left[2\omega\left(t - \frac{r}{c}\right)\right] & \cos\left[2\omega\left(t - \frac{r}{c}\right)\right] & 0\\ 0 & 0 & 0 \end{pmatrix}$$
(1.26)

So, following this simple model, the frequency at which the gravitational wave is emitted is  $2\omega$ .

Our purpose, now, is to characterize the behaviour of the wave as seen from an observer located at the Earth surface. To this end, let's introduce a second reference frame, also with its origin in the center of mass of the star, but which is rigidly rotated with respect to the previous one by an angle  $\gamma$ , so that the new axis  $\hat{e}_{z'}$  is aligned with the direction of the vector joining the pulsar to the Earth. In this new reference system, let's indicate the gravitational wave tensor in the transverse traceless gauge, so that its only non zero components are the ones orthogonal to the direction of propagation. This can be realized by using an operator which projects the wave tensor in the plane which is orthogonal to the direction of propagation  $\vec{n}$  of the wave itself. This operator  $P_{jk}$  is expressed by the following relationship:  $P_{jk} = \delta_{jk} - n_j n_k$ , where  $n_j$  are the components of  $\vec{n}$  in our reference frame. Applying this projector to the wave tensor, we have

$$h_{jk}^{TT} = P_{jl}P_{mk}h^{lm} - \frac{1}{2}P_{jk}P_{ml}h^{lm} . aga{1.27}$$

Substituting equation (1.27) in (1.26) we finally have

$$h^{TT}(t, r\hat{e}_{z'}) =$$

$$= \frac{2G}{c^4 r} \varepsilon I_3 \omega^2 \begin{bmatrix} -(1 + \cos^2 \gamma) \cos(2\omega t') & 2\cos\gamma \sin(2\omega t') & 0\\ 2\cos\gamma \sin(2\omega t') & (1 + \cos^2 \gamma) \cos(2\omega t') & 0\\ 0 & 0 & 0 \end{bmatrix}$$

For sake of simplicity, in what follows, we call  $h_0 \equiv 2G\varepsilon I_3\omega^2/(c^4 r)$  the amplitude of the wave. We can now define two polarization states:

$$h_{+}(t) \equiv h_{0} \left(1 + \cos^{2} \gamma\right) \cos\left(2\omega t'\right)$$
(1.28)

$$h_{\times}(t) \equiv h_0(2\cos\gamma)\sin\left(2\omega t'\right). \tag{1.29}$$

These are the final equations that connect the amplitudes of the gravitational wave as seen by the observer, with the geometrical properties of the rotating star, and these equations can be used to infer the typical values of h one can expect for a pulsar in a binary system. However, before that, we need good estimations of the possible values of  $\varepsilon$ . These values are provided by the models that we describe now.

As can be seen from the definition of  $h_0$ , the wave amplitude depends linearly on the ellipticity. The maximum values that this parameter can assume depend on the structural model assumed for the pulsar. Models do generally agree in predicting that there is a solid crust outside the fluid which the star is basically made of. This crust is formed during the early evolutional states of the star, when its temperature drops below some threshold. Later, the pulsar keeps on cooling and slowing its rotation (due to magnetic braking), and so decreasing its angular rotation [9], while the solid crust cannot adapt to the new shape of the star core and keeps its own original configuration. This causes the internal stresses into the crust. An estimation of  $\varepsilon_{\text{max}}$  can be done if we think that, when ellipticity has a particular value  $\varepsilon_0$ , there are no internal stress into the crust. The potential energy of the star can be written as

$$E\left(\varepsilon\right) = E_{sf} + A\varepsilon^{2} + B\left(\varepsilon - \varepsilon_{0}\right)^{2}, \qquad (1.30)$$

where  $E_{sf}$  is the energy that the stars would have if it was spherical,  $A\varepsilon^2$  is an addictive term due to the non-sphericity of the mass distribution, and  $B(\varepsilon - \varepsilon_0)^2$  is the potential energy trapped into the crust. The structure obviously tends to assume the minimum energy configuration, namely

$$\varepsilon = \frac{B}{A+B}\varepsilon_0 \ . \tag{1.31}$$

The orders of magnitude for A and B can be found in literature [10]: we can assume  $B \simeq 10^{47}$ erg and  $A \simeq 10^{52}$ erg. Substituting these values in equation (1.31) we have  $\varepsilon \simeq 10^{-5} \varepsilon_0$ . Then, the value of  $\varepsilon_{\text{max}}$  is determined by the breaking strength  $f_{\text{max}}$  of the crust

$$\varepsilon_{\max} = \frac{B}{A+B} f_{\max} \; ; \tag{1.32}$$

unfortunately the value of  $f_{\text{max}}[11]$  is known with an uncertainty of two orders of magnitude. The final result we get is

$$10^{-7} \le \varepsilon_{\max} \le 10^{-9}$$
 (1.33)

At this point we need an estimate of the order of magnitude for  $h_0$ . In the expression of  $h_0$ , we substitute the value of the constant  $G/c^4 = 8 \cdot 10^{-45} s^2 kg^{-1}m^{-1}$  and assume a typical value for the moment of inertia of a neutron star  $I_3 \simeq 10^{38} \text{kg} \cdot \text{m}^2$ ; for what concerns the distance r from the star to the Earth we can assume r = 5 kpc; let's also fix  $\omega/2\pi \simeq 1$  kHz. If we take for  $\varepsilon$  the upper limit of the interval (1.33) we find a dimensionless wave amplitude at the Earth of the order of  $h_0^{\text{max}} \simeq 10^{-25}$ . These order of magnitude are interesting because they are close enough to sensitivities of the existing gravitational wave detectors, so that an experimental result, both a detection of a true signal or an upper limit, can bring informations.

A second mechanism we can invoke as a cause of a crust deformation is the presence of a magnetic field. In order to estimate its importance, we compare the magnetostatic energy of the field B, which is trapped into the pulsar with radius R, with the gravitational potential energy. In the approximations of spherical star and uniform field B we have

$$\varepsilon \simeq \frac{B^2 R^3}{G M^2 / R} , \qquad (1.34)$$

and for a typical neutron star  $(M \simeq 1, 4M_{\odot}, R \simeq 10 \text{km})$ 

$$\varepsilon \simeq 10^{-12} \left(\frac{B}{10^{12} \text{ Gauss}}\right)^2. \tag{1.35}$$

In order to have an effect on  $\varepsilon$  by means of B of the same order of the one found in (1.33) it's necessary that  $B \simeq 10^{14}$  Gauss [12]. When the stellar fluid is a type-I superconductor this could be possible. In this case the field lines are confined into the crustal region, and the resulting magnetic field would be very strong. As discussed in [13], some studies about X-ray pulsars indicate that most neutron stars are born with magnetic fields between  $10^{14}$ and  $10^{15}$  Gauss. In this case, by substituting these values in Eq. 1.35, we can expect for ellipticity  $\varepsilon_{\max} \simeq 10^{-6}$ . Following [12], we can introduce an adimensional coefficient  $\beta$  which describes the efficiency of how the magnetic configuration distorts the star, by means of the relation  $\varepsilon \simeq \beta \mathcal{M}^2$ , where  $\mathcal{M}$ is the magnetic dipole moment. Numerical studies [12] show that, expressing  $\mathcal{M}$  in units of  $2.6 \cdot 10^{32} A \cdot m^2$  one has  $\beta \simeq 1$  for a non-superconducting star, and  $\beta \simeq 100$  for a type-I superconductor.

Another model which has been proposed to distort a neutron star [10] is the following: if the star core is a superfluid, while it rotates it generates some vortices, and the amount of these vortex is proportional to the rotational frequency. These vortex are attached to the neutron star crust and therefore they tend to rotate at the same frequency at which the crust rotates: vortex slide respect to the fluid, as it happens for Magnus effect, when an object moves through the air. The force acting on the crust is given by [11]

$$\vec{F} = 2 \vec{\omega} \times \left(\vec{\Omega} \times \vec{r}\right) \rho , \qquad (1.36)$$

where  $\overrightarrow{\Omega}$  is the angular velocity of the pulsar,  $\overrightarrow{\omega}$  is the difference between the angular velocity of the superfluid and the one of the crust, and  $\rho$  is the density of the superfluid. The stellar deformation produced by this mechanism has the same order of magnitude of the ratio between the expression 1.36 and the gravitational force  $GM^2/R^2$  at the surface, and so

$$\varepsilon \simeq \frac{2\omega\Omega R^3 \Delta M}{GM^2} , \qquad (1.37)$$

where  $\Delta M$  è is the mass contained in the volume  $\Delta V$  of the crust. Parametrizing (1.37) we have

$$\varepsilon \simeq 5 \cdot 10^{-7} \left(\frac{\omega}{1 \text{Hz}}\right) \left(\frac{\Omega}{1 \text{kHz}}\right) \left(\frac{R}{10 \text{km}}\right)^3 \left(\frac{\Delta M}{10^{-2} M_{\odot}}\right)^2.$$
 (1.38)

As we can see, the presence of a very massive crust and a big value of  $\omega$  would make the star to have very interesting ellipticity values. So, for some pulsars, this third mechanism could be the most relevant. However, ve must always remember that, as we have seen talking about the consequences of

the Birchoff theorem in section 1.1, the only possibility for an isolated pulsar to emit gravitational waves is if the star is not spherically symmetric. The most simple configuration to describe, for a non spherically symmetric star, is if the star is axially symmetric. Also an axially symmetric body will not emit gravitational waves, if its angular momentum lies along the symmetry axis. The only case in which we can have an emission of gravitational waves from such an object is if the so called "wobble angle", i.e. the one between the direction of the angular momentum and the symmetry axis is not equal to zero. In this configuration, the body can precess. It is important to notice that there are some observational evidences in the radio field that precession can occur [14] and that these evidences also concern the pulsar formed from the explosion of SN1987A. The phenomenon of precession is widely observed in rigid bodies like planets, and it can arise in neutron stars because, as we have already seen, they probably have a solid crust. The most simple case to deal with is the one of a spheroid. Let  $\vec{e_3}$  be the symmetry axis, and let's suppose that the principal moments of inertia are defined so that  $I_1 = I_2 > I_3$ . Let  $\vec{\omega}$  be the angular velocity of the star and let's suppose that the system is isolated, meaning that J is a constant. Let's call  $\vartheta$  the "wobble angle" and let be  $\gamma$  the angle between  $\omega$  and  $\vec{e_3}$ . From the classical equations of the motion of a rigid body without any external force, if  $\omega_1, \omega_2$ and  $\omega_3$  are the projections of the angular velocity over the symmetry axes, we have

$$(I_1 - I_3) \omega_2 \omega_3 - I_1 \dot{\omega_1} = 0$$

$$(I_3 - I_1) \omega_3 \omega_1 - I_1 \dot{\omega_2} = 0$$

$$I_3 \dot{\omega_3} = 0.$$
(1.39)

From the third equation we have that  $\omega_3$  is constant; the first and the second ones give

$$\dot{\omega_1} = -\frac{(I_3 - I_1)\omega_3}{I_1}\omega_2$$

$$\dot{\omega_2} = \frac{(I_3 - I_1)\omega_3}{I_1}\omega_1 .$$
(1.40)

Introducing the constant quantity

$$\Omega = \frac{I_3 - I_1}{I_1} \omega_3 \tag{1.41}$$

and defining  $A = \sqrt{\omega_1^2 + \omega_2^2}$  we finally get the solution

$$\begin{aligned}
\omega_1(t) &= A \cos \Omega t \\
\omega_2(t) &= A \sin \Omega t .
\end{aligned}$$
(1.42)

This shows that the projection of  $\vec{\omega}$  on the plane containing  $\vec{e_1}$  and  $\vec{e_2}$  describes a circle, and this is what a precession is. The ellipticity is defined by  $\varepsilon = (I_1 - I_3)/I_3$ , and it's possible to show that  $\varepsilon = \gamma - \vartheta$ . One could also demonstrate that, if  $\Omega$  is the angular frequency of precession, it is

$$\Omega = \varepsilon \cdot \omega \cdot \cos \vartheta \,. \tag{1.43}$$

The expressions for the amplitude of the emitted gravitational wave (respectively, for the two polarizations) are [15]

$$h_{+} = \frac{G}{c^{4}} \frac{2I_{1}\omega^{2}\varepsilon\sin\vartheta}{r} \left[ \left(1 + \cos^{2}i_{s}\right)\sin\vartheta\cos2\omega t + \cos i_{s}\sin i_{s}\cos\vartheta\cos\omega t \right]$$
(1.44)

and

$$h_{\times} = \frac{G}{c^4} \frac{2I_1 \omega^2 \varepsilon \sin \vartheta}{r} \left[ 2\cos i_s \sin \vartheta \sin 2\omega t + \sin i_s \cos \vartheta \sin \omega t \right]$$
(1.45)

where r is the distance between the source and the observer,  $i_s$  is the angle between  $\overrightarrow{J}$  and the sight line.

#### 1.4 Accreting N.S.: models and expected amplitudes

We would like to obtain an estimate of the GW amplitude an accreting neutron star can generate. We begin, for simplicity, by describing the case of a non magnetized star. Although this is not the case with the sources we are interested in, we will apply these results later when we will consider the effect of the magnetosphere.

The keplerian velocity at the NS surface is [16]

$$\Omega_{\rm K}(R_{\rm NS}) = \left(\frac{GM_{\rm NS}}{R_{\rm NS}^3}\right)^{1/2} \simeq 13.6 \times 10^3 \ {\rm s}^{-1},\tag{1.46}$$

where  $M_{\rm NS}$  and  $R_{\rm NS}$  are respectively the mass and radius of the NS. This corresponds to a period of  $P_* \simeq 0.461$  ms. This is the value at which a nonmagnetized NS cannot be spun up anymore by accretion. If  $P < P_*$ , and the star is not magnetized, accretion will occur exactly at the surface, so the angular momentum transferred to the star by accretion will be

$$\tau = M\Omega_{\rm K}^2 R_{\rm NS},\tag{1.47}$$

since matter is accreted at the rate  $\dot{M}$  to the NS surface will have angular velocity exactly equal to  $\Omega_{\rm K}$ . If the star is a rigid body and the accretion disk strongly couples with the NS crust, then the angular acceleration will be  $\alpha = \tau/I$  where I is the NS moment of inertia. Substituting the numerical values, we get  $\alpha \simeq 2 \times 10^{-12} \, {\rm s}^{-2}$ , using the typical value  $I = 10^{38} \, {\rm kg} \cdot {\rm m}^2$  for a standard NS. Given the angular acceleration  $\alpha$ , the corresponding period derivative is

$$\dot{P} = \frac{2\pi\alpha}{\omega^2}.\tag{1.48}$$

For a rapidly rotating star, i.e. assuming  $\nu = 500$  Hz, we find  $\dot{P} \simeq 10^{-18}$ . The energy derivative, still assuming the NS is a rigid body, will classically be

$$\dot{E} = 4\pi^2 \frac{I}{P^3} \dot{P} = 2 \times 10^{30} \text{ J/s.}$$
 (1.49)

If we assume that the NS doesn't spin up because the whole energy is radiated in gravitational waves, the corresponding strain amplitude at a distance rfrom the source to the Earth is

$$h = \left[G\frac{\dot{E}}{c^3\omega_{\rm GW}^2}\right]^{1/2} \cdot \frac{1}{r} \simeq 2 \times 10^{-26} \left(\frac{\omega_{GW}}{(2\pi)1\rm kHz}\right)^{-1} \left(\frac{10\rm kpc}{R}\right), \quad (1.50)$$

with  $\omega_{\rm GW} = 2\pi \cdot 1$  kHz and r = 10 kpc. This model, even with some modifications, has been considered viable in the literature, see e.g. [17] and it has been improved, although the general idea remains the same, in ref. [18]. In fact, in ref. [17] the authors perform the calculations neglecting the possible presence of an elastic response of the crust, treating it as perfectly rigid. However, what is shown in [18] is that, for resonable models of the crust, the general results are still robust.

Now, we consider the case of a magnetized star. Following [19], we will assume the magnetic field B to be that of a dipole:  $B(r) = (R_0/r)^3 \cdot B_0$ , where  $B_0$  is the field at the surface  $R_0$  of the NS. We will also assume the accretion to be spherically symmetric. First, we have to calculate the distance from the NS where matter will be accreted. A simple way to calculate an order of magnitude for the magnetosphere radius is to calculate the distance from the NS at which the pressure due to the magnetic field on the infalling accreted charged particles is equal to the internal pressure of the infalling gas [19]. The magnetic pressure can be written as [16]

$$P_B = \frac{[B(r)]^2}{8\pi} = \frac{B_0^2 \cdot R_0^6}{8\pi} r^{-6}, \qquad (1.51)$$

while for the internal pressure of the gas we can write, if the gas falls from rest at infinity,

$$P_g = \rho v^2 = \rho G M_{\rm NS} r^{-1}, \tag{1.52}$$

where  $\rho$  is the gas density and v its velocity, here approximated with the freefall velocity. The density can be easily written in terms of the mass transfer, in fact  $\rho = \dot{M}/(4\pi r^2 v)$ , and so  $P_g = (\dot{M}v)/(4\pi r^2)$ . The value of r at which the equilbrium is reached is an approximation of the magnetosphere radius  $R_*$ :

$$R_* = G^{-1/7} B_0^{4/7} R_0^{12/7} M_{NS}^{-1/7} \dot{M}^{-2/7}, \qquad (1.53)$$

Substituting the values  $B_0 = 10^8$  Gauss,  $R_0 \simeq 10$  km,  $\dot{M} = 10^{-9} M_{\odot} \text{yr}^{-1}$ , we finally get  $R_* \simeq 2R_0$ : the magnetosphere radius, in these old, recycled NS, is only a factor of two the star's radius. This means that the angular momentum transferred to the star by accretion, and the amount of gravitational waves, will be similar to the non-magnetized case. In fact, equations 1.47,1.48,1.49 and 1.50 show that there will be only a factor of  $\sqrt{2}$  increase in h, and so we finally get

$$h \simeq 3 \times 10^{-26}$$
. (1.54)

For some years in the literature [20], the r-mode instability has been studied as an alternative mechanism to remove angular momentum from the NS. In principle, this instability can grow in all rapidly rotating NS. However, various dissipation mechanisms could damp the mode on a much shorter timescale, preventing their growth and thus eliminating them as a substantial source of gravitational waves. It seems clear now that steady r-modes are not likely to play an important role [17]. Because, as argued in [21], the equilibrium between spin-up due to accretion and spin-down due to gravitational radiation is unstable, an interesting situation occurs in which the star is characterized by a limit cycle: the star charges itself for several million years until the instability grows, and then the star slows down due to a GW emission. The relative duration of these charging and emission cycles strongly depends on the amplitude at which the mode saturates. For our "observational" point of view, it is important to have an idea of this relative duration. Unfortunately, this aspect is not well understood. The first estimation of these r-mode cycles can be found in [22]. Here the discussion takes into account that, due to different viscosities in the NS fluid, the instability can arise only in a narrow window of the NS core temperature, which is expected in the range  $1 \div 4 \cdot 10^8 K$  in LMXBs. They find that the

r-mode is active only for a small fraction of the lifetime of the system, lasting about 1 month while the time required to turn the emission on is about  $10^7$ years. However, given the fact that the total energy which is radiated at each cycle is constant, the smaller is the duration of emission, the greater is the GW amplitude. Calculations show that these signals could be seen from the distances of the galaxies in Virgo cluster, so r-modes are still interesting when one performs a blind search for unknown sources. In 2000 Lindblom et al. [23] showed that r-modes cannot become unstable (and thus emit GWs) if the crust is completely rigid because of the viscous dissipation in the boundary layer between the outer fluid and the inner crust. However, about this point, Levin showed that if the crust is not completely rigid and can "adapt" itself to the motion of the undergoing fluid [21], the relative velocity between the outer fluid and the crust can be lower, and so r-modes can grow before they are damped by viscosity. It's important to note that, in the case of neutron stars in binary systems where accretion rates are probably sufficiently stable, it is sufficient to use the linearized equations of fluid dynamics, and so the frequency at which most of the signal is emitted is exactly  $\Omega_{\rm r-mode} = 4/3\Omega_{\rm NS}$ . Current estimates show that GW emission probably lasts about  $10^3$  to  $10^4$  years every  $10^8$  years. If this is the case, we will have no more than about one emitting source every  $10^4$ , and so for our propose of a targeted search the probability of finding one of them in the emitting phase is too low, given the size of our source database to look at. We can conclude that, for few galactic pulsars, this second emission mechanism is not a likely source of measurable GWs. It has been observed (see for example [24]), that spins of NS in accreted systems tend to be clustered at high frequencies. In support of this, there are both simulations of the dynamics of such systems and observations. Among the observations, a good evidence has been found in globular clusters, in particular 47 Tuc [24]. For example, the spin of some galactic LMXBs has been indirectly measured, using phenomena such KHz QPOs or burst oscillations. Other demonstrations of this clustering, which is a very promising feature for nowadays gravitational wave detectors, come from Monte-Carlo simulations of accretion in LMXBs. We refer in particular to a simulation of [25] where a population of old NS rotating at very low frequency begin to accrete matter. Under some assumptions about the average value of the magnetic field and the amount of transferred mass (values that are sufficiently well-known from X-ray observations), and depending on the Equation of State (EoS) of the superdense neutron fluid, the authors found that, in the case of a "soft EoS", final rotation periods are clustered onto very small values, as low as 0.7 ms. By considering the frequency of the fastest discovered radio pulsar, this allows us to conclude that the frequency range at which resonant detectors are operating is probably a good choice.

In the case of "stiff EoS", final periods are found to be uniformly distributed between  $\sim 1 \div 10$  ms, and so the number of objects emitting in the band could be lower, but nonetheless interesting.

#### 1.5 Phenomenology of accreting pulsars

In accreting pulsars, very interesting variability phenomena can arise: these features are now quite well known theoretically and supported by a lot of observations facts. Here, we want to focus on these features, and see how both theoretical and observational indications can be useful to infer some properties of the gravitational waves emitted by these objects. There are substantially three classes of phenomena: i)millisecond pulsations in X-ray spectra; ii)bursts oscillations, due to nuclear burning on the matter falling from the accretion disk onto the surface of the neutron star; and iii)the kHz quasi-periodic oscillations (QPOs), due to orbital motions in the inner accretion flow [27].

i) Millisecond pulsations in the X-ray spectrum: the first one was discovered in 1998 in the soft X-ray transient SAX J1808.4-3658. In this object, the pulse frequency is 401 Hz, and the orbital period of the pulsar around the center of mass of the binary system is 2 hours. Following the models that have been proposed, the presence of these phenomena provides an explanation of why a fast rotating and accreting neutron star must have a weak magnetic field. In fact, if this statement was not true, the radius of the magnetosphere would exceed the keplerian radius, i. e. the radius of the last stable orbit around the star, and so matter corotating in the magnetosphere would not be able to overcome the centrifugal barrier. Estimations of the magnetosphere radius, and so of the magnetic field, based on this model, indicate a viable value to be  $B \simeq 2 - 6 \cdot 10^8$  Gauss.

ii) Burst oscillations: they are interpreted as thermonuclear runaways in the accreted matter on a neutron star surface. A typical spectral shape is the one shown in figure 1.1. When density and temperature in the accumulated nuclear fluid approach the ignition point, the matter ignites at one particular spot, from which a nuclear burning front spreads then it propagates around the star. This leads to a burst of X-ray emission with a rise time typically < 1 s, and a 10 - 100 s exponential decay due to cooling of the neutron star atmosphere. The total amount of emitted energy is about  $10^{39} - 10^{40}$  erg. In the initial phase, when the burning point is spreading, the energy generation is inherently very anisotropic. The occasional occurrence of multiple bursts closely spaced in time indicates that not all available fuel is burned up in each bursts, suggesting that, in some bursts, only part of the surface partic-



Figure 1.1: Energy (in the X-ray spectrum) emitted during a burst oscillation, and its power spectrum (inset) from the source 4U 1728-24 in 1996. The time is espressed in seconds, with a time resolution of 31.25 ms.

ipates. Anisotropic emission from a spinning neutron star leads to periodic or quasi-periodic observable phenomena, because, due to stellar rotation, the viewing geometry of brighter regions periodically varies (unless the pattern is symmetric around the rotation axis). The first burst oscillation was discovered by RXTE in 1996, in the object 4U 1728-34: an oscillation with a slightly drifting frequency near 363 Hz was evident in a power spectrum of 32 s of data, and the oscillation frequency increased from 362.5 to 363.9 Hz in about 10 seconds, as shown in figure 1.1. Burst oscillations have so far been detected in several different sources [27]. It's important to notice that, usually, the frequency increases by 1 or 2 Hz during the burst tail, converging to an asymptotic frequency which is stable. In a widely accepted scenario, the burst oscillations arise from a hot spot in an atmospheric layer of the neutron star rotating slightly slower than the star itself because it expanded by 5-50 m because the spot is like a mountain on the neutron stars' surface, and while this happens the angular momentum is conserved. The frequency drift towards a slightly higher limit frequency is caused by spin-up of the atmosphere, and is closest to the spin frequency of the neutron star. So, if the oscillations are due to a stable pattern in the spinning layer, then it should be possible to describe them as a frequency-modulated and coherent signal, and in this description, the coherence Q-values of oscillations are very



Figure 1.2: Power spectrum of a twin-peak QPO in Sco-X1.

high, in the order of  $Q \simeq 4000$ .

*iii)* Quasi-Periodic Oscillations: they are due to orbital motions in the inner accretion flow. Two simultaneous quasi-periodic oscillation peaks in the 300 - 1300 Hz region and roughly 300 Hz apart one from each other occur in the power-spectra of low-mass X-ray binaries with widely different X-ray luminosity, as shown in figure 1.2. An accepted interpretation of this feature is provided by the so-called "sonic-point beat-frequency model". In this scenario, if we indicate with  $\nu_1$  and  $\nu_2$  the frequencies of the lower-frequency and higher frequency peaks,  $\nu_2$  can be interpreted as the rotation frequency of an orbit at a particular radius in the inner accretion disk; often, objects showing kHz QPOs have also burst oscillations (even if at different epochs), the frequency of these oscillations arising after the burst peak  $\nu_{\text{burst}}$  is probably, as we have already shown, close to the neutron star spin frequency  $\nu_s$ , the beat-frequency model forecasts that is  $\nu_1 \simeq \nu_2 - \nu_{\text{burst}} \simeq \nu_2 - \nu_s$ , and so, if the "beat-frequency" model is correct, the frequency distance between the two peaks is equal to the frequency of the burst oscillation. A consequence of this model is that, analyzing QPOs, we can infer which is the rotation frequency of the neutron star (although this measure will not be very accurate) even if it cannot be directly observed. Up to now, tens of sources have shown kHz



Figure 1.3: Typical shapes for the behaviour of an atoll (left) and a Z (right) source. The y axis substantially indicates the frequency at which most of X-rays are emitted, the x axis contains the integral of the spectral density (total energy at a fixed time). Sources cover the diagrams as time goes by.

QPOs, even if sometimes only one peak is present, in general there are two peaks. There are basically two types of sources showing QPOs, namely the so-called "z-sources" and "atoll sources", given the different kind of shapes of the plots shown in picture 1.3. Each plot refers to a single QPO event from a particular source, and shows how the spectral integral of the energy emitted during the burst changes, related to the frequency of the peak of the X-ray emission. QPOs pulsations, when pulsations are intense and regular enough, can be used for the search of gravitational waves [26].

### Chapter 2

# Binary pulsars: kinematic of pulsar and detector

#### 2.1 Binary pulsars: description of orbits

It is useful to describe the relative motions of the source and the detector placed at a point of the Earth surface in the "SSB reference frame". This frame is centered in the center of mass of the Solar System; its x axis is along the direction of the Aries  $\gamma$  point, namely, the point of the celestial sphere where the celestial equator crosses the ecliptic; its z axis is orthogonal to the plane containing the orbit of the Earth, and the third co-ordinate axis is defined as orthogonal to the others. The Solar System Barycenter frame (SSB) is the best approximation available for an inertial reference frame. The SSB was introduced in these kind of studies by radioastronomers. In fact, to get precise observations of their sources, they have the problem of combining data sets coming from different telescopes, located at different points on the Earth surface. In fact, a system fixed with respect to the center of mass of the Earth would not be inertial, due to all the motions of the Earth around the Sun, like rotation, revolution, precessions. When radio astronomers use SSB then they are able to have, from different radiotelescopes, homogenous time arrivals  $\tau$  of pulsar wavefronts. It has been demonstrated [28] that, if two sources emit a delta signal seen from the first radiotelescope at a time 0, the same signal will reach the second radiotelescope with a delay  $\Delta \tau$  given by equation 2.1:

$$\Delta \tau = \frac{\Delta \tau_g - \frac{\vec{s} \cdot \vec{B}}{c} \left[ 1 - (1 + \gamma)U - \frac{1}{c^2} \left( \frac{E^2}{2} - \vec{E} \cdot \vec{r_2} \right) \right] \left( 1 + \frac{\vec{s} \cdot \vec{E}}{2c} \right)}{1 + \frac{\vec{s} \left( \vec{E} + \vec{r_2} \right)}{c}} , \qquad (2.1)$$

and 2.2:

$$\Delta \tau_g = (1+\gamma) \frac{GM_{\odot}}{c^3} \ln \left( \frac{\vec{s} \cdot \vec{R}_{1\odot} + |\vec{R}_{1\odot}|}{\vec{s} \cdot \vec{R}_{2\odot} + |\vec{R}_{2\odot}|} \right) , \qquad (2.2)$$

where  $\vec{s}$  is the unit vector of the direction of the source from the SSB,  $\vec{B}$  is the baseline between the two radiotelescopes,  $\gamma$  is a parametrized post-newtonian (PPN) parameter  $\gamma$  is 1 in general relativity, U is the total gravitational potential at the geocenter,  $\vec{E}$  is the velocity of the geocenter with respect to the SSB,  $r_1$  and  $r_2$  are the vector positions of the first and the second radiotelescope.  $\Delta \tau_g$  is the Einstein gravitational effect, due to the passage of the two light paths from the source to the two radiotelescope more or less close to the sun:  $\vec{R_{1\odot}}$  and  $\vec{R_{2\odot}}$  are the position vectors of the two radiotelescopes respect to the sun. In the following considerations, all the quantities about orbits, such angles, vectors and so are defined in this reference frame. The transformations which are needed in order to translate the coordinates of a point on the Earth surfaced in terms of the SSB coordinates are provided by some software routines, which are universally used by both radioastronomers and gravitational wave physicists. These routines reed some ephemeris files, in fact these files hold all needed informations about the positions of the bodies in the Solar System and other astronomical quantities, given at a certain fixed time of origin. The routines we adopt are in the NOVAS package (Naval Observatory Vector and Astrometry Subroutines) and the ephemeris files is the "DE 405" by "JPL Planetary and Lunar Ephemerides", all this material is available on the web [29]. In figure 2.1, we show the orbital plane of a binary pulsar as seen from the SSB [30]. However A is the real major semiaxis of the orbit, what we need in our calculations is "A1", which is its projection along the vector radius which identifies the center of mass of the system from the Solar System Barycenter. The point D represents a generic detector on the surface of the Earth. Figure 2.2 shows how the longitude of periastron OM is defined: it is the angle, in the pulsars' orbital plane, between the directions of the line of sight and the periastron P. It is important to notice that, for some objects, this angle is not constant in time, namely the orbit precesses, and sometimes very precise measures allow to calculate the rate of this precession. In figure 2.3, other important quantities we are going to use are defined. xma is the eccentric mean anomaly, i. e. the angle between the position of periastron and the "ficticious pulsar" at any time. "Ficticious" means that, in this case, we are not considering the real pulsar, but we consider instead a fake object that covers a circular orbit with the same A, obviously with a constant angular velocity. xma is measured at

the center of the ficticious circular orbit. On the other hand, ea is the true anomaly, namely respect to the true orbit of the pulsar, which is in general not exactly circular but elliptic, with some measured value of the eccentricity parameter *ECC*. Given the orbital parameters, the problem of calculating the velocity of the pulsar respect to the SSB is completely solved if we are able to calculate ea(t) at any time. The routines we wrote to do this start calculating xma(t) and then find ea(t) by means of the Kepler equation 2.3:

$$ea = xma + ECC \cdot \sin ea . \tag{2.3}$$

Formula 2.3 is a transcendent equation and doesn't have an exact analytical solution, and so needs to be solved numerically. The calculation is performed by a routine called Kepler, that here we have rewritten and integrated into our C++ code starting a previously existing one written in Fortran. The routine works as follows: as first guess, we put xma as the argument of the trigonometric function and equation 2.3 gives a rough first approximation of ea, say  $ea_1$ . This value is substituted as argument of the sin to get the second approximation and so by iteration:

$$ea_{n+1} = xma + ECC \cdot \sin ea_n . \tag{2.4}$$

The succession of these approximations converges to the true value, so we impose the degree of precision we need: when the difference between the last estimation and the previous goes below a fixed value, the program outputs the final result. For the pulsars of our interest, the binary model, that belongs to the ones included in the TEMPO package [31], is called ELL1, i.e. we deal with non relativistic binaries, with an elliptic orbit characterized by a low eccentricity. For these binaries, the radial velocity of the pulsar with respect to the center of mass of the binary is

$$v = \frac{2\pi}{PB} \frac{A1}{\sqrt{1 - ECC^2}} \left[ \cos\left(ea + OM\right) + ECC \cdot \cos OM \right] , \qquad (2.5)$$

where PB is the orbital period, A1 is the projection of the major semiaxis along the line of sight, ECC is the eccentricity and OM is the periastron longitude.

Parameters PB, A1, ECC, OM are what we need to fully describe the orbit, together with the epoch of a periastron passage T0. All these quantities are provided by radioastronomical observations.

In practice, instead of OM, ECC and T0, we get the quantities that describe orbits in the Laplace-Lagrange model, i.e. the parameters that radioastronomers call EPS1, EPS2 and TASC. Starting from Lagrange-Laplace



Figure 2.1: description of relative motions in the SSB reference frame

parameters we can calculate the keplerian ones by means of the following relations:

$$ECC = \sqrt{EPS1^2 + EPS2^2} , \qquad (2.6)$$

$$T0 = TASC + \frac{PB}{2\pi} \arctan \frac{EPS1}{EPS2} , \qquad (2.7)$$

$$OM = \arctan \frac{EPS1}{EPS2} . (2.8)$$

Figure 2.4 shows the two component of the velocity of a generic detector in the SSB: the total velocity is the sum of the two vectors  $v_{rot}$  and  $v_{riv}$ , the first referring to the rotation of the Earth around its polar axis, the second to the revolution around the Sun. What is shown in figure 2.4 is basically correct, but there are also other smaller effects to compute, namely the precession and nutation movements, which we also take fully into account in the routines.



Figure 2.2: definition of the longitude of periastron



Figure 2.3: true anomaly and eccentric anomaly



Figure 2.4: speed vectors of a Earth-based detector in the SSB

| Name                                                 | R.A. $(h:m:s)$    | $\delta$ (° :':") |
|------------------------------------------------------|-------------------|-------------------|
| PSR J0024-7204J                                      | 00:23:59.40738156 | -72:03:58.7914455 |
| $\mathrm{PSR}\ \mathrm{J0024}\text{-}7204\mathrm{W}$ | 00:24:06.03246948 | -72:04:48.8829686 |
| PSR J0218+4232                                       | 02:18:06.3556854  | +42:32:17.40027   |
| PSR J1939+2134                                       | 19:39:38.558720   | +21:34:59.13745   |

Table 2.1: Positions in the sky of target pulsars

#### 2.2 Target sources and their properties

Here we focus on the target sources of our search for gravitational wave signals. We anticipate that the useful band for the search with the gravitational wave detector AURIGA is  $840 \div 960$  Hz and this is our main selection criterion. The frequency of the pulsar is measured by radiotelescopes. The other relevant source characteristics measured by radiotelescopes are sky coordinates and orbital parameters. All our target pulsars are binaries, eccept PSR J1939+2134. We give their names and coordinates in the sky, along with all known quantities which are crucial in our analysis and which have been measured by radioastronomers: frequency of radio signals (and so the intrinsic frequency of the gravitational waves, if they are present) and orbital parameters. We will also review whatelse about what astronomers know about these sources, namely the environments around the sources, and which phenomena they have shown so far: glithces, QPOs, bursts. This information has been retrieved from the ATNF pulsar database catalogue [32], or from special observations which have been done by radioastronomers for our study. The informations about the positions in the sky are given in table 2.1. Following the usual astronomical co-ordinate equatorial system, R.A. is the right ascension and  $\delta$  is the declination of the source. Table 2.2 holds the information about the frequency of the radio pulsar and its derivative, and the intrinsic frequency of the gravitational signal we want to search for. In table 2.2,  $F_0$  is the frequency of the pulse arrivals,  $F_1$  is its derivative, and  $F_{\rm grav}$ is the frequency of the gravitational signal we want to look at. Following the models discussed in chapter 2, for the firsts 4 objects, this frequency is twice the electromagnetic one, because in these cases we are looking at the signal, if present, which is emitted because of a deformation of the star from sphericity; only for the last object,  $F_{\text{grav}} = (4/3)F_0$ , because we are searching a signal from r-mode instability (the only possible emission frequency which falls into the AURIGA sensitivity band. Table 2.3 gives all known informations about their orbits (orbital parameters). Notice that there was originally another promising source in the band we wanted to look at, namely

| Name                                                 | $F_0[Hz]$      | $F_1[s^{-2}]$        | $F_{\rm grav}[Hz]$ |
|------------------------------------------------------|----------------|----------------------|--------------------|
| PSR J0024-7204J                                      | 476.0468584406 | $2.2199 * 10^{-15}$  | 952.0937168812     |
| $\mathrm{PSR}\ \mathrm{J0024}\text{-}7204\mathrm{W}$ | 425.1077984117 | $7.5130 * 10^{-15}$  | 850.2155968349     |
| PSR J0218+4232                                       | 430.4610663457 | $-1.4340 * 10^{-14}$ | 860.9221326914     |
| PSR J1939+2134                                       | 641.9282611068 | $-4.3317 * 10^{-14}$ | 855.9043481424     |

Table 2.2: Frequencies of electromagnetic and gravitational wave signals from target pulsars

| Name PSR    | PEPOCH [MJD] | PB [days]      | A1 $[lt s]$ | OM $[^{\circ}]$ | ECC              |
|-------------|--------------|----------------|-------------|-----------------|------------------|
| J0024-7204J | 51600.00     | 0.120664937725 | 0.040402329 | 1.2744          | $3.85 * 10^{-5}$ |
| J0024-7204W | 51600.00     | 0.132944430397 | 0.243456050 | 169.0721        | $6.28 * 10^{-6}$ |
| J0218+4232  | 50864.00     | 2.028846083963 | 1.9844345   | 32.4150         | $6.68 * 10^{-6}$ |

Table 2.3: Orbital parameters of the 3 target binary pulsars

PSR J1701-3006F. Unfortunately, it was impossible to perform the measure for this object. In fact, this object is very weak in the radio emission, and so at present radioastronomers did not manage to build a good timing solution for this pulsar. In table 2.3, PEPOCH is the date at which the parameters are referred, expressed in units of "modified julian days" (MJD). The Modified Julian Day (MJD) is an abbreviated version of the old Julian Day (JD) dating method which has been in use for centuries by astronomers. Start of the JD count is from 0 at 12 noon 1 JAN -4712 (4713 BC). The Modified Julian Day, on the other hand, was introduced by space scientists in the late 1950's. It is defined as MJD = JD - 2400000.5. PB is the orbital period; A1 is the projected major semi-axis of the orbit, i. e. not the length of the real semi-axis in the space, but its projection in the direction which is radial respect to the celestial sphere. It is given in units of light seconds. OM is the longitude of the periastron, namely the angle between the line of sight of the orbit as seen from the Earth, and the vector lying on the orbital plane which fixes the position of the periastron. ECC is the eccentricity, and obviously it's a dimensionless parameter. All the required parameters have been provided by radioastronomers and are the results of fitting the data coming from different radio-telescopes and combining them at the SSB reference frame.

#### 2.3 Computation of all Doppler effects in the SSB

To relate the intrinsic frequency at which the signal is emitted by the pulsar, with the observed instantaneus frequency (meaning the frequency measured by the detector) we have to take into account all the effects that affect the observed frequency. The most important are the doppler shifts, due to the relative motions of the star in its orbital plane, and the detector with respect to the SSB. In radioastronomers' language, what we call a "frequency drift" are described as a delay (or an advance) in the time of arrivals of the wavefronts. They call it "Roemer delay", from the name of the Danish astronomer who first noticed that. This "time domain" description, is completely equivalent to our "frequency domain" point of view. In fact, [33], let's consider a periodic signal with frequency  $f_0$  from an object in a binary. Define R to be the radius of the orbit of the object about the barycentre of the binary and let  $\Omega$  be the orbital frequency. For simplicity, we will assume that the angle of inclination is  $i = \pi/2$  so the the line of sight lies in the orbital plane of the binary. Then the received signal h(t) at the barycenter of the Earth's orbit is given by

$$h(t) = A\sin(2\pi \int_0^t f(\tau)d\tau)$$
 (2.9)

If the frequency is Doppler shifted due to the motion of the object in its orbit (and the speeds are not relativistic), then

$$f(\tau) = f_0 (1 + \frac{v}{c}) , \qquad (2.10)$$

where  $v = v_0 \sin(\Omega t) = \Omega R \sin(\Omega t)$ . Thus

$$f(t) = f_0(1 + \frac{\Omega R}{c}\sin(\Omega t)) . \qquad (2.11)$$

Consequently, the integral of the frequency gives:

$$\int_0^t f(\tau)d\tau = ft + \frac{\Omega R}{c} \int_0^t \sin(\Omega \tau)d\tau = ft - \frac{R}{c}\cos(\Omega t) .$$
 (2.12)

Inserting 2.12 back into equation 2.9 gives

$$h(t) = A\sin(2\pi f t - 2\pi [\frac{R}{c}\cos(\Omega t)]) .$$
 (2.13)

We can interpret

$$\phi = 2\pi \left[\frac{R}{c}\cos(\Omega t)\right] \tag{2.14}$$

to be the phase shift due to time delay caused by the light travelling the addition distance  $R \cos(\Omega t)$ . Therefore, if we can assume that the orbital speeds are not relativistic and that R is much less than the distance to the binary, which is the case, then the Doppler interpretation is equivalent to the Roemer interpretation.

In general, there is another kind of Doppler effect, the so-called Shapiro time delay. It is a relativistic effect, and this has also been implemented in our routines. This results in a delay in the arrival times of the wavefronts given by

$$\Delta t = -2M2 \ln \left[ \frac{1 - \sin i \cdot \sin \left( OM + ea \right)}{1 + ECC \cdot \cos ea} \right] , \qquad (2.15)$$

where i is the inclination of the orbital plane with respect to the line of sight is the sine of the inclination of the orbital plane, and M2 is a quantity which is proportional to the mass of the companion star.

In our binaries, that are not relativistic, these parameters are not present; however, in order to set up a more general routines package, this additional effect, that can arise in some cases, is already implemented.

# 2.4 A possible dangerous disturbance: pulsar glitches

The phenomenon of glitches consists of unforeseeable deviations of the pulsar phase evolution from its usual behaviour. At some random time, the frequency of the radio pulses from some pulsars grows up, by a quantity that, in most important glitches, can reach values of

$$\frac{\Delta\Omega}{\Omega} = 10^{-6} . \tag{2.16}$$

After this period, the phase re-starts growing in time with the usual behaviour. The phenomenon can be explained in a quite convincing way with a simple model, which assumes the neutron star as composed by two phases: a superconducting fluid composed of neutrons (with moment of inertia  $I_n$ ), weakly coupled with a crust of moment of inertia  $I_c$ , that can be modelled by a crystalline structure, whose reticular centers are nuclei rich in neutrons,
and in which free electrons are present. Let's assume that the crust rotates at the angular velocity of the electromagnetic signal  $\Omega_c$ , and let's indicate with  $\Omega_n$  the frequency at which the central superconductor rotates (notice that in general a fluid has a differential rotation, and so for  $\Omega_n$  one takes a mean pulsation). Let's introduce a constant parameter T which indicates the timescale of the interactions between the two components, and so quantifies the coupling between the two phases. In this model, each glitch is due to a break and to the consequent readjustment of the crust (this crust failure is called a "starquake"), as shown below. The basic idea is that, when the crust spinup occurs, the angular momentum transferred to the superconductor depends on T. The interaction between the two components after a starquake is given by the differential equations system [34]

$$I_c \dot{\Omega}_c = -\alpha - \frac{I_c \left(\Omega_c - \Omega_n\right)}{T}$$
(2.17)

$$I_n \dot{\Omega}_n = \frac{I_c \left(\Omega_c - \Omega_n\right)}{T} , \qquad (2.18)$$

where  $\alpha$  is an external breaking due, for example, to the magnetic dipole moment; here we take  $\alpha$  to be a constant. The solution of the previous system is

$$\Omega_c(t) = -\frac{\alpha}{I}t + \frac{I_n}{I}\Omega_1 e^{-t/T^*} + \Omega_2$$
(2.19)

$$\Omega_n(t) = \Omega(t) - \Omega_1 e^{-t/T^*} + \frac{\alpha T^*}{I_c} , \qquad (2.20)$$

where  $\Omega_1$  and  $\Omega_2$  are two integration constants, which depend on the chosen initial conditions, I is the total moment of inertia of the star and  $T^*$  is defined as

$$T^* = \frac{I_n}{I}T \ . \tag{2.21}$$

We may wonder what happens when the interaction between the crust and the internal region is very efficient, namely in the limit for  $t/T \to \infty$ . In this case the solution becomes

$$\Omega_n - \Omega_c = \frac{\alpha T^*}{I_c} = \frac{I_n}{I_c} \frac{T}{T} \Omega_c , \qquad (2.22)$$

where we introduced the spindown age of the pulsar  $T = P/\dot{P}$ . Let's suppose that  $I_n \simeq I_c$ . For the typical values of mass and moment of inertia of a standard neutron star, we find, by means of Equation (2.22):

$$\frac{\Omega_n - \Omega_c}{\Omega_c} \simeq 10^{-5} , \qquad (2.23)$$



Figure 2.5: Evolution of the angular frequency of a pulsar after a glitch.

which provides the order of magnitude of the difference between the pulsations of the two components. Let's suppose now that at the time t a glitch is observed, i. e. that  $\Omega_c$  changes, let's suppose for sake of simplicity that this happens instantaneously, from  $\Omega_c(t)$  to  $\Omega_c(t) + \Delta \Omega_c$ : introducing a parameter Q which describes how rapidly the effect of the increment due to the glitch decays, the solution for the motion of the crust reads

$$\Omega(t) = \Omega_0(t) + \Delta \Omega_c \left[ Q e^{-t/T} + 1 - Q \right] .$$
(2.24)

The qualitative behaviour of the pulsation of the crust after the glitch is shown in Figure (2.5). Once we have described the evolution of the pulsation after a glitch, let's describe in more details the mechanism that drives the crustal break which generates the starquake. The crust is deformed from sphericity because of rotation. When the star slows down, the internal fluid changes its proper configuration, and this generates a stress acting on the crust, which breaks, when this stress reaches a typical value. In a newtonian description, the total energy of the star is

$$E = E_{int} + U + T + E_{crust} , \qquad (2.25)$$

where U and T are the potential and cinetic energy respectively,  $E_{int}$  is the internal energy of the fluid and  $E_{crust}$  is the one trapped into the crust because of the stress. If we assume for the equation of state the usual polytropic form, with polytropic index that we parametrize as

$$\Gamma = \frac{d\log p}{d\log \rho} = 1 + \frac{1}{n} , \qquad (2.26)$$

we can write that

$$E_{int} = C \cdot \rho^{1/n} \cdot M , \qquad (2.27)$$

where M is the mass of the fluid,  $\rho$  is its density and C is a constant. Using the same arguments we introduced in section 1.3, U can be re-written as the sum of the potential energy that the star would have if it would be exactly spherical  $U_0$  and an additive term, which is proportional to the square of the ellipticity  $\epsilon$ , depending on a proportionality constant A:

$$U = U_0 + A \left(\frac{1}{\varepsilon} - 1\right)^2 \tag{2.28}$$

The kinetic energy can be written as a function of the angular momentum J as

$$T = J^2 \cdot \frac{1}{2\varepsilon I_0} . \tag{2.29}$$

Presumably, the energy trapped into the crust is an elastic energy [34]: we can assume that the star has an initial deformation  $\varepsilon_0$  that corresponds to the absence of stress on the crust; when the rotation motion slows down,  $\varepsilon$  decreases and an elastic stress arises, which turns out to be proportional to  $(\varepsilon - \varepsilon_0)^2$ :

$$E_{crust} = N\left(\frac{1}{2}K\left(\varepsilon - \varepsilon_0\right)^2\right) , \qquad (2.30)$$

where N is the density of nuclei on the crust: introducing the ionic density of the crust n, the stellar radius R and the separation  $R_0$  between the ions, one has

$$E_{crust} = \frac{Z^2 e^2}{R_0} n R^3 \left(\varepsilon - \varepsilon_0\right)^2 = C \left(\varepsilon - \varepsilon_0\right)^2 , \qquad (2.31)$$

where we have collected in the factor C all the constants. We can associate to this energy an average elastic force

$$\sigma = \left| \frac{1}{V_{crust}} \frac{\partial E_{crust}}{\partial \varepsilon} \right| = \mu \left( \varepsilon - \varepsilon_0 \right) , \qquad (2.32)$$

where  $V_{crust}$  is the volume of the crust and the constant  $\mu$  is equal to

$$\mu = \frac{2C}{V_{crust}} . \tag{2.33}$$

Finally, the total energy can be written as

$$E = c\rho^{1/n}M + U_0 + A\left(\frac{1}{\varepsilon} - 1\right)^2 + J^2\frac{1}{2\varepsilon I_0} + C\left(\varepsilon - \varepsilon_0\right)^2 .$$
 (2.34)

The equilibrium ellipticity can be obtained by minimizing this expression. The calculations give

$$\varepsilon = \frac{I_0 \Omega^2}{4 \left( A + C \right)} + \frac{C \varepsilon_0}{A + C} \,. \tag{2.35}$$

For standard pulsars A >> C, and therefore Equation 2.35 can be further simplified:

$$\varepsilon \simeq \frac{I_0 \Omega^2}{4A}$$
 (2.36)

When the stress reaches a critical value  $\sigma_c$  the crust breaks and ellipticity decreases by an amount  $\Delta \varepsilon$  which, following Equation (2.36) can be related to the pulsation. We find that

$$\Delta \varepsilon = -\left(1 - Q\right) \frac{\Delta \Omega}{\Omega} . \tag{2.37}$$

The amount of the glitch, in terms of  $\Delta\Omega/\Omega$  can give an indication about  $\Delta\varepsilon$ : we have very low values, of the order of  $\Delta\varepsilon \sim 10^{-6}$ . The occurrence of a glitch on a particular target object can be monitored on the ATNF pulsar database. Glitches are very dangerous for our analysis in principle, because they can make impossible a continuous reconstruction of the phase evolution from time separated measurements by radiotelescopes. We anticipate that, for objects in table 2.1, and in the period of observation, no glitches occurred.

## 2.5 Amplitude modulation: the antenna pattern

The interaction between a periodic gravitational wave and the detector depends on several parameters. First of all, let's consider the effect of the relative orientation of the detector axis and the source. The maximum interaction between the wave and the detector occurs when the wave vector is orthogonal to the bar's axis, and the bar senses only the polarization component of the wave along its axis. Let's call  $\vartheta$  the angle of incidence of the wave on the bar, and  $\varphi$  the angle between the bar's axis and the direction of the "+" polarization. It's only the component of the gravitational wave tensor lying along the direction of the bar  $h_b$  that effectively interacts with the bar. The force acting on the bar can be written as

$$F(t) = \frac{1}{2}\mu L\ddot{h}_b(t) \quad . \tag{2.38}$$

The two parameters  $\mu$  and L are respectively the "effective" mass and length of the first (fundamental) longitudinal oscillation mode of the bar which can be found by solving the elastic equations of the cylinder. In order to get  $\ddot{h}_b(t)$  it is useful to define a new reference frame  $(\hat{e}''_x, \hat{e}''_y, \hat{e}''_z)$  having its origin on the detector,  $\hat{e}''_z$  axis lying along the detector's zenith, and  $\hat{e}''_x$  along the bar's axis. With these notations, projecting the vectors, one has

$$h_b = \sin^2 \vartheta \cos 2\varphi \ h_+(t) + \sin^2 \vartheta \sin 2\varphi \ h_\times(t) \ . \tag{2.39}$$

where  $\sin^2 \vartheta \cos 2\varphi$  and  $\sin^2 \vartheta \sin 2\varphi$  are the antenna patterns [35]. Now, let's try to characterize how the time dependence of the two angles  $\vartheta$  and  $\varphi$ translates onto the shape of the signal extracted by the antenna. These angles are two periodic functions of time, and the period of their variations is 1 day. Qualitatively, for what concerns the signal amplitude at the detector, this means that it is modulated with a 1 day period. In order to do quantitative considerations, let's refer to the equatorial reference frame: let  $\alpha_P$  and  $\delta_P$ respectively the right ascension and the declination of the pulsar. Let's also call  $\psi$  the angle between  $\hat{e}''_x$  and the projection on the plane  $(\hat{e}''_x, \hat{e}''_y)$  of the polarization vector. The equatorial reference frame and the one centered on the bar are related each one to the other by a rigid time-dependent rotation, which is described by this matrix:

$$\begin{array}{c} \cos\psi\sin\delta_P\cos\alpha_P + \sin\psi\sin\alpha_P & \cos\psi\sin\delta_P\sin\alpha_P - \sin\psi\cos\alpha_P & \cos\psi\cos\delta_P \\ \sin\psi\sin\delta_P\cos\alpha_P - \sin\psi\sin\alpha_P & \sin\psi\sin\delta_P\sin\alpha_P - \cos\psi\cos\alpha_P & \sin\psi\cos\delta_P \\ & -\cos\delta_P\cos\alpha_P & -\cos\delta_P\sin\alpha_P & \sin\delta_P \end{array}$$

If T is the local sidereal time, and  $\alpha$  and  $\delta$  are respectively the longitude and latitude of the detector, we can calculate [36] that the functions  $\vartheta(t)$ and  $\varphi(t)$  are so that

$$\cos \vartheta = \hat{e}''_{x} \cdot \hat{e}''_{z} = \cos \alpha \sin \delta \cos \delta_{P} \cos(\omega_{\oplus} T - \alpha_{P})$$

$$- \sin \alpha \cos \delta_{P} \sin(\omega_{\oplus} T - \alpha_{P})$$

$$- \cos \alpha \cos \delta \sin \delta_{P}$$

$$(2.40)$$

$$\cos \varphi = \hat{e}_x'' \cdot \hat{e}_x'' = -\cos \psi [\cos \alpha \sin \delta \sin \delta_P \cos(\omega_{\oplus} T - \alpha_P) \qquad (2.42)$$
  
-  $\sin \alpha \sin \delta_P \sin(\omega_{\oplus} T - \alpha_P)$   
+  $\cos \alpha \cos \delta \cos \delta_P ] + \sin \psi [\cos \alpha \sin \delta \sin(\omega_{\oplus} T - \alpha_P)$   
+  $\sin \alpha \cos(\omega_{\oplus} T - \alpha_P) ] \qquad (2.43)$ 

$$\sin \varphi = \hat{e}''_x \cdot \hat{e}''_y = -\sin \psi [\cos \alpha \sin \delta \sin \delta_P \cos(\omega_{\oplus} T - \alpha_P) - \sin \alpha \sin \delta_P \sin(\omega_{\oplus} T - \alpha_P) + \cos \alpha \cos \delta \cos \delta_P] - \cos \psi [\cos \alpha \sin \delta \sin(\omega_{\oplus} T - \alpha_P) + \sin \alpha \cos(\omega_{\oplus} T - \alpha_P)] (2.44)$$

where  $\omega_{\oplus} = 2\pi/1$  day. Taking the square of these relations we have

$$\cos\left(2\varphi\right) = \left(\hat{e}''_x \cdot \hat{e}''_x\right)^2 - \left(\hat{e}''_x \cdot \hat{e}''_y\right)^2 \tag{2.45}$$

$$\sin(2\varphi) = 2\left(\hat{e}_x'' \cdot \hat{e}_x''\right)\left(\hat{e}_x'' \cdot \hat{e}_y''\right)$$
(2.46)

At this point, it's useful to define two functions of time A(t) and B(t), that are completely determined by the co-ordinates of the source and of the detector and by the sidereal time t: let be

$$A(t) = \cos\alpha\sin\delta\sin\delta_P\cos(\omega_{\oplus}T - \alpha_P) - \sin\alpha\sin\delta_P\sin(\omega_{\oplus}T - \alpha_P) \quad (2.47)$$

$$B(t) = \cos\alpha\sin\delta\sin\left(\omega_{\oplus}T - \alpha_{P}\right) + \sin\alpha\cos\left(\omega_{\oplus}T - \alpha_{P}\right)$$
(2.48)

With these definitions one gets

$$\cos(2\varphi) = [A^{2}(t) - B^{2}(t)] \cos(2\psi) - 2A(t)B(t)\sin(2\psi) \qquad (2.49)$$

$$\sin(2\varphi) = 2A(t)B(t)\cos(2\psi) + [A^{2}(t) - B^{2}(t)]\sin(2\psi) \qquad (2.50)$$

Now we can substitute these relations in equation (2.39). Finally, formula (2.39) takes the following form:

$$h_{b} = \left[\sin^{2}\vartheta\left(A^{2}-B^{2}\right)\cos\left(2\psi\right)-2AB\sin^{2}\vartheta\sin\left(2\psi\right)\right]h_{+}(t) \\ + \left[2\sin^{2}\vartheta AB\cos\left(2\psi\right)+\sin^{2}\vartheta\left(A^{2}-B^{2}\right)\sin\left(2\psi\right)\right]h_{\times}(t) \quad (2.51)$$

This final relation shows how amplitudes are modulated in time, which gives rise to an amplitude modulation of the signal seen at the bar.

The daily modulations for our 4 target pulsars are shown in figures 2.6 and 2.7. Table 2.5 shows, on the other hand, for each pulsar, the mean value of the antenna pattern. The complement to unity of the values shown in the table give an idea of the fraction of signal that we can consider as lost because of the antenna pattern. Finally, picture 2.8 shows the positions in the sky of the 4 pulsars of interest.



Figure 2.6: PSR J0024-7204J and PSR J0024-7204W, behaviour of the antenna pattern, average over all polarizations.



Figure 2.7: PSR J0218+4232 and PSR J1939+2134, behaviour of the antenna pattern, average over all polarizations.

| Ν. | Nome                                                 | Antenna Pattern |
|----|------------------------------------------------------|-----------------|
| 1  | PSR J0024-7204J                                      | 0.519           |
| 2  | $\mathrm{PSR}\ \mathrm{J0024}\text{-}7204\mathrm{W}$ | 0.519           |
| 3  | PSR J0218+4232                                       | 0.482           |
| 4  | PSR J1939+2134                                       | 0.455           |

Table 2.4: Mean values of the antenna pattern for the 4 objects in table 2.1



Figure 2.8: Positions in the sky of our target pulsars. The yellow stars are our 4 target pulsars, the red dots are isolated pulsars, the blue triangles are binary pulsars.

# Chapter 3

# Data analysis pipeline

### 3.1 AURIGA: transfer function of the detector

As we have seen, gravitational waves are perturbations of the space-time geometry (i.e. perturbations of the Riemann tensor) that propagate at the speed of light. Their interaction with free falling particles results in a variation in their relative distances, with a frequency which is equal to the wave frequency and an amplitude which is the product of the distance of the particles and the wave amplitude (when the distance is lower than the wavelength). So, in principle, the fundamental idea to set up a gravitational wave detector is very simple: we need a system of two masses, elastically coupled one to each other, and a device which measures the displacements of the masses with respect to their equilibrium positions. The AURIGA antenna is a resonant detector, meaning that the role of the two masses is played by a solid elastic body, in our case a cylinder. It's worth noticing that resonant detectors take advantage of the phenomenon of resonance, and so their sensitivity curves have peaked maxima in correspondence to their fundamental mechanical resonance. Around this frequency, the energy captured by the bar is maximum. To model this, let's remember that, if two bodies of equal mass m are coupled by a spring of elastic constant k and rest length l and they are in the presence of a viscous force  $-2\beta mv$ , where v is the relative velocity between the masses, a polarized gravitational wave with an amplitude h(t) that hits the bar orthogonally to the direction of the vector that joins the two masses generates a variation of their distance; so, in the system of the center of mass each body moves from its equilibrium position by a



Figure 3.1: Mechanical model of the two nested systems antenna + transducer.

quantity  $\xi(t)$  given by equation

$$\ddot{\xi} + 2\beta\dot{\xi} + \omega_0^2 \xi = \frac{l}{2}\ddot{h},\tag{3.1}$$

where  $\omega_0 = \sqrt{k/m}$  is the characteristic angular frequency of the system. The transfer function is defined as

$$T(\omega) \equiv \frac{X(\omega)}{F_g(\omega)},\tag{3.2}$$

where  $X(\omega)$  is the Fourier transform of  $\xi(t)$ ,  $F_g(\omega) = -m(l/2)\omega^2 H(\omega)$  and  $H(\omega)$  is the Fourier transform of h(t). Calculating the Fourier transform one gets

$$T(\omega) = -\frac{1}{m} \left[ \frac{1}{(\omega^2 - \omega_0^2) - 2i\beta\omega} \right].$$
(3.3)

It is straightford to verify that this curve presents a resonance peak at the angular frequency  $\omega_0$ . However, in reality, the system is not a simple two point masses system, but a continuous mass distribution. The result we found can be extended to the real case, by using the Hooke law generalized to continuous systems: it's possible to demonstrate that the bar faces oscillate obeying a transfer function that is equal in form to the (3.3): the bar of length L and mass M, in a close spectral region around the fundamental longitudinal frequency, is equivalent to the two point masses system, located at a distance  $l = 4L/\pi^2$  and with a characteristic angular frequency  $\omega_0 = \pi v_s/L$ , where

 $v_s$  is the propagation speed of sound in the material (usually an Aluminium alloy) which the bar is made of. It's also possible to demonstrate that the energy that the bar absorbs from the gravitational wave is equal to the one of the two point mass system, if relation m = M/2 is satisfied. We are interested in examinating how the solution of (3.1) behaves when the Q-factor is large, so we will consider only the case in which  $Q \gg 1$ ; then, for sake of simplicity, let's take an impulsive gravitational wave, i.e.  $H(\omega) = H_0$  where  $H_0$  is a constant: substituting in (3.3), ed antitransforming, we find that

$$\xi(t) \simeq \frac{2L}{\pi^2} e^{-\beta t} H_0 \,\omega_0 \sin(\omega_o t). \tag{3.4}$$

This is a damped oscillation at the resonance frequency of the bar, which damping time  $\tau \equiv 1/\beta = 2Q/\omega_0$  is proportional to the Q-factor. The bar is a cilynder having a mass of  $2.3 \cdot 10^3$  kg, 3m length and 60 cm radius, it is made of a metallic alloy of aluminium and magnesium called Al5056. This material, if it is cooled to cryogenic temperatures, guarantees a Q-factor of the order of  $Q \simeq 10^{6\div7}$ . This property is very important, because it has been demonstrated that the minimum of the power spectral density is determined by the thermal noise and satisfies  $S_h \propto T/(QM)$ , where T is the temperature and M the mass of the bar. Such a cylinder has lots of oscillation modes and resonances, but the one that interests is the fundamental longitudinal one, because this is the one with the maximum cross section to gravitational waves.

The bar is coupled to a resonant transducer, that is a second mass which is mechanically coupled to a face of the bar. This mass is devised to have its resonance frequency equal to the bar's one. In this way, the vibration of the transducer is amplified respect to the bar's one, and the amplification factor, as we will see soon, is equal to square root of the ratio between the masses. Then, the mechanical resonant transducer represents a face of a capacitor (capacitive transducer), which is charged at a very high voltage. In this way, mechanical oscillations are translated into capacity variations and then into an electrical signal. Finally, this signal is amplified by an amplification stage (SQUID, Superconducting QUantum Interference Device) [37]. The electric signal is transformed into a magnetic flux by means of an inductance. The magnetic signal is then read by the SQUID. Between the SQUID and the transducer, there is a superconductive transformer, which works as an impedance matching device. The use of the SQUID is possible because the experiment works at the temperature of the liquid helium, and its advantage is that it offers a very low noise and very high sensitivity, better than any other traditional device at frequencies around 1 kHz.

After the first scientific runs (1997-1999), AURIGA has been re-designed

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Figure 3.2: Electromagnetical scheme of the AURIGA detector

to improve the detector performances. Figure 3.2 shows a simplified scheme of the AURIGA detector with the resonant capacitive transducer read by a double stage SQUID amplifier. The bar and the resonant frequency of the transducer were optimized for the best AURIGA sensitivity and bandwidth. The system composed by the bar, the resonant transducer and the electric oscillator is a three-mode oscillator. It's so easy to understand that its transfer function will be written as

$$T(\omega) = (i\omega)^3 A \prod_{k=1,2,3} \frac{1}{(p_k - i\omega)(p_k^* - i\omega)} , \qquad (3.5)$$

where A is the calibration constant in force, and the  $p_k$  are the poles of the transfer function, defined as

$$p_k = -\frac{\omega_k}{2Q_k} + i\omega_k \ . \tag{3.6}$$

where  $k = 1, 2, 3, \omega_k$  is the angular frequency of the k-th mode and  $Q_k$  is its quality factor. Using a capacitive calibrator, which is a device that directly acts on the bar in order to calculate experimentally the calibration constants, it is possible to "hit" the bar with a force whose spectrum is

$$F_{bar}(\omega) = E_{bias} C_{cal} \frac{\omega^2 V(\omega)}{(\omega_{cal}^2 - \omega^2)} , \qquad (3.7)$$

where  $E_{bias}$  is the electric field applied at the calibrator,  $C_{cal}$  its capacity and  $V(\omega)$  is the electric tension applied to the calibrator and  $\omega_{cal} \gg \omega$ . Measuring the current that is inducted by the force 3.7 at the SQUID input, one can verify if the theoretical formula 3.5 is correct or not. Unfortunately, the experimental results show that formula 3.5 doesn't really accurately fit

| pole | $\nu_k[\text{Hz}]$ | $Q_k$                |
|------|--------------------|----------------------|
| sp1  | 828.909            | $1.40 \times 10^{5}$ |
| sp2  | 843.552            | $1.28 \times 10^5$   |
| 1    | 865.715            | 7016                 |
| 2    | 914.421            | 551                  |
| 3    | 955.963            | 373                  |
| sp3  | 1056.32            | $1.4 \times 10^4$    |
| sp4  | 1062.43            | $2.6 \times 10^4$    |
| sp5  | 1067.73            | $3.0 	imes 10^4$     |

Table 3.1: Poles of the real transfer function and their Q-factors.

the experimental points. In fact, the real model seems to hold more than three poles, and data can be well fitted in the frequency range 800-1050 Hz with an experimental transfer function with 8 poles  $p_k$  and 4 zeroes  $z_k$ :

$$T_{exp}(\omega) = (i\omega)^4 A \frac{\prod_{k=1}^4 (z_k - i\omega)(z_k^* - i\omega)}{\prod_{k=1}^8 (p_k - i\omega)(p_k^* - i\omega)} .$$
(3.8)

The constructive parameters of the various parts of the detector, the frequencies and the Q-values are [38] listed in table 3.1. At this point, to find the power spectral density of the noise  $S_{hh}(\omega)$  we need to know the value of the constant of calibration in force A that appears in equation 3.5 which has to be experimentally measured, since the model is not fully determined a priori. We know that additional mechanical resonant modes of the transducer system are playing a role in the empirical transfer function given by eq 3.8. However, we do not have a detailed model to explain it and therefore the calibration constant A has to be measured empirically with the following procedure. If the bar is excited by an impulsive force, i. e. that can be written as  $F(t) = F_0 \delta(t)$ , the energy absorbed by the system is

$$E_{abs} = \frac{F_0^2}{2M_{eff}} , (3.9)$$

where  $M_{eff}$  is the "effective mass" of the bar, that we will assume to be  $M_{eff} = M_{bar}/2$ . The calibration constant A can be experimentally measured measuring the  $E_abs$  related to impulsive excitations of the bar produced by the calibrator.

# 3.2 Homodyne detection method for periodic signals

As first, we focus on the properties of a lock-in at a fixed frequency, which is what people commonly refer to as a lock-in; as we have already shown, our procedure of multiplying the signal by its own true phase is in practice a modified lock-in with not-constant reference frequency, and the peculiarity of our analysis will be discussed afterwards, but the results are still the same. A lock-in at a fixed frequency  $\omega_0$  is a device [39], at which input one puts the detector's output s(t) (namely, containing both noise and signal), and whose two outputs are expressed by the following relations:

$$x(t) = \frac{1}{\tau} \int_{-\infty}^{t} s(t') e^{-\frac{(t-t')}{\tau}} \operatorname{sign}(\cos \omega_0 t') dt' \text{ and}$$
(3.10)

$$y(t) = \frac{1}{\tau} \int_{-\infty}^{t} s(t') e^{-\frac{(t-t')}{\tau}} \operatorname{sign}(\sin \omega_0 t') dt'.$$
 (3.11)

In practice, they are the two parts (real and immaginary) of the Fourier component of angular frequency  $\omega_0$ , integrated over the time. The outputs carry the function sign because, in origin, the first lockin were hardware implemented by means of an electric circuit, and conceptually it's simpler to set up a device which perform a square wave instead of a trigonometric function. The parameter  $\tau$  depends on how the lockin is implemented and represents its time constant. Let be  $v(t) = s(t) \text{sign}[\cos(\omega_0 t))]$ . The factors  $\text{sign}(\cos \omega_0 t')$ and  $\text{sign}(\sin \omega_0 t')$ , which represent square waves, can be expressed with their Fourier expansion. One has

$$v(t) = \frac{4}{\pi} s(t) \left[ \cos(\omega_0 t + \frac{1}{3} \cos(3\omega_0 t) + ...) \right] .$$
(3.12)

To understand the advantages of the lockin, we need to calculate the power spectral density of its output, defined as the Fourier transform of its self-correlation. We write the self-correlation of the function v(t), defined as  $R(\tau) = \langle v(t)v(t+\tau) \rangle_t$  where  $\langle \rangle_t$  is a mean value over the time  $t \gg 1/\omega_0$ . It's important to remember that, from its definition, the self-correlation function is in practice something that identifies, in a noisy string, regular structures as the ones of a signal which repeats or that have, in general, a regular behaviour of some kind. It is

$$R(\tau) = \frac{16}{\pi^2} \langle s(t+\tau) \cos(\omega_0(t+\tau)) s(t) \cos(\omega_0 t) \rangle_t + \frac{16}{\pi^2} \cdot \frac{1}{9} \langle s(t+\tau) \cos(3\omega_0(t+\tau)) s(t) \cos(3\omega_0 t) \rangle_t + \dots$$
(3.13)

The power spectral density  $S_v$  of v(t) is the Fourier transform of the selfcorrelation R. Doing the calculations, we have that  $S_v$  is related to the power spectral density of the stocastic process s(t) by the following relation:

$$S_{v}(\omega) = \frac{4}{\pi^{2}} [S(\omega - \omega_{0}) + S(\omega + \omega_{0}) + \frac{1}{9} [S(\omega - 3\omega_{0}) + S(\omega + 3\omega_{0})] + \dots$$
(3.14)

This is an always positive, oscillating function and if a signal at  $\omega_0$  is present it reaches an abslute maximum at  $\omega = 0$  and has local maxima at each frequency  $2n\omega_0$ , where n scans on all relative integers. The function tends to 0 for large values of  $\omega$ . Following definition 3.10, we finally have to calculate the integral over the time in 3.10. The result is that the power spectral density at the lock-in output is

$$S_x(\omega) = \frac{1}{1 + \omega^2 / \tau^2} S_v(\omega) .$$
 (3.15)

An analogous relation can be written for  $S_y$ . In  $S_x$ , the only peak of  $S_v$  that survives is the one at  $\omega = 0$ . For this reason, we can say that the lock-in brings the frequencies from  $\omega_0$  to 0. So, the Fourier component of the signal at the fixed frequency  $\omega_0$  down-converts the signal frequency into the continuous component of the spectrum.

The same method can be used if the phase doesn't grow linearly with time, but can be conveniently written as  $\phi_0 + \omega_0 t + \hat{\phi}(t)$ , where  $\phi_0$  is the unknown initial phase,  $\omega_0$  is the intrinsic frequency and  $\hat{\phi}(t)$  represents the phase shift due to the Doppler effects. The signals we want to extract from the detector's data are in the form

$$s(t) = M(t)\cos[\phi_0 + \phi(t)], \qquad (3.16)$$

where we called  $\phi(t)$  the known part of the phase,  $\phi(t) = \omega_0 t + \hat{\phi}(t)$ . The timescale in which  $\phi(t)$  varies must be very larger than the period  $1/\omega_0$ , and M(t) is an amplitude modulation, in our case the one given by the antenna pattern factor for the source. Also the evolution of M(t) must satisfy  $\frac{dM}{dt} \ll \omega_0$  in order to make formula 3.16 make sense, but this is definetely our case, because the antenna pattern factor varies with a period of 1 sidereal day. In what follows, we simply call M(t) the amplitude modulation, and  $\frac{d\phi}{dt}$  the frequency modulation of the signal. Obviously, s(t) can be also expressed as

$$s(t) = A(t)\cos\phi(t) - B(t)\sin\phi(t) , \qquad (3.17)$$

where we have introduced the two time-dependent functions

$$A(t) = M(t)\cos\phi_0\tag{3.18}$$

and

$$B(t) = M(t)\sin\phi_0. \qquad (3.19)$$

Now the question is substantially to set up a lockin at the variable frequency  $\frac{d\phi}{dt}$ . Written in this way, A(t) and B(t) are the two components in quadrature with each other of the signal. In our real case, the signal can be present in both components. In fact, we don't know what could be the initial phase of the signal, because the gravitational wave is not in general in phase with the radioastronomical modulation. Taking the Fourier transform of the signal expressed in this way, we have that the spectrum of s(t) is the convolution of the spectrum of A(t) with the one of the function  $\cos(\omega_0 t)$ , minus the convolution of the spectrum of B(t) with the one of the function  $\sin \phi(t)$ : in the frequency domain, if

$$d\phi(t)/dt \ll \omega_0 , \qquad (3.20)$$

it is

$$s(\omega) \simeq \frac{1}{2} [A(\omega - \omega_0) + A(\omega + \omega_0)] - \frac{1}{2i} [B(\omega - \omega_0) - B(\omega + \omega_0)] . \quad (3.21)$$

Introducing the so-called complex signal as

$$F(t) = M(t)e^{i\phi(t)} , \qquad (3.22)$$

so that

$$s(t) = F(t)e^{i\phi(t)} \tag{3.23}$$

we have that

$$s(\omega) \simeq \frac{1}{2} [F(\omega - \omega_0) + F^*(\omega + \omega_0)] . \qquad (3.24)$$

Now, let's describe the simple software operations we have to do in order to extract the signal from the detector's output. What we do is to give the signal s(t) as input to a program which in practice does what was once done by an hardware device called mixer. In practice, the program doubles the time series holding the signal into two channels: the first one holds the signal multiplied by  $\cos \phi(t)$ , the second holds the signal multiplied by  $\sin \phi(t)$ . Let's show what happens for the first channel, meaning that, obviously, the result is analogous for the second one. The first channel  $s_1$  will so hold the following time series:

$$s_{1}(t) = A(t)\cos^{2}\phi(t) - B(t)\sin\phi(t)\cos\phi(t)$$
  
=  $\frac{1}{2}A(t)[1 + \cos 2\phi(t)] - \frac{1}{2}B(t)\sin 2\phi(t)$  (3.25)

and its Fourier transform is

$$s_1(\omega) \simeq \frac{1}{2}A(\omega) + \frac{1}{4}[A(\omega - 2\omega_0) + A(\omega + 2\omega_0)] - \frac{1}{4}[B(\omega - 2\omega_0) - B(\omega + 2\omega_0)]$$
(3.26)

Equation 3.26 shows that, now, it's possible to extract the signal: in fact, remembering that A and B are functions that slowly vary with time, the only components of their Fourier transform  $A(\omega)$  and  $B(\omega)$  are the one characterized by the condition  $\omega \ll \omega_0$ . For these frequencies,  $|\omega \pm 2\omega_0| \gg \omega$ . So, if we apply a numerical low-pass filter close enough, in the sense that we will discuss in the following paragraph, which means that the filter must have a time constant

$$\tau \gg \frac{1}{2\omega_0} , \qquad (3.27)$$

the product of the 3.26 and the transfer function of the lowpass, done in the frequency domain, is, for the case of a single pole low-pass filter

$$s_1^f(\omega) = \frac{A(\omega)}{2(1+i\omega\tau)} , \qquad (3.28)$$

which in practice, if the filter is chosen to have a rapid decay (i. e. the filter is almost a square box with a certain amplitude around the 0 bin), leads to

$$s_1^f(0) \simeq \frac{A(0)}{2}$$
 (3.29)

As we have already said, an analogous result is true for the second channel  $s_2$ : the same calculations would lead to

$$s_2^f(0) \simeq \frac{B(0)}{2}$$
 (3.30)

Finally, taking the estimation of the amplitude of the signal simply is

$$H = \sqrt{s_1^f(0)^2 + s_2^f(0)^2} . \tag{3.31}$$

The outline of the method we have now discussed is represented in figure 3.3. Our complete data analysis pipeline is the one shown in figure 3.4.



Figure 3.3: Analysis method outline



Figure 3.4: Complete pipeline of the data analysis

# **3.3** Band-pass filtering: characterization of filters

Now, let's show in detail how the band-pass filtering stage is implemented in our analysis. The most suitable band-pass filters for our case are the Butterworth filters, which are well studied in literature [40]. The filter which is used as a progenitor of all categories of filters is the low-pass filter, so we describe now how the low-pass is thought and implemented, and then we will show how, starting from a low-pass, it's possible to set up a band-pass. The low-pass Butterworth filters are are characterized by a transfer function that doesn't have zeroes but only poles  $H(\Omega) = 1/[1+i(\Omega/\Omega_c)^N]$ , and the square of their magnitude response in the frequency domain is in general given by the following relation:

$$|H(\Omega)|^{2} = \frac{1}{1 + (\frac{\Omega}{\Omega_{c}})^{2N}} = \frac{1}{1 + \epsilon^{2}(\frac{\Omega}{\Omega_{p}})^{2N}}, \qquad (3.32)$$

where N is the order of the filter,  $\Omega_c$  is its cutoff frequency, namely the frequency at which the response decreases at some fraction of the maximum peak,  $\Omega_p$  is the passband edge frequency, and  $1/(1 + \epsilon^2)$  is the band-edge value of  $|H(\Omega)|$ . In order to choose the poles of the transfer function 3.32, we use the fact that, if  $s = i\Omega$ , it is  $H(s)H(-s) = H(\Omega)^2$  and so

$$H(s)H(-s) = \frac{1}{1 + (\frac{-s^2}{\Omega_c^2})^N} , \qquad (3.33)$$

and so one immediately has that the poles are located at

$$s_k = \Omega_c e^{i\pi/2} e^{i(2k+1)\pi/2N}$$
, k=0,1,..., N-1. (3.34)

The frequency response characteristics of the class of Butterworth filters are monotonic, and the rapidity at which the cutoff occurs is obviously proportional to the order of the filter. Here we focus on the following problem: we want to find the order N which is needed, given a precise requirement about how much attenuation  $\delta$  we want to have at a fixed frequency  $\Omega^*$ . We can do this by simply using the relation 3.32. In fact, our requirement is to impose that

$$\frac{1}{1 + \epsilon^2 (\Omega^* / \Omega_p)^{2N}} = \delta^2 , \qquad (3.35)$$

and so one has that the requested order is given by

$$N = \frac{\log\left(1/\delta^2\right) - 1}{2\log\Omega^*/\Omega_c} = \frac{\log\delta/\epsilon}{\log\Omega^*/\Omega_p} , \qquad (3.36)$$

which solves the problem, given the fact that the filter is completely defined if one knows its parameters N,  $\Omega_p$  and  $\epsilon$ . The low-pass filter is the prototype we used to set up the band-pass filter, by means of the frequency transformation

$$\Omega \to \Omega_p \frac{\Omega^2 + \Omega_l \Omega_u}{\Omega(\Omega_u - \Omega_l)} , \qquad (3.37)$$

where  $\Omega_u$  and  $\Omega_l$  are the upper and lower cutoff frequencies of the band-pass filter respectively.

The above Butterworth filters are defined in the continuous time domain; however, in our case, the analysis is performed in the discrete time domain and these filters can be implemented in this domain by means of a recursive technique (ARMA, Auto Regressing Moving Average) [41]. Here we show an example of the characteristic quantities for a Butterworth filter, as the ones we want to use. The plots refer to the band-pass filter used for the band which the search for PSR J0218+4232 refers to. The order of the filter is 2, the sampling rate is 406 Hz (the one of the AURIGA h-reconstructed data), the bandwidth is about 3 Hz. We use this example to show what are the important features such a filter is characterized by. The order of the filter must be chosen keeping in mind that, the higher the order of the filters, the more unstable the filter is, this is due to the approximations of the ARMA algorythm. In this example, the filter with N = 4 was not stable, in the sense that its impulsive response doesn't tend to 0 for  $t \to \infty$ , and so we decreased the order to N = 2. Another important requirement is that the phase of the transfer function of the filter has to be linear in the sensitivity region, as can be seen in figure 3.5. It's not important if the phase covers more than one period through the sensitivity region, what is important is its linearity. Generally, this linearity is better reached when the order of the filter is low. Then we show the impulsive response of our filter in figure 3.6. The impulsive response is important because it shows if the filter's transfer function is stable or not. This impulsive response must tend to 0 as the time increases. This fact is guaranteed as a sub-result if the step response (see picture 3.7) has the same behaviour, in fact a converging step response is a more restrictive condition than a converging impulsive response. All these behaviours are shown in the pictures.

#### **3.4** Coherent analysis and spectra averaging

Considering that the stability of the detector transfer function is not guaranteed over very long timescales, we divided the observation time (10 days) in



Figure 3.5: Amplitude (red line) and phase (blue line) of one of our Butterworth bandpass filters. The plot refers to the band relative to PSR J0218+4232. The x axis is in terms of the fraction of the sample frequency.



Figure 3.6: Impulse transfer of our example Butterworth bandpass filter.



Figure 3.7: Step response of our example Butterworth bandpass filter.

sub-periods. The duration of each sub-period is 1 day. So, once we have demodulated for all frequency effects, for each sub-period we perform a Fourier Transform of the data. Each Fourier transform has the property that all the signal power is now completely helded by the first bin (which we will refer to as the "0 bin") if each Fourier-transformed vector.

So we finally deal with M = 10 independent spectra. The first bin of each spectra holds the quantity  $h^2T_c$ , in addition to the noise (which has the same properties of the noise of the contiguous bins and so fluctuate with the same variance), where  $T_c$  is the coherence time,  $T_c = 1$  day.

As the last step of the analysis, the question is how to combine, for each source, the informations held by in each spectrum. The natural way to do this is to simply averaging the spectra sample by sample. To do this, we can observe that, although the signal held by the first bin is expected to be constant through the spectra, the noise variance in the bins holding only the noise goes down proportionnally to  $M^{1/2}$ , where M is the number of averaged spectra, M = 10. This lowers the final variance of the noise, as we will see in more detail in section 3.5

# 3.5 Estimation of noise, and choice of the false alarm probability

To estimate the noise level for a single FFT, it's worth noticing that the 0 bin is the only one which holds the signal power, if present. So, we must esclude it from the description of the noise statistics. The other bins are in principle all available to be used to describe how the noise is distributed. However, the detector spectrum is not flat. Even if we are looking to a band whose bandwidth is narrow respect to the total spectrum extension, the band (about 3 Hz) is not so narrow to make possible to neglect the spectral structures inside it. The noise has thus not the same properties through all the bins of the band, and only the bins which are closer to the 0 bin can be used to infer how the noise is distributed. This is not a problem, because the number of samples we deal with is still large enough to make possible to model the distribution: even if we take 1% of the total bins, we are still dealing with about 10000 counts. Moreover, at high frequencies inside the band, it is also present the copy of the signal at twice its original frequency minus the band begin corner (this is an artifact due to the fact that we have multiplied the signal by a function which is oscillating with the same law of the signal itself, and so this is simply a consequence of the prostapheresis formulas). This is another motivation to exclude from the further steps of the analysis the bins at higher frequencies.

The distribution we expect from the theory follows an exponential law, whose variance is the estimation of the noise level.

Then, averaging the spectra, the variance of the noise computed on the final averaged spectrum is  $M^{1/2}$  times smaller than the one of a single spectrum. In this way, averaging over M spectra, the threshold on the statistics for the requested false alarm probability will reduce of a factor  $M^{1/2}$ , and thus the upper limit on the signal amplitude h will reduce of a factor  $M^{1/4}$ . If we were in the detection region, for the same considerations, the signal-to-noise ratio would increase by a factor  $M^{1/2}$ . Also the distribution of the noisy bins changes onto a centered  $\chi^2$  with M degrees of freedom. For what concerns the 0 bin, the distribution we theoretically expect for it is a non-centered  $\chi^2$ , with non-centrality parameter  $\theta$  proportional to the signal amplitude, as shown in figure 3.8.

#### **3.6** Setting confidence levels

Now we focus on the final spectrum, namely the average of the 10 1-day FFTs. A lot of the bins of these objects can be used in order to test the



Figure 3.8: statistics of bin 0 counts when the signal is not present (solid curve,  $\chi^2$  PDF with 10 degrees of freedom without non-centrality parameter) or it is present (dashed curve,  $\chi^2$  PDF with 10 degrees of freedom and non-centrality parameter  $\theta = 2$ .)

null hypothesis "h0" that the signal is not present in the data. Because of the operation of spectra averaging, the distribution we can assume (once we have verified this fitting the data, as we will see in chapter 4.2) for the 0 bin is, as discussed in section 3.5, a non-centered  $\chi^2$ , with some non-centrality parameter  $\theta$  holding the information about the signal. So,  $\theta$  is the parameter we want to estimate.

The first part of the hypothesis test consists of arbitrarily setting a false alarm probability, in our case, to be confident, we choose a false alarm probability of  $10^{-2}$ . This says that once in 100 times we say that the signal is present when it is not. For the whole search, meaning for the 3 targeted pulsars togheter, this brings to a 3 per-cent total false alarm rate. The histogram directly gives, in terms of  $h_0^2 T$ , where T is the integration time of each coherent sub-search, a false alarm threshold that we will use in the forthcoming step. Considering that  $\sigma$  assumes a different value for each pulsar, because each pulsar belongs to a different band in the spectrum and each band has its own specific variance, we end up with a different value of the threshold for each pulsar. Let's call  $x_{FA}$  this calculated threshold on x. Finally, we have to define a procedure in order to set a confidence interval, either uper limit or two-sided, on the measured on the gravitational wave amplitude  $h_{min}$ . We know from section 3.5 that, if the signal is not present, the 0 bin statistics is a centered chi-square with M degrees of freedom; as the signal amplitude grows, the statistics change onto the non-centered chi-square and, when the signal-to-noise ratio is large, tends to be gaussian. The way we choose, to measure h and to exactly know the statistical meaning of our conclusions, is to build, in the plane  $(x, \theta)$ , where x is a generic result of the experiment and  $\theta$  is the parameter that we want to estimate, the so-called "confidence belt". There are several ways to construct a confidence belt. Here, we decided to follow, and to implement, the recipe given by Feldman and Cousins in 1998 [42]. We require our confidence belt to have the property to guarantee a selected coverage over all the parameters region. The coverage of a confidence interval is defined as follows: it is the probability interpreted as the mean value of the fraction of times that, if one repeats the experiment under the same conditions, the two limits of the interval cover the true value of the parameter to estimate. This selected coverage we want to obtain is, for us, C = 0.9. In reality, our method is a little different from Feldman and Cousins' one [42], because we also choose to fix a small false alarm probability of 0.01. This choice, in fact, causes the coverage to be more than the goal-coverage in the upper-limit region, namely for  $x < x_{FA}$ . This over-coverage that is present in a region of the parameter space is the price we need to pay in order to have a small false-alarm probability. The construction of our confidence belt proceeds as follows. For each fixed  $\bar{x}$ , let's define  $\theta_{best}$  the value of  $\theta$ that maximizes the likelihood  $f(x,\theta)$ , requiring the physical constrain that  $\theta_{best} \geq 0$ . In particular, if the measured x is less that its average value  $\bar{x}$ , we impose  $\theta_{best} = 0$ , because if the result of the measurement is less than the mean value, the best estimator of the signal is 0. Now, for each possible value  $\theta$ , we calculate the likelihood ratio given by

$$R(\bar{x},\bar{\theta}) = \frac{f(\bar{x},\theta)}{f(\bar{x},\theta_{best})}.$$
(3.38)

This ratio of likelihoods is the function of  $\bar{x}$  that we use to choose the confidence intervals. In fact, for each choice of  $\bar{\theta}$ , the confidence interval  $(x_1, x_2)$ is uniquely defined by the requirements 3.39 end 3.40:

$$\int_{x_1}^{x_2} f(x,\bar{\theta}) = C .$$
(3.39)

$$R(x_1) = R(x_2) . (3.40)$$

If the condition 3.39 cannot be satisfied within the condition 3.40, we consider as good the confidence interval also if  $R(x_2) < R(x_1)$ .

The choice of this ordering principle for the choice of the confidence intervals, will result in a more regular behaviour of the confidence belt in the regions of the parameters space where x is very low. The couples of values  $(x_1, x_2)$  are taken starting from the value  $x_{max}$  which maximizes  $R(x, \bar{\theta})$  for a given  $\bar{\theta}$ . Taking all the different values of  $\theta$ , we cover all the parameter space and so we can trace the confidence belt. We have implemented all this procedure using MATHEMATICA. We start selecting a grid of values in the parameters' space  $(x, \theta)$  and to calculate the value of the likelihood ratio R over all the points of the grid. It's important to notice that, unfortunately, the problem of finding  $\theta_{best}$  can be solved only numerically, because in our case the probability density function f is very complicated: it is in fact expressed in terms of the regularized hypergeometric 0F1 functions, and the problem of finding an always viable relationship  $\theta_{best} = \theta_{best}(f, x)$  is not analytically solvable. Then, the program takes a value  $\theta$  and, for the section of the parameter grid at  $\theta = \theta$ , searches for  $x_{max}$ . Then, we move from  $x_{max}$ to larger values of x, and the program shows all the couples of values  $(x_1, x_2)$ that best satisfy equation 3.40. For each couple, the program integrates the probability density function to find the coverage. Finally, between all the coverage values, the program extracts the couple for which the coverage is the most similar to C and not less that it, thus computing the confidence belt at  $\theta$ . See section 4.3 for the result.

The confidence belt must be read in this way: given a result  $\bar{x}$  of the experiment, we trace a vertical line, which intercepts the edges of the confidence belt, thus giving the extreme values  $\theta_1$  and  $\theta_2$  of the confidence interval.

### 3.7 Tests on the doppler demodulation procedure

In order to verify that our doppler demodulation procedure is correct, we have performed a data exchange with radioastronomers. In fact, we have taken two ideal sources that we called respectively "test1" and "test 2", the first one lying on the ecliptical line, the second one quite far from that line. We simulated a gravitational wave signal from these sources, manually fixing the positions in the sky and the orbital parameters of our fake sources, as it is seen from the SSB reference frame, and then we provided these simulated waves to radioastronomers, in order to see if they could fit the waveform with their procedure, which is independent from ours. Radioastronomers required the fake sources to be two, one in the ecliptic and the second one far from it, in order to check if all the motions of the detectors and the source in

| paramater      | source "test1" | source "test2" |
|----------------|----------------|----------------|
| RA             | 0              | 0              |
| DEC            | 0              | $45^{\circ}$   |
| frequency [Hz] | 900            | 900            |
| F1             | 0              | 0              |
| PEPOCH [MJD]   | 54100          | 54100          |
| A1             | 0.1            | 0.1            |
| ECC            | 0.01           | 0.01           |
| PB             | 1.0            | 1.0            |

Table 3.2: Parameters of the simulated sources

the SSB reference are exactly taken into account. Given the fact that the frequency of the simulated gravitational wave signal is about 1 KHz, and that the sampling time of these simulated data is required by radioastronomers to be at least 1/100 of the period time or smaller, the files we produced for this check are very large. We chose to simulate the signal for 10 minutes. We anticipate that the duration we will chose for the coherent searches will be 1 day, i.e. about  $10^2$  times the one of the simulations. With these settings, the precision required in the test is sufficient to be sure that the errors are within the required demodulation precision for a time of 1-day. This results in a 60 Mbyte ASCII file for each test source. The waveform of the signal is not important at all, because all the information for the check is held by the phase. So, we chose a square waveform, namely the value of the simulated signal is 1 when the phase is contained in the interval between 0 and  $\pi$ , and 0 othwerwise. The parameters of our simulated sources are the ones in table 3.2.

### 3.8 Behaviour of AURIGA in the observation period

We chose a period called run 852, containing AURIGA data during the period between December 4, 2006 and December 22, 2006, namely 18 days. In this period, we chose 10 contiguous days between December 8 and December 17. This period has been chosen because the behaviour of the detector during these days was satisfactory, namely the detector noise was low and stable. The noise spectral density of the detector during the run 852 is shown in figure 3.9, along with the position in the spectrum of the expected signal from each source. The plot represents the quantity  $\sqrt{\frac{|Fourier(A(t))|^2}{|T(\omega)|^2}}$ , where A(t) is the time series of the output data of the detector. The plotted quantity is the one that can directly compared with the dimensionless amplitude of a gravitational wave applied to the bar input. In fact, for example, for a signal with amplitude  $h_0$ , constant over a bandwidth  $\Delta \nu$ , the signal-to-noise ratio is

$$SNR = h_0 S_h^{-1/2} \Delta \nu^{-1} . aga{3.41}$$

Unfortunately, in the case of PSR J0024-7204W, the frequency of the expected gravitational signal is in a region of the spectrum where a spurious line due to an electromagnetic interfernce is present. This line is an harmonic of the 50 Hz frequency of the power line. If the only problem was a higher value of the noise here, the analysis could be done despite this. However, it was not possible to exactly model this line, because its frequency jitters as time goes by, and the amplitude is also non stationary. So, during the production of the h-reconstructed data, all the band holding this line has been vetoed, making impossible to perform the further steps of the analysis for PSR J0024-7204W. In figure 4.1 we show a randomly chosen set of samples od the antenna decimated data. This gives an idea of the dispersion of the plots and the order of magnitude we have to deal with.

#### **3.9** Evaluation of frequency shifts

For our 10 days of interest, we have calculated the frequency correction due to all motions of the detector and the source in the SSB reference frame. We show the behaviours for the source PSR J0218+4232 in some plots. Figure 3.10 shows the velocity of the detector respect to the center of mass of the binary; in figure 3.11 we show the velocity of the pulsar with respect to the center of mass of the binary system; and finally in figure 3.14 shows how this relative motion brings to the variations to the intrinseic signal frequency.

In order to perform the real analysis, we choose to divide the observational time into short sub-periods. This is primarly due to the fact that, as we have seen before, the instability of the transfer function of the detector brings us to the fact that a 10-days coherent search is problematic, and also to the fact that, if we have some independent coherent sub-searches, we can better address the problem of how is the statistics of the frequency bins we have to finally analize to find a robust upper limit for the gravitational wave amplitude. In table 3.3 we show the time of beginning of all the sub-searches we have so far performed, both showing the data and the corresponding value of the Co-ordinate Universal Time, which is the one used in the codes.



Figure 3.9: Detector noise spectral density during the run 852. The plotted quantity is  $S_h 1/2$ .

| sub-search number | time begin          | UTC       |
|-------------------|---------------------|-----------|
| 1                 | dec 8, 2006, h 0    | 849571214 |
| 2                 | dec 9, 2006, h $0$  | 849657614 |
| 3                 | dec 10, 2006, h 0   | 849744014 |
| 4                 | dec 11, 2006, h 0   | 849830414 |
| 5                 | dec 12, 2006, h 0   | 849916814 |
| 6                 | dec 13, 2006, h 0   | 850003214 |
| 7                 | dec 14, 2006, h 0   | 850089614 |
| 8                 | dec 15, 2006, h $0$ | 850176014 |
| 9                 | dec 16, 2006, h 0   | 850262414 |
| 10                | dec 17, 2006, h $0$ | 850348814 |

Table 3.3: Time of beginning of the coherent sub-searches.



Figure 3.10: detector's velocity respect to the binary's center of mass, PSR J0218+4232



Figure 3.11: pulsar's velocity respect to the binary's center of mass, PSR J0218+4232



Figure 3.12: the frequency of the signal as observed in the SSB, PSR J0218+4232



Figure 3.13: the frequency of the signal as observed in the SSB, PSR J0024-7204J  $\,$ 



Figure 3.14: the frequency of the signal as observed in the SSB, PSR J0024-7204W

## Chapter 4

## Analysis results

### 4.1 From the h-reconstructed data to the demodulate vectors

Here, we show examples of all the features of the real analysis through AU-RIGA run 852 data. Examples of the behaviour of the demodulated vectors, extracted from the real analysis for PSR J0218+4232, are shown in figure 4.1. This is just to give an idea of the number of the orders of magnitude we deal with when we use the h-reconstructed data. In figure 4.2 there is the effect of the sine modulation on the vector in 4.1.

# 4.2 Noise distribution and result for the noise level

Once we have done all the corrections due to the Doppler shiftes, which is done by several routines we have written in C, we choosen to process the demodulated vectors using MATLAB.

In plot 4.3 we show how the typical fft of our coherent sub-searches appear. As we saw in section 3.5, the spectrum is not flat through the band and only the first bins, where the spectrum can be considered locally flat within a good approximation, can be used for the forthcoming steps.

Figure 4.4 shows how the counts of a single FFT are distributed. The stetistics is well fitted by an exponential curve 4.5.

In figure 4.6, we whow how the noisy bins of the averaged spectrum is distributed. The histogram is well fitted by a  $\chi^2$  with M degrees of freedom.



Figure 4.1: a typical noise string as seen by the detector



Figure 4.2: the effect of the sine modulation on the previous string


Figure 4.3: one of the short ffts



Figure 4.4: an histogram of fft bin counts



Figure 4.5: fit of the istogram of the noisy bins of the first 1-day fft, PSR J0218+4232. The fitting function is  $f(x) = 42970.7 \cdot e^{-0.700449x}$ , and the normalized  $\chi^2$  of the fit is  $\chi^2 = 1.077$ 



Figure 4.6: histogram of the noisy bins of the final averaged spectrum, for PSR J0218+4232.

| PSR          | $x_0[Hz^{-2}]$        | $std[Hz^{-2}]$        | $x_0/std[]$ | $	heta^{u.l.}/std[]$ | $\theta^{u.l.}[Hz^{-2}]$ |
|--------------|-----------------------|-----------------------|-------------|----------------------|--------------------------|
| J0024-7204J  | $5.43 \cdot 10^{-36}$ | $3.12 \cdot 10^{-36}$ | 1.74        | 8.8                  | $2.74 \cdot 10^{-35}$    |
| J0128 + 4232 | $1.34 \cdot 10^{-36}$ | $1.00 \cdot 10^{-36}$ | 1.33        | 8.0                  | $2.50 \cdot 10^{-35}$    |
| J1939 + 2134 | $1.58 \cdot 10^{-36}$ | $9.62 \cdot 10^{-37}$ | 1.64        | 8.6                  | $2.68 \cdot 10^{-35}$    |

Table 4.1: Values of the statistical quantities for 3 target pulsars.  $x_0$  is the value of the 0 bin of the averaged spectrum; *std* is the standard deviation of the noisy bins of the same spectrum;  $\theta^{u.l.}$  is the upper limit on the parameter  $\theta$  to estimate.

| PSR         | upper limit $(h^{u.l.})$ |
|-------------|--------------------------|
| J0024-7204J | $6.00 \cdot 10^{-23}$    |
| J0218+4232  | $3.27 \cdot 10^{-23}$    |
| J1939+2134  | $3.33 \cdot 10^{-23}$    |

Table 4.2: Measured upper limits for 3 targeted pulsars. Here the goal coverage is fixed to C = 0.9.

#### 4.3 Upper limits and confidence intervals

The computed confidence belt for C = 0.9 is shown in picture 4.7

The measured quantities needed to reach the estimation of  $\theta$  are summarised in table 4.1.

We finally give the results for the upper limit of the gravitational wave emission for our 3 target pulsars, see table 4.2.

We plotted several confidence belts for different values of the coverage one may want to choose. In figure 4.8 we plot three confidence belts, referring to three different coverage values. For each coverage, we calculate the upper limit for the parameter  $\theta$  to estimate, the results are shown in table 4.3

Finally, in table 4.4, we show what are the corresponding values we find for our final upper limits on h.

| PSR          | $	heta_{68\%}^{u.l.}/std[]$ | $	heta_{90\%}^{u.l.}/std[]$ | $	heta_{95\%}^{u.l.}/std[]$ |
|--------------|-----------------------------|-----------------------------|-----------------------------|
| J0024-7204J  | 5.4                         | 8.8                         | 10.8                        |
| J0218 + 4232 | 4.3                         | 8.0                         | 9.7                         |
| J1939 + 2134 | 5.1                         | 8.6                         | 10.5                        |

Table 4.3: Values of  $\theta^{u.l.}$  for different coverages.



Figure 4.7: setting confidence intervals: the confidence belt. The red dashed line indicates the most likely value of  $\theta$  for each x. The parameter  $\theta$  we want to estimate bring the information about h, in fact  $\theta = h_0^2 \cdot T^2$ , where T is the coherent integration time. The plotted quantities are then re-scaled by means of the standard deviation of the noise *std*, in order to have unit variance, so we finally deal with dimensionless quantities. The dimensionless confidence belt so described is good for each analysis; it is sufficient to substitute for each sub-band its proper noise level.

| PSR          | $h^{u.l.}_{68\%}$     | $h^{u.l.}_{90\%}$     | $h^{u.l.}_{95\%}$     |
|--------------|-----------------------|-----------------------|-----------------------|
| J0024-7204J  | $4.75 \cdot 10^{-23}$ | $6.00 \cdot 10^{-23}$ | $6.71 \cdot 10^{-23}$ |
| J0218+4232   | $2.40 \cdot 10^{-23}$ | $3.27 \cdot 10^{-23}$ | $3.60 \cdot 10^{-23}$ |
| J1939 + 2134 | $2.56 \cdot 10^{-23}$ | $3.33 \cdot 10^{-23}$ | $3.68 \cdot 10^{-23}$ |

Table 4.4: Calculated upper limits for 3 different coverages



Figure 4.8: three confidence belts for three coverages: C = 0.68(blue curve), C = 0.9(red curve), C = 0.95(yellow curve) .

### 4.4 Comments about the measured upper limits

The final values 4.4 fit with the spectrum of the detector during run 852, see figure 3.9. In fact, the worst upper limit is the one for PSR J0024-7204J, that is in a spectral zone (952 Hz) where a noisy line is present. For the other 2 sources the upper limits are very similar, and this respects the fact that the two sources are very close one to each other in the spectrum (they are only 5 Hz far away one from each other) and so the properties of the noise in the 3 Hz bands containing the signal are almost the same.

## Chapter 5

## Conclusions

### 5.1 Future perspective: applying our method to other experiments

Even if our results didn't lead to a detection of gravitational waves from the targeted binary pulsars, but to upper limits, and these upper limits are not so restrictive to set important constraints on astrophysical models of these sources, we have to focus on the following fact, which is very important. Our method can be applied, without in fact any differencies or corrections, to any other gravitational wave experiment, such as the new generation of Earth based detectors Advanced LIGO [43] and Advanced VIRGO [52]. The selections of the sub-bands holding each signal, and the whole data analysis will be done exactly in the same way, but with two important improvements, that can be seen directly by simply looking at figure 5.1 and 5.2.

These detector promise to reach much better sensitivities that the ones we have used in this work. Moreover, the bandwidth of these interferometric detectors are much larger than the ones available for resonant-mass detectors like AURIGA and this band is optimally located respect to the possible sources of interest, thus resulting in a lot of possible sources to look at with Advanced LIGO and Advanced VIRGO. So, the science that one could do with such detectors can be much more interesting, and the complete analysis pipeline and implementations are the ones that have been implemented and tested in this work, so this could be the natural future development of this work.

Before the advanced LIGO detector, a preliminary step is planned, i.e. Enhanced LIGO, which will be available in 2010. The design curve reaches its maximum sensitivity around 200  $\simeq$  300 Hz, with  $S_h^{1/2} \simeq 10^{-23} \text{Hz}^{-1/2}$ . Performing a fully coherent analysis using 1 year of data, the minimum mea-



Figure 5.1: Sensitivity curve of the LIGO advanced detector.



Figure 5.2: Sensitivity curve of the planned advanced VIRGO detector [52].

surable amplitude is about  $h_{min} \simeq 3 \cdot 10^{-27}$ , corresponding to values of the ellipticity of  $\varepsilon \simeq 3 \cdot 10^{-7}$ , which are very interesting from an astrophysical point of view.

Comparing these values with the one shown in the chapter about astrophysical models, we see that these orders of magnitude are likely to lead to a detection, or to very strict upper limits about the astrophysical parameters of these sources, and so make possible to really do interesting science. In fact, even if it is difficult to think it's possible to reach a detection of a real signal, the upper limits are close enough to the expected values of the amplitude which is expected by astrophysical models, and so some close enough constraints can be set on some unknown astrophysical properties of rapidly rotating neutron stars.

It is moreover important to see that, if a new detector as the Advanced LIGO will be available in a few years, the probability of a detection is good, in fact, using the sensitivity curve of the projected Advanced LIGO, in the spectral region in which our targeted sources are clustered the  $h_{min}$  that could be revelated is in the order of  $h_{min} \simeq 3 \cdot 10^{-28}$  (for 1 year of fully coherent integration). Continuing in this kind of searches is so very promising for the future.

#### 5.2 SNR formula and its consequences

The low that describes how the sensitivities of our search increases with the time of coherency  $T_c$  and with the number of averaged spectra M is

$$h_{\min} = S_h^{1/2} (T_c^{1/2}) M^{1/4} .$$
(5.1)

This relation has two important consequences, the first one referring to the robustness of our method, the second one referring to the sensitivities that would be possible to reach with advanced interferometric detectors. The fact that, if a real signal is present in the data, it has to be the exact signature of the template we used to extract it from the noise, dramatically reduces the possible of false detections. In fact, every signal or structure in the noise with another shape and not the exact one used in our filters would be automatically destroyed by the extreme accuracy of the filter itself.

Then, if a signal is present, equation 5.1 shows that its SNR has to grow as the analyzed time increases following this exact law. So, another strong evidence of a detection would be the one of repeating the analysis through shorter or larger observation time, also in different epochs, calculate the SNR and see if the relation 5.1 holds. The present AURIGA design guarantees a very good duty cicle, so one could think to keep coherently the phase information for a very long time. With 1 year of fully coherent observation, and for PSR J1939+2134 (the one characterized by a lower noise in its spectral band), we could be able to reach an upper limit of  $h_{min} = 10^{-24}$ .

## 5.3 Comparison of our results with other similar works in literature

Now, let's see which are the searches that have so far been done and reported in the literature, about the attempt to measure gravitational waves from neutron stars. We want to understand how our work fits with the other studies in this field. Historically, the first study concerned the data taken in 1991 by the Explorer detector [44]. This is a search focusing only in the Galactic Center direction, and restricted to the very narrow frequency band  $921.32 \div 921.38$  Hz around the sensitivity peak of the antenna. They used 3 months of effective data (because of the low detector duty cicle), and the analysis method applied the optimal filtering strategy to the simple case of isolated neutron stars without any spindown. It provided to reach an upper limit of  $h^{u.l.} = 2.9 \cdot 10^{-24}$ . It was a very preliminary and limited approach: the chosen direction to observe was certainly privileged and very interesting; however, as we have discussed in chapter 2, isolated neutron stars, even if they are the most simple case to deal with, are the less promising as gravitational wave sources. Then the ROG group presented the first study with an approach of "all-sky" kind, but still only looking at isolated sources, using the data taken by Explorer in the period between September 2001 and November 2002 [45]. The search is limited to the frequency band  $921.00 \div$ 921.76 Hz around the antenna peak sensitivity, and takes for the first time in account the neutron star spindown. Using a maximum likelihood method, they reached an upper limit of  $h^{u.l.} = 2 \cdot 10^{-23}$ , namely about 1 order of magnitude worse than the one presented in the paper [44] about the galactic center, but now, for the first time, scientists looked to all the positions in the sky. With the beginning of the interferometers era, for the first time scientists started searching (as we do here) for signals from existing pulsars. known from observations in the radio field. The first attempt is a work that uses the data taken by the LIGO detectors in their first scientific run, called S1 [46], providing an upper limit  $h^{u.l.} = 1.4 \cdot 10^{-22}$  for one of the targeted pulsars of our work, PSR J1939+2134. With the interferometers second scientific run (S2), two other important works have been published. The

first one, whose approach is the most similar to our one, concerns 28 isolated pulsars [47]. The stricter upper limit is the one about PSR J1910-5959D and it is equal to  $h^{u.l.} = 1.7 \cdot 10^{-24}$ . The orders of magnitude of all these 28 upper limits are similar to the values that we found here. However, we are still only dealing with isolated objects. On the other and, the paper about all-sky [48], still about isolated sources, uses the Hough transform method for the pattern recognition, and reaches a best upper limit of  $h^{u.l.} = 4.43 \cdot 10^{-23}$  in the spectral region  $200 \div 400$  Hz. An analogous study (all-sky for isolated neutron stars), using the data taken by Explorer in 2005 [49], placed a similar upper limit  $(h^{u.l.} = 3 \cdot 10^{-23})$  in the band  $885 \div 925$  Hz. The last one that has been published, in chronological order, is the one that can better be compared with our study, and it's the one realized by the LIGO Scientific Collaboration using the data of the two scientific runs S3 and S4 [50]. For the first time, scientists realized a search targeted also to binary pulsars, the most promising ones (they are more that half the total of 78 considered objects). The stricter upper limit they give is  $h^{u.l.} = 2.6 \cdot 10^{-25}$ , about PSR J1603-7202. In the paper, they look also to the 3 sources we measure here. In particular, the upper limits are:  $h^{u.l.} = 7.41 \cdot 10^{-25}$  for PSR J0024-7204J;  $h^{u.l.} = 1.14 \cdot 10^{-24}$ for PSR J0218+4232;  $h^{u.l.} = 1.65 \cdot 10^{-24}$  for PSR J1939+2134. Comparing these values with the ones of table 4.1, we see that for PSR J0218+4232 and PSR J1939+2134 the improved sensitivity of interferometers allowed to reach upper limits about 10 times better that what we could do. To conclude, the experimental and analysis efforts that have been so far done in this field are several, and they begin now to give their results. In fact, even if a detection has not so far been achieved, the upper limit estimation begin to be very close to the expected amplitudes (see the results we found in Equations 1.50) and 1.54): at present, measurements can play the important task of placing limits on several important and not well known astrophysical parameters of these objects, as the magnetic field, the ellipticity, and the nature of the equation of state of the hyperdense matter which neutron stars are made of.

### 5.4 Another application: transients in X-ray pulsars

Finally, it is very important to notice that, very recently, X-ray observatories have done a new kind of observations, which bring to another, both very promising and very simple, application of our work. In some transients in X-ray pulsars, they managed to exactly track the phase of the signal at the SSB for the duration of the emission [51]. Now, these X-ray emission are very interesting from an energetic point of view, and they arise because of matter falling onto the star from the accretion disk, so during these transients the ellipticity of the star probably changes, and also emitting mechanisms such the r-modes are likely to become instable and so available as gravitationalwave emission channels. So, using the method presented in this work to see if a signal is present during these phenomena is in principle very interesting. It's also worth noticing that, recently, the scientists of tge LIGO Scientific Collaboration published a paper of this kind [53]: they searched through the data a possible gravitational wave associated with the gamma-ray burst 070201. This is to show that this kind of approaches is currently used and considered promising by the scientific community.

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# Bibliography

- [1] M. van der Klis, A review of rapid X-ray variability in X-ray binaries, Compact stellar X-ray sources, Lewin van der Klis (eds), Cambridge University Press (2004).
- [2] T. Piran, The Physics of Gamma-Ray Bursts, Rev.Mod.Phys. 76 1143-1210 (2004).
- [3] R. D'Inverno, *Introducing Einstein's Relativity*, Oxford University Press (1992).
- [4] L. Taylor, Binary pulsars and relativistic gravity, Rev. of Mod. Phys., 66, 711-719 (1994).
- [5] M. Kramer et al, Tests of general relativity from timing the double pulsar, Science **314** 97-102 (2006)
- [6] C. Baraldo, A. Hosoya, T. Nakamura, Gravitational induced interference of gravitational waves by a rotating massive object, Phys. Rev. D 59 083001 (1999).
- [7] L. Landau, E. Lifshitz, *The classical theory of fields* (1962).
- [8] S. Weinberg, Gravitation and Cosmology: principles and applications of the general theory of relativity, Wiley (1972).
- [9] C.-A. Faucher-Giguere and V. M. Kaspi, 40 Years of Pulsars: The Birth and Evolution of Isolated Radio Pulsars, proceedings of "40 Years of Pulsars: Millisecond Pulsars, Magnetars, and More", McGill University, Montreal, Canada (2007)
- [10] D. I. Jones, Gravitational waves from rotating neutron stars, Class. Quant. Grav. 19, 1255-1266 (2002).
- [11] M. Ruderman, Pulsar Spin, Magnetic Fields, and Glitches, arXiv:astroph/0610375v1 (2006).

- [12] E. Gourgoulhon and S. Bonazzola, *Gravitational waves from isolated neutron stars*, Proceedings of the International Conference on Gravitational Waves: Sources and Detectors (1996).
- [13] C. Cutler, Gravitational waves from neutron stars
- [14] I. H. Stairs, A. G. Lyne and L. Shemar, Evidence of free precession in a pulsar, Nature, 406/3, 466-484 (2000)
- [15] M. Zimmerman and E. Szedentis Jr., Gravitational waves from rotating and precessing bodies. Simple models and applications to pulsars, Phys. Rev. D. 20, 351-355 (1979).
- [16] A. Mion et al, Searching for quasi-periodic gravitational waves from Low Mass X-ray Binaries with AURIGA, Class. Quant. Grav. 22 S1021-S1026 (2005)
- [17] G. Ushomirsky et al, Proceedings of 3rd Edoardo Amaldi conference on gravitational waves (2000).
- [18] G. Ushomirsky, C. Cutler and L. Bildsten, Deformations of Accreting Neutron Star Crusts and Gravitational Wave Emission, Mon. Not. Roy. Astron. Soc. **319** 902 (2000).
- [19] T. Padmanabhan, Theoretical Astrophysics Volume II: Stars and Stellar Systems, Cambridge University Press (2001).
- [20] N. Andersson and K. D. Kokkotas, The r-mode instability in rotating neutron stars, Int. J. Mod. Phys. D10, 381-442 (2001).
- [21] Y. Levin and G. Ushomirsky, Crust-core coupling and r-mode damping in neutron stars: a toy model, Mon. Not. Roy. Astron. Soc. 324 917 (2001).
- [22] N. Andersson et al, *R-mode runaway and rapidly rotating neutron stars*, Astroph. Journal **534** L75 (2000).
- [23] L. Lindblom et al, Effect of a neutron-star crust on the r-mode instability, it Phys. Rev. D 62 084030 (2000).
- [24] G. Ushomirsky, L. Bildsten, C. Cutler, Gravitational Waves from Low-Mass X-ray Binaries: a Status Report, Proceedings of 3rd Edoardo Amaldi Conference on Gravitational Waves (2000).

- [25] A. Possenti et al, Recycling neutron stars to ultra short periods: a statistical analysis of their evolution in the mu-P plane, Astrophys. Journal Suppl. 125 463-478 (2000).
- [26] Search for gravitational wave radiation associated with the pulsating tail of the SGR 1806-20 hyperflare of 27 December 2004 using LIGO, arXiv:astro-ph/0703419v2, submitted to Phys. Rev. D
- [27] M. Van der Klis, Millisecond Oscillations in X-ray Binaries, Ann. Rev. Astron. Astrophys. 38 717-760 (2000).
- [28] I. Stairs, Testing General Relativity with Pulsar Timing, Liv. Rev. Rel. 6 5 (2003)
- [29] http://aa.usno.navy.mil/software/novas/novas\_c/novasc\_info.php
- [30] C. Barbieri, *Lezioni di astronomia*, Zanichelli (1999).
- [31] http://www.atnf.csiro.au/research/pulsar/tempo
- [32] www.atnf.csiro.au/research/pulsar/catalogue
- [33] Prof. Matthew Benacquista, private communication, 24-1-2007.
- [34] S. L. Shapiro e S. A. Teukolsky, Black Holes, White Dwarfs, and Neutron Stars: the physics of compact objects, Wiley (1983).
- [35] Y. Gursel and M. Tinto, Phys. Rev. D 40, 3884 (1989)
- [36] E. Mauceli, *Data analysis of the ALLEGRO gravitational wave detector*, PhD thesis, Lousiana State University, (1997).
- [37] P. Falferi et al, 27 ħ SQUID amplifier operating with high-Q resonant input load, Appl. Phys. Lett. 88 062505 (2006)
- [38] Jean-Pierre Zendri, private communication, 23-1-2007.
- [39] G. Pizzella, *Fisica Sperimentale del Campo Gravitazionale*, Carocci (1993).
- [40] J. G. Proakis, D. G. Manolakis, *Digital Signal Processing*, Macmillan (1992).
- [41] http://www-users.cs.york.ac.uk/ fisher/mkfilter/
- [42] G. J. Feldman and R. D. Cousins, Unified approach to the classical analysis of small signals, Phys. Rev. D 57 3873 (1998).

- [43] P.Saulson, Status of ground-based gravitational wave detectors.
- [44] P. Astone et al, Search for Periodic Gravitational Wave Sources with the Explorer Detector, Phys.Rev. D 65 022001 (2002)
- [45] P. Astone et al, All-sky upper limit for gravitational radiation from spinning neutron stars, Class.Quant.Grav. 20 S665-S676 (2003)
- [46] R. J. Dupuis (for the LIGO Scientific Collaboration), Targeted searches for gravitational waves from radio pulsars, J. Phys. Conf. Ser. 32 52-57 (2006).
- [47] B. Abbott et al (the LIGO Scientific Collaboration), Limits on gravitational wave emission from selected pulsars using LIGO data, Phys.Rev.Lett. 94 181103 (2005).
- [48] B. Krishnan (for the LIGO SCientific Collaboration), Wide parameter search for isolated pulsars using the Hough transform, Class.Quant.Grav. 22 S1265-S1276 (2005).
- [49] P. Astone et al, All-sky incoherent search for periodic signals with Explorer 2005 data, arXiv:0708.4367v1 [gr-qc] (2007).
- [50] B. Abbott et al (the LIGO Scientific Collaboration), Upper limits on gravitational wave emission from 78 radio pulsars, arXiv:grqc/0702039v5 (2007).
- [51] M. Falanga et al, *High-Energy pulsar profile of the Transient X-ray pulsar SAX J2103.5+4545*, Accepted for publication as a research Note in Astronomy Astrophysics (2005).
- [52] A. Viceré for the VIRGO Collaboration, talk at conference PAFT 2007.
- [53] B. Abbott et al (the LIGO Scientific Collaboration), Implications for the Origin of GRB 070201 from LIGO Observations, arXiv:0711.1163v2, to be submitted to the Astroph. Journal (2007).