SLAC-PUB-3000
October 1982
(T)

TRINUCLEON BOUND STATE CALCULATIONS WITH GRAZ SEPARABLE POTENTIALS*

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ABSTRACT

The results of trinucleon bound state calculations using with the recent version (Graz II) of Graz separable $N-N$ potentials are presented. Graz II potentials fit not only the latest phenomonological phases rather accurately in the energy range $0-600 MeV(E_{lab})$, but also provide a correct description of the deuteron data (E_d, p_d, Q, η). Their off-shell behavior is similar to that of the Paris potential. The results of calculations reported here are discussed and compared with those obtained for Graz I separable potentials and Yamaguchi forces used by Doleschall, and Sitenko and Karchenko. The results show that Graz II potentials provide no improvement when compared with aforementioned forces used in a trinucleon system. The $^3S_1$ Graz II force seems to be unreasonably weak.

Submitted to Physical Review C

* Work supported by the Department of Energy under contract number DE-AC03-76SF00515.

Recently a new version (Graz II)[1-2] of the Graz separable potentials (Graz I)[3-4] for the $N-N$ system was proposed. The construction of Graz II forces arose from the need to overcome the shortcomings of extant separable force models, which provide a technically convenient description of the $N-N$ forces when used in a Faddeev-type calculation of a few-body system. The advantages of the Graz II forces over the Graz I, Doleshall,[5] and Pieper[6] separable forces are discussed in detail in Refs. 1 and 2. The off-shell behavior was studied by means of Noyes[7] -Kowalski[8] half-off-shell function and compared with Reid-soft-core[9] and Paris[10] potential. A reasonable agreement between the separable and local forces was found only for the case of Graz II potentials.

It is known that separable forces which reproduce low energy two-nucleon observables tend to overbind the trinucleon system (triton for example by $3-4MeV$).[11]

The investigation of the problem of three nucleons serves as a source of additional information on the interactions between nucleons. In contrast to the problem of two nucleons at low energies the three-nucleon motion problem proves to be very sensitive to the form (off-shell behavior) of the two-particle potential. Therefore it is quite important to find a potential representing the interaction between nucleons which, besides explaining the known experimental data of the deuteron bound state and two nucleon scattering will describe correctly the motion of three-nucleons.

Since the Graz II separable potentials achieved significant improvement in fitting the on-shell two-nucleon data according to the recent $N-N$ phase shift analysis[12-13] and also care has been taken in constructing a reasonable off-shell behavior, it became interesting to use these forces within a trinucleon system. In this paper we report on several trinucleon bound state calculations with different separable forces. The forces used are listed in Table I.

Below analytic formulae for these forces are given (numerical values for the corresponding parameters can be found in references as indicated in Table I). The forces $TNPSK$, $SNPSK$, $SNPDO$ and $SNNDO$ are rank one separable forces of Yamaguchi type:

$$V(p,p') = g(p)\lambda g(p')$$

with

$$g(p) = (p^2 + \beta^2)^{-1} . \quad (1.1)$$

All other forces are rank two separable potentials:

$$V(p, p') = g_1(p)\lambda_1 g_1(p') + g_2(p)\lambda_2 g_2(p') \quad (1.2)$$

with

$$g_1(p) = (p^2 + \beta_1^2)^{-1}$$

and

$$g_2(p) = p^2(p^2 + \beta_2^2)^{-2}$$

for $TNPG1$, $SNPG1$ and $SPPG1$,

$$g_1(p) = (p^2 + \beta_{11}^2)^{-1} + \gamma_1 p^2 (p^2 + \beta_{12}^2)^{-2}$$

and

$$g_2(p) = p^2(p^2 + \beta_{21}^2)^{-2} + \gamma_2 p^4 (p^2 + \beta_{22}^2)^{-3}$$

for $SNPG2$ and $SNNG2$, and

$$g_1(p) = \alpha_1(1 + \gamma_0 p^2)(p^2 + \beta_1^2)^{-2} + \beta_1 p^2 (p^2 + \beta_2^2)^{-2}$$

and

$$g_2(p) = -\beta_1(1 + \gamma_0 p^2)(p^2 + \beta_1^2)^{-2} + \alpha_1 p^2 (p^2 + \beta_2^2)^{-2}$$

for $TNPG2$. In the case of $TNPG1$ and $TNPG2$ we use only an s-wave projection of the original forces.

We are solving a Faddeev equations of the following type[14] for the amplitudes A:

$$A_i(p) = \tau_i\left(\sqrt{K^2 + \frac{3}{4}p^2}\right) \sum_j \chi_{ij} \int_0^\infty J_{ij}(K; p, p') A_j(p') p'^2 \, dp', \quad (1.3)$$

where

$$J_{ij}(K;p,p') = \int_{-1}^{+1} \frac{g_i\left(\sqrt{p^2+\frac{1}{4}p'^2+pp'y}\right)g_i\left(\sqrt{\frac{1}{4}p^2+p'^2+pp'y}\right)}{K^2+p^2+p'^2+pp'y}dy \tag{1.4}$$

The summation in Eq. (1) runs over the spin constants

$$\chi^{(S=\frac{1}{2},I=\frac{1}{2})} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & -1 \\ -\frac{3}{2} & -\frac{1}{2} & 1 \\ -\frac{3}{2} & \frac{1}{2} & 0 \end{pmatrix}$$

in the case of charge-dependence of $N-N$ forces,

$$\chi^{(S=\frac{1}{2})} = \begin{pmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

in the case of charge-independence, and $\chi = 2$ for three identical zero-spin particles. Here S denotes total spin, I total isospin, K^2/M is the binding energy of the system of three particles, and M is the mass of the particle. The matrices χ have different multiplicities according to the rank index of the used separable potentials.

In Table II we give results for the binding energy for three identical zero-spin particles interacting via the forces as listed in Table I. The binding energy of the three-nucleon system in case of $TNPG1$ (s-wave projection) would be larger if the parameter would fit the $n-p$ triplet effective range parameter as in the case of $TNPSK$, for example. Thus we can conclude that Graz I triplet force gives too strong attraction in the three nucleon system, and the overbinding is comparable with that obtained by Yamaguchi force ($TNPSK$). In the case of Graz II triplet force we find surprisingly that for this force the trinucleon system does not bind. It is probable that the force including D-wave interaction would bind the trinucleon system. However this binding will be very small. We conclude that the $^3S_1$ Graz II force is too weak and therefore not reasonable.

In the case of $^1S_0$ Graz I forces we find that the binding energy agrees within the first two digits following the decimal point for the full potential Eq. (2) and for the same potential in which the second repulsive term is neglected. The repulsion of the second term is strong but is active at higher momenta than the action of the attractive first term. This finding indicates that the three-nucleon wave function takes full advantage of the attractive part while avoiding the repulsive part at higher momenta. Thus the apparent modification of the two-nucleon wave function at short distances (off-shell behavior) due to the repulsive part of the potential has almost no impact on the three-nucleon binding energy. This is the reason why the binding energy of the trinucleon system for the $^1S_0$ Graz forces is larger than in the case of the simple parametrizations by Sitenko-Karchenko and Doleshall. We thus conclude that the introduction of a repulsive part of the potential at higher momenta does not cure the off-shell behavior of two-nucleon wave functions in a proper way.

In Table III we present the results for the triton binding energy for a calculation with charge-independent forces. As already mentioned before, Graz I and Yamaguchi forces gives similar results. It can be expected that with a correct fit of the triplet effective range for the s-wave projection the binding of the triton (charge independence) would give even larger overbinding than the Sitenko-Karchenko parametrization. We find again that the repulsive part of the Graz I forces almost does not affect the triton binding energy.

In Table IV we give the results for the triton binding energy for charge-dependent nucleon-nucleon forces. The combination of Graz I and Doleshall forces seems to be very close to the experimental value ($E_{triton} = 8.48$). One has however to bear in mind that $^3S_1$ force (s-wave projection) would be stronger if it would fit the deuteron binding pole exactly.

Our results show that the two-nucleon off-shell ambiguity is still an open problem. Since the Graz II potentials incorporate an off-shell behavior suggested by relativistic models of the nuclear force, this finding is quite disappointing. Three alternatives can be considered:

1. The appropriate off-shell behavior has still to be found, which would provide an adequate description of a three-nucleon system.

2. There is appreciable effect of a genuine three-body force.

3. The off-shell ambiguity cannot be resolved within the present quantum mechanical models.

The third alternative gave rise to the zero-range approach by Noyes.[15] In this approach the two-body empirical on-shell information only is used to calculate the three-body observables. Calculations in this approximation for the three-nucleon problem will be presented in a paper to come.[16]

The author is indebted to Professor H. Pierre Noyes for helpful and valuable suggestions.

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Table Captions

I. List of $N-N$ separable forces used in this paper.

II. Calculated binding energy of a system of three identical zero-spin particles for different forces.

III. Triton binding energy, assuming charge-independent $N-N$ forces.

IV. Triton binding energy, assuming charge-dependent $N-N$ forces.

TABLE I

Partial Wave	Force	Notation used in the text
$^3S_1$	Graz II[1,2]	TNPG2
$^3S_1$	Graz I[2,3]	TNPG1
$^3S_1$	Sitenko-Karchenko[14]	TNPSK
$^1S_0$ (n-p)	Graz II[1,2]	SNPG2
$^1S_0$ (n-p)	Graz I[2,3]	SNPG1
$^1S_0$ (n-p)	Sitenko-Karchenko[14]	SNPSK
$^1S_0$ (n-p)	Doleshall[2,5]	SNPDO
$^1S_0$ (p-p)	Graz II[1,2]	SPPG2
$^1S_0$ (p-p)	Graz I[2,3]	SPPG1
$^1S_0$ (n-n)	Doleshall[2,5]	SNNDO

TABLE II

Partial Wave	Force	Binding energy (MeV)
$^3S_1$	TNPG2	no binding
	TNPG1	19.42
	TNPSK	25.40
$^1S_0$	SNPG2	5.40
	SPPG2	8.52
	SNPG1	3.94
	SPPG1	5.04
	SNPSK	2.49
	SNPDO	2.23
	SNNDO	1.23

TABLE III

Forces		Binding Energy (MeV)
$^3S_1$	$^1S_0$	
TNPSK	SNPSK	11.98
TNPG1	SNPG1	10.51

TABLE IV

Forces			Binding Energy
$^3S_1$	$^1S_0$(n-p)	$^1S_0$(n-n)	
TNPKS	SNPKS	SNNDO	10.80
TNPG1	SNPDO	SNNDO	8.4