

# OPTIMIZATION OF THE GEOMETRIC BETA FOR THE SSR2 CAVITIES OF THE PROJECT X\*

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## Abstract

Project X based on the 3 GeV CW superconducting Linac and is currently in the R&D phase. The cw SC Linac starts from a low-energy SCRF section (2.1 - 165 MeV) containing three different types of resonators. HWR  $f=162.5$  MHz (2.1 - 11 MeV) having  $\beta=0.11$ , SSR1  $f=325$  MHz (11 - 35 MeV) having  $\beta=0.21$ . In this paper we present the analysis that lead to the final design of SSR2  $f=325$  MHz cavity (35 - 165 MeV). We present the results of optimization of the geometric beta and the comparison between single, double and triple spoke resonators used in Project X frontend.

## INTRODUCTION

Project-X is the proposed high intensity proton facility to be built at Fermilab [1]. The facility is based on 3 GeV 1 mA CW and on a 8 GeV pulsed superconducting Linac. The CW Linac is made of one half wave resonator section and two families of single spoke resonators. HWR cavities, operating at 162.5 MHz and having optimal  $\beta=0.11$ , accelerate the  $H^+$  ions from the exit of RFQ to the first SSR1 cryomodule (2.1 MeV – 10.8 MeV). After that there is a single spoke resonator part at 325 MHz, SSR1 having  $\beta=0.21$  (10.8 MeV - 35 MeV). The last part operating at 325 MHz contains SSR2 single spoke resonator, having optimal  $\beta=0.47$  (35 MeV to 165 MeV). The high energy part of the CW Linac operates at 650 MHz: two families of five cells elliptical cavities having  $\beta=0.61, 0.9$  accelerate the beam in this section up to the final CW energy (165 MeV - 3 GeV). After the CW part there is a pulsed Linac made of ILC having  $\beta=1$  and operating at 1.3 GHz. Project X scheme is shown in figure 1.

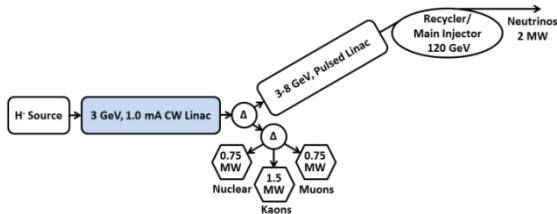


Figure 1: Project X scheme.

During the last year at Fermilab SSR2 cavity has been re-optimized to achieve better performance [2 paper EM design], moreover beta optimal of this cavity has been changed from 0.4 to 0.47. This paper presents the results of this beta optimization of SSR2, in addition the

performance of single, double and triple spoke resonators are compared for usage in Project X front end (35 to 165 MeV).

## SSR2 BETA OPTIMIZATION

Simulations of the low energy of Project X front end have been run using a simple model: beam dynamic of the particle was not simulated entirely just longitudinal dynamic was taken into account. The aim of this process was to check how far from the optimal value  $\beta_{opt}$  of SSR2 was.

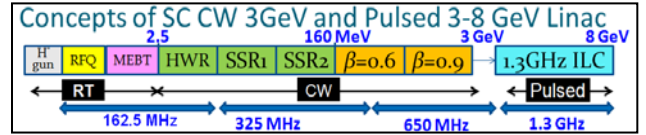
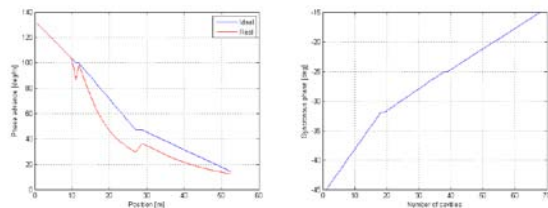


Figure 2: Project X CW and pulsed Linacs [2].

The part of Project X simulated comprehends HWR, SSR1 and SSR2 cavities. The first two sections were maintained the same while SSR2 number of cavities and  $\beta_{opt}$  were varied: the output parameter to satisfy was the final energy of the part of the CW Linac ( $\geq 160$  MeV). The physics simulated is very simple and includes:

- cavity voltage
- synchronous phase of the particle
- normalized transit time factor.

Cavity voltage is the maximum energy gain achievable in a cavity, it is limited by: surface peak electric and magnetic fields (40 MV/m and 60 mT) and longitudinal phase advance constraints. The cavity voltage can be expressed by  $\Delta W_{max} = E_{acc} L_{eff}$ , where  $E_{acc}$  is the accelerating gradient and  $L_{eff} = \beta_{opt} \lambda$  is the effective length of the cavity. The phase advance over meter is smooth in the whole portion of the Linac simulated, at the beginning of SSR2 section it matches SSR1 final value and in the end the first value for 650 MHz five cells cavities. Synchronous phase is linear in each section, fig. 3 shows the phase advance over meter on the left and the synchronous phase on the right.



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Figure 3: phase advance over meter (left), synchronous phase (right).

Considering a particle traveling at constant  $\beta$  through a cavity, a part from the synchronous phase the energy gain can be written:

$$\Delta W(\beta) = \int_{-L}^L E_z(z) \sin\left(\frac{kz}{\beta}\right) dz \quad (1)$$

where  $-L$  and  $L$  are the cavity gap  $z$  boundaries, considering the center in  $z = 0$ .

$\beta_{opt}$  is defined as the  $\beta$  corresponding to the maximum of  $\Delta W(\beta)$ . If one calculates the energy gain in the whole range of energy in which a cavity is used, it is possible to evaluate the normalized transit time factor (NTTF):

$$T(\beta) = \frac{\Delta W(\beta)}{\Delta W(\beta_{opt})}. \quad (2)$$

It is possible to express this quantity as a function of  $\beta$  or to scale it and plot it as a function of  $\beta/\beta_{opt}$ .

Figure 4 shows the normalized transit time factor vs  $\beta/\beta_{opt}$  for HWR ( $\beta_{opt} = 0.11$ ), SSR1 ( $\beta_{opt} = 0.21$ ) and SSR2 ( $\beta_{opt} = 0.4$ ), despite the difference of the optimal beta the NTTF dependence on  $\beta/\beta_{opt}$  is the same for all the cavities.

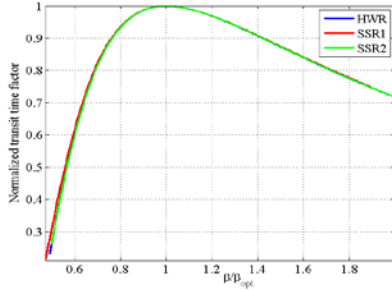


Figure 4: normalized transit time factor comparison

This characteristic of the NTTF allows to interpolate it using a polynomial function, then scale the transit time factor vs  $\beta/\beta_{opt}$  for any  $\beta_{opt}$  to get the NTTF vs  $\beta$  curve. During the  $\beta$  optimization the normalized transit time factor of SSR2 was scaled to obtain the NTTF vs  $\beta$  curve for a cavity having a certain  $\beta_{opt}$  which was not designed yet. Once defined the normalized transit time factor the energy gain can be written as

$$\Delta W(\beta) = \Delta W(\beta_{opt}) T(\beta) \cos \varphi_s \quad (3)$$

where  $\varphi_s$  is the synchronous phase of the particle.

Since the optimization process does not take into account any transverse particle dynamic, in each Linac section the solenoid was replaced by a drift space and the cavity is considered as an accelerating gap. The period length for SSR2 cavity is assumed linearly dependent on  $\beta_{opt}$ , having just two estimations for  $\beta_{opt} = 0.4$  and  $\beta_{opt} = 0.47$ . Segmentation of SSR2 section is not taken into account also, the only allocation constraint is due to period length, considering two cavities and one solenoid each period.

## OPTIMIZATION RESULTS

The parameters of the front end version prior to the optimization improvement of SSR2 section are reported in table 1.

Table 1: old Project X front end parameters.

N. HWR( $\beta_{opt} = 0.11$ )	8
N. SSR1( $\beta_{opt} = 0.21$ )	20
N. SSR2 ( $\beta_{opt} = 0.4$ )	40
SSR2 initial energy	40 MeV
SSR2 Final energy	160 MeV
B surface field limitation	60 mT

Parameters of table 1 are the starting point of this optimization,  $\beta_{opt}$  of SSR2 has been checked in a wide range to find the most optimal value. For each value of  $\beta_{opt}$  the minimum number of SSR2 cavities needed to achieve the target final energy was found. Final energy was set to 160 MeV as the original design. Results reported in figures below are number of SSR2 cavities, figure 5, and total front end length, figure 6, both of them are plot as function of SSR2  $\beta_{opt}$  from 0.4 to 0.6.

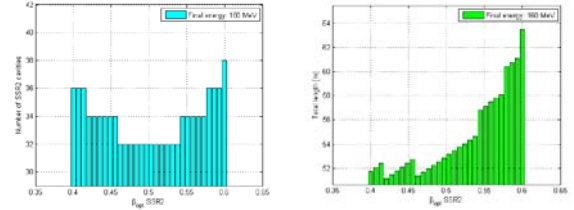


Figure 5: number of SSR2 cavities (left) and total length of the front end (right) vs SSR2  $\beta_{opt}$ .

Looking at these pictures it is clear that the choice of  $\beta_{opt} = 0.4$  for SSR2 cavity can be improved: an higher value will reduce both number of cavities and total length. The best range of  $\beta_{opt}$  is in between 0.46 and 0.54, but from the length point of view the higher the beta the longer the period, hence  $\beta_{opt} = 0.47$  is the final choice; the parameters of the optimized front end are reported in table 2. The actual design of Project X includes 4 more SSR2 cavities [2], this is due to cryomodule segmentation constraints not taken into account during the optimization process.

Table 2: optimized front end parameters.

N. HWR( $\beta_{opt} = 0.11$ )	8
N. SSR1( $\beta_{opt} = 0.21$ )	20
N. SSR2 ( $\beta_{opt} = 0.47$ )	32
SSR2 initial energy	40 MeV
SSR2 Final energy	160 MeV
B surface field limitation	60 mT

According to table 2 the total saving is 8 SSR2 cavities, this gain is partially due to the improvement made on SSR2 geometry: the  $\beta_{opt} = 0.4$  version has a magnetic field enhancement factor  $B_{max}/E_{acc} = 6.93 \text{ mT/(MV/m)}$

while the new  $\beta_{opt} = 0.47$  version can achieve higher energy gain because the cavity was re-designed and  $B_{max}/E_{acc}$  was lowered to 6.24 mT/(MV/m) [3].

## SSR2, DSR AND TSR COMPARISON

In the previous section the optimization of  $\beta_{opt}$  for SSR2 cavity of Project X is described, this cavity is a single spoke resonator but double (DSR) and triple spoke resonators (TSR) could be an alternative to the single one. The whole front end was re-optimized again using double and triple spoke instead of SSR2 cavity, to see if this cavity is actually the right choice for Project X. The method used is the same described above, the NTTF was first calculated for a designed DSR and a TSR, then it was interpolated polynomially and it was scaled for any  $\beta_{opt}$ . Since DSR and TSR RF design is not optimal, because these two cavities are not currently used in Project X, the field enhancement factors were, by assumption, considered very low ( $B_{max}/E_{acc} = 6.24$  mT/(MV/m) and  $E_{max}/E_{acc} = 3$ ). These values of  $B_{max}/E_{acc}$  and  $E_{max}/E_{acc}$  were considered to not limit the cavities performance by the design, but to see what can be theoretically achieved with optimized resonators. The NTTF vs  $\beta$  becomes narrower while the number of accelerating gaps increases [4], the energy gain around  $\beta_{opt}$  increases because of the effective length is  $L_{eff} = n\beta_{opt}\lambda/2$ , where  $n$  represents the number of accelerating gaps and  $\lambda$  the RF field wavelength. These two effects can compensate each other or one can dominate upon the other. Figure 7 compares NTTF and figure 8 shows the energy gain of a single, double and triple spoke resonator. On the left the normalized transit time factor is plot; plotting the fig. on the right it has been assumed that SSR, DSR and TSR cavities have all the same field enhancement factors, the same frequency and the same  $\beta_{opt}$ , the energy gain is normalized over the SSR gain. DSR and TSR show a much sharper NTTF vs  $\beta$  behaviour than SSR cavity. Table 3 presents the results of the  $\beta$  optimization of the front end, comparing SSR, DSR and TSR.

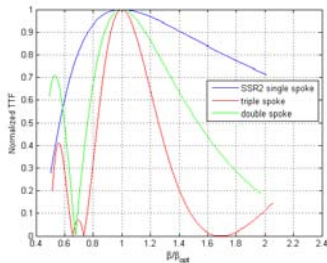


Figure 7: SSR, DSR and TSR NTTF comparison.

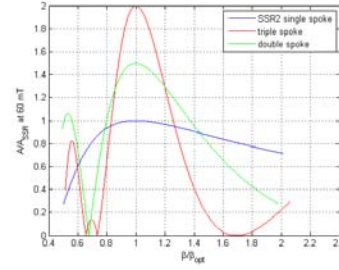


Figure 8: SSR, DSR and TSR voltage comparison.

Table 3: SSR, DSR and TSR  $\beta$  optimization comparison

	SSR	DSR	TSR
$\beta_{opt}$	0.47	0.375	N/A
N. cavities	32	30	N/A
Initial energy	40 MeV	40 MeV	40 MeV
Final energy	160 MeV	159 MeV	N/A
Tot. length	51.7 m	62.8 m	N/A
$B_{max}$	60 mT	60 mT	60 mT
$B_{max}/E_{acc}$ mT/(MV/m)	6.24	6.24	6.24

Table 3 does not report any number of cavities in the TSR column, the reason is TSR acceleration is not efficient in a sufficiently wide range of energies, so it has not been possible to find a combination of  $\beta_{opt}$  and number of cavities that leads to a final energy of 160 MeV. To consider triple spoke cavities one should increase the transition energy between SSR1 section and TSR section, but that would mean increase the number of SSR1 cavities.  $\beta_{opt}$  found for double spoke resonator is lower than the one of SSR2, this leads to a lower energy gain per cavity than expected initially, the number of cavities is lower than for SSR2 but the energy is slightly lower and the total length increases significantly.

## CONCLUSIONS

A  $\beta$  optimization has been carried out for the last spoke cavity section of Project X front end. The optimization process of  $\beta_{opt}$  for a single spoke resonator family SSR2 shown that  $\beta_{opt} = 0.47$  looks better than the previous choice, which is  $\beta_{opt} = 0.4$ . This change can save some cavities and provide the same final energy for this section, 160 MeV. Single double and triple spoke resonator performances have been compared. The best option is the single spoke resonator SSR2 because the NTTF of a multi-spoke resonator is much narrower than a single one. In the energy range considered (40-160 MeV) the most efficient resonator is the single spoke one.

## REFERENCES

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