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NONPERTURBATIVE PHYSICS FROM INTERPOLATING ACTIONS

Anthony Duncan
Department of Physics and Astronomy[1]
University of Pittsburgh, PA 15260, USA
and
Department of Physics
Technion-Israel Institute of Technology, Haifa 32000, Israel

Moshe Moshe
Department of Physics[1]
Technion - Israel Institute of Technology, Haifa 32000, Israel
and
Fermi National Accelerator Laboratory[2]
P.O. Box 500, Batavia, Illinois 60510

Abstract

We study the expansion in an artificial parameter δ which interpolates between a solvable theory at $\delta = 0$ and the desired theory at $\delta = 1$. The interpolating actions are of the form $\delta S + (1-\delta)S_0$; and augmented by an optimization procedure which introduces nonperturbative features into our results. This procedure relies on the freedom in choosing the best S_0 without affecting the convergent results at $\delta = 1$. Our linear interpolation is similar in spirit but differs in details from the novel δ expansion that was recently formulated for scalar theories where the parameter $2(1+\delta)$ was the power of the field in the interaction lagrangian. Here we use interpolating actions for the first time in fermionic and gauge theories.

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[1] Permanent address

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1. INTRODUCTION

One of the obstacles to extracting nonperturbative physics in quantum field theory is the paucity of alternatives to conventional Feynman-Dyson perturbation theory, which provides, even in the best of cases, at most an asymptotic expansion. Large N methods[1] have yielded much insight but have various limitations; primarily, the difficulty of computing higher order corrections, and the fact that in cases of direct physical interest, N is not particularly large. Since the 1/N expansion is itself only asymptotic, we are led back in the case of small N to much the same situation that is obtained in a weak coupling perturbation theory when the effective coupling is strong. Recently,[2] there has been some interest in exploring expansions based on an artificial parameter (called generically "δ" below), which interpolates between a solvable theory at $\delta = 0$ and the desired theory at $\delta = 1$. In some cases the parameter δ is basically the power of a scalar field in the action. In others the interpolation is linear, corresponding to an action $\delta S + (1 - \delta)S_0$.

In this paper we discuss the application of the linear interpolation technique, augmented by an optimization procedure, to theories containing fermionic and gauge degrees of freedom. The optimization used relies on the obvious fact that we are free to introduce arbitrary parameters in S_0 which cannot affect results at $\delta = 1$, provided the delta expansion is convergent. However, these parameters can strongly affect the radius of convergence of this expansion. By maximizing the radius of convergence we can minimize the error incurred by working only to finite order in delta, at the same time minimizing the dependence of the final results on the arbitrary parameters in S_0. In fact, the optimization also introduces directly important nonperturbative dependencies on the coupling constants in the theory (cf Section 2).

In Section 2 we show that the above ideas can be applied to the Gross-Neveu model,[3] and a convergent series of approximants obtained for the effective potential for the auxiliary σ field, even for N (= no. of fermion species) as small as 2. In this case the delta expansion is not merely asymptotic, but convergent. In Section 3 lattice gauge theories (Z_2 and U(1)) in 3 spacetime dimensions are studied. We find that in the case of U(1) an appropriate choice of S_0 leads to considerable improvement over strong-coupling Padé extrapolations[4] for plaquette energies at large values of β. We also find good agreement with published Monte-Carlo data[5] for these systems.

2. OPTIMIZED INTERPOLATION FOR THE GROSS-NEVEU MODEL

In this section we shall concentrate on the problem of extracting nonperturbative behavior from a model with at least one limit where explicit analytic results are available. In the limit of large number of fermion species (N), the $(\bar{\psi}\psi)^2$ model in 1 space - 1 time dimension is solvable and provides a beautiful example of nonperturbative chiral symmetry breaking.[3] The procedure described below will in fact allow us to calculate nonperturbative effects even for finite N.

The basic idea of the interpolation is that the $\delta = 0$ theory should (a) be solvable, and (b) reflect, at least crudely, the physics of the true ($\delta = 1$) theory. In the case of the Gross-Neveu model the chiral breaking gives a mass to the fundamental fermions in the theory, which suggests that the following interpolation might be useful:

$$\mathcal{L}_\delta = \bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}\sigma^2 + g\sigma\delta\bar{\psi}\psi + g\mu\Lambda(1-\delta)\bar{\psi}\psi \qquad (.1)$$

Here σ is the usual auxiliary field, Λ an ultraviolet cutoff (we shall see below that it is essential to work in the cutoff theory) and μ an arbitrary parameter which we can choose to optimize the convergence of the expansion in δ. It is precisely at this last step that nonperturbative features enter the calculation.

In the large N limit, one may easily obtain the effective potential for the σ-field following from Eq. 1, for arbitrary values of the parameters μ, δ. Namely, one finds

$$V = \frac{1}{2}\sigma^2 - \frac{N}{4\pi}X^2\left(\ln(1+\frac{\Lambda^2}{X^2}) + \frac{\Lambda^2}{X^2}\ln(1+\frac{X^2}{\Lambda^2})\right) \qquad (.2)$$

where

$$X = g(\sigma\delta + \mu\Lambda(1-\delta)) \qquad (.3)$$

The obstruction to a convergent expansion of (2) in δ arises from the branch points of the logarithms at

$$\delta = \frac{\Lambda}{\mu\Lambda - \sigma}(\mu, \mu \pm \frac{i}{g}) \qquad (.4)$$

The radius of convergence of the expansion is therefore maximized by choosing $\mu = \frac{\sigma}{\Lambda}$ for any given preassigned value of the auxiliary field σ at which one is expanding the effective potential. The interesting physics in this model occurs for σ close to the minimum of $V(\sigma)$ so we shall get rapid convergence in this region by choosing μ to *coincide* with $\frac{\bar{\sigma}}{\Lambda}$, where $\frac{\partial V}{\partial \sigma}|_{\sigma=\bar{\sigma}} = 0$. This will mean that the parameter μ becomes implicitly a function of g, and depends also on the order to which we work in the delta expansion. This procedure will be seen below to give good convergence of the effective potential for large as well as finite N. The convergence is very good, of course, especially in the vicinity of $\sigma = \bar{\sigma}$.

It is straightforward to calculate the expansion of V for general N by ordinary perturbation theory starting from Eq.1. If we denote $V^{(n)}$ the coefficient of δ^n, the first four orders in δ yield

$$V^{(0)}(\phi) = \frac{1}{2}\phi^2 \qquad (.5)$$

$$V^{(1)}(\phi) = -\lambda\mu \ln(1 + \frac{1}{g^2\mu^2})(\phi - \mu)$$

$$V^{(2)}(\phi) = -\frac{1}{2}\lambda(\frac{2}{1+g^2\mu^2} - \ln(1 + \frac{1}{g^2\mu^2}))(\phi - \mu)^2 \qquad (.6)$$

$$V^{(3)}(\phi) = -\frac{1}{3}\lambda\frac{1}{\mu}\frac{g^2\mu^2 - 1}{(1+g^2\mu^2)^2}(\phi - \mu)^3 - \frac{\lambda^2}{2N}\mu\ln(1 + \frac{1}{g^2\mu^2})(\frac{2}{1+g^2\mu^2} - \ln(1 + \frac{1}{g^2\mu^2}))(\phi - \mu) \qquad (.7)$$

$$V^{(4)}(\phi) = -\frac{1}{12}\lambda\frac{1}{\mu^2}(1 + 6g^2\mu^2 - 3g^4\mu^4)(1+g^2\mu^2)^{-3}(\phi - \mu)^4 \qquad (.8)$$

$$+ \frac{\lambda^2}{4N}\left(\frac{2}{1+g^2\mu^2} - \ln(1 + \frac{1}{g^2\mu^2})\right)^2(\phi - \mu)^2 + \frac{\lambda^2}{4N}\frac{g^2\mu^2 - 1}{(1+g^2\mu^2)^2}\ln(1 + \frac{1}{g^2\mu^2})(\phi - \mu)^2 \qquad (.9)$$

We have introduced the following dimensionless quantities

$$\lambda \equiv \frac{g^2 N}{2\pi} \qquad (.10)$$

$$\phi \equiv \frac{\sigma}{\Lambda} \qquad (.11)$$

As explained previously, the condition for fixing μ is

$$\frac{\partial V}{\partial \phi}|_{\phi=\mu} = 0 \tag{.12}$$

To first order in δ, at $\delta = 1$ (i.e. setting $V = V^{(0)} + V^{(1)}$) this condition implies

$$\lambda \ln(1 + \frac{1}{g^2\mu^2}) = 1$$

or

$$\mu^2(g^2) = \frac{1}{g^2}\frac{1}{exp(\frac{2\pi}{g^2 N}) - 1} \tag{.13}$$

A glance at Eq. 6 shows that this condition does not change at order δ^2, as $V^{(2)}(\phi)$ is quadratic in $(\phi - \mu)$. However, at order δ^3, Eq. 13 is corrected in consequence of the (subdominant in N) linear term in $(\phi - \mu)$. This pattern persists in higher order: at every second order in δ, we must change $\mu(g)$. It is apparent from (13) that the optimization condition (12) has resulted in a completely nonperturbative expression for V as a function of g: in particular, the characteristic essential singularity at g=0 is now present, even though the calculation is perturbative in δ. Moreover, the first order result (13) for the location of the minimum of the effective potential agrees completely with the large N result.

In Figure (1) we show the convergence of the δ expansion of the exact large N effective potential (2). The convergence in the region of the minimum is excellent for values of bare coupling g essentially at the continuum limit ($g\mu = 1.9 \times 10^{-3}$ in units of the cutoff). As we reduce N to finite values, the convergence is found to worsen at smaller values of λ. Figure (2) shows the situation of $\lambda = 0.3$, $N = 2$ up to fourth order in δ. In the region of the minimum we have good convergence: the 3rd and 4th order curves agree to within a few percent for $0.5\mu < \phi < 1.5\mu$. Outside this region the convergence is worse. In particular, a convincing demonstration of chiral breaking for $N = 2$ would require us to go to higher order in δ to make absolutely sure that the central value (at $\phi = 0$) does not decrease continually until the symmetry is restored. This seems highly unlikely on the basis of the "visual evidence" presented in Figure (2).

The method described above also allows the calculation of a nonperturbative β-function of the theory. Although we are prevented from renormalizing the **entire** effective potential order by order in δ by the failure of the convergence of the expansion once the bare coupling is forced to zero, we can extract a sensible convergent sequence approximants to the β function by staying at the point where the expansion (at least for large N) is optimal, i.e. at $\sigma = \bar{\sigma}$, $\frac{\partial V}{\partial \sigma} |_{\sigma=\bar{\sigma}} = 0$. At this point, the renormalization group equation for V reads simply

$$(\Lambda \frac{\partial}{\partial \Lambda} + \beta \frac{\partial}{\partial g}) V = 0 \qquad (.14)$$

as the $\gamma \sigma \frac{\partial}{\partial \sigma}$ term is absent at the minimum of V. To order, δ, we have simply (at $\delta = 1$) in virtue of (12).

$$V = \frac{1}{2}\sigma^2 - \mu(g^2)\sigma\Lambda \qquad (.15)$$

so

$$\beta^{(1)} = -\left(\frac{\partial \ln \mu}{\partial g}\right)^{-1} \qquad (.16)$$

Using (13) one finds

$$\beta^{(1)}(g) = -\frac{g}{\frac{2\pi}{g^2 N}[1 - exp(-\frac{2\pi}{g^2 N})]^{-1} - 1} \qquad (.17)$$

This behaves correctly $(-\frac{g^3 N}{2\pi})$ in the weak coupling limit but there are also nonperturbative corrections. One may easily compute the corrections to (17) in higher order in δ.

3. OPTIMIZED INTERPOLATION FOR LATTICE GAUGE THEORY

As a second example of the procedure of optimized interpolation, we describe in this section an approach to the computation of observables in lattice gauge theory with more rapid convergence properties in the crossover region from strong to weak coupling than the usual extrapolations from strong coupling expansion.

Consider the lattice action

$$S = \beta\delta \sum_p s_p + \beta'(1-\delta) \sum_p{}' s_p \qquad (.18)$$

where s_p denotes $\prod_{\ell \epsilon p} s_\ell$ and $s_\ell = \pm 1$ for each link in plaquette p for Z_2 gauge theory, $cos(\sum_{\ell \epsilon p} \theta_\ell)$ for U(1) theory, and the usual $tr(\Pi_{\ell \epsilon p} U_\ell)$ for SU(N) theory. This is evidently the conventional action at $\delta = 1$. The $\sum'_p$ in the second term runs over **a maximal tree of plaquettes.**[6] This is simply a subset of plaquettes with the property that the addition of any single plaquette to the set gives rise to a closed surface in the lattice tiled by plaquettes in the (new) set. A moment's thought shows that a maximal tree of plaquettes also has the property that, forcing the plaquette actions to their maximal values for all plaquettes on the tree also forces the nontree plaquettes to maximal action. For simplicity, consder the case of U(1). Any nontree plaquette p' forms a closed surface with a set of tree plaquettes p_i. Thus we have a kinematic constraint $\theta_{p'} = -\sum_i \theta_{p_i}$ for the plaquette angles. If all the θ_{p_i} are forced to zero, as in the weak-coupling regime $\beta' \to \infty$, so is $\theta_{p'}$. So the behavior of the $\delta = 0$ and $\delta = 1$ actions in (18) are crudely similar in the weak-coupling (large β, β') regime. The other motivation for choosing a maximal tree in the $\delta = 0$ action is the possibility of evaluating the resulting graphs in terms of explicit analytic expressions. The insertion of nontree plaquettes as one expands in δ results in closed surfaces of finite volume embedded in an (infinite volume) tree; the latter may be integrated out trivially, much as one does in $1+1$ dimensional lattice gauge theory, leaving a finite cluster of plaquettes which can be evaluated straightforwardly in terms of the usual modified Bessel functions. Finally, we see from (18) that β' like μ in the case of Gross-Neveu, is an arbitrary parameter (irrelevant at $\delta = 1$) so we may choose it freely to optimize the convergence of the expansion.

It will be convenient to rewrite (18) in the equivalent form

$$S = \beta\delta \sum_p{}'' s_p + (\beta' + \delta\beta_t) \sum_p{}' s_p \tag{.19}$$

where $\sum''_p$ runs over all nontree plaquette, $\beta_t \equiv \beta - \beta'$. The expansion will be made with respect to the first term. Later, the Bessel functions of argument $\beta' + \delta\beta_t$ can be further expanded in δ to give the final series. For U(1) theory, the "zeroth" order contribution to the partition function then (no nontree plaquettes) is just (N_t = number of tree plaquette, N_p = number of plaquettes)

$$Z_0 = I_0(\beta' + \delta\beta_t)^{N_t} \tag{.20}$$

and the free energy per plaquette is

$$W_0 = \frac{1}{N_p}\ell n Z_0 = \frac{N_t}{N_p}\ell n I_0(\beta' + \delta\beta_t) \qquad (.21)$$

At this point, it is convenient to specify more precisely the structure of the maximal tree that we have used. There is of course a great deal of freedom here and it may be the case that different choices give different rates of convergence. We have found it convenient to use a maximal tree in which (in 3 space time dimensions) all (xz) and (yz) plaquettes are included, as well as all (xy) plaquettes for $z = 0$, say. The results presented below for Z_2 and U(1) theory in 3 dimensions all refer to this choice of tree. Evidently, $\frac{N_t}{N_p} = \frac{2}{3}$ in the infinite volume limit.

At zeroth order in δ, we have

$$W_0 = \frac{2}{3}\ell n I_0(\beta') \underset{\beta'\to\infty}{\longrightarrow} \frac{2}{3}\beta' + O(\ell n(\beta')) \qquad (.22)$$

whereas the correct large β behavior for the exact W is

$$W(\beta) \underset{\beta\to\infty}{\sim} \beta + O(\ln\beta) \qquad (.23)$$

The match is best at large β by choosing $\beta' = \frac{3}{2}\beta$. We shall see below that the optimal $\frac{\beta'}{\beta}$ decreases from $\frac{3}{2}$ as higher order in the δ expansion are included.

Insertions of one or more nontree plaquettes can be computed by standard cluster expansion methods. Odd numbers of nontree plaquettes give rise to connected parts with factors $\left(\frac{I_1(\beta'+\delta\beta_t)}{I_o(\beta'+\delta\beta_t)}\right)^{n_z}$ where n_z is the z coordinate of the furthest plaquette from the $z = 0$ level of the tree. Since $I_1 < I_0$, such contributions vanish exponentially fast as $n_z \to \infty$ and cannot give a volume contribution to W.

The contribution arising from two nontree plaquettes corresponds to graphs of the type shown in Fig. (3). Again, in order to get a volume term, the plaquettes must lie at the same (xy) location, with arbitrary values for z_1, z_2. The value of the graph is $\frac{1}{2}c_1^{4|z_1-z_2|}$, where $c_1 \equiv \frac{I_1(\beta'+\delta\beta_t)}{I_o(\beta'+\delta\beta_t)}$. Thus, the contribution to W from such graphs is

$$\begin{aligned} W_2 &= \frac{\beta^2\delta^2}{6}\lim_{N_z\to\infty}\frac{1}{N_z}\sum_{z_1,z_2=0}^{N_z}\frac{1}{2}c_1^{4|z_1-z_2|} \\ &= \frac{\beta^2\delta^2}{12}\frac{1+c_1^4}{1-c_1^4} \end{aligned} \qquad (.24)$$

Of course, when c_1 is expanded out, W_2 contains contributions of order δ^2, δ^3, etc. One feature of the expansion which is apparent in (24) is the appearance of an **infinite** sum over graphs at a **finite** order in δ (rather as one has to sum an infinite number of Feynman graphs to obtain the full contribution at a given order of a large N expansion). The resulting geometric sum in (24) exhibits a turnover of behavior from β^2 ($\beta << 1$) to β^3 ($\beta > 1$) and we may hope that this resumation will allow us to approach more rapidly the correct behavior (e.g. for specific heats) near transition regions where conventional strong-coupling Padé's begin to fail.

The fourth order contribution (in nontree plaquettes) arises from the graphs shown in Fig. (4). The calculations analogous to (24) are straightforward and will not be repeated here. The graph in Fig. (4a), for example, is found to contribute

$$W_{4a} = \frac{\beta^4\delta^4}{3}\left(\frac{1}{8}f_a(c_1^4, c_2^4, c_1^4) - \frac{1}{2}f_a(c_1^4, c_1^8, c_1^4)\right) \qquad (.25)$$

where

$$c_2 \equiv \frac{I_2(\beta' + \delta\beta_t)}{I_0(\beta' + \delta\beta_t)} \text{ and}$$

$$\begin{aligned} f_a(x,y,z) \equiv & \frac{xyz}{(1-x)(1-y)(1-z)} + \frac{1}{2}\frac{yz}{(1-y)(1-z)} \\ & + \frac{1}{2}\frac{xz}{(1-x)(1-z)} + \frac{1}{2}\frac{xy}{(1-x)(1-y)} + \frac{1}{4}\frac{y}{1-y} \\ & + \frac{1}{6}\frac{z}{1-z} + \frac{1}{6}\frac{x}{1-x} + \frac{1}{24} \end{aligned} \qquad (.26)$$

After expanding (22), (24), (25) to fourth order in δ, one may obtain diagonal (1,1) and (2,2) Padés, **in delta**, for the plaquette energy $\frac{\partial W}{\partial \beta}$. This has been done by setting $\beta' = \alpha\beta$, and searching for a regoin where the dependence on α is flat or extremal. For the (1,1) Padé the optimal choice of α is still close to the value 1.5 which we saw previously gives the best **large** β behavior at the zeroth order level. For $0 < \beta \leq 0.6$ Padé gives a broad maximum in α with the maximum value about 15% below the Monte Carlo results[5], while for $\beta > 0.8$ the broad maximum disappears and thus the optimization procedure fails. There is a dramatic improvement as we go to fourth order in δ and use the (2,2) Padé. Now the optimal value for α is found to be about 0.8 and an extremum is found all the way out to $\beta = 2$. The α dependence is flat for a wide range of values around the optimal α. The resulting values lie very close (see Fig. (5)) to the Monte Carlo measurements for $0 < \beta < 2.0$ The results are a clear improvement over the strong-coupling Padés, also shown in Fig. (5).

Fig. (6) shows the results obtained by the same procedure for Z_2 gauge theory. Here there is an actual phase transition and a much sharper turnover from strong to weak coupling. Again, the (2,2) Padé, after optimizing α, seems to do a good job up to $\beta = 0.8$, which is past the critical point at $\beta = 0.76$. In contrast to the U(1) case, here the (3,4) Padé of the strong coupling expansion is as good as the δ expansion. Note however, that different levels of calculations are compared when we are comparing the (3,4) Padé of the strong coupling expansion with the (2,2) Padé of the δ expansion which is only at the second nontrivial level in this expansion.

4. CONCLUSION

We employed an expansion in a parameter δ which interpolates between different actions. Other arbitrary parameters that appear in these actions (μ in the Gross-Neveu model in Eq. (1) and β' in the gauge theory action in Eq. (18)) were chosen by the optimization procedure described above. Our results in Figs. 1,2 and Figs. 5,6 show the very good convergance of the δ expansion. The good quality of the expansion and its extension into the transition regime in the case of the gauge theory shows that the subtle nonperturbative features of the theory are very well reproduced by our optimization procedure already at low order of the expansion. Clearly, the extension of these calculations to four dimensional gauge theories and other physical problems is now very desirable and promissing.

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FIGURE CAPTIONS

Fig. 1. Convergence of the δ expansion of the exact large N effective potential. (Here $g\mu = 1.9 \times 10^{-3}$ in units of Λ).

Fig. 2. δ expansion for the effective potential up to order δ^4 (Here $\lambda = 0.3$, N = 2). The y axis is: $\ln(V_{eff} + \frac{\epsilon}{2})$

Fig. 3. Contribution to order δ^2 from non-tree plaquettes.

Fig. 4. Contributions to order δ^4 from non-tree plaquettes.

Fig. 5. U(1) gauge theory in 3 dimensions : Plaquette energy in δ expansion - (1,1) and (2,2) Padé. The (3,4) Padé extrapolation of the strong coupling expansion is from results of ref. 4. Monte Carlo data and the conventions for definning the plaquette energy are from ref. 5 .

Fig. 6. Z_2 gauge theory in 3 dimensions : Plaquette energy in δ expansion - (1,1) and (2,2) Padé. The (3,4) Padé extrapolation of the strong coupling expansion is from results of ref. 4. Monte Carlo data and the conventions for definning the plaquette energy are from ref. 5 .

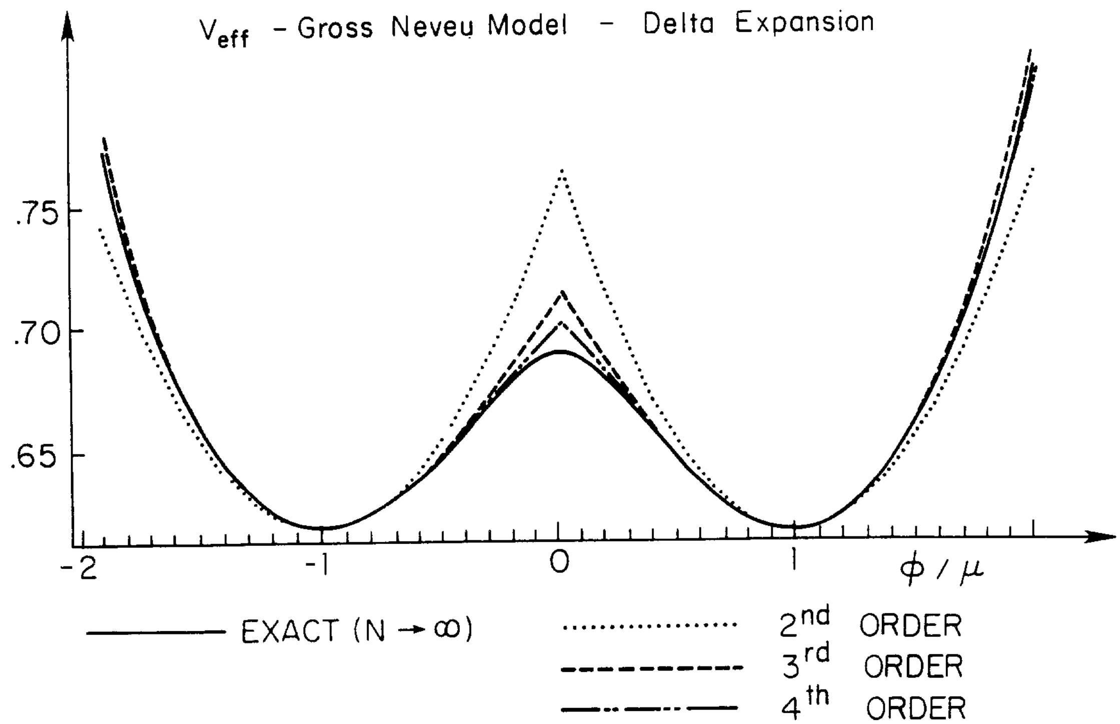


FIGURE 1

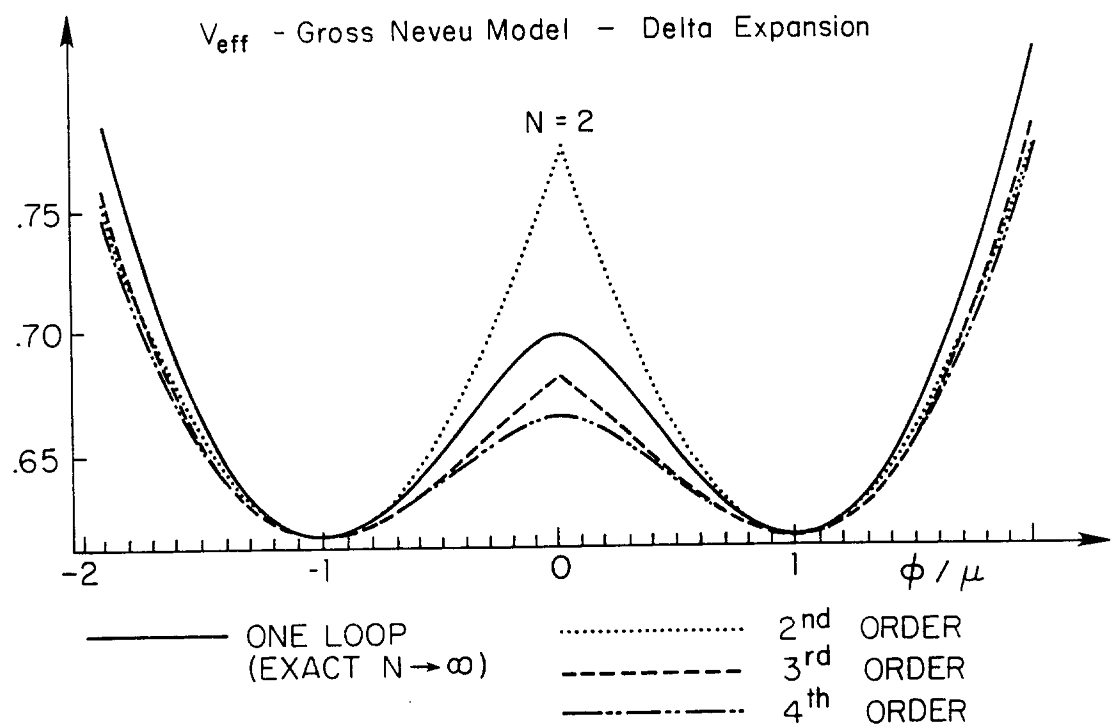
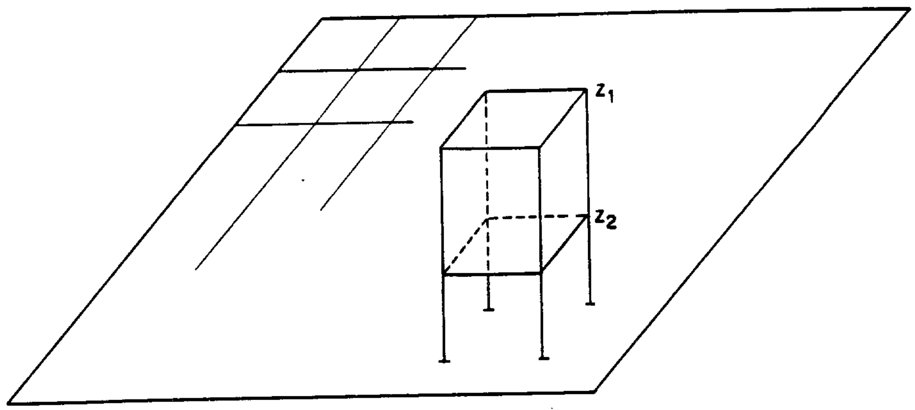


FIGURE 2



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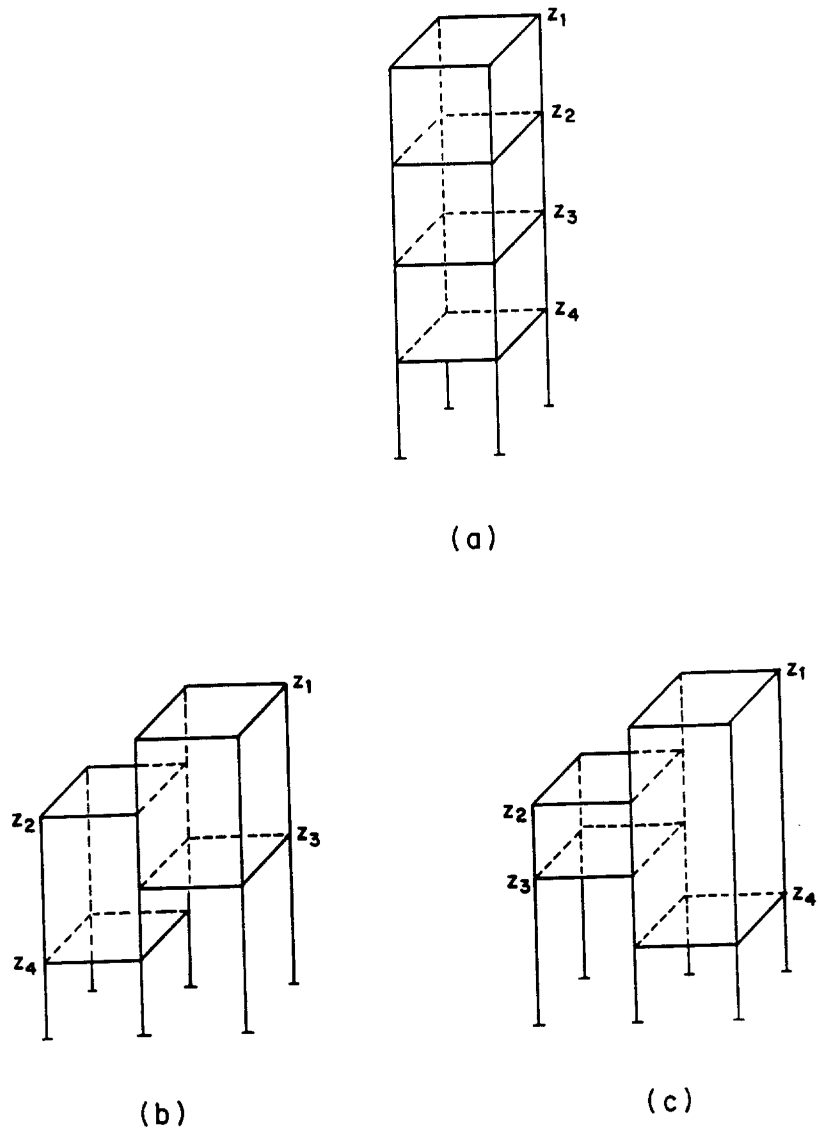

FIGURE 4

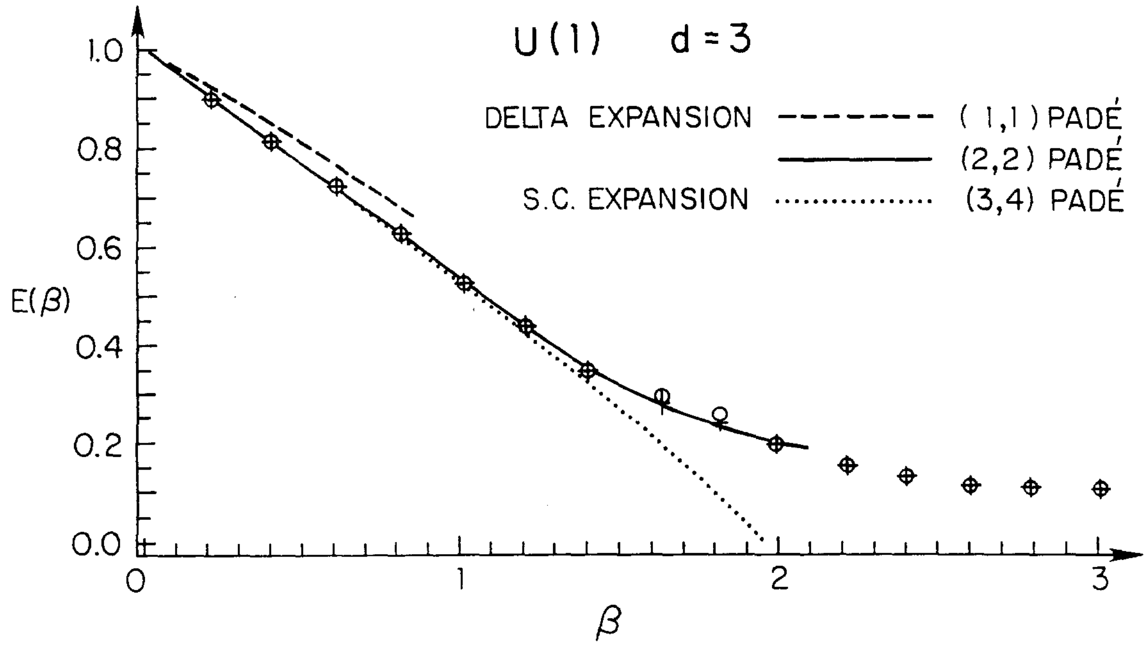

FIGURE 5

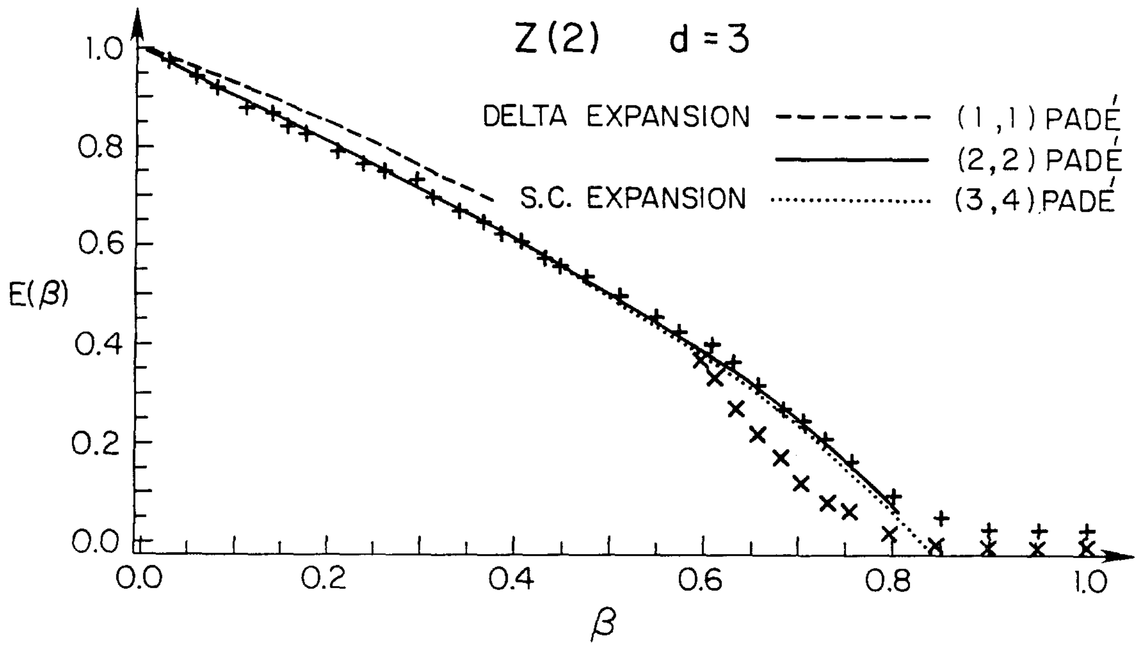

FIGURE 6