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T. Muta and K. Yamawaki

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PREFACE

In recent years we have gained insights into much wider possibilities for the dynamical symmetry breaking than ever thought of. In a word, field theories with a nontrivial ultraviolet fixed point and/or a large anomalous dimension have become rather serious candidates for the framework of the dynamical electroweak symmetry breaking.

In December of 1989, on the occasion that Professor Y. Nambu, the very originator of dynamical symmetry breaking, was visiting Japan, we organized 1989 Workshop on Dynamical Symmetry Breaking at Nagoya University. The purpose of this workshop was to have intensive discussions on the new development of this exciting field, particularly that after our preceding workshop, 1988 International Workshop on New Trends in Strong Coupling Gauge Theories(SCGT 88), Nagoya, August 24-27. More than 60 people gathered at the workshop. About 30 papers were presented, which are contained in this volume.

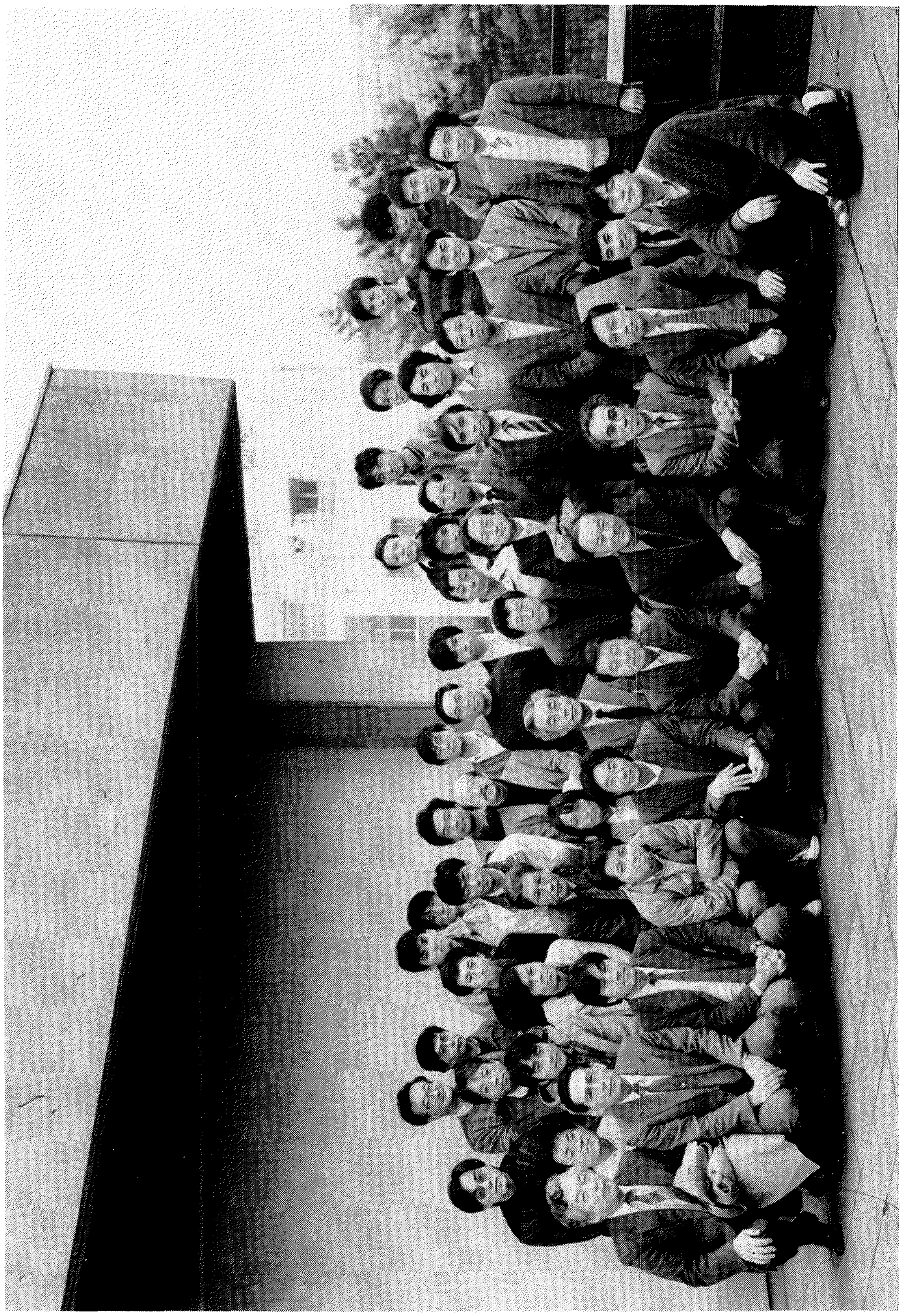
The workshop was sponsored by the Particle Theory Group at Nagoya University and was supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture(No.63540221 and No.62540202). We would like to express our sincere thanks to the Ishida Foundation for financial support and to the High Energy Experiment Groups at Nagoya University. Special thanks are due to young physicists at Nagoya University for their devoted assistance to make the workshop a success. Finally but not the least, we would like to thank Mrs. M. Kitajima for her patient help in preparing the workshop.

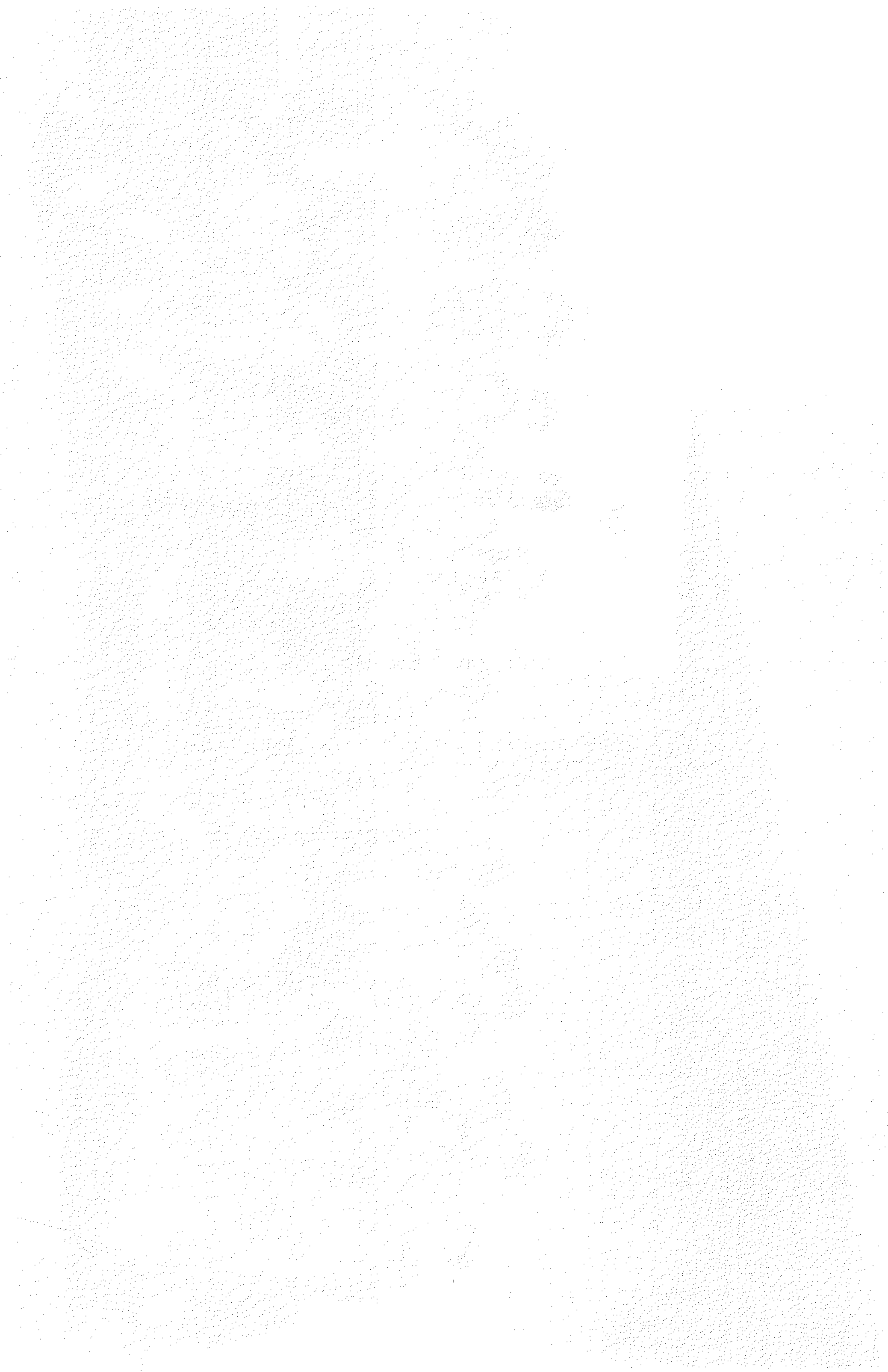
April 1990

Editors

T. Muta

K. Yamawaki





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Model Building Based on Bootstrap Symmetry Breaking*

Yoichiro Nambu

University of Chicago

1. At the Nagoya Workshop last year, I presented my ideas concerning the nature of the Higgs boson in the standard model [1]. It was motivated by my long-standing interest in the BCS mechanism as the origin of masses. In essence it boils down to the proposition that the Higgs boson is a top-antitop bound state.

This idea has also been advanced by the Nagoya-Kiev group [2], and some detailed calculations have have been carried out by the Fermilab group [3]. Independtly of the motivation that led me to the model, it looks very natural now that the experimental searches for the top quark are raising the top quark mass higher and higher. (I understand the current lower limit for the top mass from the Fermilab experiment to be in the W-Z mass range.) This makes the Yukawa coupling of the top quark larger and larger, and therefore the picture emerges that the Higgs field is largely made up of the top-antitop component.

I will briefly recapitulate here my ideas about the BCS mechanism, which consist of several ingredients. By the BCS mechanism I mean dynamical generation of the fermion mass due to a short range interaction, together with the Goldstone (pi) and Higgs (sigma) bosons as collective modes. The mass scale is usually small compared to the energy scale at which the original interaction is considered. Remarkably the masses of these low energy modes satisfy a simple relation

$$m_H : m_f : m_G = 2 : 1 : 0, \qquad (1)$$

in the short range, or bubble approximation, limit. Actually this is a special case of a more general sum rule

$$m_1{}^2 + m_2{}^2 = 4\, m_f{}^2, \qquad (2)$$

which is essentially a completeness relation for the composite two-fermion operators. In nonrelativistic examples of superconductivity and ^{3}He superfluidity, these relations are known to be satisfied.

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Once the low energy modes are identified, one can write down an effective Ginsburg-Landau theory for them. Because of the mass relation (1), the Higgs self-coupling λ and the Yukawa coupling f are related by $\lambda = m_f$. The vacuum expectation value v, or the "pion decay constant", represents the high energy scale of the original BCS Hamiltonian, and f is by definition the ratio between the two energy scales.

The simple mass relations suggest a kind of broken supersymmetry inherent in the BCS mechanism. In fact the effective GL Hamiltonian can be factorized as a product of fermionic opeerators as in the real supersymmetry, provided one ingnores the kinetic terms. I will discuss relativistic generalizations of this "quasi-supersymmetry" at the end.

There are also two physical concepts that I have proposed to add to the BCS mechanism. One is tumbling, borrowing from the work of Dimopoulos, Raby and Susskind [4]. The other is what I would call bootstrap. Tumbling means that the Higgs boson associated with the chiral symmetry breaking in turn provides a new attractive interaction that can trigger a second symmetry breaking at a lower energy than the original, and so on. Indeed one such example of tumbling is the formation of nuclei and nucleon pairing in nuclei, due to the existence of the sigma meson which is the "Higgs" boson associated with the chiral symmetry breaking in QCD. One could say that superconductivity is another example:it is induced by the phonons which are the Goldstone bosons associated with the breaking of translational and rotational invariance in crystals.

Instead of a hierarchical chain of symmetry breakings, one can contemplate a theoretical possibility of bootsrtap, namely the possibility that the Higgs boson is the cause of attraction bewteen fermions that is responsible for its very existence. I proposed the hypothesis that this applies to the standard model, which implies that the fermions, gauge bosons, and the Higgs fields that appear in the standard model are dynamically closed among themselves without a need to refer to the possible underlying structure. The condition for bootstrap was expressed by the requirement that the standard theory be free from quadratic divergences on the ground that the theory should be insensitive to the cut-off, which presumbaly represent the energy scale of the underlying structure. Unfortunaely the formulas I used to predict the Higgs and top masses were not correct. Later I will discuss a modified version of the hypothesis.

2. Dyson-Schwinger equation, hard mass, and soft mass

The mass of a fermion can be spontaneouly generated when there is a sufficiently strong attractive interaction, as is known from various examples.

In a renormalizable theory, the interaction may be due to a vector or a scalar boson exchange. There are two types of Feynman diagrams that contribute to the fermion self-energy viewed as the potential energy due to fermions in the vacuum, corresponding to the direct (tadpole) and exchange (usual self-energy) terms which are respectively quadratically and logarithmically divergent in the lowest order. Pursued to all orders, the latter leads to the Dyson-Schwinger equation which may be treated nonperturbativley at various levels of approximation.

For the vector interaction, only the exchange term is nonzero. For the scalar interaction, both terms are present, but the quadratically divergent direct term (tadpole) is dominant, whereas the exchange term has the wrong sign to generate a mass by itself. (In general the sign alternates with the degree of divergence.) The latter fact was the basis for the theory of Sakata and Pais [5], who independently proposed to render the electron self-energy finite by the cancellation between electromagnetic forces and the cohesive force due to a hypothetical scalar field. Their theory is not relevant here because of the neglect of the tadpole, as well as the lack of chiral invariance.

Speaking of old theories, Weisskopf [6] was the first to show the logarithmic nature of electron's self-energy, but he also interpreted it as the result of a cancellation between two quadratically divergent physical effects. In terms of Feynman diagrams, one corresonds to a loop made up of on-shell electron and off-shell photon, and the other made up of off-shell electron and on-shell photon. Actually these quadratic terms belong only to the wave function renormalization, so again this analysis is not of interest here.

A similar but more instructive way of analyzing the self-energy may be the following formal manipulation: Set the external electron momentum to zero (this will not change the divergence properties), and split the two progpagators in the loop as

$$1/[k^2 - m_f^2)(k^2 - m_v^2)]$$

$$= [1/(k^2 - m_f^2) - 1/(k^2 - m_v^2)]/(m_f^2 - m_v^2). \qquad (3)$$

When inserted into the Dyson-Schwinger equation, each term yields a quadratically divergent integral like that for a tadpole or a four-fermion interaction. In fact the two terms look like the fermion loop and boson loop contributions to the tadpole, and their signs are correct for such an interpretation provided that $m_v > m_f$, i. e., a sufficiently short range interaction.

The above excercise was to compare the vector interaction with the scalar and four-fermion interaction cases, and to see how the latter might be interpreted as effective low energy theories of the former.

The Dyson-Schwinger equation for the exchange term gives a running mass $m(p)$ as a nonperturbative solution, whereas the tadpole supplies a bare mass m0 which gets dressed up and becomes a running mass by the exchange term. The bare mass serves as the boundary condition for $m(p)$ at large momentum. If m0 is absent as in the vector interaction, $m(p)$ goes to zero like $\sim 1/p^2$ (up to log factors) as is well known, so the mass is soft. Are the masses (current masses) of quarks and leptons soft or hard? This should be an important question which has a bearing on whether the Higgs field is elementary, or comes from a gauge theory like technicolor.

One way in which the dressing effect manifests itself is in the mass ratio between the fermion and the Higgs. In the bubble approximation of the four-fermion interaction, it was 1:2. In general one has to solve a Bethe-Salpeter type equation in the scalar channel, but even in the context of bubble approximation, one can see the effect of the softness of the fermion mass in the bubble. It is easy to derive the expression

$$m_H{}^2 = 4\langle m_f{}^2\rangle, \tag{4}$$

where the right-hand side is an average with respect to a weight $dp^4/(p^2 + m_f{}^2)^2 \sim dp/p$. Clearly $m_H \leq 2m_f$ in general. This is in agreement with the more elaborate calculations by Yamawaki et al. [7] and by suwa and So [8]. If one adopts the standard behavior

$$m(p) \sim 1/p^\gamma,\ \gamma = 1 - \surd(1 - \lambda), \tag{5}$$

where λ characterizes the coupling strength in the Dyson-Shwinger equation, one gets

$$m_H{}^2/m_f{}^2 = 4(1 - \exp(-x))/x,$$

$$x = 4/(1 + \surd(1 - \lambda)). \tag{6}$$

Even in the limit $\lambda = 0$, this gives $m_H{}^2/m_f{}^2 = 2(1 - \exp(-2))$.

3. Models of bootstrap

First consider nonrelativistic cases like supercondcuctivity. The typical gap equation can be written as

$$1 = (4f^2v^2/m^2)\ 2sh^{-1}(\Lambda/m_f). \tag{7}$$

Usually the factor in front of $2sh^{-1}$ is <V>N, the product of the average potential <V> and the density of states N of fermions. Here the short range potential is represented by a propagator $1/(m^2 + \nu^2k^2) \sim 1/m^2$ (ν = velocity) and a coupling constant f, whereas N is related to the Higgs condensate v by $N = 4v^2$ in the Ginzburg-Landau translation.

Now apply the bootstrap concept, and say that the propagator is that of the sigma (or Higgs) boson, in which case $m_\sigma = 2m_f = 2fv$. So the factor in Eq.(7) = 1, and the solution is $\Lambda/m_f \sim 1/2$, $\Lambda/m_\sigma \sim 1$. In other words, bootstrap is a self-consistent picture that makes m_σ the only available scale parameter.

Let us next turn to relativistic dynamics. Assume a U(1) * U(1) set of massless fermion fields and massive spin 0 fields (bare mass m_0) with Yukawa coupling f. As was shown above, the fermion mass is generated by the tadpole and exchange diagrams. The equation for m_0 from the tadpole takes the form

$$m_f = m_fF, \text{ or } 1 = F, \text{ where}$$
$$F = (f^2/m_\sigma{}^2)[\Lambda^2 - m_f{}^2 \ln(\Lambda^2/m_\sigma 2)]/(4\pi^2). \tag{8}$$

The exchange diagram dresses m_f, but this can be interpreted as the dressing of the vertex f. The scalar and pseudoscalar masses m_σ and m_π must also include their own self-energies:

$$m_i 2 = m_0{}^2 + S_i(p)\ |p^2{=}m_i{}^2,$$

$$S_\sigma = -(f^2/4\backslash\pi^2)[\Lambda^2 + (p^2/2 - 3m_f{}^2)\ln(\Lambda^2/m_f{}^2)]$$

$$S_\pi = -(f^2/4\pi^2)[\Lambda^2 + (p^2/2 - m_f 2)\ln(\Lambda^2/m_f{}^2)] \tag{9}$$

The condition $m_\pi = 0$ serves to relate m_0 to Λ, or to eliminate it between m_π and m_σ:

$$m_\sigma{}^2 - m_\pi{}^2 = m_\sigma{}^2 = S_\sigma(m_\sigma) - S_\pi(0), \text{ or}$$

$$m_\sigma{}^2\ (1 + (f^2/8\pi^2)\ln(\Lambda^2/m_f{}^2)) = 2m_f{}^2\ (f^2/4\pi^2)\ln(\Lambda^2/m_f{}^2)$$

(10)

This gives the mass ratio m_σ/m_f in terms of f and Λ/m_f, which can be seen to be ≤ 2. When m_σ is substituted in Eq.(9), one gets the gap equation determining m_f. Or one may regard it as an equation for the "vacuum expectation value" v defined by $v = m_f/f$.

The above model is not satisfactory for two reasons: It has a bare mass, and it does not have a quartic coupling, which is not "natural". If the masses are to be generated dynamically in a renormalizable theory, one may allow all dimensionless parameters but no bare mass. The mass scale will then be related only to a scale parameter or a cut-off.

One is thus led back to the conventional Higgs (or Ginzburg-Landau-Gell-Mann-Levy) Lagrangian, except that the vacuum expectation value v must be purely dynamical, i.e., no bare v_0, only the tadpoles. Equating v with the tadpoles, one gets the gap equation

$$1 = (1/m_H^2)\,\Sigma\, c_i\, g_i^2\, (\Lambda^2 - m_i^2 \ln(\Lambda^2/m_i^2))/(16\pi^2) \qquad (11)$$

The sum runs over all fermions and bosons that can couple to the Higgs boson. One may conveniently define the coupling constants in such a way that the masses are given by $m_i = g_i\, v$. The numerical coefficeients c_i are then 4 for each Dirac fermion, 3/2 for the Higgs scalar, 1/2 for each pseudoscalar, and 3 for each gauge boson. Eq.(11) is an equation for v, given the coupling constants g_i.

Assuming the Λ^2 terms to dominate the sum, one first gets the inequality

$$\Sigma\, c_i\, g_i^2 \gtrsim 0, \text{ or } \Sigma\, c_i\, m_i^2 \gtrsim 0. \qquad (12)$$

The presence of quadratic terms means, however, a fine tuning of Λ. It was also argued before that, from the bootstrap point of view, the gap equation should not sensitively depend on Λ since the low energy parameters should be self-consistent among themselves, without a need to refer to an unknown high energy scale. With this ansatz, Eq.(12) becomes an equality

$$\Sigma\, c_i\, m_i^2 = 0. \qquad (13)$$

One is then left with

$$1 = -(1/m_H^2)\,\Sigma\, c_i\, g_i^2\, m_i^2 \ln(\Lambda^2/m_i^2)/(16\pi^2), \text{ or}$$

$$\Sigma\, c_i\, m_i^4 + 16\pi^2\, m_H^2\, v^2/\ln(\Lambda^2/\mu^2) = 0. \qquad (14)$$

where a common mass μ was inserted in the logarithms for simplicity. One first observes from Eq.(14) that

$$\Sigma\, c_i\, m_i^4 \leq 0. \qquad (15)$$

Eqs.(13) and (15) constrain the mass values. For fixed Λ, Eqs.(13) and coupling constants g_i^2 in general behave like $1/\ln(\Lambda/\mu)$, so all the terms are of the same order.

The specific application of these conditions to the standard model lead to the following results.

Eqs.(13) and (15) read

$$m_t^2 = (m_H^2 + 2\, m_W^2 + m_Z^2)/4,$$

$$m_t^4 \leq (m_H^4/2 + 2\, m_W^4 + m_Z^4)/4. \qquad (16)$$

Wtih the known values m_W = 80 Gev, M_Z = 91 Gev, there are two regions of compatibility,

$$m_t \leq 80 \text{ Gev}, m_H \leq 64 \text{ Gev};$$

$$m_t \geq 150 \text{ Gev}, m_H \geq 195 \text{ Gev}. \qquad (17)$$

The first region seems excluded by experiment. If Eqs.(13) and (14) are used, one can solve for m_t and m_H as a function of Λ. The mass values turn out to be pushed considerablly higher for standard choices of Λ: m_t = 230 Gev, m_H = 440 Gev for $\Lambda = 10^{19}$ Gev (Planck); m_t = 260 Gev, m_H = 500 Gev for $\Lambda = 10^{15}$ Gev (GUTS); and getting even higher as Λ is further lowered. The general trend is similar to the results of Bardeen et al. [3].

There remain questions of principle and questions of numerical reliability. These have not been addressed yet. The most serious one may be that concerning the quadratic divergence condition. For it to make sense, one must have a prescription for handling higher order terms as well. (See [9] for computations using dimensional regularization, but the physical meaning of such a procedure is not clear.) If it makes sense at all, it may perhaps be understood in terms of a dynamical supersymmetry like quasi-supersymmetry. The quantitative results like those given above of course will change when renormalization correctons are included, but one must also have a definite prescription for them.

4. Quasi-supersymmetry

I will now briefly dicuss a different topic. Quasi-supersymmetry was found in non-relativistic BCS mechanisms. Whatever the orign of the symmetry, can it be made relativistic? This question has led to the following results [10].

The static part of the effective Landau-Ginzburg Hamiltonian satisfying the BCS mass ratios can in general be factorized in terms of fermionic operators

$$Q = \Pi\,\psi + iW\,\psi^+,$$

$$Q^+ = \Pi^+\psi^+ - iW\,\psi,$$

$$W = G(\phi^+\phi - v\hat{}2),$$

$$H = \{\underline{Q}, \underline{Q}^+\}/n. \qquad (18)$$

ψ and ψ^+ are n-component fermion fields, ϕ and ϕ^+ are n by n complex matrix Higgs fields acting on the former, $\Pi = \partial\phi/\partial t$ and $\Pi^+ = \partial\phi^+/\partial t$ are their canonical cojugates. The underline indicates spatial integral.

The fermionic currents in (extended) supersymmetry, on the other hand, are made up of the following types of pieces in general.

$$Q^\mu{}_R(x) = \partial_\nu\,\phi(x)\,\sigma^\nu\,\sigma^{\sim\mu}\,\psi_R(x),$$

$$Q^\mu{}_R(x) = F_{\lambda\rho}(x)\,\sigma^\lambda\,\sigma^{\sim\rho}\,\sigma^\mu\,\psi_L(x),$$

$$Q^\mu{}_R(x) = W\,\sigma^\mu\,\psi_L(x), \qquad (19)$$

and similar forms with L and R interchanged. The first and second lines represent kinetic currents for chiral and gauge multiplets, the third one represents the potential for the scalar field. The Q's have internal indices which are suppressed. Anticommutators $\{Q, Q^+\}$ generate the Poincare algebra, $\{Q, Q\}$ and $\{Q^+, Q^+\}$ generate central charges.

Comparing Eqs.(18) and (19), one can see the correpondence: $\psi \to \psi_R$, $\psi^+ \to \psi_L$; the first term of Eq.(18a) -> Eq.(19a), the second term -> Eq.(19c).

So the relativistic generalization seems easy. However, the Higgs potential in Eq.(18) is not the Kaehler potential in Eq.(19c), the main reason the masses do not come out equal. Since one does not have exact supersymmetry, one has to decide which part of supersymmery relations to keep and which part to give up. I have proposed to keep one Poincre algebra (N=1 subalgebra), and give up all others. That means, in addition to the energy part of $\Sigma \{Q_i, Q_i^+\}$, the momentum part of it must come out right. It turns out that the follwoing conditions must be met:

a. Matching of fermionic and bosonic degrees of freedom, in order that kinetic energies of the various fields in the Hamiltonian have the same weight;

b. Absence of interaction pieces in the momentum part of the Poincare agebra. For this, one must have fermion, Higgs, and gauge fields all present in such a way that the interactions arising from various cross terms in the anticommutators cancel each other for the momentum algebra, but not for the energy algebra. The Yukawa and gauge couplings must then be related, but the relation does not seem to be unique. It is due to the fact that the currents in our cases carry gauged quantum numbers, and there are ambiguities in the gauge-invariant definition of anticommutators.

The physical meaning of relativistic quasi-supersymmetry thus defined is unclear, but some models satisfying the above criteria can be constructed. They seem to have some resemblance to the hidden symmetry scheme in chiral dynamics. It should be interesting if Higgs and gauge fields both are found to play dynamical roles in the BCS mechanism and lead to quasi-supersymmetry. One would also hope that the quasi-supersymmetry eliminate the quadratic divergences like in real spersymmetry, but this does not seem to be the case in general.

A final comment concerns an observation on quasi-supersymmmetry in SU(5) grand unification. The degree matching beween fermions and bosons works out fine if there are three generations of fermions $(1 + 5 + 10^{*}) * 2 * 3 = 96$ against the SU(5) gauge fields $(24 * 2)$ and a set of complex adjoint Higgs fields $(24 * 2)$. The Higgs fields can break SU(5) down to SU(3)*SU(2)*U(1), but there are no Yukawa couplings, so fermions remain massless at this energy scale. The Higgs fields of the standard model presumbaly will arise later as composites.

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FUN WITH LARGE ANOMALOUS DIMENSION IN DYNAMICAL SYMMETRY BREAKING*)

Koichi Yamawaki

Department of Physics, Nagoya University
Nagoya 464-01, Japan

Abstract

We advocate dynamical symmetry breaking with large anomalous dimension for triggering the electroweak symmetry breaking. Based on the ladder Schwinger-Dyson equation, we explicitly obtain spontaneous-chiral-symmetry-breaking solution of QED-like theories, QCD-like theories and these gauge theories plus four-fermion interactions ("gauged Nambu–Jona-Lasinio models"), and argue the phase structure associated with them. Although all such theories equally yield the sigma model as the low energy effective theory, the parameters of the effective theory are determined to be different depending on the high energy behavior of the original theories, i.e., the anomalous dimension plays a crucial role. Implications of the large anomalous dimension for technicolor and "top-mode standard model" (dynamical electroweak symmetry breaking due to a top quark condensate) are fully discussed. In particular, the top-mode standard model predicts a very large mass for the top quark, typically of $m_t \simeq 250$GeV, without affecting the weak isospin relation $\rho \equiv m_W^2/m_Z^2 \cos^2\theta_W \simeq 1$, and a Higgs boson as a $\bar{t}t$ composite with a mass $m_H \simeq \sqrt{2}m_t \simeq 350$GeV.

1. Introduction

As it stands now, the standard model is a very successful framework for describing elementary particles in the low energy region, say, less than 100GeV. However one of the most mysterious part of the theory, *the Origin of Mass*, has long been left unexplained. Actually, mass of *all* particles in the standard model is attributed to a *single* order parameter, the vacuum expectation value (VEV) of the Higgs doublet. Thus the problem of the origin of mass is simply reduced to understanding the dynamics of the Higgs sector.

Here we note that formation of the Higgs VEV is a second order phase transition. The situation thus very much resembles the Ginzburg-Landau (GL)'s *macroscopic* theory for the superconductivity, the mysterious parts of which were eventually explained by the *microscopic* theory of Bardeen-Cooper-Schrieffer (BCS): The GL's phenomenological order parameter was replaced by the Cooper pair condensate due to the short range attractive forces.

A similar thing has also happened to the hadron physics where the sigma model description by Gell-Mann and Levy (GML) works very well as far as the low energy (macroscopic) phenomena are concerned, while the deeper understanding of it was first given by Nambu and Jona-Lasinio (NJL)[1] based on the analogy with the BCS dynamics (short range four-fermion attractive interaction). Nowadays people believe that essentially the same phenomena as described by the NJL paper takes place in the microscopic theory for hadrons, QCD, where the VEV of σ, the GML's

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order parameter $\langle\sigma\rangle = f_\pi = 93$MeV, has been replaced by the quark-antiquark pair condensate $\langle\bar{q}q\rangle = O(f_\pi^3)$, an analogue of the Cooper pair condensate, formed by the attractive color forces. The Nambu-Goldstone (NG) boson, the pion, is now a composite state of the quark and antiquark. This is actually the prototype of the dynamical symmetry breaking (DSB).

In fact Higgs sector in the standard model is precisely the same as the sigma model except that $\langle\sigma\rangle = f_\pi = 93$MeV is now replaced by the Higgs VEV= $F_\pi = 250$GeV, roughly a 2600 times scale-up. One is thus naturally lead to speculate[2] that there should exist a microscopic theory for the Higgs sector, with the Higgs VEV being replaced by the fermion-antifermion pair condensate due to yet another strong interaction called Technicolor (TC). It is well known that the original version of TC was too naive to survive the FCNC (flavor-changing neutral current) syndrome[3]. It was not the end of the story, however. QCD-like theories (simple scale-up's of QCD) turned out not to be the unique candidate for the underlying theory of the Higgs sector. In fact, a TC model with a large anomalous dimension, $\gamma_m = 1$, was proposed to solve the FCNC problem, based on the ladder Schwinger-Dyson (SD) equation for "QED-like" theories (non-asymptotically gauge theories with a nontrivial ultraviolet fixed point or "walking" gauge theories)[4,5,6,7].

It was further pointed out[8] that "gauged NJL models" (gauge theories plus four-fermion interactions) in the ladder approximation[9] possess an even larger anomalous dimension, $2 > \gamma_m > 1$, near the critical line[10,11]. Such a large anomalous dimension, particularly $\gamma_m \simeq 2$ (in QCD plus four-fermion interactions), enables one to construct a model of a top quark condensate $\langle\bar{t}t\rangle \neq 0$ as an underlying theory of the Higgs sector without destroying the weak isospin relation $\rho \equiv m_W^2/m_Z^2\cos^2\theta_W \simeq 1$ ("top-mode standard model")[12,13].

The top-mode standard model predicted[12] a very large mass of the top quark

$$m_t \simeq 250\text{GeV} \tag{1.1}$$

as a typical value ($\delta\rho \equiv \rho - 1 \simeq 0.02$), and also a spinless $\bar{t}t$ bound sate (Higgs boson) with a mass

$$m_H \simeq 2m_t. \tag{1.2}$$

Although these values seemed to be absurd at the time when the paper[12] was written, recent experimental situation seems to be getting more and more closer towards the above prediction. Similar ideas were also developed by Terazawa[14] and Nambu[15], with the same result as (1.2) and somewhat different values for m_t. Several groups[16,17,18] have further studied consequences of the top-mode standard model and confirmed the large $m_t (> 200\text{GeV})$ similar to that in Ref.12.

In this talk I would like to emphasize that there in fact exist many varieties of microscopic theories with DSB, i.e., QCD-like theories, QED-like theories, QCD/QED-like theories plus four-fermion interactions, etc., which yield the *same* kind of low energy effective (macroscopic) theory, the sigma model (or the Higgs sector). How can we, then, distinguish among these varieties of theories by only looking at the low energy phenomena? We shall argue that this is the very place where the anomalous dimension comes into play.

One might immediately raise an objection that the anomalous dimension is only related to the high energy behavior of the theory and has little to do with the low energy parameters we are talking about. However, a crucial point is that important low energy parameters in DSB are often given in terms of integral of a certain function of the dynamical mass of the fundamental fermion $\Sigma(p^2)$, and hence can be affected by the high energy behavior of $\Sigma(p^2)$,

$$\Sigma(p^2) \overset{p \gg m}{\sim} \frac{m^3}{p^2}\left(\frac{p}{m}\right)^{\gamma_m}, \tag{1.3}$$

with γ_m being the anomalous dimension (Eq.(1.3) is a consequence of the operator-product expansion (OPE) and the renormalization-group equation)[7]. Typical examples of such are the "decay constant" of composite NG boson F_π (see (3.16)) and the quark/lepton masses in the TC models (see (2.15)). Quite recently we have calculated[19] the "sigma model parameters", F_π, $g_Y = m/F_\pi$ (Yukawa coupling of π and σ) and m_σ (mass of σ), in terms of $m \equiv \Sigma(0)$ (or more precisely $m \equiv \Sigma(m^2)$) in the (QED-like) gauged NJL model in the ladder approximation. Actually these parameters do vary according to the change of γ_m along the critical line. Thus we might be able to "measure" the anomalous dimension or discriminate the microscopic DSB theories through the low energy parameters in the effective theory.

Phenomenological implications of the DSB with a large anomalous dimension will also be discussed. Special emphasis will be placed on the top-mode standard model in which the above calculation[19] implies

$$m_H \simeq \sqrt{2} m_t \simeq 350\text{GeV} \tag{1.4}$$

instead of the original prediction (1.2), due to the effect of a small (but non-zero) QCD gauge coupling. The prediction (1.1) and (1.4), although crude because of the ladder approximation, could be tested experimentally in 1990's, which I hope will open the window to deeper understanding of *the Origin of Mass.*

2. Dynamical Symmetry Breaking with $\gamma_m = 1$ (QED-like Theories)

In order to demonstrate virtue of the large anomalous dimension, we start with a brief review of TC with $\gamma_m = 1$[4] which simultaneously resolved the problems of FCNC and light technipions (pseudo NG bosons)*). The dynamical model we are based on is the ladder SD equation of QED, which takes the form (in Landau gauge and in Euclidean space)

$$\Sigma(p^2) = m_0 + 3e^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{(p-q)^2} \frac{\Sigma(q^2)}{q^2 + \Sigma^2(q^2)}, \tag{2.1}$$

where the m_0 and g are the bare mass of the fermion and the gauge coupling constant, respectively. Eq.(2.1) has been extensively studied by many authors. In particular, it was clearly demonstrated in the cutoff version[20] that there exists a critical point $\alpha(\equiv e^2/4\pi) = \alpha_c(\equiv \pi/3)$ above which ($\alpha > \alpha_c$) the chiral symmetry is spontaneously broken, with $\Sigma(0)$ being non-zero for $m_0 = 0$.

This is easily seen by converting (2.1) (after angular integration) into a differential equation and the infrared (IR) and ultraviolet (UV) boundary conditions (BC's):

$$(x\Sigma(x))'' + \frac{\lambda\Sigma(x)}{x + \Sigma(x)^2} = 0 \qquad (x \equiv p^2, \lambda \equiv \frac{3\alpha}{4\pi}), \tag{2.2}$$

$$\lim_{x\to 0} x^2 \Sigma(x)' = 0 \qquad \text{(IRBC)}, \tag{2.3}$$

$$(x\Sigma(x))'\Big|_{x=\Lambda^2} = m_0(\Lambda) \qquad \text{(UVBC)}. \tag{2.4}$$

A useful analytical solution to (2.2) with (2.3) have been obtained[21] in a linearized approximation where $\Sigma(x)$ in the denominator of (2.2) is replaced by a constant mass m; $\lambda/(x+\Sigma^2(x)) \to \lambda/(x+m^2)$. This is a very good approximation particularly for large $\Lambda(\gg m)$ which we are interested in. Out of two linearly independent solutions to (2.2) in this approximation, we have a unique solution written in terms of the hypergeometric function $\mathrm{F}(1/2-\gamma', 1/2+\gamma', 2; -x/m^2)$,

*) For detailed discussions see Ref.7.

with $\gamma' \equiv i\gamma \equiv (1/2)\sqrt{1-\lambda/\lambda_c}$ ($\lambda_c \equiv 3\alpha_c/4\pi = 1/4$), which satisfies the IRBC (2.3), while the other solution is not (divergent at $x=0$). Thus we have[21]

$$\Sigma(x) = \xi m \mathrm{F}(\frac{1}{2}-\gamma', \frac{1}{2}+\gamma', 2; -\frac{x}{m^2}), \tag{2.5}$$

where ξ is a normalization constant of order $O(1)$ ($\Sigma(0)=\xi m$)*). The asymptotic form of (2.5) is given by

$$\Sigma(x) \overset{x\gg m^2}{\simeq} m\frac{\Gamma(2\gamma')}{\Gamma(\frac{1}{2}+\gamma')\Gamma(\frac{3}{2}+\gamma')}\left(\frac{x}{m^2}\right)^{\gamma'-\frac{1}{2}} + (\gamma' \leftrightarrow -\gamma') \qquad (0<\lambda<\lambda_c), \tag{2.6}$$

$$\frac{2}{\pi}m\left(\frac{x}{m^2}\right)^{-\frac{1}{2}}\left(\ln\frac{x}{m^2}+2(\ln 4-1)\right) \qquad (\lambda=\lambda_c), \tag{2.7}$$

$$m\left(\frac{x}{m^2}\right)^{-\frac{1}{2}}\left[\frac{\coth\pi\gamma}{\pi\gamma\left(\gamma^2+\frac{1}{4}\right)}\right]^{\frac{1}{2}}\sin\left(\gamma\ln\frac{x}{m^2}+2\gamma(\ln 4-1)\right) \qquad (\lambda>\lambda_c). \tag{2.8}$$

It is evident that (2.4) for $m_0\equiv 0$ is not satisfied by (2.6), behaving as $\Sigma(p^2)\sim p^{-1+\sqrt{1-\lambda/\lambda_c}}$, nor by (2.7). (These solutions are actually the explicit chiral-symmetry breaking solutions with $m_0(\Lambda)\neq 0$ (and $m_0(\Lambda)\to 0$ as $\Lambda\to\infty$))[20]. Only the oscillating solution for $\lambda>\lambda_c$ satisfies the UVBC (2.4) for $m_0(\Lambda)\equiv 0$, to be identified with the spontaneous chiral-symmetry breaking ($S\chi SB$) solution. Eq.(2.4) with $m_0(\Lambda)\equiv 0$ now reads (scaling relation)

$$m = \Lambda\cdot 4e^{-\frac{n\pi}{\sqrt{\lambda/\lambda_c-1}}} \quad (n\geq 1), \tag{2.9}$$

with $n=1$ being the ground state solution.

Eq.(2.9) requires a nontrivial dependence of λ on the cutoff Λ if m is required to be kept finite for $\Lambda\to\infty$;

$$\begin{aligned}\frac{\lambda(\Lambda)}{\lambda_c} &= 1+\pi^2\left(\ln\frac{4\Lambda}{m}\right)^{-2} \qquad (n=1)\\ &\to 1 \qquad (\Lambda/m\to\infty).\end{aligned} \tag{2.10}$$

This cutoff dependence of the $S\chi SB$ solution has been nicely interpreted by Miransky[21] in the sense of the continuum limit of lattice gauge theories, leading to a new field-theoretical insight; the critical point $\lambda=\lambda_c$ should be regarded as a nontrivial UV fixed point, defining the continuum limit of the theory. In fact, from (2.10) we obtain

$$\beta(\lambda(\Lambda)) \equiv \Lambda\frac{\partial\lambda(\Lambda)}{\partial\Lambda} = -\frac{1}{2\pi}\left(\frac{\lambda(\Lambda)}{\lambda_c}-1\right)^{3/2} \qquad (\lambda>\lambda_c), \tag{2.11}$$

while $\beta(\lambda)=0$ for $\lambda<\lambda_c$. Eq.(2.9) now takes a characteristic form to the nonperturbative mass generation; $m=\Lambda\cdot\exp\left(\int^{\lambda(\Lambda)}\frac{d\lambda'}{\beta(\lambda')}\right)$. Mass scale m has been acquired through the dimensional transmutation[4]. If this β-function can be identified with that of the continuum theory, we may set $\beta(\lambda(\mu)) = -\frac{1}{2\pi}\left(\frac{\lambda(\mu)}{\lambda_c}-1\right)^{3/2}$, where $\lambda(\mu)$ is the renormalized coupling at the renormalization point μ**).

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*) ξ may be determined by requiring $\Sigma(m^2)=m$, which yields $\xi = F(1/2-\gamma', 1/2+\gamma'; -1)^{-1}$ ($\simeq 1.1$ for $\gamma'\simeq 0$). For simplicity we shall set $\xi=1$ hereafter.

**) This would contradict a formal proof that $\beta(\lambda(\mu))\geq 0$ based on the spectral function representation[22]. It is actually a highly dynamical problem how $\beta(\lambda(\Lambda))$ is related to $\beta(\lambda(\mu))$. We shall come back to this point later.

Still another amazing fact is that the anomalous dimension of the fermion bilinear operator, $\gamma_m(\lambda) = -\Lambda\frac{\partial}{\partial\Lambda}\ln m_0(\Lambda)$, becomes unity at the fixed point ($\Lambda/m \to \infty$) in the strong coupling regime;[4]

$$\gamma_m = 1 \qquad (\lambda \searrow \lambda_c), \tag{2.12}$$

while $\gamma_m = 1 - \sqrt{1-\lambda/\lambda_c}$ for $\lambda < \lambda_c$,[23] where $m_0(\Lambda)$ can easily be obtained from (2.4) with the solution (2.8) and (2,6). Accordingly, the asymptotic form of $\Sigma(p^2)$, "renormalized" *à la* Miransky, can be rewritten through (2.10) into the form of (2.7);

$$\Sigma(p^2) \sim \frac{m^2}{p}\ln\frac{p}{m}, \tag{2.13}$$

which in fact agrees with the OPE with (2.12) up to logarithm (see (1.3))[4,24].

Now in the *non*-asymptotically free TC model, the ladder SD equation remains the same as (2.2)-(2.4) except that λ is replaced by $C_2(F)\lambda$, with $C_2(F)$ being the quadratic Casimir of the technifermion representation F.*) Thus the nontrivial UV fixed point does exists at $\lambda = \lambda_c = 1/4C_2(F)$, with the same anomalous dimension $\gamma_m = 1$. Accordingly, we have the same asymptotic behavior of the $S\chi SB$ solution $\Sigma(p^2)$ as that in the $U(1)$ case (2.13), with m being the dynamical mass of technifermion.

This is to be compared with the asymptotically free TC in which we have[25]

$$\Sigma(p^2) \sim \frac{m^3}{p^2}\left(\ln\frac{p}{m}\right)^{A/2-1}, \tag{2.14}$$

with $A \equiv 3C_2(F)/\pi b = \frac{9(N^2-1)}{N}\frac{1}{33-2N_f}$ for N_f-flavored $SU(N)$ TC. Eq.(2.14) is a $S\chi SB$ solution to the "improved" ladder SD equation with the above fixed coupling constant simply replaced by the running one $\lambda(p^2) = \frac{3C_2(F)}{4\pi}\alpha(p^2) = \frac{A}{4t}$ $(t \equiv \ln\frac{p}{m})$. Eq.(2.14) is also consistent with OPE, (1.3), where $\left(\frac{p}{m}\right)^{\gamma_m}$ in this case is understood as $\exp\int_0^t \gamma_m(t')dt' = \left(\ln\frac{p}{m}\right)^{A/2}$ for the vanishing anomalous dimension $\gamma_m \simeq 2\lambda(p^2) \to 0$ (additional $\left(\ln\frac{p}{m}\right)^{-1}$ comes from the Wilson coefficient).

$\Sigma(p^2)$ in (2.13) is then communicated, through ETC (or preonic) gauge interactions having a scale $\Lambda_S(\gg m)$, down to the ordinary fermions (quarks/leptons) mass m_f[7]

$$m_f \simeq \frac{1}{\Lambda_S^2}\frac{N}{4\pi^2}\int_0^{\Lambda_S^2} dx\frac{x\Sigma(x)}{x+\Sigma^2(x)}, \tag{2.15}$$

where we have taken $SU(N)$ TC for simplicity. Substituting (2.13) into (2.15), we have (up to $N/4\pi^2$ and logarithm) [4,5,6,7]

$$m_f \simeq \frac{m^2}{\Lambda_S}, \tag{2.16}$$

which is much enhanced compared with the asymptotically free case (2.14); $m_f \simeq m^3/\Lambda_S^2$[3].

Now, the most stringent FCNC is the K^0-$\bar{K}^0$ invoking s quark mass, from which we have $\Lambda_S > 350$TeV. This yields $m_f \sim 10^2$MeV for $m \simeq 250$GeV (a typical value of TC models), 10^3 times enhancement of the asymptotically free case, in agreement with the realistic value. Moreover, (2.13) simultaneously raises the mass of problematic light technipions (pseudo NG bosons). This in fact gave rebirth to TC.[7]

*) $U(1)$ TC model is not excluded, though not quite successful.

Actually the above TC model is the first explicit dynamical model realizing the fixed point scenario that Holdom and others hoped for before[26]. Also a similar idea of "walking" TC[6] imitates the effect of (2.13) (or $\gamma_m = 1$) within the terminology of asymptotically free theories.*) Note that m_f is indeed a typical example of the low energy parameters which are influenced by the anomalous dimension; (2.15) with (1.3) yields $m_f \simeq \frac{m^3}{\Lambda_S^2}\left(\frac{\Lambda_S}{m}\right)^{\gamma_m}$.[26]

Although the above result is based on a very crude approximation, ladder approximation, it is remarkable that recently a similar phase structure to (2.9)–(2.11) has also been observed by Kogut et al. through the Monte Carlo simulation of the non-compact lattice QED.[28] Apart from the application to TC or other model buildings, it is certainly very interesting whether we can construct a nontrivial continuum field theory (QED, quantum gravity, etc.) at the nontrivial UV fixed point. This direction I hope will become one of the central activities on field theory in 1990's.

3. Dynamical Symmetry Breaking with a Very Large Anomalous Dimension

So far we have considered four-technifermion interaction due to ETC as merely a perturbative effect. This may not be true when the four-technifermion interaction becomes strong enough to trigger the $S\chi SB$. We here consider nonperturbative effect of this interaction in the framework of ladder SD equation for QED plus chiral-invariant four-fermion interaction, $G_0/2[(\bar\psi\psi)^2 - (\bar\psi\gamma_5\psi)^2]$, (gauged NJL model) which was first studied by Bardeen et al[9]. This dynamics, especially for $\gamma_m \simeq 2$, is also useful in models of dynamical electroweak symmetry breaking other than TC, as will be seen in the next section.

In this case the ladder SD equation (2.1) is replaced (after angular integration) by

$$\Sigma(x) = m_0 + \frac{g}{\Lambda^2}\int_0^{\Lambda^2} dy \frac{y\Sigma(y)}{y+\Sigma^2(y)} + \lambda \int_0^{\Lambda^2} dy \frac{y\Sigma(y)}{y+\Sigma^2(y)}\left[\frac{1}{x}\theta(x-y) + \frac{1}{y}\theta(y-x)\right], \tag{3.1}$$

where we have defined a dimensionless coupling $g \equiv G_0\Lambda^2/4\pi^2$. Eq.(3.1) may be converted into precisely the same differential equation as (2.2) and the same IRBC (2.3), with only the UVBC (2.4) being changed into

$$\left.(x\Sigma(x))' + \frac{g}{\lambda}x\Sigma(x)'\right|_{x=\Lambda^2} = m_0(\Lambda). \tag{3.2}$$

It is very important that the four-fermion coupling constant enters only through UVBC (3.2). Thanks to the additional term there exists a $S\chi SB$ solution even for the non-oscillating solution ($\lambda < \lambda_c$), which was first obtained by Ref.10 and 11. Existence of this $S\chi SB$ solution at $\lambda < \lambda_c$ is due to the attractive four-fermion interaction. Substituting dominant three terms in the asymptotic expansion of Eq.(2.5), $(x/m^2)^{\pm\gamma'-1/2}$ and $(x/m^2)^{\gamma'-3/2}$, into the UVBC (3.2) with $m_0(\Lambda) = 0$, we obtain a scaling relation among λ, g and Λ/m**):

$$g = \frac{(\frac{1}{2}+\gamma')^2 - (\frac{1}{2}-\gamma')^2(P+Q)}{1 - P + \frac{3/2-\gamma'}{1/2+\gamma'}Q}, \tag{3.3}$$

$$P \equiv \frac{\Gamma(1-2\gamma')\Gamma(3/2+\gamma')^2}{\Gamma(1+2\gamma')\Gamma(3/2-\gamma')^2}\left(\frac{m^2}{\Lambda^2}\right)^{2\gamma'}, \tag{3.4}$$

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*) Detailed comparison between TC's of Ref.4 and Ref.6 is given in Ref.24 and Ref.27.

**) This particular form was obtained in Ref.29.

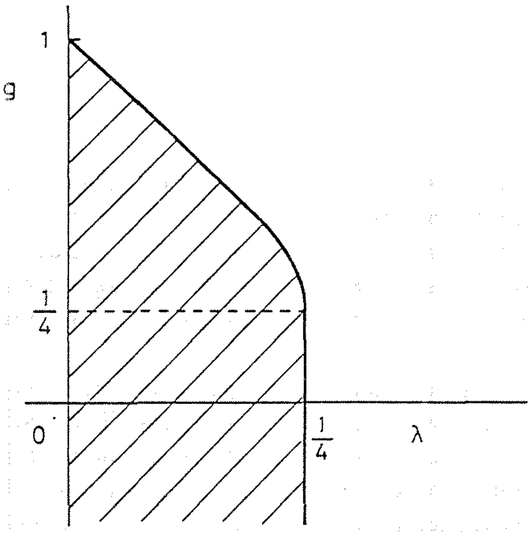

Fig.1.

Critical line in (λ, g) plane is depicted by the solid line, which separates the spontaneously unbroken phase (shaded region) and the broken phase of chiral symmetry.

$$Q \equiv \frac{(1/2+\gamma')^2}{1-2\gamma'}\frac{m^2}{\Lambda^2}, \tag{3.5}$$

with Q being the contribution from the $(x/m^2)^{\gamma'-3/2}$ term which becomes of the same order as the $(x/m^2)^{-\gamma'-1/2}$ term in the pure NJL limit, $\lambda \to 0$ ($\gamma' \to 1/2$). Note that Q is crucial to reproducing the correct NJL limit; at $\lambda \to 0$ ($\gamma' \simeq 1/2-\lambda \to 1/2$) we have $P \simeq (m^2/\Lambda^2)^{1-2\lambda}/2\lambda$ and $Q \simeq (m^2/\Lambda^2)/2\lambda$, which cancel each other in the denominator of (3.3), yielding the well-known NJL result

$$g = \frac{1}{1-\dfrac{m^2}{\Lambda^2}\ln\dfrac{\Lambda^2}{m^2}}. \tag{3.6}$$

Were it not for the Q term, (3.3) would be divergent at $\lambda \simeq 2\lambda_c m^2/\Lambda^2$.

More explicitly, we write (3.3) into[29,30,31]

$$m = \Lambda \cdot \left[\frac{\Gamma(1+2\gamma')\Gamma(\frac{3}{2}-\gamma')^2}{\Gamma(1-2\gamma')\Gamma(\frac{3}{2}+\gamma')^2}\cdot\frac{(\frac{1}{2}+\gamma')^2-g}{(\frac{1}{2}-\gamma')^2-g}\right]^{\frac{1}{4\gamma'}} \quad (\lambda < \lambda_c), \tag{3.7}$$

$$m = \Lambda \cdot 4e^{-\frac{2g}{g-1/4}} \quad (\lambda = \lambda_c), \tag{3.8}$$

$$m = \Lambda \cdot 4\exp\left[-\frac{n\pi + 2\tan^{-1}2\gamma - \tan^{-1}\left(\frac{\gamma}{1/4-\gamma^2-g}\right)}{2\gamma}\right] \quad (\lambda > \lambda_c), \tag{3.9}$$

where $n = 1$ is the ground state solution. It is clear that these scaling relations take a generic form $m = \Lambda \cdot G(\lambda, g)$, with $G(\lambda, g)$ being a certain function of λ and g.

Actually, the scaling relation (3.3) or (3.7)-(3.9) is nothing but the generalization of Miransky's scaling[21] in the pure QED ($g = 0$), (2.9). Taking $\Lambda/m \to \infty$ limit, we obtain the critical line[10,11] (Fig.1):

$$g = (\frac{1}{2} + \gamma')^2 \qquad (0 < \lambda < \lambda_c),$$
$$\lambda = \lambda_c \qquad (g < \frac{1}{4}). \tag{3.10}$$

This is the line of second-order phase transition separating the spontaneous broken ($m/\Lambda \neq 0$) and unbroken ($m/\Lambda = 0$) phases of the chiral symmetry. Then the renormalization in the spontaneously broken phase is performed *à la* Miransky now for the two couplings (λ, g) according to the above scaling relation. Thus, as in the pure QED we expect existence of a sensible continuum theory (cutoff $\Lambda \to \infty$, m =finite) at the nontrivial UV fixed point on the critical line (3.10).

In order to identify the fixed point, however, we further need to know the renormalization-group flow, since this time we have two couplings instead of one. This may be obtained through the scaling law of $\Sigma(x)$, an analogue of the scaling law of the correlation function; we require that $\Sigma(x, \lambda, g) = C\Sigma(x/\kappa^2, \lambda', g')$ under rescaling $x \to x/\kappa^2$, with κ and C being constants, which determines the line, the renormalization-group flow, connecting (λ, g) and (λ', g'). In the case at hand, our solution (2.5) only depends on λ but not on g at all and hence the flow satisfying the above scaling law is the fixed-λ line (upward direction); this would suggest that the whole critical line is the fixed point (may be called a "fixed line")[10,29]. Once the renormalization-group flow is so identified, the β-functions are readily obtained from (3.7)-(3.9). $\beta(g)$ has an UV fixed point only at $\lambda \leq \lambda_c$, i.e.,

$$\beta(g) = \begin{cases} -2\left[g - (\frac{1}{2} + \gamma')^2\right]\left[g - (\frac{1}{2} - \gamma')^2\right] & (g > (\frac{1}{2} + \gamma')^2),^{[31]} \\ 0 & (g < (\frac{1}{2} + \gamma')^2), \end{cases} \tag{3.11}$$

while

$$\beta(\lambda) = 0 \tag{3.12}$$

for *all* λ. This is a rather different situation from the pure QED case, (2.11), and seems to suggest that (2.11) may not correspond to a β-function of the continuum QED in accord with Ref.22.

Now we turn to the anomalous dimension in this model. Substituting the asymptotic form (2.5) into the UVBC and *using the scaling relation* (3.7), we obtain[8]

$$m_0(\Lambda) \sim Z_m \sim (\Lambda^2/m^2)^{-\gamma'-1/2} \qquad (0 < \lambda < \lambda_c), \tag{3.13}$$

which implies *)

$$\gamma_m = 1 + 2\gamma' = 1 + \sqrt{1 - \lambda/\lambda_c} > 1. \tag{3.14}$$

This is substituted into the OPE expression (1.3), yielding $\Sigma(p^2) \sim p^{-1-\sqrt{1-\lambda/\lambda_c}}$, which agrees with (2.6), the $S\chi SB$ solution in this case[10,11]. Thus we find a large anomalous dimension

*) This is to be compared with the result in the symmetric phase[23], $\gamma_m = 1 - 2\gamma' = 1 - \sqrt{1 - \lambda/\lambda_c} < 1$. The difference of the sign is due to the lack of the scaling relation (gap equation) in the symmetric phase. This discontinuity of the anomalous dimension between the symmetric and the broken phases may be an artifact of the ladder approximation. In fact, in the renormalizable four-fermion theory in less than four dimensions, renormalization of the four-fermion coupling in the symmetric phase fill in the gap of the anomalous dimension[32].

$1 < \gamma_m < 2$ for $\lambda_c > \lambda > 0$. In particular, we have a very large anomalous dimension $\gamma_m \simeq 2$ at $\lambda \simeq 0$. This corresponds to the very slowly damping behavior of $\Sigma(p^2)$ in (2.6);

$$\Sigma(p^2) \simeq c_1(\lambda) m \left(\frac{p^2}{m^2}\right)^{-\lambda} + c_2(\lambda) m \left(\frac{p^2}{m^2}\right)^{-1+\lambda} . \tag{3.15}$$

with $c_1(\lambda), c_2(\lambda) \simeq 1$ $(\lambda \simeq 0)$. It is very important to note that thanks to the four-fermion interaction such a very slowly damping solution (irregular asymptotics) can be the $S\chi SB$ solution as well as the explicit breaking solution.

It is also noted that the large anomalous dimension ($\gamma_m > 1$) would suggest the four-fermion interaction might be renormalizable, since in ladder approximation $\dim(\bar{\psi}\psi)^2 = 2(3 - \gamma_m) < 4$, i.e., $(\bar{\psi}\psi)^2$ might be a relevant operator[8]. In fact, in the gauged NJL model ($\lambda \neq 0$) we obtain finite fermion-antifermion scattering amplitude, with the effective Yukawa coupling $g_Y = m/F_\pi$ being finite in the $\Lambda/m \to \infty$ limit, in contrast to the pure NJL model where $g_Y = 0$ ($F_\pi \to \infty$) in that limit. In fact, F_π is evaluated through the formula[33]:

$$F_\pi^2 = \frac{1}{4\pi^2}\int_0^{\Lambda^2} dx x \frac{\Sigma^2(x) - \frac{x}{4}\frac{d}{dx}\Sigma^2(x)}{(x + \Sigma^2(x))^2}, \tag{3.16}$$

which is convergent for Σ in (3.6) as far as $\lambda \neq 0$. Calculation of g_Y through (3.16) along the whole critical line is given in Fig.2[19]. *g_Y does depend on the anomalous dimension γ_m or the gauge coupling λ.*

We also obtain a non-zero mass $m_\sigma (< \infty)$ for the scalar bound state σ ("massive dilaton") [9,10,29,30,34]. This is actually a composite Higgs boson in the model for dynamical electroweak symmetry breaking. The absolute value of m_σ^2 may be calculated through the Partially Conserved Dilatation Current (PCDC) relation. The result[19] is given in Fig.3 for both the solution of the linearized SD equation and that of the full nonlinear equation (3.1). *The ratio m_σ/m also varies according to the anomalous dimension γ_m or the gauge coupling λ.* A striking feature of the result based on the (numerical) solution of the full *nonlinear* SD equation is that[19]

$$m_\sigma/m \simeq \sqrt{2} \tag{3.17}$$

for very small λ (but non-zero), in contrast to the pure NJL result, $m_\sigma/m = 2$.

Finally the above discussion can easily be applied to QCD (QCD-like theories) plus four-fermion interaction by replacing the fixed λ by the running one in the ladder SD equation (3.1) ("improved" ladder SD equation):

$$\Sigma(x) = m_0 + \frac{g}{\Lambda^2}\int_0^{\Lambda^2} dy \frac{y\Sigma(y)}{y + \Sigma^2(y)} + \int_0^{\Lambda^2} dy \frac{y\Sigma(y)}{y + \Sigma^2(y)}\left[\frac{\lambda(x)}{x}\theta(x-y) + \frac{\lambda(y)}{y}\theta(y-x)\right], \tag{3.18}$$

where $\lambda(x)$ is parametrized as[24,27,35]

$$\lambda(p^2) = \begin{cases} \frac{\rho}{4} & (p^2 < \mu^2) \\ \frac{\rho}{4}\left(1 + \frac{\rho}{2A}\ln\frac{p^2}{\mu^2}\right)^{-1} & (p^2 > \mu^2) \end{cases}, \tag{3.19}$$

with A being $24/(33 - 2N_f)$ and ρ a free parameter which is chosen so as to trigger the $S\chi SB$ already in the pure QCD (μ is the chiral symmetry breaking scale of order $O(\Lambda_{QCD})$). $S\chi SB$ solution to (3.18) takes the asymptotic form

$$\Sigma(p^2) = c_1 m \left(\ln\frac{p^2}{m^2}\right)^{-A/2} + c_2\frac{m^3}{p^2}\left(\ln\frac{p^2}{m^2}\right)^{-1+A/2}, \tag{3.20}$$

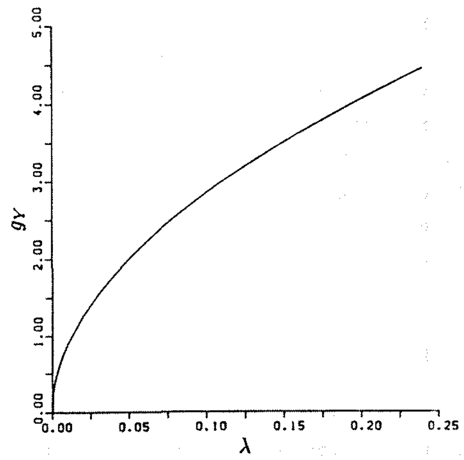

Fig.2.
Dependence of Yukawa coupling g_Y on the gauge coupling along the critical line.

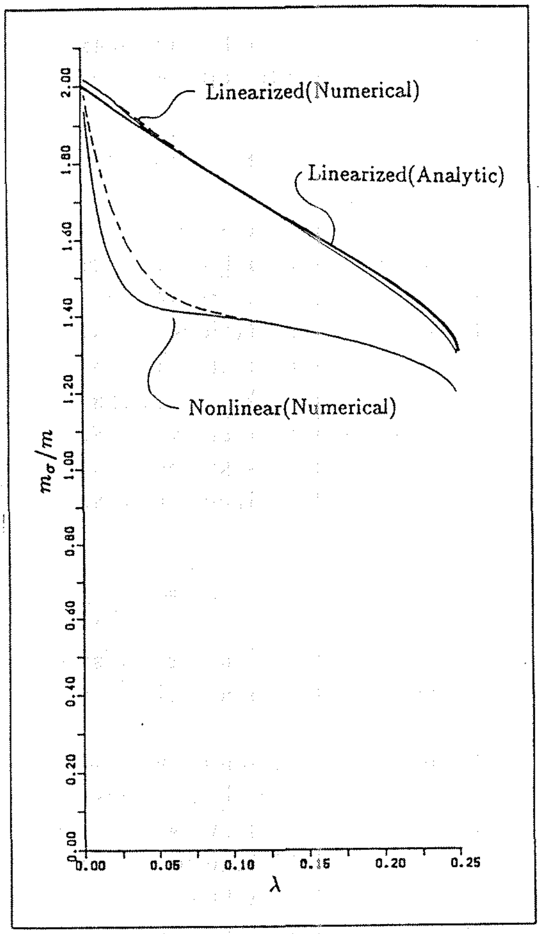

Fig.3.
Dependence of m_σ/m on the gauge coupling λ along the critical line. Linearized analytical result is depicted by the bold solid line. Linearized numerical result is depicted by the dashed line ($\Lambda^2/m^2 = 10^{10}$) and by the solid line ($\Lambda^2/m^2 = 10^{20}$). Full nonlinear numerical result is depicted by the dashed line ($\Lambda^2/m^2 = 10^{10}$) and by the solid line ($\Lambda^2/m^2 = 10^{20}$).

where $c_1, c_2 \simeq 1$ as far as we take $\rho \equiv$ constant (Λ-independent) or $g \simeq 1 + (2/A - 1)\lambda(\Lambda^2) - (\mu/\Lambda)^2/\lambda(\Lambda^2)$.[35] Note that $g \to 1 \equiv g_{cr}$ $\quad(\Lambda \to \infty)$. As in (3.15) the slowly damping solution (irregular asymptotics) can be a $S\chi SB$ solution with $\gamma_m \simeq 2 - 2\lambda(\Lambda^2) \to 2$ due to the presence of four-fermion interaction[8,35]. This property is an essential ingredient to the top-mode standard model in the next section.

4. What is Higgs After All?

Having been equipped with the basic machinery of explicit dynamics of DSB with a large anomalous dimension $\gamma_m \simeq 2$, we now attack our central problem addressed in the beginning of this talk,i.e., *the Origin of Mass*.

Recent experiment has shown that the number of light neutrino species is three, which strongly suggests that the number of families is also three. This implies that top quark is the last quark to be found. Also the lower bound of the mass of the top quark is nearly a hundred GeV. Some indirect measurements suggest even much larger value for m_t than a hundred GeV. This implies only the top quark has a mass close to the weak scale 250GeV: top quark is a special quark. Thus we may infer a distinguished role of the top quark in the mass generation of *all* particles. In the standard model language the large top quark mass in fact corresponds to large Yukawa coupling, which is nothing but a reminiscent of Goldberger-Treiman relation in hadron physics.

A simple picture[12,13] to understand the large top quark mass will then be to regard the top quark mass as the *dynamical mass* due to a top quark condensate $\langle \bar{t}t \rangle \neq 0$. This top quark condensate actually produces composite NG bosons which give rise to the mass of W and Z bosons, and at the same time to the mass of other quarks and leptons. Thus the top quark condensate entirely takes the place of elementary Higgs VEV (top-mode standard model). This picture predicted[12] a very large mass of the top quark, $m_t \simeq$ 250GeV, without affecting the weak isospin relation ($\delta\rho \simeq 0.02$), and also the existence of a spinless $\bar{t}t$ bound state (Higgs boson) with a mass $m_H \simeq 2m_t$. Actually, this prediction came out much earlier than the recent experiments mentioned above. If this picture is indeed correct, the circle will be closed in the standard model, with the identification of the two missing ingredients, the top quark and the Higgs boson.

At first sight the obstacle to this scenario is obvious. We would need a large weak isospin violation in the condensate $\langle \bar{t}t \rangle \neq \langle \bar{b}b \rangle$, to give a realistic mass difference $m_t \gg m_b$, which would then lead to a large deviation of the ρ parameter $\rho \equiv m_W^2/m_Z^2 \cos^2\theta_W \neq 1$ in contradiction to the present experimental limit $\delta\rho \lesssim O(10^{-2})$. However, we shall see that this is not the case for the theory with a large anomalous dimension $\gamma_m \simeq 2$. Such a theory, QCD plus four-fermion interaction, was already described in the previous section. Presence of such additional four-fermion interactions is very common to unified theories beyond the standard model.

Let us now consider the simplest version of our model which consists of the standard three families of quarks and leptons with the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge interactions but without Higgs doublet. Instead of the standard Higgs sector we introduce the $SU(3)_C \times SU(2)_L \times U(1)_Y$-invariant four-fermion interactions among quarks and leptons, the origin of which is not specified at this moment (may be due to the ETC, GUT, preon dynamics, gravity, etc.). The general form of such four-fermion interactions among quarks may be written as[12]

$$\begin{aligned}\mathcal{L}_{4f} = \frac{4\pi^2}{N_c\Lambda^2}\Big[& g^{(1)}_{\alpha\alpha'\beta\beta'}(\bar\psi_L^{\alpha i}\psi_R^{\alpha' j})(\bar\psi_R^{\beta j}\psi_L^{\beta' i}) \\ &+ g^{(2)}_{\alpha\alpha'\beta\beta'}(\bar\psi_L^{\alpha i}\psi_R^{\alpha' j})(i\tau_2)^{ik}(i\tau_2)^{jl}(\bar\psi_L^{\beta k}\psi_R^{\beta' l}) \\ &+ g^{(3)}_{\alpha\alpha'\beta\beta'}(\bar\psi_L^{\alpha i}\psi_R^{\alpha' j})(\tau_3)^{jk}(\bar\psi_R^{\beta k}\psi_L^{\beta' i})\Big] + \text{h.c.},\end{aligned} \tag{4.1}$$

where $N_c(=3)$ is the number of color and the sum of color indices is understood, $g^{(1)}_{\alpha\alpha'\beta\beta'}$, $g^{(2)}_{\alpha\alpha'\beta\beta'}$ and $g^{(3)}_{\alpha\alpha'\beta\beta'}$ are the dimensionless four-fermion couplings, with Λ, (α, α', β, β') and (i, j, k, l) being the ultraviolet cutoff, the family and the weak isospin indices, respectively.

The symmetry structure (besides $SU(3)_C$) of those four-fermion interactions in (4.1) is $SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$, $SU(2)_L \times SU(2)_R \times U(1)_V$ and $SU(2)_L \times U(1)_Y \times U(1)_V \times U(1)_A$, respectively. In the absence of the $g^{(2)}$-term, (4.1) possesses the $U(1)_A$ symmetry which is explicitly broken only by the anomaly due to the gauge interaction and plays the role of the Peccei-Quinn symmetry[13]. It is straightforward to include the leptons into the form of (4.1).

Out of the terms in (4.1) we are particularly interested in those associated with the third family quarks $\Psi \equiv \delta_{\alpha 3}\psi_\alpha = (t, b)$ and $g^{(a)} \equiv g^{(a)}_{3333}$ $(a = 1, 2, 3)$. Here we study the dynamical symmetry breaking of the (t,b) quarks in our model on the basis of the ladder SD equation. In order to illustrate the essential feature of our dynamics, we first study the ladder SD equation for the fixed gauge coupling λ. It is possible to show that the ladder SD equation possesses the solution without $U(1)_{em}$ breaking ($\langle \bar{t}b\rangle = \langle \bar{b}t\rangle = 0$). Actually in such a case, in the lowest order of $g^{(2)}$, the ladder SD equations take the same form as (3.1);

$$\begin{aligned}\Sigma_i(x) =& \frac{g_i}{\Lambda^2}\int_0^{\Lambda^2} dy \frac{y\Sigma_i(y)}{y+\Sigma_i(y)^2} \\ &+ \lambda \int_0^{\Lambda^2} dy \frac{y\Sigma_i(y)}{y+\Sigma_i(y)^2}\left[\frac{1}{x}\theta(x-y)+\frac{1}{y}\theta(y-x)\right],\end{aligned} \tag{4.2}$$

where $\Sigma_i(x)$ ($x \equiv p^2$ and $i = t, b$) are the dynamical mass functions of t and b quarks, respectively and $g_t \equiv g^{(1)} + g^{(3)}$ and $g_b \equiv g^{(1)} - g^{(3)}$.

In view of the critical line (3.10), there exists a $S\chi SB$ solution $\Sigma_i(p^2)$ having a large anomalous dimension (3.14) and asymptotically behaving as (3.15), when $g_i > g_{cr} = (1/2 + \gamma')^2$:

$$\Sigma_i(p^2) \underset{p\gg m_i}{\simeq} \frac{m_i^3}{p^2}\left(\frac{p}{m_i}\right)^{\gamma_m} \simeq m_i\left(\frac{p}{m_i}\right)^{-1+\sqrt{1-\lambda/\lambda_c}} \underset{\lambda \ll 1}{\simeq} m_i\left(\frac{p}{m_i}\right)^{-2\lambda}. \tag{4.3}$$

Our SD equation (4.2) includes the weak isospin-violating four-fermion couplings $g_t \neq g_b$ ($g^{(3)} \neq 0$). It is easily seen that when $g_t > g_{cr} > g_b$, we have a $S\chi SB$ solution with maximal isospin violation, $m_t \neq 0$ and $m_b = 0$, which indeed yields a standard symmetry breaking pattern $SU(2)_L \times U(1)_Y \to U(1)_{em}$ to feed the mass of W and Z bosons. The relevant NG bosons contents are $\pi^+ \sim i\bar{t}(-1+\gamma_5)b/\sqrt{2}$, $\pi^- \sim i\bar{b}(1+\gamma_5)t/\sqrt{2}$ and $\pi^0 \sim i\bar{t}\gamma_5 t$.

In the actual case where λ is the running QCD coupling (3.20), the SD equation takes the same form as (3.18);

$$\Sigma_i(x) = \frac{g_i}{\Lambda^2}\int_0^{\Lambda^2} dy \frac{y\Sigma_i(y)}{y+\Sigma_i^2(y)} + \int_0^{\Lambda^2} dy \frac{y\Sigma_i(y)}{y+\Sigma_i^2(y)}\left[\frac{\lambda(x)}{x}\theta(x-y)+\frac{\lambda(y)}{y}\theta(y-x)\right]. \tag{4.4}$$

Eq.(4.4) has a very slowly damping $S\chi SB$ solution (irregular solution with $\gamma_m \simeq 2$), (3.20), at $g \simeq g_{cr} = 1$ and $\lambda(\Lambda^2) \simeq A/2\ln(\Lambda^2/\mu^2)$ (ρ = const.):

$$\Sigma_i(p^2) \simeq m_i\left(\frac{\ln\frac{p^2}{\mu^2}}{\ln\frac{m_i^2}{\mu^2}}\right)^{-\frac{A}{2}}, \quad A = \frac{8}{7} \quad (N_f = 6), \tag{4.5}$$

where we have parametrized $\Sigma(x)$ such that $\Sigma(m^2) = m$. As in the above fixed-λ case, we have a $S\chi SB$ solution with maximal isospin violation, $m_t \neq 0$ and $m_b = 0$, for $g_t \simeq g_{cr} = 1 > g_b$.

We now come to the central part of this model, determining the top quark mass m_t and the ρ parameter in terms of the cut off Λ. In the lowest order in the electroweak coupling, ρ is given by $\rho = F_{\pi^\pm}^2/F_{\pi^0}^2$, where $F_{\pi^\pm}$ and F_{π^0} are the decay constants of the composite NG bosons to be absorbed into W and Z, respectively. We can compute the decay constants through modification of (3.16) into the $SU(2)$-asymmetric case $m_t \neq m_b$.[12,36] In the general case $m_{t,b} \neq 0$, F_π's take the form[12]

$$F_{\pi^\pm}^2 = \int_0^{\Lambda^2} dx I_\pm(\Sigma_t, \Sigma_b) = \frac{N_c}{8\pi^2}\int_0^{\Lambda^2} dx x \cdot$$
$$\cdot \frac{(\Sigma_t^2+\Sigma_b^2) - \frac{x}{4}(\Sigma_t^2+\Sigma_b^2)' + \frac{x}{2}(\Sigma_t^2-\Sigma_b^2)\left[\frac{1+(\Sigma_t^2)'}{x+\Sigma_t^2} - \frac{1+(\Sigma_b^2)'}{x+\Sigma_b^2}\right]}{(x+\Sigma_t^2)(x+\Sigma_b^2)}, \tag{4.6}$$

$$F_{\pi^0}^2 = \int_0^{\Lambda^2} dx I_0(\Sigma_t, \Sigma_b)$$
$$= \frac{N_c}{8\pi^2}\int_0^{\Lambda^2} dx x \cdot \left[\frac{\Sigma_t^2 - \frac{x}{4}(\Sigma_t^2)'}{(x+\Sigma_t^2)^2} + \frac{\Sigma_b^2 - \frac{x}{4}(\Sigma_b^2)'}{(x+\Sigma_b^2)^2}\right]. \tag{4.7}$$

Let us consider the extreme case of the maximal isospin violation mentioned above, $\Sigma_t(p^2) \neq 0$ and $\Sigma_b(p^2) \equiv 0$. In this case one would expect a large isospin violation $F_{\pi^\pm} \neq F_{\pi^0}$. For $\gamma_m \simeq 2$, however, with a slowly damping $\Sigma_t(p^2)$, (4.3) or (4.5), the integrals (4.6) and (4.7) are dominated by the ultraviolet region $p^2 \gg m_t^2$. This is in sharp contrast to the case of smaller $\gamma_m < 2$ (e.g., $\gamma_m = 1$) where those integrals are dominated by the infrared region. In the "weak-coupling" limit, $\lambda \longrightarrow 0$, the difference in the denominators in (4.6) and (4.7) can be ignored, and the contribution from the terms with derivative Σ_t' is suppressed. (An extreme case would be $\Sigma_t =$ const. (NJL limit), for which (4.6) and (4.7) are both logarithmically divergent with the same coefficient, thus giving $\rho = 1$ in a trivial sense.) Consequently, one obtains

$$\rho = \frac{F_{\pi^\pm}^2}{F_{\pi^0}^2} \simeq 1. \tag{4.8}$$

More specifically, (4.6) and (4.7) are rewritten as

$$F_{\pi^\pm}^2 = \int_0^{\Lambda^2} dx I_\pm(\Sigma_t(x), 0) = m_t^2 \int_0^{\frac{\Lambda^2}{m_t^2}} dx I_\pm(\sigma_t(x), 0) \equiv \frac{m_t^2}{g_Y^{(\pm)2}}, \tag{4.9}$$

$$F_{\pi^0}^2 = \int_0^{\Lambda^2} dx I_0(\Sigma_t(x), 0) = m_t^2 \int_0^{\frac{\Lambda^2}{m_t^2}} dx I_0(\sigma_t(x), 0) \equiv \frac{m_t^2}{g_Y^{(0)2}}, \tag{4.10}$$

where $\sigma_t(x) \equiv \Sigma_t(x)/m_t$, and the effective Yukawa couplings $g_Y^{(\pm)}$ and $g_Y^{(0)}$ are slowly decreasing functions of Λ/m_t (finite at $\Lambda/m_t \to \infty$) for (4.5), i.e.,

$$g_Y^{(\pm)}, \quad g_Y^{(0)} \searrow \quad \text{as} \quad \Lambda \nearrow, \tag{4.11}$$

which implies

$$m_t \searrow \quad \text{as} \quad \Lambda \nearrow, \tag{4.12}$$

since we fixed $F_{\pi^\pm} \simeq F_{\pi^0} \simeq 250$ GeV. On the other hand, $\delta\rho = \rho - 1$ is given by

$$\delta\rho \equiv \frac{m_t^2}{F_{\pi^0}^2}\int_0^{\frac{\Lambda^2}{m_t^2}} dx \left[I_\pm(\sigma_t(x), 0) - I_0(\sigma_t(x), 0)\right], \tag{4.13}$$

whose integral is quite insensitive to change of Λ^2/m_t^2. (Eq. (4.13) is identical to the one-loop contribution of the top quark in the standard model with the QCD effects included). Thus we have

$$\delta\rho \searrow \quad \text{as} \quad \Lambda \nearrow . \tag{4.14}$$

The largest (physically sensible) Λ would be the Planck scale $\Lambda = 10^{19}$GeV, which yields [12]

$$m_t \simeq 250\text{GeV}, \tag{4.15}$$

$$\delta\rho \simeq 0.02, \tag{4.16}$$

$$g_Y \simeq 1. \tag{4.17}$$

These values (4.15) (4.16) are little bit larger than the present experimental limit, < 200GeV at 90 % CL and < 230GeV at 95 % CL. However, Bardeen *et al.*[17] have recently commented that ambiguity of the charm threshold in the analysis of the deep inelastic ν scattering might relax the constraint to $m_t < 250$GeV even at 90% CL.

Now that we have resolved the weak isospin problem, we are in a position to discuss the full implication of the simplest version of our model. Our basic assumption here is that only the g_t is large enough ($g_t \simeq g_{cr} = 1$) to trigger the spontaneous chiral symmetry breaking, while all other four-fermion couplings in (4.1) are small; g_b, $g^{(a)}_{\alpha\alpha'\beta\beta'} \ll g_{cr}$. Then the solution of the ladder SD equation mentioned above implies that only the t quark acquires the dynamical mass m_t, whereas the b quark (as well as other quarks) does not, besides a small mass due to the (long- distance) color forces.

The b quark can only acquire the mass from the $\bar{t}t$ condensate

$$\langle\bar{t}t\rangle_\Lambda = -\frac{N_c}{4\pi^2}\int^{\Lambda^2} dx \frac{x\Sigma_t(x)}{x+\Sigma_t(x)^2} \simeq -\frac{N_c}{4\pi^2}\left(\frac{m_t}{\Lambda}\right)^{2-\gamma_m} \cdot \Lambda^2 m_t, \tag{4.18}$$

through the perturbative $g^{(2)}$ interaction in the same manner as the ETC in (2.15);

$$m_b = -g^{(2)}\frac{4\pi^2}{N_c\Lambda^2}\langle\bar{t}t\rangle_\Lambda \simeq g^{(2)}\left(\frac{m_t}{\Lambda}\right)^{2-\gamma_m} \cdot m_t \simeq g^{(2)} m_t, \tag{4.19}$$

for $\gamma_m \simeq 2$. This also coincides with the result of the nonperturbative $g^{(2)}$ interaction.[13]

Similarly, quarks of the first and the second families also acquire mass from the $\bar{t}t$ condensate through the perturbative $g^{(a)}_{\alpha\alpha'\beta\beta'}$ interactions. In fact, mass matrices of the "up" and the "down" quarks are given by $m_{(u)} \simeq (g^{(1)}_{\alpha\alpha'33} + g^{(3)}_{\alpha\alpha'33})m_t$ and $m_{(d)} \simeq g^{(2)}_{\alpha\alpha'33}m_t$, respectively. It is now evident that our dimensionless four-fermion couplings $g^{(a)}_{\alpha\alpha'33}$ play the role of Yukawa couplings of the standard model. Obviously the above mass spectrum leads to the realistic Kobayashi-Maskawa matrix. Also the lepton masses are acquired in the same way as those of the "down" quarks through the $g^{(2)}$ type couplings $m_{(\ell)} \simeq g^{(2)'}_{\alpha\alpha'33}m_t$. Due to the large Λ (could be 10^{15}GeV-10^{19}GeV) and the small anomalous dimension of $\bar{\psi}\psi$ other than $\bar{t}t$, our four-fermion interactions of order $O(1/\Lambda^2)$ are perfectly free from the FCNC's problem.

Thus the ad hoc Higgs sector of the usual standard model (we may call this "Higgs-mode" standard model) has been successfully replaced by the new very short range interactions, the four- fermion interactions, of quarks and leptons themselves, the origin of which is not specified at this moment. Among them only one quark is assumed to have a sufficiently large coupling to produce its own mass, the dynamical mass. This we identify with the t quark ($g_t \simeq g_{cr} = 1$), which is then responsible for the mass of W and Z bosons (we may call it a "top-mode" standard model in contrast to the usual one, the "Higgs-mode" standard model).

A striking feature of our model is a definite prediction[12] of a tightly bound spinless $\bar{t}t$ state (Higgs boson) due to the very short range dynamics of the four-fermion interactions. We have already argued in section 3 that mass of the Higgs boson ("massive dilaton") can be much affected by the small gauge coupling,(3.17),

$$m_H \simeq \sqrt{2}m_t \simeq 350\text{GeV}, \tag{4.20}$$

in contrast to the original prediction $m_H \simeq 2m_t$[12]. This can clearly be distinguished from the softly bound (non-relativistic) $\bar{t}t$ state due to color forces.

Our discussions here were based on the crude approximation, the ladder SD equation. We needed a nontrivial UV fixed point in g (see (3.11)) to obtain a natural mass hierarchy $m_t \ll \Lambda$. It is actually an open question whether or not the fixed point described in the above really exist beyond the ladder approximation. However, it is important that the main qualitative results do not depend on the details of such a dynamics but are based on merely the fact that the anomalous dimension $\gamma_m \simeq 2$.

Of course the values we obtained , (4.15)-(4.17) and (4.20), are also based on the very crude approximation, ladder approximation, and should not be taken so seriously. More elaborate calculation may change the concrete values substantially. Recently several authors [16,17] made further studies of the top quark condensate and confirmed the same tendency as (4.12) and (4.14), and also the large value for m_t through somewhat different computational method. (Method of Ref.16 is the same as ours but was supplemented by the arguments on some relation to the coupling reduction).

Finally, the dynamics with large anomalous dimension can also be applied to TC. The mechanism described here would give a simple answer, an alternative to the custodial $SU(2)$, to the question [37] concerning the weak isospin violation in TC with $\gamma_m \simeq 1$. In TC models other effects of large anomalous dimension, particularly $\gamma_m \simeq 2$, are obvious: Suppression of FCNC's and enhancement of the pseudo NG bosons masses are much more dramatic than in the $\gamma_m = 1$ case (note that the TC of this kind now can be an asymptotically free gauge theory with normal (or even fast) running coupling constant in contrast to the $\gamma_m = 1$ case). [12,13,38]

5. Conclusion

Under the motivation to understand the *Origin of Mass* of *all* particles, we have discussed various new aspects of DSB. We have seen a variety of theories having different short distance behavior / anomalous dimension yield the same type of low energy effective theory, the sigma model. It was explicitly demonstrated that the low energy parameters in the latter theory, m_f, F_π, m_σ, g_Y, etc., can be different depending on the difference in the anomalous dimension of the original theories. We have argued virtue of the large anomalous dimension in DSB, which is realized at the nontrivial UV fixed point. Phase structure of such fixed-point theories, i.e., QED-like theories, QED-like/QCD-like theories plus four-fermion interactions (gauged NJL models) were studied based on the ladder SD equation. Particular emphasis was placed on a possibility that the four-fermion interactions may become renormalizable due to the presence of gauge interactions in these theories.

The applications of these theories to the models of dynamical electroweak symmetry breaking were discussed in some details, TC with $\gamma_m = 1$ and the top-mode standard model with $\gamma_m \simeq 2$. Influence of the large anomalous dimension on the low energy parameters was in fact remarkable. Particularly, the top-mode standard model would be the simplest idea to understand the *Origin of Mass* by identifying the two missing ingredients of the Standard Model, the top quark and the Higgs boson, as the same object.

Although the results discussed here are totally based on the ladder approximation, a rather crude approximation, it may be clarified in 1990's whether or not such new types of field theories really exist. Also 1990's may become an era when the Origin of Mass will be eventually revealed by the experiment, and hopefully by the DSB with large anomalous dimension (LAD). Actually, DSB with small anomalous dimension (SAD), motivated by the asymptotically free gauge theories (a paradigm in 1970's through 1980's), has been really *sad* to account for the *Origin of Mass*. TeV physics in 1990's may become a turning point where the old paradigm will be taken over by a new one for younger physicists, LAD DSB.

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DYNAMICAL SYMMETRY BREAKING DUE TO STRONG COUPLING YUKAWA INTERACTION *

Kei-Ichi Kondo

Department of Physics, Chiba University
1-33, Yayoi-cho, Chiba 260, Japan

Masaharu Tanabashi and Koichi Yamawaki**

Department of Physics, Nagoya University
Nagoya 464-01, Japan

Abstract

Motivated by the top quark condensation scenario of the electroweak symmetry breaking ("top-mode standard model"), dynamical chiral symmetry breaking (χSB) due to strong coupling Yukawa interaction is studied in the framework of Schwinger-Dyson equations. In quenched approximation, we show existence of the dynamical χSB phase($\langle 0|\sigma|0\rangle = 0$, $\langle 0|\bar{\psi}\psi|0\rangle \neq 0$) in strong Yukawa coupling region. Introducing dynamical fermion (tadpole) in our framework, we still have a parameter region where χSB has its origin in the fermion condensate.

1. Introduction

The origin of electroweak symmetry breaking, which explains the masses of the weak bosons and fermions, is one of the most important problems in modern particle physics. In the standard model we introduce Higgs field $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\pi_2 + i\pi_1 \\ \sigma - i\pi_3\end{pmatrix}$ which is tuned to have a non-zero vacuum expectation value (VEV) $\langle 0|\sigma|0\rangle = v$. Here v is an order parameter of the electroweak symmetry breaking, through which the weak gauge bosons $W^{\pm}, Z^0$ become massive due to Higgs mechanism. Masses of fermions, though being in principle independent order parameters of the electroweak symmetry breaking, are also explained by v through Yukawa couplings with Higgs boson,

$$m_f = \frac{\eta_f}{\sqrt{2}} v.$$

This scenario is reasonable when all fermions have small masses, $m_f \ll v$. However if there exists a heavy fermion, $m_f \gtrsim v$, it seems rather awkward to assume that the fermion gets its large mass from a small VEV of Higgs field. In this case it would be more natural to consider a converse, i.e., the origin of $v \neq 0$ comes from a large m_f, the dynamical mass of the fermion.

In fact, in the low energy effective theory of the technicolor models, the mass of technifermion determines the value of the order parameter v.

More exciting possibility will be the top quark which may have a large mass $m_t \gtrsim v$ in the recent experimental situation. Actually, two of the authors (M.T. and K.Y.) and Miransky

* Reported by M. Tanabashi

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proposed some time ago the top quark condensation scenario[1] ("top-mode standard model"). In this model we no longer need elementary Higgs boson. We instead regard the large top quark mass as the result of certain short range dynamics of unspecified origin which breaks the chiral symmetry dynamically through the top quark condensate $\langle 0|\bar{t}t|0\rangle \neq 0$. Because of dynamical χSB we obtain composite Nambu-Goldstone (NG) bosons, which give rise to masses of the weak gauge bosons through dynamical Higgs mechanism. We predicted a very large mass of the top quark $m_t \sim 250\mathrm{GeV}$ and also a composite Higgs boson $H \sim \bar{t}t$ with a mass $m_H \simeq 2m_t$.* Similar ideas were also advanced by Terazawa[2] and Nambu[3] in somewhat different terminologies and with different results for the value of m_t. Further studies of the top-mode standard model have recently been done by various groups[4,5,6,7] and confirmed the very large top quark mass $m_t > 200\mathrm{GeV}$ in this model.

In our previous paper[1], we considered the case where the four-fermion interactions are responsible for triggering the top quark condensation at very high energy scale ($\gtrsim$ GUT scale), and in fact our arguments were based on the explicit solution of the gap equation for spontaneous χSB in the gauged Nambu–Jona-Lasinio model (four-fermion interaction plus gauge interaction).[7]

What is the origin of the four-fermion interactions, then? One might immediately think of exchange of heavy spin 1 bosons with mass m_V. In fact one finds[9,10,11] that the behavior of the χSB solution in this system is similar to that of the NJL model for $m_V \sim \Lambda$, based on the ladder SD equation[10] for $iS^{-1}(p) \equiv A(-p^2)\not{p} - B(-p^2)$ (with "gauge parameter" ξ $D_{\mu\nu}(p) = -i(p^2 - m_V^2)^{-1}[g_{\mu\nu} - (1-\xi)p_\mu p_\nu(p^2 - m_V^2)^{-1}]$);

$$B(x) = \frac{e^2}{(4\pi)^2}\int_0^{\Lambda^2} dy\, y\mathcal{K}_B(x,y)\frac{B(y)}{A^2(y)y + B^2(y)}, \tag{1a}$$

$$A(x) = 1 + \frac{e^2}{2(4\pi)^2}\int_0^{\Lambda^2} dy\, \frac{y}{x}\mathcal{K}_A(x,y)\frac{A(y)}{A^2(y)y + B^2(y)}, \tag{1b}$$

where

$$\mathcal{K}_B(x,y) = K_B(x,y;m_V^2)\left[(3+\xi) + \frac{(1-\xi)m_V^2}{\sqrt{(x+y+m_V^2)^2 - 4xy}}\right], \tag{2a}$$

$$\mathcal{K}_A(x,y) = 2K_A(x,y;m_V^2)\left[\xi + \frac{(\xi-1)m_V^2}{\sqrt{(x+y+m_V^2)^2 - 4xy}}\right], \tag{2b}$$

with K_A and K_B being defined in Eq.(6). Including gauge interaction in Eq.(1), we obtain a solution which is similar to that of the gauged NJL model in view of the top-mode standard model. (For detailed analysis, see Ref.[11].)

However, in the case of spin 1 boson exchange, we cannot obtain such an effective four-fermion interaction as

$$\epsilon_{ij}(\bar{\psi}_L^i t_R)(\bar{\psi}_L^j b_R),$$

through which ($g^{(2)}$ term in Ref.[1]) the bottom quark acquires its mass from a top quark condensation. We then must assume a bottom condensate independently of a top quark condensate

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* Our recent analysis[8], including the effect of gauge interaction on the spectrum, implies $m_H \simeq \sqrt{2}m_t \simeq 350\mathrm{GeV}$.

in order to feed the mass to "down"-like fermions. It would be simple that the masses of fermions other than the top quark are also explained by the top quark condensate alone. So it does not seem to be the case that the interaction is mediated by spin 1 bosons.

Here, we wish to discuss another possibility that an attractive force due to a heavy spinless boson exchange through Yukawa interaction causes the top quark condensation. Even if we write the same $SU(2)_L \times U(1)_Y$ symmetric Yukawa interaction as the usual standard model, there will be in this picture an essential difference that the $SU(2)_L \times U(1)_Y$ breaking is mainly not due to the VEV of the spinless boson but to the top quark condensate caused by the attractive force of the strong Yukawa coupling.

Then, our task is to investigate the phase structure of the standard model with very large Yukawa coupling. In the following sections, we will investigate chiral phase transition of standard model in the framework of the Schwinger-Dyson (SD) equation and find the phase where χSB is dynamical.

2. The SD equations

In this section, we derive the SD equation for fermion propagator in the form of integral equation, which we can solve numerically and analytically. We discuss here $SU(2)_L \times SU(2)_R$ symmetric Yukawa interaction, for simplicity. Extension to other types of Yukawa interaction is straightforward. The lagrangian is given by

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi - \frac{\eta_0}{\sqrt{2}} \left[\bar{\psi}\psi\sigma + \bar{\psi} i\gamma_5 \vec{\tau}\psi \cdot \vec{\pi}\right] + \frac{1}{2}\left[(\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2\right] - \frac{m_0^2}{2}\left[\sigma^2 + \vec{\pi}^2\right] - \frac{\lambda_0}{4}\left[\sigma^2 + \vec{\pi}^2\right]^2 .$$

From the equation of motion for ψ, $i\not{\partial}\psi - \frac{\eta_0}{\sqrt{2}}[\psi\sigma + i\gamma_5\vec{\tau}\psi \cdot \vec{\pi}] = 0$, we obtain the SD equation for fermion propagator,

$$i\not{\partial}\langle 0|T\psi(x)\bar{\psi}(0)|0\rangle = i\delta^{(4)}(x) + \frac{\eta_0}{\sqrt{2}}\langle 0|T(\sigma(x) + i\vec{\tau}\cdot\vec{\pi}(x)\gamma_5)\psi(x)\bar{\psi}(0)|0\rangle .$$

Assuming $\langle 0|\vec{\pi}|0\rangle = 0$, we can rewrite the SD equation in momentum space

$$\begin{aligned} \not{p} - iS^{-1}(p) = \frac{\eta_0}{\sqrt{2}}\langle 0|\sigma|0\rangle + \frac{\eta_0}{\sqrt{2}}\int \frac{d^4k}{(2\pi)^4}\Big[&D_\sigma(k)S(p-k)\Gamma_\sigma(p-k,p) \\ &+ D_\pi(k) i\gamma_5 \vec{\tau} S(p-k)\vec{\Gamma}_\pi(p-k,p)\Big], \end{aligned} \tag{3}$$

where D_σ, D_π and S are full propagators of σ, π and fermion ψ, respectively, and Γ_σ and $\vec{\Gamma}_\pi$ are full vertices of $\psi\bar{\psi}\sigma$ and $\psi\bar{\psi}\vec{\pi}$.

Under the approximation of boson propagators and vertices

$$D_\sigma = \frac{iZ_3}{k^2 - m_\sigma^2}, \qquad D_\pi = \frac{iZ_3}{k^2 - m_\pi^2},$$
$$\Gamma_\sigma = -i\frac{\eta_0}{\sqrt{2}}, \qquad \vec{\Gamma}_\pi = -i\frac{\eta_0}{\sqrt{2}}\vec{\tau} i\gamma_5,$$

we obtain the integral equation (SD equation), which reads after Wick rotation

$$B(x) = \frac{\eta}{\sqrt{2}}v + \frac{C}{2}\int_0^{\Lambda^2} dy\, y\mathcal{K}_B(x,y)\frac{B(y)}{A^2(y)y + B^2(y)}, \tag{4a}$$

$$A(x) = 1 + \frac{C}{4}\int_0^{\Lambda^2} dy\, \frac{y}{x}\mathcal{K}_A(x,y)\frac{A(y)}{A^2(y)y + B^2(y)}, \tag{4b}$$

where $C \equiv \eta^2/(4\pi)^2, x \equiv -p^2$, $y \equiv -k^2$ and $v \equiv Z_3^{-\frac{1}{2}}\langle 0|\sigma|0\rangle$, $\eta^2 \equiv Z_3\eta_0^2$. Here an ultraviolet (UV) cutoff Λ is introduced. The integral kernels $\mathcal{K}_B$ and $\mathcal{K}_A$ are defined as

$$\mathcal{K}_B(x,y) = 3K_B(x,y;m_\pi^2) - K_B(x,y;m_\sigma^2), \tag{5a}$$

$$\mathcal{K}_A(x,y) = 3K_A(x,y;m_\pi^2) + K_A(x,y;m_\sigma^2). \tag{5b}$$

K_B and K_A are given by

$$\begin{aligned} K_B(x,y;m^2) &\equiv \frac{2}{\pi}\int_0^\pi d\theta \frac{\sin^2\theta}{x+y-2\sqrt{xy}\cos\theta+m^2} \\ &= \frac{2}{x+y+m^2+\sqrt{(x+y+m^2)^2-4xy}}, \end{aligned} \tag{6a}$$

$$\begin{aligned} K_A(x,y;m^2) &\equiv \frac{4}{\pi}\int_0^\pi d\theta \frac{\sqrt{xy}\cos\theta\sin^2\theta}{x+y-2\sqrt{xy}\cos\theta+m^2} \\ &= \frac{4xy}{\left[x+y+m^2+\sqrt{(x+y+m^2)^2-4xy}\right]^2}. \end{aligned} \tag{6b}$$

Note that from Eq.(5a) σ gives repulsive force while π does attractive one.

For the case of Yukawa interaction with a discrete chiral symmetry, $\mathcal{L}_{Yuk} = -\frac{\eta_0}{\sqrt{2}}\bar{\psi}\psi\sigma$, the SD equations are given by Eq.(4) with the integral kernels

$$\mathcal{K}_B(x,y) = -K_B(x,y;m_\sigma^2), \tag{7a}$$

$$\mathcal{K}_A(x,y) = K_A(x,y;m_\sigma^2). \tag{7b}$$

For the case of $U(1)_L \times U(1)_R$ symmetric Yukawa interaction, $\mathcal{L}_{Yuk} = -\frac{\eta_0}{\sqrt{2}}[\bar{\psi}\psi\sigma + \bar{\psi}i\gamma_5\psi\pi]$, the SD equations are given by Eq.(4) with the integral kernels

$$\mathcal{K}_B(x,y) = K_B(x,y;m_\pi^2) - K_B(x,y;m_\sigma^2), \tag{8a}$$

$$\mathcal{K}_A(x,y) = K_A(x,y;m_\pi^2) + K_A(x,y;m_\sigma^2). \tag{8b}$$

In the case of the massless boson exchange, i.e., $m^2 = 0$, K_B and K_A become simple;

$$\begin{aligned} K_B(x,y;0) &= \frac{1}{x}\theta(x-y) + \frac{1}{y}\theta(y-x), \\ K_A(x,y;0) &= \frac{y}{x}\theta(x-y) + \frac{x}{y}\theta(y-x). \end{aligned}$$

Then, in this case the integral kernels are the same as that of QED in ladder approximation.

3. Solution within Quenched Approximation

First we consider $v = 0$ phase. In quenched approximation, $v = 0$ does not mean $\langle 0|\bar{\psi}\psi|0\rangle = 0$. In fact, as we will see in the following, there exists a chiral phase transition even in the $v = 0$ phase at strong Yukawa coupling region for $SU(2)_L \times SU(2)_R$ symmetric Yukawa interaction.

In the $v = 0$ phase, σ and π have degenerate masses $m_\sigma = m_\pi \equiv m$. Then the integral kernels of the SD equation Eq.(4) are written simply as

$$\mathcal{K}_B(x,y) = 2K_B(x,y;m^2), \tag{9a}$$

$$\mathcal{K}_A(x,y) = 4K_A(x,y;m^2). \tag{9b}$$

Note that the integral kernels Eq.(9) are the same up to factor as those of the case of the massive vector boson exchange Eq.(2) in "Feynman gauge", $\xi = 1$. Then we obtain the χSB solution at strong coupling Yukawa region in the same way as the case of the massive vector boson exchange[10].

Note that we cannot obtain the χSB solution within this approximation (only ladder, without tadpole) in the cases of discrete chiral symmetric Yukawa interaction ($U(1)_L \times U(1)_R$ symmetric Yukawa interaction) because of absence (cancellation) of attractive force. (See Eq.(7a), Eq.(8a).)

Following Ref.[10], we approximate wave function renormalization $A \equiv 1$ for analytical calculation. This approximation is good if $\Lambda \sim m \gg B(0)$. Here we make a simple approximation for $(6a)$;

$$K_B(x, y; m^2) = \frac{1}{x + m^2}\theta(x - y) + \frac{1}{y + m^2}\theta(y - x).$$

To study a scaling relation near the critical point of chiral phase transition, it is sufficient to study the linearized integral equation (bifurcation technique)[12]. Then we obtain a simple integral equation;

$$B(x) = C\left[\int_{M^2}^{x} dy \frac{B(y)}{x + m^2} + \int_{x}^{\Lambda^2} dy \frac{B(y)}{y + m^2}\right], \tag{10}$$

where an infrared (IR) cutoff $M \simeq B(0)$ was introduced. Solving Eq.(10), we obtain a scaling relation[10];

$$\frac{M^2}{\Lambda^2 + m^2} = \exp\left[\frac{-4}{\sqrt{4C - 1}}\left(\frac{\pi}{2} - \tan^{-1}\sqrt{4C - 1}\right)\right] - \frac{m^2}{\Lambda^2 + m^2}. \tag{11}$$

Critical coupling constant C_c which separates χSB phase from the symmetric one corresponds to the solution of Eq.(11) in the limit of $M \to 0$.

Let us next consider $v \neq 0$ phase. In this phase m_π is zero because of the Goldstone theorem. Then our integral equation is given by the kernels

$$\mathcal{K}_B(x, y) = 3K_B(x, y; 0) - K_B(x, y; m_\sigma^2), \tag{12a}$$
$$\mathcal{K}_A(x, y) = 3K_A(x, y; 0) + K_A(x, y; m_\sigma^2). \tag{12b}$$

In this phase, we can define a renormalized ϕ^4 coupling λ as $\lambda \equiv m_\sigma^2/(2v^2)$. In the case of $\lambda = \infty$, $K_B(x, y; m_\sigma^2)$ can be neglected. Then the SD equations are

$$B(x) = \frac{\eta}{\sqrt{2}}v + \frac{3}{2}C\left[\frac{1}{x}\int_0^x dy \frac{yB(y)}{A^2(y)y + B^2(y)} + \int_x^{\Lambda^2} dy \frac{B(y)}{A^2(y)y + B^2(y)}\right], \tag{13a}$$
$$A(x) = 1 + \frac{3}{4}C\left[\frac{1}{x^2}\int_0^x dy \frac{y^2 A(y)}{A^2(y)y + B^2(y)} + \int_x^{\Lambda^2} dy \frac{A(y)}{A^2(y)y + B^2(y)}\right]. \tag{13b}$$

These integral equations are the same as those of QED in ladder approximation with gauge parameter $\xi = 3$. Here, we will discuss the behavior of solution only in the $\lambda = \infty$ case.

It is convenient to rewrite Eq.(13a) and Eq.(13b) into differential equations and boundary conditions;

$$\left[x\frac{d^2}{dx^2} + 2\frac{d}{dx} + \frac{3C}{2}\frac{1}{A^2(x) + B^2(x)}\right]B(x) = 0, \tag{14a}$$
$$\left[x\frac{d^2}{dx^2} + 3\frac{d}{dx} + \frac{3C}{2}\frac{1}{A^2(x) + B^2(x)}\right]A(x) = 0, \tag{14b}$$

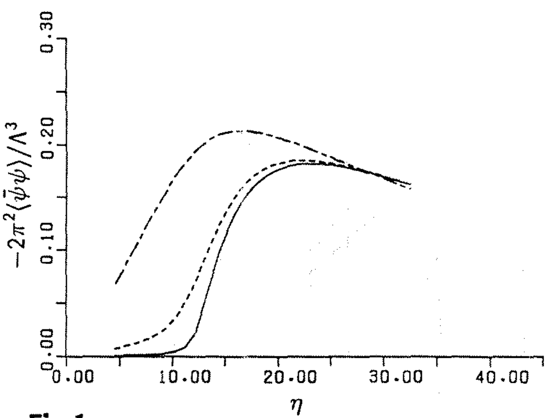

Fig.1a

Yukawa coupling dependence of the fermion pair condensation $\langle 0|\bar{\psi}\psi|0\rangle$ in the $v \neq 0$ phase. Dashed dotted line, dotted line and solid line correspond to $v/\Lambda = 2\times10^{-2}, 2\times10^{-3}, 2\times10^{-4}$, respectively.

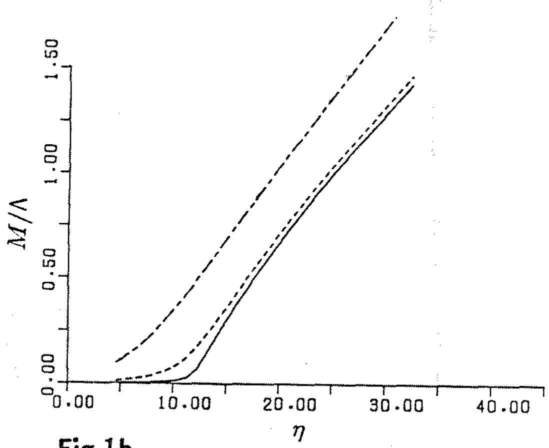

Fig.1b

Yukawa coupling dependence of the fermion mass $M \equiv B(0)/A(0)$ in the $v \neq 0$ phase. Dashed dotted line, dotted line and solid line correspond to $v/\Lambda = 2\times10^{-2}, 2\times10^{-3}, 2\times10^{-4}$, respectively.

$$
\begin{aligned}
&x^2\frac{d}{dx}B(x)\Big]_{x=0}, \quad \left(1+x\frac{d}{dx}\right)B(x)\Big]_{x=\Lambda^2} = \frac{\eta}{\sqrt{2}}v,\\
&x^3\frac{d}{dx}A(x)\Big]_{x=0}, \quad \left(1+\frac{x}{2}\frac{d}{dx}\right)A(x)\Big]_{x=\Lambda^2} = 1.
\end{aligned}
\tag{15}
$$

We define a local order parameter of the chiral phase transition:

$$
\langle 0|\bar{\psi}\psi|0\rangle \equiv -\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}S(p) = -\frac{1}{2\pi^2}\int_0^{\Lambda^2}dx\frac{xB(x)}{A^2(x)x+B^2(x)}. \tag{16}
$$

$-\langle 0|\bar{\psi}\psi|0\rangle$ is a positive definite function of η. In the case of $\eta = 0$, we obtain $\langle 0|\bar{\psi}\psi|0\rangle = 0$. In the strong coupling limit $\eta \to \infty$, $B(x)$ becomes large and dominates the denominator of Eq.(16) and we obtain $-\langle 0|\bar{\psi}\psi|0\rangle \sim 1/B(0)$ in that region. (This behavior is consistent with the strong coupling expansion which says $-\langle 0|\bar{\psi}\psi|0\rangle \sim 1/\eta$.) Then we have a turning-over point where the function $\langle 0|\bar{\psi}\psi|0\rangle$ takes the maximum value*. We in fact investigate the behavior of this function using a numerical solution of Eq.(14). The result is given at Fig.1a and Fig.1b. The turning-over point appears when the dynamical mass of fermion $M \equiv B(0)/A(0)$ has its value $M \sim \Lambda$. Note also $\langle 0|\bar{\psi}\psi|0\rangle$ is nonvanishing at the strong Yukawa coupling region even in the limit of $v/\Lambda \to 0$ (continuum limit).

Note that the behavior of $\langle 0|\bar{\psi}\psi|0\rangle$ is consistent with the result of lattice MC simulation[13].

We next investigate the "renormalized Yukawa coupling"[13] η_R, defined by $\eta_R \equiv \sqrt{2}M/v$. The result is shown in Fig.2. Because of nonvanishing M in the continuum limit $(v/\Lambda \to 0)$, this value diverges at the strong Yukawa coupling region.

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* Note here that this property of $-\langle 0|\bar{\psi}\psi|0\rangle$ (existence of a turning-over point and a maximum value) is universal[14] in our framework, i.e., it does not depend on details of the interaction which breaks the chiral symmetry. In fact, we can explicitly show the existence of a maximum value of $-\langle 0|\bar{\psi}\psi|0\rangle$ also in the cases of strong coupling QED and the NJL model in ladder approximation.

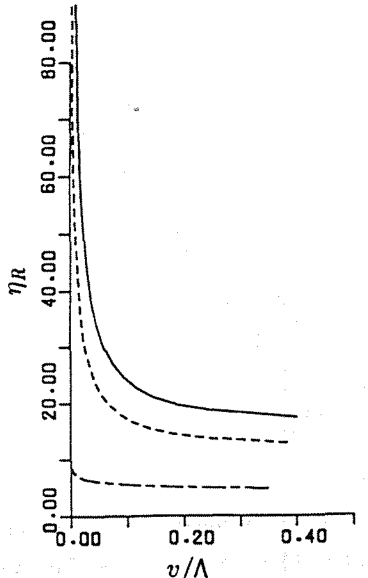

Fig.2

"renormalized Yukawa coupling" as the function of v. Dashed dotted line, dotted line and solid line correspond to $\eta = 4.59, 12.14, 16.54$, respectively.

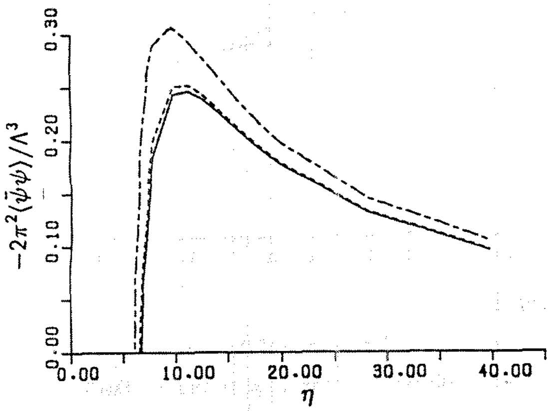

Fig.3

Scaling relation of the fermion pair condensation $\langle 0|\bar{\psi}\psi|0\rangle$ without quenched approximation when $Z_3 m_0^2 = \Lambda$. Dashed dotted line, dotted line and solid line correspond to $m = \Lambda, 10^{-1}\Lambda, 10^{-2}\Lambda$, respectively.

4. Effect of Dynamical Fermion

Finally, we discuss the effect of dynamical fermion (tadpole) on the above analysis using the SD equation.

Using the equation of motion of σ,

$$\Box\sigma + m_0^2\sigma + \lambda_0(\sigma^2 + \vec{\pi}^2)\sigma + \frac{\eta_0}{\sqrt{2}}\bar{\psi}\psi = 0,$$

we obtain the SD equation for VEV of σ,

$$m_0^2\langle 0|\sigma|0\rangle + \lambda_0\langle 0|\sigma(\sigma^2 + \vec{\pi}^2)|0\rangle + \frac{\eta_0}{\sqrt{2}}\langle 0|\bar{\psi}\psi|0\rangle = 0. \tag{17}$$

We wish to discuss χSB due to the effect of η_0, hence we disregard the effect of λ_0 here. Then, the value of v is determined by $\langle 0|\bar{\psi}\psi|0\rangle$;

$$v = Z_3^{-\frac{1}{2}}\langle 0|\sigma|0\rangle = -\frac{\eta}{\sqrt{2}}\frac{\langle 0|\bar{\psi}\psi|0\rangle}{Z_3 m_0^2}. \tag{18}$$

Unlike the case of quenched approximation, $v = 0$ means $\langle 0|\bar{\psi}\psi|0\rangle = 0$ in this unquenched case. From Eq.(4), Eq.(16) and Eq.(18), we obtain the SD equation

$$B(x) = \frac{4C}{Z_3 m_0^2}\int_0^{\Lambda^2} dy \frac{yB(y)}{A^2(y)y + B^2(y)} + \frac{C}{2}\int_0^{\Lambda^2} dy\, y\mathcal{K}_B(x,y)\frac{B(y)}{A^2(y)y + B^2(y)}, \tag{19}$$

where the kernel $\mathcal{K}_B$ is defined in Eq.(5a) and the SD equation for A is the same as Eq.(4b). Here we must note that m_σ and m_π are not independent quantities of η. For example, in the strong coupling phase of η where χSB occurs, we have $m_\pi = 0$ because of the Goldstone theorem. On the other hand, in the weak coupling phase of η where chiral symmetry is unbroken, m_π and m_σ should be degenerate. Such an η dependence of mass spectrum of bosons comes from the loop effect of fermion in the vacuum polarization in the σ and π propagators.

Especially in the strong coupling phase, the massless pole of π propagator comes from mixing with massless bound states of fermions,i.e., composite NG bosons. Then we must solve the SD equations for σ and π propagators and Nambu-Bethe-Salpeter equation for the bound state in a self-consistent manner. This is very difficult technically, however. We simply disregard the effect of dynamical fermion on the propagators of σ and π. We only consider the effect of dynamical fermion on the VEV of σ. Here we use the integral kernels $\mathcal{K}_B$, $\mathcal{K}_A$ given in Eq.(9).

In such an approximation we obtain the SD equation with the effect of the dynamical fermion,

$$B(x) = \frac{4C}{Z_3 m_0^2}\int_0^{\Lambda^2} dy \frac{yB(y)}{A^2(y)y + B^2(y)} + C\int_0^{\Lambda^2} dy\, y K_B(x,y;m^2)\frac{B(y)}{A^2(y)y+B^2(y)}, \tag{20a}$$

$$A(x) = 1 + C\int_0^{\Lambda^2} dy\, \frac{y}{x} K_A(x,y;m^2)\frac{A(y)}{A^2(y)y+B^2(y)}. \tag{20b}$$

We calculate this integral equation numerically. Fig.3 is the result of the chiral phase transition of this system. In this case it is difficult to say whether the χSB is dynamical or not, because we always have non-zero value of v whenever χSB occurs. Hence we next discuss a criterion of dynamical χSB.

The NG bosons couple to the axialvector current through its "decay constant" F_π,

$$\langle 0|J_{5\mu}^a(x)|\pi^b(q)\rangle = iq_\mu F_\pi \delta^{ab} e^{-iqx}.$$

The axialvector current is written as

$$J_{5\mu}^a = \bar\psi \frac{\tau^a}{2}\gamma_\mu\gamma_5\psi + \sigma\partial_\mu\pi^a - \pi^a\partial_\mu\sigma.$$

We divide the NG boson decay constant into two parts;

$$\langle 0|\bar\psi\frac{\tau^a}{2}\gamma_\mu\gamma_5\psi|\pi^b(q)\rangle = iq_\mu F_\pi^f \delta^{ab} e^{-iqx},$$
$$\langle 0|\sigma\partial_\mu\pi^a - \pi^a\partial_\mu\sigma|\pi^b(q)\rangle = iq_\mu F_\pi^b \delta^{ab} e^{-iqx}.$$

We call the χSB is dynamical, when *the fermionic part of the NG boson decay constant* F_π^f *is sufficiently larger than the bosonic part of the NG boson decay constant* F_π^b.

In this case the bosonic part F_π^b is written in terms of the VEV of σ, $F_\pi^b = Z_3 v$. On the other hand, the fermionic part F_π^f is written in terms of the mass function of the fermion and its value is order of M, $F_\pi^f \sim M$. Then our criterion of dynamical χSB is

$$M \gg Z_3 v = -\frac{\eta}{\sqrt{2}}\frac{\langle 0|\bar\psi\psi|0\rangle}{m_0^2}. \tag{21}$$

$\langle 0|\bar\psi\psi|0\rangle$ is given by

$$\langle 0|\bar\psi\psi|0\rangle \sim M^3\left(\frac{\Lambda}{M}\right)^\gamma,$$

where γ is determined from the high energy behavior of the fermion mass function (an analog of anomalous dimension[7]).

In our numerical calculation we obtain $\gamma \simeq 1.61$ for $m^2 = 10^{-3}\Lambda^2$, $Z_3 m_0^2 = \Lambda^2$, and $\gamma \simeq 1.97$ for $m^2 = \Lambda^2$, $Z_3 m_0^2 = \Lambda^2$. Noting $Z_3 < 1$, we find our criterion of dynamical χSB is fulfilled for small m and sufficiently large UV cutoff Λ.

5. Conclusions and Discussion

We have investigated the dynamical χSB due to strong coupling $SU(2)_L \times SU(2)_R$ symmetric Yukawa interaction in the framework of the SD equations. Within the quenched approximation we found the phase where χSB occurs while the VEV of elementary scalar field vanishes. In the approximation where the loop effect of the dynamical fermion affects the value of v, we discussed the criterion for dynamical χSB. We found the region where our criterion is fulfilled.

We discussed here only $SU(2)_L \times SU(2)_R$ symmetric Yukawa interaction. However, the Yukawa interaction with large isospin violation is important for the top quark condensation. More detailed analysis including the case of isospin violation will appear elsewhere.

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Color-Sextet Condensation*

K. FUKAZAWA, T. MUTA†, J. SAITO, I. WATANABE and M. YONEZAWA‡

Department of Physics, Hiroshima University, Hiroshima 730

and

M. INOUE

Hiroshima National College of Maritime Technology
Higashino-cho, Hiroshima 725-02

Abstract

The condensation of the color-sextet quarks is considered. We present the dynamical electroweak symmetry breaking model with high-color effect and four-fermion interactions where the color-sextet quark condensate $\overline{Q}Q$ acts as the Higgs field. The masses of sextet quarks are predicted to be 210∼280 GeV for the lighter sextet quark of the iso-spin doublet U and D, and 280∼330 GeV for the heavier partner. In the simplest model, the sextet quarks belong to $\underline{6}^*$ of $SU(3)_C$, and their charges are $+2/3$ for U, $-1/3$ for D.

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I. Motivation and Basic Assumptions

As is well known, the standard model (SM) of strong, weak and electromagnetic interactions is successful with $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetries. However, there are some unsatisfactory features in this theory.

One of these features concerns with the representation of $SU(3)_C$. The strong sector of SM is described by Quantum Chromodynamics (QCD) with an SU(3) gauge group. Quarks belong to $\underline{3}$ the fundamental representation of SU(3), and gluons to $\underline{8}$ the adjoint representation. Mathematically there are also an infinite number of higher representations in SU(3): $\underline{6}$, $\underline{10}$, $\underline{15}$, $\underline{15}'$,$\cdots$. It has not been known whether the particles belonging to such representations exist. If these particles are not in nature, the reason which forbids such particles should exist. Marciano[1] suggested that the high-color effect of quark–anti-quark binding potential may increase in the energy scale of QCD confinement to a few hundred GeV or a few TeV and proposed the sextet quark condensation model.

Another unsatisfactory feature is the mass hierarchy problem. In SM, masses of all fermions and weak gauge bosons are originated from the Higgs field. Because the Higgs is a scalar field, the perturbative correction to masses is quadratically divergent. One can evade this problem by replacing the Higgs field by some fermion-field condensate in the dynamical electroweak symmetry breaking just as the technicolor model,[2] or Marciano's high-color quark model mentioned above. To produce the weak boson masses, the vacuum expectation value $\langle\overline{\Psi}\Psi\rangle$ of these fermion fields Ψ must be about 250 GeV. Then, in general, masses of fermions Ψ may lie in the range between a few hundred GeV and a few TeV.

Though once the technicolor-like models were nearly abandoned because of the well-known flavour changing neutral current problem, it has been shown that the models may be revived if the composite operator $\overline{\Psi}\Psi$ has a large anomalous dimension γ^* in these models.[3] In the technicolor model however, we were forced to introduce an extra fundamental fermion by hand. In the present paper we would like to circumvent this unsatisfactry situation by appearing to the color-sextet quarks.

On the other hand, being stimulated by recent experimental consequences that the top-quark mass may be very heavy as the W boson mass or more, several authors pointed out that the top quark may condense to be a substitute for the Higgs particle.[4],[5] Miransky, Tanabashi and Yamawaki[4],[6] observed that the composite operator $\bar{t}t$ acquires a large γ^* by using the Schwinger–Dyson equation in ladder approximation. This implies that the four-fermion operators such as $\bar{t}t\bar{t}t$, $\bar{t}t\bar{q}q$ and $\bar{t}t\bar{\ell}\ell$ become relevant and have to be included in the original Lagrangian, where q means u, d, s, c and b quark and ℓ the lepton. Their model is attractive because of the large γ^* and the economy: less particles and less free parameters than Higgs' scenario.

We pursue an alternative possibility that the color-sextet quark condensate to trigger the electroweak symmetry breaking. We start with the following assumptions to examine this possibility:

- The color-sextet quarks Q belonging to $\underline{6}$ (or $\underline{6}^*$) in $SU(3)_C$ exist.
- The bound states $\overline{Q}Q$ condensate.

We choose Q as an weak iso-doublet (U, D) for the second assumption. Q should be heavy as the weak boson masses to induce them. This may explain why the sextet quarks have not been found by experiments yet.

II. Dynamical Electroweak Symmetry Breaking by Sextet Quark Condensation

We start with the Lagrangian including a Nambu-Jona-Lasinio[7] type four-fermion interaction term to and study the mechanism of the dynamical electroweak symmetry breaking by the sextet quarks,

$$\mathcal{L} = \mathcal{L}_{\mathrm{QCD}} + \mathcal{L}_{\mathrm{Electro\text{-}Weak}}(\text{without Higgs terms}) + \mathcal{L}_{4\mathrm{f}} \ ,$$

$$\mathcal{L}_{4\mathrm{f}} = \frac{1}{2} G^{(0)} \left[(\overline{Q}Q)^2 - (\overline{Q}\gamma_5 Q)^2 \right] \ . \tag{1}$$

One gets the self energy or the dynamical mass $\Sigma(p)$ of Q by solving the Schwinger–Dyson equation in the quenched planner approximation,

$$\begin{aligned} \Sigma(p) \ \simeq \ & m^{(0)} + \frac{g_{4\mathrm{f}}}{\Lambda^2} \int_{m_Q^2}^{\Lambda^2} dq^2 \frac{\Sigma(q)}{q^2 + \Sigma(q)^2} \\ & + \int_{m_Q^2}^{\Lambda^2} dq^2 \frac{q^2 \Sigma(q)}{q^2 + \Sigma(q)^2} \left[\frac{\lambda(q)}{q^2}\, \theta(q^2 - p^2) + \frac{\lambda(p)}{p^2}\, \theta(p^2 - q^2) \right] \ , \end{aligned} \tag{2}$$

where

$$g_{4\mathrm{f}} = \frac{G^{(0)} N_c N_f \Lambda^2}{4\pi^2} \quad ,$$
$$\lambda(q) = \frac{3}{4\pi} \, \alpha_3^{(0)}(q) \quad .$$

Here N_c and N_f are the numbers of colors and flavours, respectively, Λ the ultraviolet cut-off, α_3 the QCD running coupling, and by the superscription (0) we mean the bare quantity. As shown in ref. [4], a non-trivial solution for $\Sigma(p)$ is obtained in the symmetry-broken phase and the anomalous dimension of the composite operator $\overline{Q}Q$ can be 2, if $g_{4\mathrm{f}}$ and $\lambda(q)$ approach along a certain path to the critical point $g_{4f} = 1$ and $\lambda(q) = 0$ in the continum limit $q \to \infty$. Therefore the four-fermion terms like $(\overline{\psi}\psi)(\overline{Q}Q)$ where ψ is any fermion field become relevant and it is justified that we have introduced the four-fermion terms into the original Lagrangian. According to the condensation of $\overline{\mathrm{Q}}\mathrm{Q}$, the vacuum expectation value $\langle \overline{Q}Q \rangle$ is non-vanishing, and the four-fermion terms play the role of the mass terms as

$$g_{4\mathrm{f}(\psi)} \, \langle \overline{Q}Q \rangle \, \overline{\psi}\psi$$

where $g_{4\mathrm{f}(\psi)}$ is a coupling corresponding to the four-fermion term $\overline{Q}Q\overline{\psi}\psi$. Note that $\overline{Q}Q$ behaves just like the Higgs field at low energy owing to the decrease of its scale dimension by γ^*.

III. Masses of Sextet Quarks

We will now evaluate the masses of Q. As in the Technicolor-like models[2], the W-boson mass m_W is induced by the condensation of the mass-generating fermions,

$$m_W{}^2 \; = \; \frac{1}{4} \, g_2(m_W)^2 \, F_{\pi^\pm}{}^2 \; , \tag{3}$$

where $g_2(m_W)$ is the weak coupling constant and $F_{\pi^\pm}$ the 'pion' decay constant of $\overline{\mathrm{Q}}\mathrm{Q}$. To estimate $F_{\pi^\pm}$, we use the following formula derived in ref. [4].

$$F_{\pi^\pm}{}^2 = \frac{N_c}{8\pi^2}\int_{m_{Q_1}^2}^{\Lambda^2} \frac{x\,dx}{(x+\Sigma_1^2)(x+\Sigma_2^2)}\left[(\Sigma_1^2+\Sigma_2^2) - \frac{1}{4}(\Sigma_1^2+\Sigma_2^2)'\right.$$
$$\left. + \frac{x}{2}(\Sigma_1^2-\Sigma_2^2)\left\{\frac{1+(\Sigma_1^2)'}{x+\Sigma_1^2} - \frac{1+(\Sigma_2^2)'}{x+\Sigma_2^2}\right\}\right] \quad , \tag{4}$$

where $x \equiv p^2$, $' \equiv d/dx$, $m_{Q_1} = \Sigma_1(m_{Q_1})$, and by the subscript 1 and 2 we mean the heavier and lighter color-sextet quark in an iso-doublet, respectively.

At the energy scale $p \geq m_{Q_1}$, the behavior of Σ_i of with $i = 1, 2$ is computed by solving eq. (??).[6]

$$\Sigma_i(p) \simeq m_{Q_i}\left[\frac{\alpha_3(p)}{\alpha_3(m_{Q_i})}\right]^{A/2} . \tag{5}$$

The quantity A is 60 for present sextet quark model, while 8/7 for the top condensation model.

By neglecting all fermion masses, the asymptotic form of $\alpha(p)$ at the energy scale above m_{Q_1} is

$$\alpha_3(p)^{-1} \cong \frac{1}{6\pi}\ln\frac{p}{M_Q} , \tag{6}$$

where M_Q is a scale parameter. Note that this expression for $\alpha_3(p)$ is applicable only for $p \geq m_{Q_1}$ and is different from the ordinary $\alpha_3(p)$ in low energy scale.

We would like to evaluate M_Q in terms of the low energy QCD parameter M_4 (for four light quarks). For this purpose, we take into account the quark mass effects so that we employ the beta function for massive quarks given by Georgi and Polizer,[8]

$$\beta_3(g_3, \tfrac{m}{p}) \cong -\frac{g_3{}^3}{16\pi^2}\left(11 - \frac{2}{3}\sum_{\text{triplets}}\frac{1}{1+\frac{5m^2}{p^2}} - \frac{10}{3}\sum_{\text{sextets}}\frac{1}{1+\frac{5m^2}{p^2}}\right) , \tag{7}$$

where $g_3{}^2 = 4\pi\,\alpha_3$. This equation leads another expression for $\alpha_3(p)$ applicable to the energy scale $p \geq m_c$,

$$\alpha_3(p)^{-1} \cong \frac{1}{6\pi}\left(25\ln\frac{p}{M_4} - \sum_{i=b,t}\ln\frac{p^2+5m_i{}^2}{M_4{}^2+5m_i{}^2}\right.$$
$$\left. -5\sum_{i=Q_1,Q_2}\ln\frac{p^2+5m_i{}^2}{M_4{}^2+5m_i{}^2}\right) . \tag{8}$$

For $p \geq m_{Q_1}$, eq. (??) should reproduce eq. (??) and we have the relation

$$\ln M_Q = 25\ln M_4 - 2\sum_{i=b,t}\ln m_i - 10\sum_{i=1,2}\ln m_{Q_i} - 12\ln 5 . \tag{9}$$

There have been reported many experimental estimations for M_4. Since our

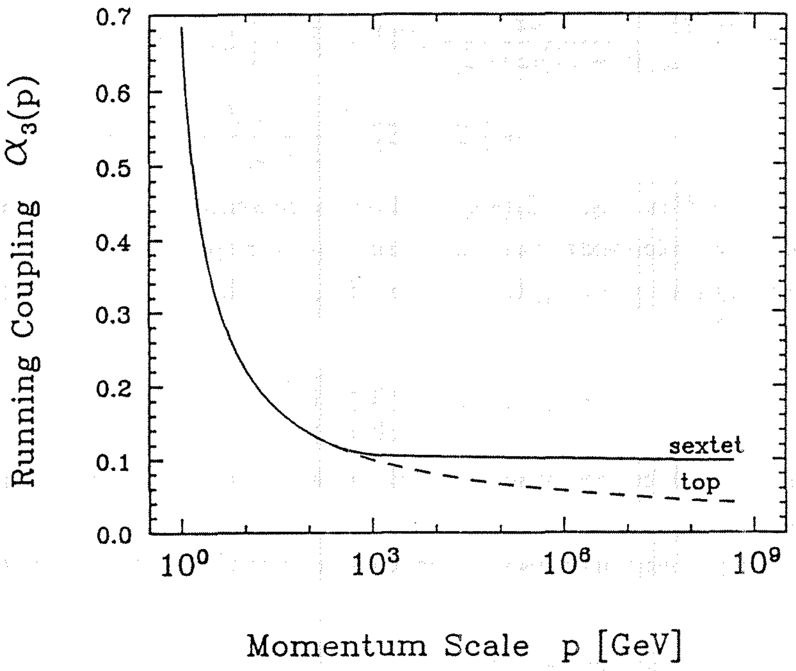

Fig. 1 The behavior of QCD running coupling $\alpha_{3(p)}$. The solid line corresponds to the present sextet model, while the dashed line represents the case without sextet quarks. Here m_{Q_1}, m_{Q_2} and m_t are taken to be 300 GeV, 250 GeV and 150 GeV, respectively.

argument is based on the leading order approximation in α_3, we adopt as M_4 the average value of the scale parameter determined by using the leading order QCD predictions.[9] Our value taken here is

$$M_4 \;=\; (\;330\;\pm\;^{210}_{130}\;)\;\;\text{MeV}\;^{1)} \tag{10}$$

The behavior of $\alpha_3(p)$ is shown in fig. 1. Note that α_3 walks very slowly in the energy region above m_{Q_1}.

Now let us go back to eq. (3),

$$m_W{}^2 \;=\; \frac{1}{4}\;g_2(m_W)^2\;F_{\pi^\pm}(m_{Q_1}, m_{Q_2}, m_t; M_4, \Lambda)^2\;. \tag{11}$$

1) Its five-light-quark version is $M_5 = (\;240\;\pm\;^{170}_{100}\;)$ MeV with $m_b = 4.9$ GeV.

The recent experimental values for the input parameters are

$$m_W = (\ 80.0 \pm 0.6\ )\ \text{GeV}\ ,^{[10]}$$

$$g_2(m_W)\ :\ \alpha_2(m_W) = \frac{g_2(m_W)^2}{4\pi} = 0.0344 \pm 0.0007\ ,^{[11]} \tag{12}$$

and M_4 given in eq. (10). We take Λ to be 10^{15} GeV for calculation, but it makes no significant change, even if it is 10^{12} GeV or 10^{19} GeV. Remaining unknown masses are m_{Q_1}, m_{Q_2} and m_t to which eq. (11) imposes a constraint.

There is another experimental restriction coming from the ρ parameter,

$$\rho \equiv \frac{F_{\pi^\pm}{}^2}{F_{\pi^0}{}^2}, \tag{13}$$

with

$$F_{\pi^0}{}^2 = \frac{N_c}{8\pi^2}\int_{m_{Q_1}^2}^{\Lambda^2} dx\ x\left[\frac{\Sigma_1^2 + \frac{x}{4}(\Sigma_1^2)'}{(x+\Sigma_1^2)^2} + \frac{\Sigma_2^2 + \frac{x}{4}(\Sigma_2^2)'}{(x+\Sigma_2^2)^2}\right] . \tag{14}$$

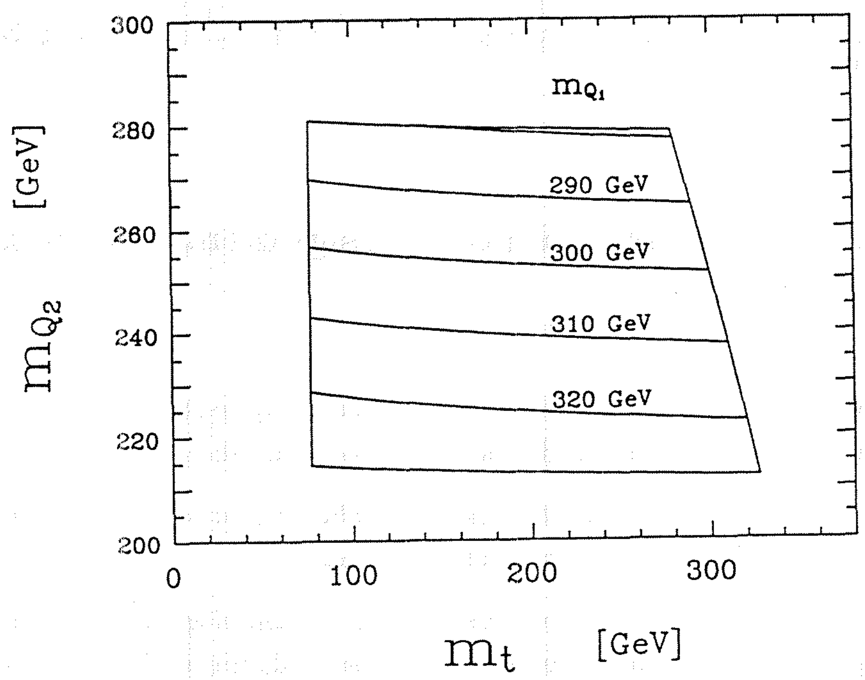

Fig. 2 The allowed region of three masses m_{Q_1}, m_{Q_2} and m_t. Here m_{Q_1} is represented by the iso-mass lines in the plane of m_{Q_2} and m_t.

Its experimental value is

$$\rho \;=\; 0.998 \pm 0.0086\,.^{[11]} \tag{15}$$

Three quark masses must be chosen to reproduce this value.

The contour map of m_{Q_1} is shown in fig. 2. The weak dependence of m_{Q_1} on m_t comes through eq. (??). The upper bound of the graph corresponds to $m_{Q_1} \geq m_{Q_2}$, the right one to $m_{Q_1} \geq m_t$ (assumed), the lower one to the ρ parameter and the left boundary to the recent $p\bar{p}$ collider experiment at Fermi Lab.[12]

The allowed ranges of the masses are

$$\begin{aligned} m_{Q_1} &: 280 \sim 330 \text{ GeV}\,, \\ m_{Q_2} &: 210 \sim 280 \text{ GeV}\,, \\ m_t &: \;\;77 \sim 330 \text{ GeV}\,. \end{aligned} \tag{16}$$

The uncertainties caused by the errors of input data in eqs. (??) and (??) are about 10 GeV for each of boundaries except for the lower limit of the top quark mass.

These predictions are consistent with the lower limit of the sextet quark mass given by the recent experiment: $m_Q \geq 84$ GeV (95% C.L.) for the long lifetime case.[13]

IV. Quantum Number Assignments for Sextet Quarks

Now we discuss quantum number assignments for Q briefly. For this purpose, we need a new assumption in addition to the ones given before:

- The sextet quarks can decay into the ordinary quarks and/or leptons via $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant interactions.

This requirement may protect our scenario from the cosmological constraints. Although there are numerous sets of quantum numbers and decay modes of Q satisfying the present assumptions, we choose the simplest solution among many possibilities:

- The sextet quarks belong to $\underline{6}^*$ and the anti-sextet-quarks belong to $\underline{6}$.

- The sextet quarks are an $SU(2)_L$ doublet.
- The charges of the sextet quarks are $+2/3$ for U and $-1/3$ for D, respectively, just as the ordinary quarks.
- The decay modes of the sextet quarks are

$$Q \longrightarrow q + q + \overline{q} \qquad \text{or} \qquad Q \longrightarrow \overline{q} + \overline{q} + \overline{\ell}. \tag{17}$$

  The baryon number and the total lepton number of Q are 1/3 and 0 for the former mode, respectively, $-2/3$ and -1 for the latter mode, respectively, if they are conserved. The heavier sextet quark Q_1 decays into the lighter one through the weak interaction, too. (If the mass splitting permits, real $W^{\pm}$ will be produced.)

- It is needed two additional iso-doublet leptons for the chiral anomaly cancellation. The additional leptons are sequential, i.e. the same quantum numbers as the ordinary leptons except for the lepton numbers associated with the generation. (These additional neutrinos must have masses heavier than 45 GeV to be consistent with the Z-decay experiments.[14])

Any additonal quarks will violate the QCD asymptotic freedom.

Possible bound states are $\overline{Q}Q$, $\overline{Q}qg$, etc. for bosons and QQQ, Qqq ,etc. for fermions where g represents gluon. Note that the charges of these bound states are integral.

V. Conclusion

The consequences of the dynamical electroweak symmetry breaking by the color-sextet-quark condensate with the four-fermion interactions is presented. Due to the large anomalous dimension of the composite operator, the four-fermion operators like $\overline{Q}Q\overline{\psi}\psi$ become relevant and the condensate $\overline{Q}Q$ play the role of the Higgs field. The masses of two color-sextet iso-doublet quarks are 280~330 Gev and 210~280 GeV. The simplest quantum number assignments is that the sextet quarks belong to $\underline{6}^*$ of $SU(3)_C$ and have charge $+2/3$ and $-1/3$ for the iso-spin up and down particle, respectively.

In this talk, we have given just a sketch of our work. For the details, please, see ref. [0].

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IS THERE ANY RELATION BETWEEN DYNAMICAL SYMMETRY BREAKING AND REDUCTION OF COUPLING CONSTANTS?

Jisuke KUBO

Department of Physics,
College of Liberal Arts, Kanazawa University
920 Kanazawa, Japan

ABSTRACT

In applying the concept of the reduction of coupling constants to the standard theory for the strong and electro-weak interactions, we have made assumptions, which can be motivated in the asymptotic behavior of the theory. Those assumptions are clarified in detail, for I believe that these may be where one could find a relation to the dynamical breaking of the electro-weak gauge symmetry.

I Introduction

In formulating realistic quantum field theories, renormalizability has played undoubtedly an important role. In particular, the structure of the independent parameters in a given theory is essentially fixed by renormalizability.

The concept of the reduction of coupling constants (RCC)[1] is to reduce the number of the independent parameters of a theory without imposing symmetries, and hence it generalizes the usual notion of renormalizability. During the last years, the theoretical nature of the RCC has been intensively investigated[1-9]. But the physical principle, which the RCC is based on, has always been lacking. And so the RCC has been often seen as something outside of phenomenology. The contact to phenomenology slowly began as the lower bound of the top quark mass became higher with time[10]. The recent theoretical observation[11], moreover, might suggest that the RCC when applied[5,8] to the standard theory for the strong and electro-weak interactions is related to the dynamical breaking of the electro-weak gauge symmetry by the top condensation[12,13].

It is the purpose of my talk to come close to an answer to the question of whether there is any relation between RCC and dynamical symmetry breaking (DSB). I would like to start, in sect.II, by reviewing briefly the concept of the RCC. In applying this concept to the standard theory, we have made assumptions[5,8] which can be motivated in the asymptotic behavior of the theory. So I shall first discuss the asymptotic behavior of the standard theory in sect.III. Then, in sect.IV, I would like to clarify our assumptions, for I believe it is this set of the assumptions that will be crucial to find the relation between RCC and DSB if any.

It is also worth mentioning that the compositeness conditions of Ref.14 - in the certain limit of parameters[15] - are exactly satisfied in the solution of the non-trivial reduction.

II What is the reduction of coupling constants?

Let me begin by considering a renormalizable massless theory with n coupling constants, g_i $(i = 1,...,n)$. The g_i s are renormalized at a renormalization scale μ and so depend on μ in general. A remarkable feature of renormalizability is that, once g_i s are known at some μ, we can determine g_i s at any different renormalization scale. This change of g_i s with respect to μ (which defines the renormalization group flow) can be studied via non-linear first-order differential equations,

$$\frac{d\vec{\alpha}}{dt} = \vec{\beta}(\vec{\alpha}) \tag{1}$$

where $\vec{\alpha} = (\alpha_1, \cdots, \alpha_n)$, $\alpha_i = g_i^2/4\pi$,

$$t = \ln \mu .$$

Here β_i s are the Callan-Symanzik β -functions, and can be calculated in perturbation theory. We assume, therefore, that all the partial derivatives of β_i s exist near the origin $\vec{\alpha} = \vec{0}$.

The differential equation (dfe) (1) is equivalent to

$$dt = \sum$$

where $\sum$ stands for

$$\frac{d\alpha_1}{\beta_1} = \frac{d\alpha_2}{\beta_2} = \cdots = \frac{d\alpha_n}{\beta_n} . \tag{2}$$

Note that Eq.(2) is nothing but the characteristic system of the partial dfe,

$$\vec{\beta} \cdot \vec{\nabla} \phi = 0 . \tag{3}$$

The solutions of (3) define n-dimensional surfaces - there are (n-1) independent surfaces in general - which do not ascend in the direction of the "velocity" $\vec{\beta}$. That is, a renormalization group(RG) trajectory corresponds to a contour line on the surface defined by a ϕ , and so the altitude on the surface along the trajectory remains constant:

$$\frac{d\phi}{dt} = 0 \quad . \tag{4}$$

(n-1) dimensional surfaces defined by $\phi_i = C_i$ may be called the RG invariant surfaces. They are analogous to the constrained surfaces in the phase space of a constrained Hamiltonian system. As in that case, it is possible to eliminate (n-1) couplings by using the (n-1) RG invariant "constraints", ϕ_i .

Suppose we have eliminated the (n-1) couplings in favor of α_3 . (The subscript 3 has no deep meaning here.) That is, α_i s(i=3) are now functions of α_3 . It could happen that these functions can be written as power series expansions

$$\alpha_i = \sum_{n=1}^{\infty} C_n{}^i \alpha_3{}^i \quad , \tag{5}$$

for sufficiently small α_3 . Then we say, the system can be reduced and the reduced system is renormalizable[1,17]. In practice, of course, one is restricted to work at a finite order in perturbation theory, and so it is not possible in general to find the exact functions. But the coefficients, $C_n{}^i$ s, can be calculated at any desired order. It is remarkable that the uniqueness of $C_n{}^i$ s can be investigated at the one-loop level[2] (see Ref.4 for a more exhaustive analysis of the problem), except for some special cases[3].

Thus the concept of RCC is to exhaustively find (perturbatively) renormalizable theories for a given set of quantum fields. Interestingly, the reduction solutions include even solutions to which one can not assign any symmetry. Can one give any physical interpretation to these solutions? We do not know the answer yet.

Let me, however, ignore this question and wonder how to apply the concept of the RCC to the standard theory[5,8]. One immediately realizes that this can not be done so simply. For 1) the theory involves couplings with opposite asymptotic behaviors, e.g. the $SU(3)_C$ and the $U(1)_Y$ couplings, and 2), even if the problem 1) is solved, one expects contradictions with phenomenology because then all the Yukawa couplings, for instance, would be related.

To overcome these problems, we have proposed the partial reduction with perturbations[5,8]. In order to carry out this program, we have made assumptions that can be motivated in the asymtotic behavior of the standard theory. This will be explained in the next section, and the notion of the partial reduction with perturbations will become clear in sect. IV.

III Asymptotic behavior of the standard theory

Here I am interested particularly in the asymptotic behavior of the couplings α_i [18], and I consider the dfe (1) for large t. For a given initial value, one obtains in principle an unique trajectory. There may be initial values which belong to the trajectories that, as t goes to infinity, approach asymptotically the origin of the n-dimensional space of couplings. This set of the initial values is called the stable manifold[19]. Since trajectories lie on RG invariant surfaces, we can talk about stable surfaces. It is then clear that the system can be asymptotically free (AF)[20] only if the stable manifold is not trivial. Therefore, if asymptotic freedom is a physical requirement, the physical trajectories have to lie on some stable (or so to say asymptotically free) surface. This is very similar to the case of constrained Hamiltonian systems where the physical trajectories lie on the constrained surfaces. Thus the requirement of asymptotic freedom may imply reduction of coupling constants[1,2].

Now I come to the question of how to find stable manifold. It is certainly possible to do this in perturbation theory at least near the origin. There is an useful theorem by Liapunov, which can be used to study the stability of the solutions of ordinary dfes at a fixed point. Let me briefly explain the idea by considering a dfe of the form,

$$\frac{d\vec{x}}{dt} = \vec{f}(\vec{x}), \tag{6}$$

with some regularity properties for $\vec{f}$. We assume that $\vec{f}(\vec{0}) = \vec{0}$, so the origin is a fixed point of (6). We are interested in the solutions near the origin. Therefore, we may expand (6) around $\vec{x} = 0$:

$$\frac{dx_i}{dt} = \sum_j \frac{\partial f_i}{\partial x_j}(\vec{0})\, x_j + \cdots \quad . \tag{7}$$

The stability of the solutions can be exactly investigated by studying the eigenvalues of the nxn matrix

$$\frac{\partial f_i}{\partial x_j}(\vec{0}) \quad . \tag{8}$$

That is, the non-linear problem can be suitably linearized as far as the stability problem is concerned. (This fact is related to that the uniqueness of (5) can be investigated at the one-loop level.)

However, in our case the problem can not be simply linearized because the β -functions are at least quadratic in the couplings. Fortunately, there is a way out. We look for a coupling which is obviously asymptotically free at the one-loop level- this coupling is denoted by α_3 - and assume that, to all orders,

$$\alpha_3 \to 0 \quad \text{as } t \to \infty \quad . \tag{9}$$

Then we look for the stable manifold under this assumption.

If this is possible, the assumption is certainly self-consistent.

How to do this job in practice? Let me make a change of variables[21]:

$$\tilde{\alpha}_i = \frac{\alpha_i}{\alpha_3} \quad (i \neq 3) . \tag{10}$$

(As mentioned previously, the α_3 is the coupling which is obviously asymtotically free at the one-loop order.)
The dfe (2) become, in the new set of variables,

$$\alpha_3 \frac{d\tilde{\alpha}_i}{d\alpha_3} = - \tilde{\alpha}_i + \beta_i / \beta_3 \tag{11}$$

Then we look for solutions which satisfy

$$\tilde{\alpha}_i \to \rho_i \quad \text{as} \quad \alpha_3 \to 0 \qquad (0 \leq \rho_i < \infty) . \tag{12}$$

If there is no solution which satisfies (12) for the choice of the variables (10), we try to choose another one which is asymptotically free at the one-loop level and denote it by α_3 , and so forth. So, the α_3 is the one which approaches the origin most slowly.

This problem can be linearized, with the price that we have introduced singularities at the origin in (11), due to (10). It is now clear how to find the stable manifold in perturbation theory:

1) We solve the algebraic equations

$$-\rho_i + \beta_i^{(1)} / \beta_3^{(1)} \Big|_{\text{at } \tilde{\alpha}_i = \rho_i} = 0 \tag{13}$$

The solutions of (13) - the fixed points of (11) - correspond to the origin of the original n-dimensional space.

2) We then analyse the stability of the solutions near the fixed points by looking at the eigenvalues of

$$M_{ij} = - \delta_{ij} + \frac{\partial}{\partial \alpha_j} \left(\frac{\beta_i^{(1)}}{\beta_3^{(1)}} \right) \Big|_{\text{at } \tilde{\alpha}_i = \rho_i} \tag{14}$$

$(i, j \neq 3)$.

Here $\beta_i^{(1)}$ s are the β -functions of the one-loop order.

With those primary discussions, let me come to the standard theory[22]. The theory involves a lot of couplings: We have three gauge couplings, α_i (i=1,2,3), corresponding to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, three Yukawa couplings for the leptons, α_l ($l = e, \mu, \tau$), and six Yukawa couplings for the quarks, α_q (q=u,d,c,s,b,t). We have also the Higgs coupling, α_λ [23]. It can be shown that points for $\alpha_1 \neq 0$ and/or α_l s $\neq 0$ can not belong to the stable manifold.
At this stage, there are two options:
1) We stop here because the physical point does not lie on a stable surface, and hence perturbative analysis on the asymptotic behavior of the theory may not be meaningful.
2) It is reasonable to study the stable manifold whilst α_l s and α_1 may be regarded as some perturbation.

The choice 2) is the first assumption which we have made in applying the RCC to the standard theory. So, in the following, we consider the system with $\alpha_1 = \alpha_l$ s $= 0$, and look for the stable manifold by using the method explained previously. There are many solutions[24] of the algebraic equations (13) for that system. It can be shown that there is, however, no solution for which all the eigenvalues of the stability matrix (14) are non-negative. This indicates the existence of some not-asymptotically free manifold near the origin, and the asymptotic freedom requirement to that system implies reduction (as mentioned at the beginning of this section). We are, of course, interested to see how the stable surfaces look like, particularly near the (semi)physical point, which is supposed to be close to the origin, and can be perturbatively reached. Since, for that point, $\alpha_t \gg \alpha_q$ $(q \neq t)$ must be satisfied - this is an experimental constraint -, we make an approximation by setting $\alpha_q (q \neq t)$ equal to zero. Thus we have arrived at a four dimensional problem; we must study the asymptotic behavior of α_2, α_3, α_t and α_λ, while all the other couplings are set equal to zero.

The corrections due to non-vanishing α_q $(q \neq t)$ can be consistently taken into account, showing that the approximation is rather good[5].

As the " α_3 ", we choose the QCD coupling, α_3 , and find for the system the stable manifold, which, once it is found, is clearly independent of this particular choice for the " α_3 ". We have[20,21,25]:

$$
\begin{aligned}
4\pi\beta_3^{(1)} &= -14\alpha_3^2 , \\
4\pi\beta_2^{(1)} &= -\frac{19}{3}\alpha_2^2 , \\
4\pi\beta_t^{(1)} &= 9\alpha_t^2 - 16\alpha_t\alpha_3 - \frac{9}{2}\alpha_t\alpha_2 , \\
4\pi\beta_\lambda^{(1)} &= 6\alpha_\lambda^2 + 12\alpha_\lambda\alpha_t - 24\alpha_t^2 \\
&\quad + \frac{9}{2}\alpha_2^2 - 9\alpha_\lambda\alpha_2 .
\end{aligned}
\tag{15}
$$

And there are three solutions of (13)[24]:

$$
\begin{aligned}
&\text{(a)}\quad \rho_2 = \rho_t = \rho_H = 0, \\
&\text{(b)}\quad \rho_2 = 0,\quad \rho_t = 2/9,\quad \rho_\lambda = (\sqrt{689} - 25)/18 = a, \\
&\text{(c)}\quad \rho_2 = 42/19,\quad \rho_t = 227/171,\quad \rho_\lambda = 1.18\ldots = d,
\end{aligned}
\tag{16}
$$

with the eigenvalues of the stability matrix (14),

$$
\begin{aligned}
&(-1 , 1/7 , -1) \text{ for (a)}, \\
&(-1 , -1/7 , -(25 + 18a)/21) \text{ for (b)},
\end{aligned}
\tag{17}
$$

and

$$(1 , -227/266 , -(6d/7 + 286/399)) \text{ for (c)}.$$

The signs of the eigenvalues (17) indicate that there is only one AF trajectory which satisfies (b) of (16) at the origin[26], and that, in the case (a) ((c)), we have two dimensional stable surfaces on which all the trajectories satisfy (a) ((c)) at the origin.

IV Assumptions and conclusion

1. The first assumption

As mentioned, the non-vanishing α_1 and α_ℓ s does not correspond to a point on a stable (AF) surface. Nevertheless, we have looked for the stable manifold (by setting $\alpha_1 = \alpha_\ell$ s = 0). How can a point on the stable manifold be related to a point in another regime? Remember there are in general (n-1) independent RG invariant surfaces defined by ϕ_i s. It may happen that a RG invariant surface contains asymptotically free, as well as not asymptotically free surfaces. That is, two points, one on an AF surface and the other on a not AF surface, may lie on the same RG invariant surface. One can, therefore, reach the one point from the other one by solving the partial dfe (4).

2. The second assumption

This assumption is related to the first one. So far we have treated α_2 , differently from α_1 . We did this because it was not necessary to set α_2 = 0 to find the stable manifold. However, this is somewhat unnatural, though technically possible. It is certainly more reasonable if the $SU(2)_L xU(1)_Y$ gauge interactions are treated on the same footing. Our second assumption is, thus, that we regard α_2 also as perturbation. Then there are exactly two solutions of (13) - (a) and (b) of (16) - for which α_2 can be treated as perturbation.

The assumptions, (1) and (2), give the unique extension of the stable surface[27]. More precisely, the AF surface of the undisturbed system (near the origin) is defined by

$$\phi_t^0 = \frac{\alpha_3^{1/7}}{\tilde{\alpha}_t}\left(\frac{2}{9} - \tilde{\alpha}_t\right) = C \quad (\geq 0), \tag{18}$$

$$\phi_h^0 = (\tilde{\alpha}_t)^7\left(\tilde{\alpha}_\lambda - \frac{4}{3}\tilde{\alpha}_t^2 + \frac{75}{2}\tilde{\alpha}_t\tilde{\alpha}_\lambda + \cdots\right) = 0 \quad \text{for } \tilde{\alpha}_t \simeq 0, \tag{19a}$$

or

$$\phi_h^0 = \left(\frac{2}{9} - \tilde{\alpha}_t\right)^{-A/2}\left((a - \tilde{\alpha}_\lambda) + \frac{12a - 32/3}{12a + 44/3}\left(\frac{2}{9} - \tilde{\alpha}_t\right) + \cdots\right) \quad \text{for } \tilde{\alpha}_t \approx \frac{2}{9}, \tag{19b}$$

where $a = (\sqrt{689} - 25)/18$ and $A = 12a + 50/3$. "..." denotes terms like $\tilde{\alpha}_\lambda^m \tilde{\alpha}_t^n$ in (19a) and $(a - \tilde{\alpha}_\lambda)^m (\frac{2}{9} - \tilde{\alpha}_t)^n$ in (19b). And the extended surface is defined by

$$\phi_t = \frac{\alpha_3^{1/7}}{\tilde{\alpha}_t}\left(\frac{2}{9} - \tilde{\alpha}_t - \frac{1}{14}\tilde{\alpha}_2 - \frac{17}{630}\tilde{\alpha}_1 + \cdots\right) = C \quad (\geq 0) \tag{20}$$

$$\phi_h = (\tilde{\alpha}_t)^7\left(\tilde{\alpha}_\lambda - \frac{4}{3}\tilde{\alpha}_t^2 + \frac{75}{2}\tilde{\alpha}_t\tilde{\alpha}_\lambda + \cdots\right) = 0 \quad \text{for } \tilde{\alpha}_t \simeq 0, \tag{21}$$

or for $\tilde{\alpha}_t$ 2/9, we have

$$\phi_t = \frac{\alpha_3^{1/7}}{\tilde{\alpha}_t}\left(\left(\frac{2}{9} - \tilde{\alpha}_t\right) - \frac{1}{12}\tilde{\alpha}_2 - \frac{17}{540}\tilde{\alpha}_1 + \cdots\right) = C \quad (\geq 0) \tag{21a}$$

$$\begin{aligned}\phi_h = {} & \left\{\left(\frac{2}{9} - \tilde{\alpha}_t\right) - \frac{1}{12}\left(\tilde{\alpha}_2 + \frac{17}{45}\tilde{\alpha}_1\right)\right\}^{-A/2} \times \\ & \left\{(a - \tilde{\alpha}_\lambda) + \frac{12a - 32/3}{12a + 44/3}\left(\frac{2}{9} - \tilde{\alpha}_t\right)\right. \\ & \left. + \frac{1}{12a + 8/3}\left(9a\tilde{\alpha}_2 + \frac{9}{5}a\tilde{\alpha}_1 + \frac{12a - 32/3}{12a + 12 + 8/3}\left(\tilde{\alpha}_2 + \frac{17}{45}\tilde{\alpha}_1\right)\right) + \cdots\right] .\end{aligned} \tag{21a}$$

3. The reduction[5,8]

The physical point is supposed to lie on the not AF surface which is defined in (20) and (21), which we call the trivial reduction. If C = 0, we call the non-trivial reduction.

Since all the couplings, except for α_t and α_λ, are experimentally known (with some uncertainties), the trivial reduction requires α_λ to be a function of α_t, while, for the non-trivial reduction, α_t and α_λ are fixed.

It can be, furthermore, shown that the surface defined by the non-trivial reduction is a boundary of the surface of the trivial reduction; the solution of the non-trivial reduction gives the upper bound for the trivial reduction. Beyond the non-trivial surface is presumably something we do not know in perturbation theory.

Since the masses in the standard theory are generated by the Higgs mechanism, reduction of couplings implies certain mass relations. For the non-trivial reduction, for instance, we obtain[5,8,28]

$$m_t \simeq 100 \text{ GeV and } m_h \simeq 68 \text{ GeV},$$

where we have used: $\alpha_3 = 0.123$, $\sin^2\theta_W = 0.228$, $\alpha_{em} = 1/128$, and $M_W = 81$ GeV.

The trivial and non- trivial reductions are technically different: In the trivial reduction all the Yukawa couplings for the quarks are equally treated, and the zeroth order system contains only QCD interactions. The fact that the contributions of α_q $(q \neq t)$ to α_λ are negligibly small is an experimental consequence.

As for the non-trivial reduction, the role of α_t is singled out, and the zeroth order system involves α_3 along with α_t and α_3 . If we thus remember the lowest order approximation of the DSB by the top condensation[12] and our assumption for the non-trivial reduction, we recognize that the both schemes share certain similarities. So, in order to establish the relation between the RCC and the DSB, it is certainly necessary to observe that the asymptotic behavior of the standard theory is influenced by the top condensation or vice versa.

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15) The limit is defined by setting all the Yukawa couplings (except for the top quark), and the $SU(2)_L \times U(1)_Y$ gauge couplings equal to zero. And we require the conditions at Λ (the compositness scale) = infinity. This is the same limit which is also considered in Ref.11.

16) There may be couplings for which we must define α as $g/4\pi$, as in the case of the Higgs coupling.

17) Reduction of coupling constants has been suggested also in: N.P. Cheng, Phys. Rev. D10 (1974) 2706, and N.P. Cheng, A. Das and J. Perez-Mercader, Phys. Rev. D22 (1980) 1829

18) We do not consider the regime of negative

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22) Here I assume three generations, but ignore the Kobayashi-Maskawa angles. They are discussed in Ref.7.

23) $\alpha_\lambda = \frac{\lambda}{4\pi}$ where λ is the Higgs coupling constant.

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26) This point is ultraviolet-unstable (according to Liapunov´s stability), and so infrared-stable in the lowest approximation. (See for instace; B. Pendleton and G.G. Ross, Phys.Lett. 98B (1981) 291, and W. Marciano, Phys. Rev. D41(1990)219.) However, there is a crucial difference in treating this point. This ultraviolet-unstable point exists to all orders in perturbation theory,i.e. the existence of the trajectory that asymptotically approaches (b) is mathematically ensured, while this point as an infrared fixed point may be an artifact of the lowest order approximation, and may vanish as such in higher orders.

27) This is known as the Cauchy initial value problem. The case at hand correspond to the characteristic initial data, and so the extension may not be unique in general. However, It can be shown, via a regularity transformation of the ordinary differential equations (11), that there is a one to one correspondence between a trajectory on the stable surface and one on the extended surface (See Ref.8).

28) Two loop effects are partially included.

Compositeness Condition in Renormalization Group Equation *

NOBUHIRO MAEKAWA

Department of Physics, Kyoto University
Kyoto 606, JAPAN

The top quark mass is now known to be heavier than 89Gev. This might imply that the Higgs boson is composite of the top quarks; that is called top condensation. Miransky, et al.[2] have elaborated this idea with the four-fermion interaction. Recently, Bardeen, et al.[3] conjectured that the divergence of the Yukawa coupling constant at some scale Λ shows the compositeness of the Higgs boson, and predicted the top quark mass and the Higgs mass. However, their compositeness condition has a few problems. Here we discuss this problem, and give some improvements to their treatment of renormalization group equations.

To begin with, we must define a composite particle. When a field has no kinetic term in a bare lagrangian, but obtains it at the low energy, we call that field composite. Fields are usually normalized such that their kinetic terms have weight 1. Therefore this definition is consistent with the ordinary compositeness condition Z=0.

We can regard a field as composite when all bare couplings of the field are infinite and the field has a pole at the low energy, because it implies that the lagrangian loses kinetic term, when the field is rescaled. For example, we take the following lagrangian;

⋆ This work is collaborated by M.Bando, T.Kugo, N.Sasakura, K.Suehiro, Y.Watabiki[1]

$$\mathcal{L}' = \mathcal{L}_k + |\partial_\mu \phi_0|^2 - m_0^2 \phi_0^\dagger \phi_0 - \frac{\lambda_0}{2}(\phi_0^\dagger \phi_0)^2 + (g_{t0} \bar{L}_0 t_{R0} \phi_0 + g_{b0} \bar{L}_0 b_{R0} \phi_0^c + h.c.) \quad (1)$$

where $\mathcal{L}_k$ is the kinetic term of quarks and $L = (t, b)_L$. When $m_0, g_{t0}, g_{b0}, \lambda_0$ are all very large of order ϵ^{-1} ($\epsilon \ll 1$), the field redefinition $\phi_0 = \epsilon^{\frac{1}{2}} \tilde{\phi}$ renders the lagrangian (1) into

$$\mathcal{L}' = \mathcal{L}_k + \epsilon |\partial_\mu \tilde{\phi}|^2 - \tilde{m}^2 \tilde{\phi}^\dagger \tilde{\phi} - \frac{\tilde{\lambda}}{2}(\tilde{\phi}^\dagger \tilde{\phi})^2 + (\tilde{g}_t \bar{L}_0 t_{R0} \tilde{\phi} + \tilde{g}_b \bar{L}_0 b_{R0} \tilde{\phi}^c + h.c.). \quad (2)$$

This indeed loses the kinetic term in the limit $\epsilon \to 0$. And if the field ϕ has a pole at low energy, ϕ can be called composite. Note here that, if the starting lagrangian is given by (2) with $\tilde{m}$, $\tilde{g}_t$, $\tilde{g}_b$, $\tilde{\lambda}$ finite and $\epsilon = 0$, then all the bare couplings in the corresponding lagrangian (1) must be of the same order of infinities.

Let us consider the theory with cut off Λ. Then the running couplings $g(\mu) \equiv (m(\mu), g_t(\mu), g_b(\mu), \lambda(\mu))$ are regarded as the bare couplings g_0 at $\mu \sim \Lambda$. Therefore it is legitimate to impose the following boundary conditions for the renormalization group equations as the compositeness condition:

$$g_t(\Lambda),\ g_b(\Lambda),\ m(\Lambda) = \infty \quad (3)$$

[Here we are considering renormalization group equation in the theory in which ϕ has its kinetic term.] For simplicity, we calculate the β function in the system (1) without cut off, and obtain

$$\begin{aligned} 16\pi^2 \frac{d \ln g_t}{dt} &= K_h + K_f^{(t)} + K_v(t) + G_t, \\ 16\pi^2 \frac{d \ln g_b}{dt} &= K_h + K_f^{(b)} + K_v(b) + G_b, \qquad t = \frac{1}{2}\ln(\mu^2/\mu_0^2) \end{aligned} \quad (4)$$

where K_h, K_f and K_v correspond to the one loop correction terms to Higgs self-energy, fermion self energy and the vertex, respectively, excluding the contributions from the gauge fields which are all included in G.

$$\begin{aligned}
&K_h = 3(g_t^2 + g_b^2),\\
&K_f^{(t)} = \frac{3}{2}g_t^2 + \frac{1}{2}g_b^2;\ \ K_f^{(b)} = \frac{3}{2}g_b^2 + \frac{1}{2}g_t^2,\\
&K_v(t) = -2g_b^2;\ \ K_v(b) = -2g_t^2,\\
&G_t = -8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2;\ \ G_b = -8g_3^2 - \frac{9}{4}g_2^2 - \frac{5}{12}g_1^2;
\end{aligned} \tag{5}$$

where g_1, g_2, and g_3 are the gauge couplings of $U(1)$, $SU(2)$ and $SU(3)$.

Let us consider running of the ratio $R = g_t/g_b$, which is given from (4) by

$$\begin{aligned}
16\pi^2\frac{d\ln R}{dt} &= K_f^{(t)} - K_f^{(b)} + K_v^{(t)} - K_v^{(b)} + G_t - G_b,\\
&= 3(g_t^2 - g_b^2) - g_1^2.
\end{aligned} \tag{6}$$

This indicates that R also diverges when $g_t, g_b \to \infty$ unless $g_t = g_b$, and therefore we cannot have a situation in which two couplings $\tilde{g}_t$ and $\tilde{g}_b$ have a finite ratio. In other words, if one starts from the low energy side with $R^{-1}(\mu = M_Z) = 0.1$, for instance, we are inevitably led to $R^{-1}(\Lambda) = 0$. This is an absurd conclusion, since we are free to consider the models possessing any values of $\tilde{g}_t/\tilde{g}_b$.

Does this indicate that the compositeness condition in renormalization group equations is inconsistent? The answer is "No" and we here propose an improverd renormalization group equation compatible with the compositeness condition with finite ratio g_t/g_b. If one recalls the fact that the Higgs mass m^2 diverges at $\mu = \Lambda$ also, one thinks that the contributions of the loop diagrams with the internal lines of Higgs particle, which appear in the terms K_f and K_v, should be suppressed. Notice that the right-hand side of eq.(6) does not include the Higgs self energy term K_h for which no suppression works. Therefore if we can take account of the infinity of the Higgs mass properly, all must go well.

So, instead of the above mass-independent renormalization, we now employ mass-dependent renormalization devised by Georgi-Politzer ,[4] in which the contributions of the heavy mass particles are automatically suppressed. We set the renormalization conditions as follows;

$$
\begin{aligned}
&\frac{\partial \Gamma_H^{(2)}}{\partial p^2}\Big|_{p^2=-\mu^2} = 1 \\
&\Gamma_H^{(2)}|_{p^2=-\mu^2} = -\mu^2 - m^2 \\
&\frac{\partial \Gamma_f^{(2)}}{\partial p^\mu \gamma_\mu}\Big|_{p^2=-\mu^2} = 1 \\
&\Gamma_{Hff}^{(3)}|_{\substack{p_f^2=-\mu^2 \\ p_H^2=0}} = g_f
\end{aligned}
\tag{7}
$$

where $\Gamma_{H,f}^{(2)}$ are the inverse propagators of Higgs and fermion, respectively, and $\Gamma_{Hff}^{(3)}$ is the vertex function of the Higgs and fermions. Here we take "$p_H^2 = 0$" instead of "$p_H^2 = -\mu^2$" in the vertex renormalization condition only for the convenience of calculation. The results are expressed as follows:

$$
\begin{aligned}
16\pi^2 \frac{d\ln g_t}{dt} &= K_h + K_f^{(t)} f(a) + K_v^{(t)} \tilde{f}(a) - G_U \\
16\pi^2 \frac{d\ln g_b}{dt} &= K_h + K_f^{(b)} f(a) + K_v^{(b)} \tilde{f}(a) - G_D \\
16\pi^2 \frac{da}{dt} &= 2a[16\pi^2 - (1+a)K_h]
\end{aligned}
\tag{8}
$$

where

$$
\begin{aligned}
f(a) &= 1 + \frac{2}{a}[(1 + \frac{3}{a} - 3] \simeq a - \frac{5}{6}a^2 + O(a^3), \\
\tilde{f}(a) &= 1 - \frac{2}{a}[1 - \frac{1}{a} ln(1+a)] \simeq \frac{2}{3}a - \frac{a^2}{2} + O(a^3)
\end{aligned}
\tag{9}
$$

with $a = \mu^2/m_H^2$. Note here that we get suppression factors $f(a)$ and $\tilde{f}(a)$ in front of K_f, K_v terms in (8) as expected. Thanks to this, these renormalization group equations can now have some solutions possessing the desired properties,

$$
\frac{g_t(\Lambda)}{m(\Lambda)} \sim O(1), \quad \frac{g_b(\Lambda)}{m(\Lambda)} \sim O(1) \tag{10}
$$

Note that the divergences of the running couplings are now governed by the contribution of the Higgs self energy term, K_h. This result is consistent with the fact that the divergences of couplings are solely the redefinition of Higgs field.

Unfortunately, the relation among m_t, m_b and the scale at which the couplings diverge depends on the renormalization scheme. But it is natural, when one recalls that a renormalization scheme decides a definition of couplings. For example, if we change the renormalization condition of the fermion propagator as follows,

$$\Gamma_f^{(2)}|_{p^2=-\mu^2} = p^\mu \gamma_\mu, \tag{11}$$

then the suppression factor becomes,

$$f(a) = 1 - \frac{2}{a}[(1+\frac{1}{a})\ln(1+a) - 1] \simeq \frac{a}{3} - \frac{a^2}{6} + O(a^3). \tag{12}$$

However, this is also a suppression factor, therefore the above properties (10) well remain to be satisfied.

As conclusions, when we impose the compositeness condition as the boundary conditon of the renormalization group equation, we should use the mass dependent renormalization, since the Higgs mass also becomes infinite.

We apply these conditions to the Standard Model, and obtain

Λ	m_t(GeV)	$\frac{g_b(\Lambda)}{g_t(\Lambda)} / \frac{g_b(M_Z)}{g_t(M_Z)}$
10^{15}	234	0.28
10^{10}	259	0.33
10^4	405	0.50

But the value of $\frac{g_b(\Lambda)}{g_t(\Lambda)} / \frac{g_b(M_Z)}{g_t(M_Z)}$ is fairly dependent on the renormalization scheme.

Finally we add a remark on an interesting fact; when the top quark mass is heavy, the mass ratio m_t/m_H is decided by the infra fixed point of the renormalization group equation.

$$16\pi^2\frac{d\lambda}{dt} = 12\lambda^2 + 4N_c\lambda g_t^2 - 4N_c g_t^4,$$
$$16\pi^2\frac{dg_t}{dt} = g_t(N_c g_t^2 + \frac{3}{2}g_t^2), \tag{13}$$

for N_c-colored fermions and the ratio $r = 2\lambda/g_t^2$ obeys the equation

$$16\pi^2\frac{d\ln r}{dt} = 12\lambda + (2N_c - 3)g_t^2 - 4N_c g_t^4\lambda^{-1}. \tag{14}$$

Thus the solution of the equation

$$12\lambda^2 + (2N_c - 3)g_t^2\lambda - 4N_c g_t^4 = 0 \tag{15}$$

gives an infrared fixed point;

$$r = \frac{1}{12}[\sqrt{(2N_c-3)^2 + 192N_c} - (2N_c - 3)] = 4 + O(\frac{1}{N_c}). \tag{16}$$

Since $2\lambda/g_t^2 = m_H^2/m_t^2$, this fixed point gives $m_H = 2m_t$, the same mass relation as Nambu-Jona-Lasinio's, in the limit $N_c \to \infty$. When we put $N_c = 3$, then we obtain $r = 1.77$. The situation does not change drascally even if one includes gauge interaction effects.

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Extra Weak Bosons Implied by Complementarity in a Confining Gauge Theory

Masaki Yasuè

Institute for Nuclear Study, University of Tokyo
Midori-cho, Tanashi, Tokyo 188, Japan

Abstract

Extra W and Z bosons together with W and Z as composite particles are introduced in a confining gauge model based on a "color" $SU(2)_L^{loc} \times SU(2)_L^{loc} \times SU(2)_R^{loc}$ symmetry. Within the framework of complementarity, the vector meson (such as Z) dominance of the photon is naturally implemented and quarks (q_i^A for A=1,2,3 and i=1,2) and leptons (ℓ_i) are composites of scalars, $\tilde{w}_i$, carrying the weak charge and spinors, c^α (α = 0,1,2,3), carrying the three colors (α = 1,2,3) and the lepton number ($\alpha = 0$): $q_i^A \sim \tilde{w}_i c^A$ and $\ell_i \sim \tilde{w}_i c^0$. The confined gauge model is shown to be equivalent to the conventional model realized in the Higgs phase as far as the scalar degrees of freedom are frozen. Phenomenological implications of these extra W and Z bosons are discussed.

The experimental determination of m_W = (80.00 ± 0.56) GeV[1] and m_Z = (91.09 ± 0.06) GeV[2] has confirmed the mass relation, $m_W = \cos\theta m_Z$, with $\sin\theta$ evaluated in low - energy weak interactions.[3] It has implied the validity of the standard electroweak model of the Glashow - Weinberg - Salam (GWS) type[4] based on $SU(2)_L^{loc} \times U(1)_Y^{loc}$. However, there is a theoretical belief that new physics beyond the standard model manifests itself above the energy scale specified by the Fermi mass, $G_F^{-1/2}$, of ~300 GeV. Among them are new phenomena due to compositeness of the "elementary" particles such as quarks, leptons and weak bosons.[5] If underlying dynamics for composite particles are provided by a non - abelian gauge theory, the useful notion called complementarity[6] can be used to examine low - energy physics for composites.[7] When it is applied to weak bosons, the GWS model turns out to be (almost) equivalent to the model on $U(1)_{em}^{loc}$ with the confined "color" $SU(2)_L^{loc}$ symmetry, *i.e.*, the Bjorken-Hung-Sakurai (BHS) model for the kinetic γ - Z mixing scheme.[8] The weak bosons, $W^\pm$ and Z, are made as[9] $W_\mu^\pm \sim Tr(\tau^{(\pm)}\tilde{w}_L^\dagger D_\mu \tilde{w}_L)$ and $Z_\mu \sim Tr(\tau^{(3)}\tilde{w}_L^\dagger D_\mu \tilde{w}_L)$, where $\tilde{w}_L$ is a scalar carrying the weak charge and is represented by the Higgs scalar ϕ as $\tilde{w}_L = (\phi^G, \phi)$. At the same time, L-handed quarks (q_{Li}^A for A=1,2,3 and i=1,2) and leptons (ℓ_{Li}) are regarded as composites described by $q_{Li}^A = \tilde{w}_{Li}^a c_{La}^A$ and $\ell_{Li} = \tilde{w}_{Li}^a c_{La}^0$. Starting with the lagrangian of the GWS model, one can derive the BHS model with the kinetic mixing parameter, λ, for γ - Z, $\lambda = e/g$ under the constraint of $\langle \tilde{w}_L^\dagger \tilde{w}_L \rangle = I$.[10] This equality can be regarded as a result of vector meson (such as Z) dominance of the photon.[11)]

One may wonder what happens in QCD, which is certainly based on the confining color $SU(3)_c^{loc}$ symmetry. Nucleons, scalar mesons and vector mesons are composites of quarks. Complementarity will state that composite nucleons are regarded as (constituent) quarks and composite vector mesons as massive gluons.[12)] Scalar mesons are described by the Nambu - Goldstone bosons, which are not absorbed into the massless gluons. Let the flavor group be $SU(3)_f$ for q_A^i (A =1, 2, 3 for three colors; i = 1, 2, 3 for three flavors), *i.e.*, u, d and s, which comes from the symmetry breaking of the chiral $SU(3)_L \times SU(3)_R$ symmetry. To faciliate the symmetry breaking, we introduce two scalars, ξ_{Li}^A: $(\mathbf{3}, \mathbf{1})$ and ξ_{Ri}^A: $(\mathbf{1}, \mathbf{3})$ for $(SU(3)_L, SU(3)_R)$, which are decomposed as $\xi_{Li}^A = \xi_i^j \xi_{Vj}^A$ and $\xi_{Ri}^A = \xi_i^{\dagger j} \xi_{Vj}^A$. It is suggested that ξ_V is identified with scalar diquarks, $\xi_{Vi}^A = \varepsilon^{ABC} \varepsilon_{ijk} q_B^j q_C^k / f_\Pi^2$.[12)] The remaining scalar, ξ, represents the Nambu - Goldstone modes, Π_i^j as $\xi = \exp(\Pi/f_\Pi)$. QCD gets broken completely as fas as diquarks are condensed to develop $< \xi_{Vi}^A > = f_\Pi \delta_i^A$. In this phase, *i.e.*, the Higgs phase of QCD, the gluons, G_A^B, become massive and serve as the octet vector mesons including ρ and quarks act as the octet baryons including P, N and Λ. While, in the confining phase, color-singlet composites are supplied by $\xi_{Vi}^A q_A^j / f_\Pi$ ($\sim qqq$ for $\xi_V \sim qq$) as the octet baryons and by $\xi_V^\dagger i D_\mu \xi_V / f_\Pi^2$ ($\sim \bar{q}\bar{q}qq$) as the octet vector mesons. Then, the both phases at low-energies contain the octet baryons and vector mesons. The transmutation of gauge bosons (*i.e.*, gluons) into massive vector mesons (*i.e.*, ρ etc.) arises.[13)] The similar suggestion has been lately advocated on the basis of the non-linear sigma model with a dummy hidden symmetry,[14)] where gauge bosons are regarded as composites and scalar mesons like π are taken into account but without the baryons as qqq.

Along this line of the compositeness of "elementary" particles, a possible new physics beyond the standard model is investigated by introducing extra W and Z bosons. The confining "color" gauge group to be studied in the present article is specified by $SU(2)_L^{loc}$ for the W and Z bosons as well as $G^{loc} = SU(2)_V^{loc} \times SU(2)_A^{loc}$ (equivalently, $SU(2)_L^{loc} \times SU(2)_R^{loc}$) for extra W and Z bosons. The QCD - like "color" (vectorial) $SU(2)_V^{loc}$ symmetry as G^{loc} is not suitable for the L - R asymmetric weak interactions. If the effects from the composite vector mesons coupled to the right - handed currents (corresponding to the $SU(2)_R^{loc}$ - gauge bosons) are neglected, the extra W and Z bosons, related to $SU(2)_L^{loc}$, are allowed to be as light as 100 GeV as far as the low-energy weak interaction phenomenology is concerned.[15)] It is because the couplings to quarks and leptons are of the V - A form, which does not alter low-energy charged-current interactions. The lagrangian with extra composite weak bosons is characterized by vector meson dominances, which are described by the kinetic mixing terms among the photon (A^0), W and Z (mainly V) and extra W and Z (mainly L and R)[16)]

$$\mathcal{L}_{mix} = -\frac{1}{2}(\lambda_{\gamma V} V_{\mu\nu}^{(3)} + \lambda_{\gamma L} L_{\mu\nu}^{(3)} + \lambda_{\gamma R} R_{\mu\nu}^{(3)}) A^{0\mu\nu} - \frac{\lambda_{VL}}{2} L_{\mu\nu}^{(i)} V^{(i)\mu\nu} \tag{3·9}$$

We will demonstrate how the kinetic mixings are generated in the confining phase of $SU(2)_L^{loc} \times SU(2)_L^{loc} \times SU(2)_R^{loc} (\equiv \mathcal{G}^{loc})$ and obtain the effective lagrangian for composite quarks, leptons, W, Z and extra weak bosons.[17]

The particles contained are 1) "color" gauge bosons, $(G_\mu)_a^b$ of $SU(2)_L^{loc}$ with the gauge coupling g, $(G_{L\mu})_m^n$ of $SU(2)_L^{loc}$ with g_L and $(G_{R\mu})_m^n$ of $SU(2)_R^{loc}$ with g_R, and a "flavor" gauge boson, B_μ, of $U(1)_Y^{loc}$ with g'; 2) "color" $SU(2)_{L,R}$ doublet fermions with the "flavor" suffix α (= 0,1,2,3) for the three colors (α = 1,2,3) and the lepton number ($\alpha = 0$), c_{mL}^α: $(\mathbf{1}, Y; \mathbf{2}, \mathbf{1})$ and c_{mR}^α: $(\mathbf{1}, Y; \mathbf{1}, \mathbf{2})$, for $(SU(2)_L^{loc}, U(1)_Y^{loc}; SU(2)_L^{loc}, SU(2)_R^{loc})$, where $m(=1, 2)$ denotes the $SU(2)_{L,R}^{loc}$ - "color" and Y (= B -L)= -1 for c^0; = 1/3 for c^A (A = 1, 2, 3),and; 3) three kinds of "color" scalars, $\tilde{w}_{Li}^m$: $(\mathbf{1}, \tau^{(3)}; \mathbf{2}, \mathbf{1})$, $\tilde{w}_{Ri}^m$: $(\mathbf{1}, \tau^{(3)}; \mathbf{1}, \mathbf{2})$ and ξ_m^a: $(\mathbf{2}, 0; \mathbf{2}, \mathbf{1})$, where $i(=1,2)$ and $a(=1,2)$, respectively, denote the "flavor" and $SU(2)_L^{loc}$ - "color".

Let us demand that $\mathcal{G}^{loc}$ be confined to generate composite particles and to form the following scalar condensates: $\langle \tilde{w}_{L(R)i}^m (\tilde{w}_{L(R)}^\dagger)_m^j \rangle = \delta_i^j$, $\langle (\tilde{w}_{L(R)}^\dagger)_m^i \tilde{w}_{L(R)i}^n \rangle = \delta_m^n$, $\langle (\xi^\dagger)_a^m \xi_m^b \rangle = \delta_a^b$ and $\langle \xi_m^a (\xi^\dagger)_a^n \rangle = \delta_m^n$. Also defined are "color" - singlet composite fermions for quarks (q) and leptons (ℓ) and composite vector mesons, V_μ, L_μ and R_μ for W, Z and extra weak bosons, according to:

$$q_{iL}^A = \sum_m \tilde{w}_{Li}^m c_{mL}^A, \qquad \ell_{iL} = \sum_m \tilde{w}_{Li}^m c_{mL}^0, \tag{2a, b}$$

$$q_{iR}^A = \sum_m \tilde{w}_{Ri}^m c_{mR}^A, \qquad \ell_{iR} = \sum_m \tilde{w}_{Ri}^m c_{mR}^0, \tag{2c, d}$$

$$f(V_\mu)_i^j = (\tilde{w}_L i D_\mu \tilde{w}_L^\dagger)_i^j, \quad f_L(L_\mu)_i^j = [\tilde{w}_L (\xi i D_\mu \xi^\dagger) \tilde{w}_L^\dagger]_i^j, \tag{2e, f}$$

$$f_R(R_\mu)_i^j = (\tilde{w}_R i D_\mu \tilde{w}_R^\dagger)_i^j \tag{2g}$$

as well as $f' A_\mu^0 = g' B_\mu$. Hereafter, quarks and leptons are denoted by $\psi_{iL}^\alpha = \ell_{iL}$ (α=0); $= q_{iL}^A$ (α (=A) = 1,2,3).

By noticing that $gG_{\mu\nu} = (\tilde{w}_L \xi)^\dagger v_{1\mu\nu} (\tilde{w}_L \xi)$, $g_L G_{L\mu\nu} = \tilde{w}_L^\dagger v_{2\mu\nu} \tilde{w}_L$ and $g_R G_{R\mu\nu} = \tilde{w}_R^\dagger v_{3\mu\nu} \tilde{w}_R$ for $v_{\mu\nu} = \partial_\mu v_\nu$ - $\partial_\nu v_\mu$ - $i[v_\mu, v_\nu]$ etc., where

$$v_{1\mu} = fV_\mu + f_L L_\mu + e(\tau^{(3)}/2) A_\mu^0, \tag{3a}$$

$$v_{2\mu} = fV_\mu + e(\tau^{(3)}/2) A_\mu^0, \tag{3b}$$

$$v_{3\mu} = f_R R_\mu + e(\tau^{(3)}/2) A_\mu^0, \tag{3c}$$

we find, from the lagrangian for the gauge theory evaluated in the confining phase,[18] $\mathcal{L}_{conf}$ to be:

$$\mathcal{L}_{conf} = -\frac{1}{2g^2} Tr(v_{1\mu\nu} v_1^{\mu\nu}) - \frac{1}{2g_L^2} Tr(v_{2\mu\nu} v_2^{\mu\nu}) - \frac{1}{2g_R^2} Tr(v_{3\mu\nu} v_3^{\nu\mu})$$

$$
\begin{aligned}
&-\frac{e^2}{4g'^2}A^0_{\mu\nu}A^{0\mu\nu}+\Lambda^2 Tr(fV_\mu)^2+\Lambda^2_L Tr(f_L L_\mu)^2+\Lambda^2_R Tr(f_R R_\mu)^2,\\
&+\overline{\psi_L}\gamma^\mu(i\partial_\mu+fV_\mu+eQ_{em}A^0_\mu)\psi_L+\overline{\psi_R}\gamma^\mu(i\partial_\mu+f_R R_\mu\\
&+eQ_{em}A^0_\mu)\psi_R,
\end{aligned}
\tag{4}
$$

as long as the radial scalar excitations are neglected. The mass - dimensions, Λ and $\Lambda_{L(R)}$, are associated with the scalars, $\tilde{w}_L$ and $\xi(\tilde{w}_R)$. Note that the extra boson, L_μ, does not couple to quarks and leptons. The coupling strengths, f, f_L, f_R and f', satisfy $1/f^2 = 1/g^2 + 1/g_L^2$, $f_L = g$, $f_R = g_R$ and $1/f'^2 = 1/g^2 + 1/g'^2 + 1/g_L^2 + 1/g_R^2$ for the canonical kinetic terms of V_μ, L_μ, R_μ and A^0_μ. For "color" singlet composites, the unbroken $U(1)_Y^{loc}$ symmetry is coincident with the $U(1)_{em}^{loc}$ symmetry. The third - isospin is provided through the $U(1)_Y^{loc}$ charge of $\tilde{w}$, which ensures $Q_{em} = (\tau^{(3)} + Y)/2$.The kinetic mixings are now characterized by e/f $(= \lambda_{\gamma V})$ for A^0 and V, $e/f_{L(orR)}$ $(= \lambda_{\gamma L(orR)})$ for A^0 and L (or R) and f/f_L $(= \lambda_{VL})$ for V and L. The kinetic mixings cause the following field - redefinition:

$$\mathcal{V}^{(3)}_\mu = \sqrt{1-\lambda^2_{\gamma V}}(V^{(3)}_\mu + \lambda_{VL}L^{(3)}_\mu - \frac{\lambda_{\gamma V}\lambda_{\gamma R}}{1-\lambda^2_{\gamma V}}R^{(3)}_\mu), \tag{5a}$$

$$\mathcal{V}^{(\pm)}_\mu = V^{(\pm)}_\mu + \lambda_{VL}L^{(\pm)}_\mu, \quad \mathcal{L}^{(i)}_\mu = \sqrt{1-\lambda^2_{VL}}L^{(i)}_\mu, \tag{5b, c}$$

$$\mathcal{R}^{(3)}_\mu = \sqrt{(1-\lambda^2_{\gamma V}-\lambda^2_{\gamma R})/(1-\lambda^2_{\gamma V})}R^{(3)}_\mu, \quad \mathcal{R}^{(\pm)}_\mu = R^{(\pm)}_\mu. \tag{5d, e}$$

It is not difficult to show the equivalence of the interactions in the confining and Higgs phase as far as the scalar degrees freedom are frozen. The vector fields, A_μ, $\mathcal{V}_\mu$, $\mathcal{L}_\mu$ and $\mathcal{R}_\mu$, defined in Eqs.(5a $\sim$ e), are also expressive in terms of fields with the orthogonal mixings in the Higgs phase, which reflect $SU(2)_L^{loc} \times \mathcal{SU}(2)_L^{loc} - SU(2)_D^{loc}$ with the gauge coupling $g_D = gg_L/\sqrt{g^2+g_L^2}$ $(= g\cos\theta_L = g_L\sin\theta_L)$, $U(1)_Y \times \mathcal{SU}(2)_R^{loc}$ $\rightarrow U(1)_D^{loc}$ with $g'_D = g'g_R/\sqrt{g'^2+g_R^2}$ $(= g'\cos\theta_R = g_R\sin\theta_R)$ and $SU(2)_D^{loc} \times U(1)_D^{loc}$ $\rightarrow U(1)_{em}^{loc}$ with $e = g_D g'_D/\sqrt{g_D^2+g'^2_D}$ $(= g'_D\cos\theta = g_D\sin\theta)$:

$$A_\mu = \sin\theta a^{(3)}_\mu + \cos\theta b_\mu, \quad \mathcal{V}^{(3)}_\mu = \cos\theta a^{(3)}_\mu - \sin\theta b_\mu, \tag{6a, b}$$

$$
\begin{aligned}
&\mathcal{V}^{(\pm)}_\mu = \sin\theta_L G^{(\pm)}_{L\mu} + \cos\theta_L G^{(\pm)}_\mu, \quad \mathcal{L}^{(i)}_\mu = \cos\theta_L G^{(i)}_{L\mu} - \sin\theta_L G^{(i)}_\mu,\\
&\mathcal{R}^{(3)}_\mu = \cos\theta_R G^{(3)}_{R\mu} - \sin\theta_R B_\mu, \quad \mathcal{R}^{(\pm)}_\mu = G^{(\pm)}_{R\mu},
\end{aligned}
\tag{6c $\sim$ f}
$$

where $a^{(3)}_\mu = \sin\theta_L G^{(3)}_{L\mu} + \cos\theta_L G^{(3)}_\mu$ and $b_\mu = \sin\theta_R G^{(3)}_{R\mu} + \cos\theta_R B_\mu$. Following these relations together with the identification of $f = g_D$, $f_L = g$ and $f_R = g_R$ leading to

$\lambda_{\gamma V} = \sin\theta$, $\lambda_{\gamma L} = \sin\theta\sin\theta_L$, $\lambda_{\gamma R} = \cos\theta\sin\theta_R$ and $\lambda_{VL} = \sin\theta_L$, it is shown that the lagrangian evaluated in the Higgs phase is exactly same as the one in the confining phase, $\mathcal{L}^A_{conf}$. The similar argument can be applied to models with an extra Z bosons based on $\mathcal{G}^{loc} = SU(2)_L^{loc} \times U(1)^{loc}$.[19]

For the $SU(2)_L^{loc} \times U(1)_Y^{loc} \times \mathcal{SU}(2)_L^{loc}$ model, where the contribution from the right - handed bosons, R_μ, are neglected, we find that the low - energy phenomenology is controlled by

$$\mathcal{L}^{ch}_{eff} = 2\sqrt{2}G_F J^{(-)}_{L\mu} J^{(+)\mu}_L, \tag{7a}$$

$$\mathcal{L}^{n}_{eff} = 4\sqrt{2}G_F[(J^{(3)}_L - \sin^2\theta J^{em})^2 + C_{em}J^{em}J^{em}], \tag{7b}$$

where $4\sqrt{2}G_F m_V^2 = f^2$ for $m_V = f\Lambda$ and $C_{em} = (m_V^2/m_L^2(f^2/f_L^2)\sin^4\theta$. The weak boson masses, $m_{W,Z}$, are fixed to be: $m_Z = 91.09$ GeV (as the central value of the averaged data) The constraints on $\sin^2\theta$ and C_{em}, respectively, come from ν - induced reactions[20] and the Bhabha - scattering with $\Lambda^c_- > 7.1$ TeV (for vector coupling),[21] which result in $\sin^2\theta = (0.22 \sim 0.24)$ and $C_{em} < 0.002$. Computation of C_{em} shows that $C_{em} < 0.002$ is satisfied. Another constraints are based on the experimental results on $p\bar{p} \to W'$ (or Z') $+ \cdots$ followed by W' (Z') $\to e\nu$ (e^+e^-).[22] These impose $m_{W'} \gtrsim$ (290, 240, 200) GeV for $\sin^2\theta = (0.22, 0.2225, 0.2235)$ and $m_{Z'} \gtrsim$ (440, 330, 200) GeV for $\sin^2\theta = (0.22, 0.225, 0.228)$ but no restriction for the case with $\sin^2\theta \gtrsim 0.2235$ (W') and 0.228 (Z'). The prediction on $p\bar{p} \to W'$ (or Z') $+ \cdots \to jj + \cdots$ is so far consistent with the data.[23] The theoretical constraint dectates $m_W m_{W'} = \cos\theta m_Z m_{Z'}$. Under these constraints, we evaluate various quantities and show

1) the dependence of the Z decay widths on $m_{Z'}$ ($= m_{Z*}$ in the Figures) for $\sin^2\theta =$ 0.22, 0.225 and 0.23: $\Gamma(Z \to \text{all})$ (in *Fig.*1) and $\Gamma(Z \to e^+e^-)$ (in *Fig.*2) together with the standard model predictions at $\sin^2\theta = 0.2313$ (for $m_t = 100$ GeV),
2) the coupling constant of f_L ($= g^*$ in the Figure) divided by e (in *Fig.*3) and the Z' decay width $\Gamma(Z' \to \text{all})$ (in *Fig.*4) and
3) the cross section of $\sigma(e^+e^- \to \mu^+\mu^-)$ as functions of $\sqrt{s}$ for $m_{Z'} = 250$, 500, 1000, 1500, 2000 and 3000 GeV at $\sin^2\theta = 0.225$ (in *Fig.*5).

The expected deviations are to be detected by the precise determination of the Z properties. Furthermore, TeV - e^+e^- colliders such as JLC, CLIC and so on will see the extra weak boson as heavy as 1 TeV or even heavier than the beam energy owing to the broader width of Z' of $\mathcal{O}(100$ GeV) as long as the extra boson acts as a "elementary" particle. However, since the compositeness scale can be as low as the order of $G_F^{-1/2} \cong$ 300 GeV, the electron itself will manifest the substructure perhaps through (unknown) form - factor effects around $E = \mathcal{O}(1$ TeV), which even distort the behavior of e^+e^- annihilation via the photon and Z.

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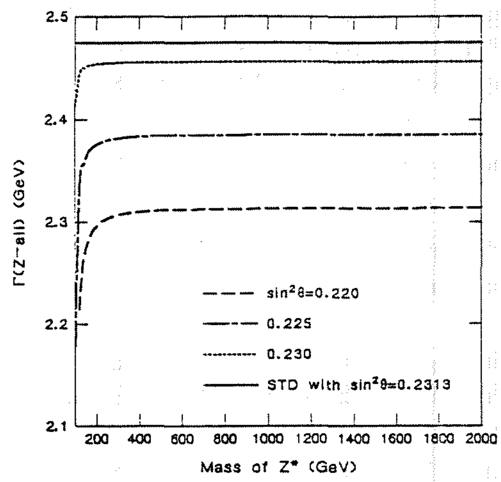

Figure 1

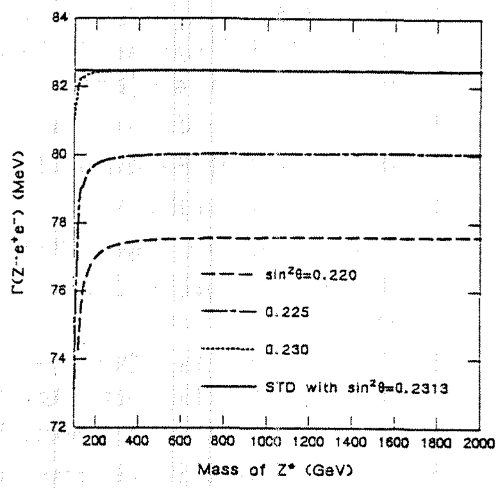

Figure 2

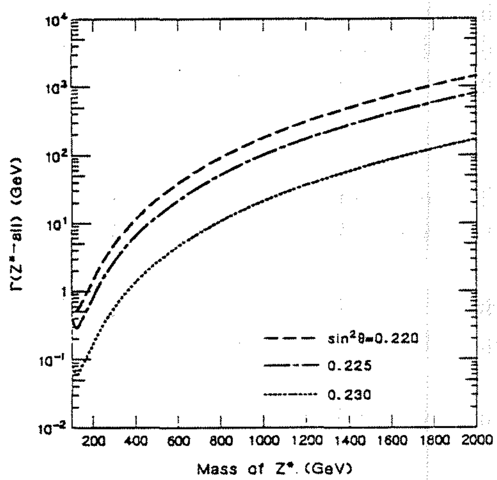

Figure 3

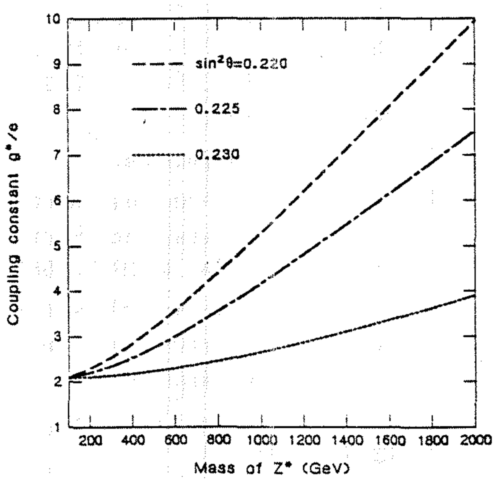

Figure 4

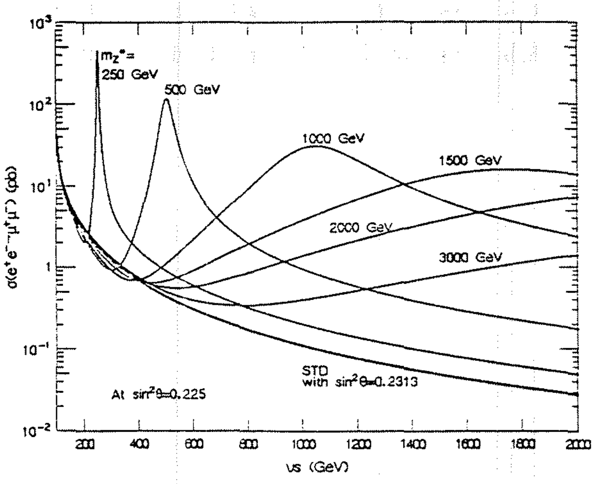

Figure 5

DYNAMICS OF THE NAMBU-JONA-LASINIO TYPE FOR SUBQUARKS

Keiichi Akama

Department of Physics, Saitama Medical College

Kawakado, Moroyama, Saitama, 350-04

abstract

We present an improved version of the dynamical subquark model of the Nambu-Jona Lasinio type. The six-fermion interaction to form the composite quarks and leptons is incorporated. The four fermion interaction to form Higgs scalar is omitted. It allows us a parameter assignment which guarantees light quarks and leptons and heavy weak bosons.

This talk is based on the recent work in collaboration with T. Hattori.[1] Proliferations of the color triplets and weak iso-doublets seem to suggest a further fundamental layer of matter, the subquark (or preon).[2),3),4] In this picture, quarks q and leptons ℓ are composite such that

$$q \sim wc \;\; or \;\; whc, \qquad \ell \sim wc_\ell \;\; or \;\; whc_\ell, \tag{1}$$

where w, h, c, and c_ℓ are the subquarks carrying the weak isospin, the generation quantum number, the color, and the leptonic color, respectively. The weak bosons W^i_μ, Higgs scalars ϕ, and even photon A_μ and gluon G^a_μ could also be composite.[3]

$$W^i_\mu \sim \overline{w}_L \gamma_\mu \tau^i w_L, \;\; \phi \sim \overline{w}_L w_R, \;\; G^a_\mu \sim \overline{c}\gamma_\mu \lambda^a c, \;\; A_\mu \sim \sum_s \overline{s}\gamma_\mu Q_s s, \tag{2}$$

where $\sum_s$ indicates the summation over the subquark species $s = (w, h, c, c_\ell)$, and Q_s is the electric charge of the subquark s. About a decade ago, we proposed the dynamical subquark model[3] of the Nambu-Jona-Lasinio type.[5] Let us summarize the main features of the model. The basic Lagrangian is given by

$$
\begin{aligned}
\mathcal{L} = & \, i\overline{w}\,\partial\!\!\!/ w + \overline{h}(i\,\partial\!\!\!/ - m_h)h + \overline{c}(i\,\partial\!\!\!/ - m_c)c + \overline{c}_\ell(i\,\partial\!\!\!/ - m_{c\ell})c_\ell \\
& + F_1\left(\sum_s \overline{s}\gamma_\mu Y_s s\right)^2 + F_2\left(\overline{w}\gamma_\mu \tau^i w_L\right)^2 + F_3\left(\overline{c}\gamma_\mu \lambda^i c\right)^2 \\
& + F_H| - a_1 \overline{w}_R^G w_{1L}^c + a_2 \overline{w}_L w_{2R}|^2
\end{aligned}
\tag{3}
$$

where m_s ($s = h, c, c_\ell$) denotes the mass of the subquark s, F_i ($i = 1,2,3,H$) denotes the coupling constant, and Y_s denotes the weak hypercharge of the subquark s. The F_i ($i = 1,2,3$) is finally taken as infinity to guarantee $U(1)_Y \otimes SU(2)_L \otimes SU(3)_c$ gauge symmetry. Quantum effects due to the four fermion interactions in (3) give rise to the composite bosons B_μ, W_μ^i, G_μ^a, and ϕ, which are interpreted as the gauge bosons of $U(1)_Y \otimes SU(2)_L \otimes SU(3)_c$ and the Higgs scalar. The derived effective Lagrangian is nothing but that of the standard model for subquarks.

$$
\begin{aligned}
\mathcal{L}_{eff} = & \, \overline{w} i D\!\!\!\!/\, w + \overline{h}(iD\!\!\!\!/ - m_h)h + \overline{c}(iD\!\!\!\!/ - m_c)c + \overline{c}_\ell(iD\!\!\!\!/ - m_{c\ell})c_\ell \\
& - \frac{1}{4}\left(B_{\mu\nu}\right)^2 - \frac{1}{4}\left(W_{\mu\nu}^i\right)^2 - \frac{1}{4}\left(G_{\mu\nu}^a\right)^2 + |D_\mu\phi|^2 + \mu^2|\phi|^2 - \lambda|\phi|^4 \\
& + G\phi^\dagger\left(-a_1 \overline{w}_R^G w_{1L}^c + a_2 \overline{w}_L w_{2R}\right) + \text{h.c.},
\end{aligned}
\tag{4}
$$

where D_μ are the covariant derivative, $V_{\mu\nu}$ is the field strength of the vector boson V_μ, and μ, λ, and G are coupling constants. The gauge, Higgs, and Yukawa coupling constants are written in terms of the compositeness scale Λ_{sub}, which serves as the momentum cutoff of the quantum loop integral. Eliminating Λ_{sub} from them leads to the following relations among the coupling constants and masses.

$$
\sin^2\theta_W = 1/4\left[Q_{w_1}^2 + Q_{w_2}^2 + 3Q_c^2 + Q_{c_\ell}^2 + N_g Q_h^2\right] \le \frac{3}{10}, \tag{5a}
$$

$$
g = \sqrt{2} g_s, \tag{5b}
$$

$$
M_\phi^2 = 4\left(m_{w_1}^4 + m_{w_2}^4\right) / \left(m_{w_1}^2 + m_{w_2}^2\right), \tag{5c}
$$

$$
M_W^2 = \frac{3}{2}\left(m_{w_1}^2 + m_{w_2}^2\right), \tag{5d}
$$

where θ_W is the Weinberg angle, g and g_s are the weak and the strong coupling constants, respectively, and M_ϕ and M_W are the masses of the Higgs scalar and the W boson, respectively. In deriving (5a), we assumed that the compositeness scale Λ_{sub} dose not depend on the subquark species. If $m_{w_1} \sim m_{w_2}$, Eqs. (5c) and (5d) implie that $m_w \sim 46\text{GeV}$ and $M_\phi \sim 92\text{GeV}$.

This model, however, involves some unsatisfactory aspects. i) Since the effective theory is a gauge theory, the coupling constants are expected to vary with the energy scale according to the renormalization group equation below Λ_{sub}, and the relations (5a)–(5d) hold at Λ_{sub}. If we use the measured values of $\sin^2\theta_W$, g, and g_s, the relation (5a) indicates that $\Lambda_{sub} \lesssim 10^{10}\text{GeV}$, while the relation (5b) indicates Λ_{sub} much larger than the Planck mass. ii) It is natural to take the chiral symmetry for the subquark w as the origin of the lightness of the quarks and leptons. Then, the w-subquark mass m_w which is related to the W-boson mass M_W by the relation (5d) is too large. iii) The model includes no interactions to form composite quarks and leptons dynamically. They are also necessary to make definite arguments on the lightness of quarks and leptons. In this talk, we would like to present an improved version of the model in Ref. 3) without the above mentioned drawbacks.

The basic Lagrangian is given by

$$\begin{aligned}\mathcal{L} = {} & \overline{w}(i\,\partial\!\!\!/ - m_w)w + \overline{h}(i\,\partial\!\!\!/ - m_h)h + \overline{c}(i\,\partial\!\!\!/ - m_c)c + \overline{c}_\ell(i\,\partial\!\!\!/ - m_{c_\ell})c_\ell \\ & + F_1\left(\sum_s \overline{s}\gamma_\mu Q_s s\right)^2 + F_2\left(\overline{w}\gamma_\mu\tau^i w_L\right)^2 + F_3\left(\overline{c}\gamma_\mu\lambda^i c\right)^2 \\ & + \sum_q F_q\overline{P}(w,h,c)P(w,h,c) + \sum_\ell F_\ell\overline{P}(w,h,c_\ell)P(w,h,\ell),\end{aligned} \tag{6}$$

where $P(\psi_1, \psi_2, \psi_3)$ is a projection from the direct product of the three spinors ψ_1, ψ_2, and ψ_3 to a spin $\frac{1}{2}$ state. We added to the basic Lagrangian (3) the six-fermion interactions to form the composite quarks and leptons. We discarded the F_H-term which is to form Higgs scalar, and replace the hypercharge Y_s in the F_1-term by the electric charge Q_s. The F_1 and F_3 are finally taken as infinity to guarantee $U(1)_{em} \otimes SU(3)_c$ gauge symmetry, while F_2 is taken as finite, since $SU(2)$ symmetry is explicitly broken by $Q_{w_{1,2}}$ in the F_1 term in (6). The F_q and F_ℓ are taken as infinity to guarantee chiral symmetry. Again quantum effects due to the Lagrangian (3) give rise to the composite bosons A'_μ, W^i_μ, G^a_μ, q, and ℓ, which are interpreted as the photon (to be diagonalized), weak boson, gluon, quark, and lepton, respectively. In evaluating the quantum effects, we adopt the regularization scheme which respects the $U(1)_{em} \otimes SU(3)_c$ gauge symmetry and the chiral symmetry for $m_w \to 0$. See Ref. 1) for the further details. The effective Lagrangian for composite particles is given by

$$\begin{aligned} \mathcal{L}_{eff} &= \overline{q}(i\not{D} - m_q)q + \overline{\ell}(i\not{D} - m_\ell)\ell \\ &\quad - \frac{1}{4}\left(A'_{\mu\nu}\right)^2 - \frac{1}{4}\left(W^i_{\mu\nu} + e\epsilon^{ij3}A'_{[\mu}W^j_{\nu]}\right)^2 - \frac{1}{4}\left(G^a_{\mu\nu}\right)^2 \\ &\quad - \frac{e}{2g}A'_{\mu\nu}W^3_{\mu\nu} + \frac{1}{2}M_W^2(W^i_\mu)^2, \end{aligned} \tag{7}$$

where

$$D_\mu q = (\partial_\mu + ieQ_q A'_\mu + \frac{i}{2}g\gamma_L \tau^i W^i_\mu + \frac{i}{2}g_s \lambda^a G^a_\mu)q, \tag{8a}$$

$$D_\mu \ell = (\partial_\mu + ieQ_\ell A'_\mu + \frac{i}{2}g\gamma_L \tau^i W^i_\mu)\ell, \tag{8b}$$

$$A'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu, \tag{8c}$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon^{ijk}W^j_\mu W^k_\nu, \tag{8d}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc}G^b_\mu G^c_\nu. \tag{8e}$$

The coupling constants and the masses are given as follows.

$$e = 1/2\sqrt{\left(Q^2_{w_1} + Q^2_{w_2}\right)I_w + 3Q^2_c I_c + Q^2_{c_\ell}I_{c_\ell} + N_g Q^2_h I_h}, \tag{9a}$$

$$g = 1/\sqrt{I_w}, \qquad g_s = 1/\sqrt{2I_c}, \tag{9b}$$

$$m_q = m_w K_q/J_q, \qquad m_\ell = m_w K_\ell/J_\ell, \tag{9c}$$

$$M_W^2 = 3m_w^2 - 1/8F_2 I_w, \tag{9d}$$

where I_s ($s = w, h, c, c_\ell$) is the logarithmically divergent integral of the loop diagram with internal s-subquark line, and J_q, K_q, J_ℓ, and K_ℓ are the quartically divergent two-loop integrals which are precisely defined in Ref. 1). The Lagrangian (6) is that for the current mixing scheme of Hung and Sakurai.[6] It is well known that this scheme is equivalent to the standard model except for the part concerned with the Higgs scalar, which has not yet been established phenomenologically. Thus, the present model can be an alternative to that in Ref. 3).

The divergent integrals depend on the momentum cutoff at Λ_{sub} and the number of subcolor. Unlike in Ref. 3), we assume that the cutoff and the number of subcolor depend on the species of the subquarks. Then, the relations (5a) and (5b) do not hold any longer, but only the following sum rule is left.

$$\frac{1}{e^2} = \frac{4\left(Q_{w_1}^2 + Q_{w_2}^2\right)}{g^2} + \frac{6Q_c^2}{g_s^2} + \frac{Q_{c\ell}^2}{g_\ell^2} + \frac{2N_g Q_h^2}{g_h^2}, \tag{10}$$

where $g_\ell = 1/2\sqrt{I_{c\ell}}$ and $g_h = 1/\sqrt{2I_h}$. If we incorporate the hypothetical particles $G_\mu^\ell \approx \bar{c}_\ell \gamma_\mu c_\ell$ (leptonic gluon) and $H_\mu^a \approx \bar{h}\lambda^a\gamma_\mu h$ (horizontal gauge boson), g_ℓ and g_h respectively become the coupling constants of their interactions. Unlike in the model in Ref. 3), the relation (5b) dose not lead to contradictory restriction on Λ_{sub}. On the other hand, the relation (5d) is replaced by (9c) and (9d), which allows the option with small $m_w \sim \mathrm{O}(m_q), \mathrm{O}(m_\ell)$ and large M_W, since F_2 is finite. Thus, we have shown that the present model overcomes the above mentioned drawbacks of the model in Ref. 3).

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Baryon Physics based on the Nambu-Jona-Lasinio Model

M. Wakamatsu

Department of Physics, Faculty of Science,
Osaka University,
Toyonaka, Osaka 560, JAPAN

The mean-field treatment of the Nambu-Jona-Lasinio lagrangian is shown to provide us with a powerful means to study the structure of baryons. It enables us to calculate various nucleon observables in a nonperturbative way, with full inclusion of the sea-quark effects.

Witten's identification [1]) of Skyrme solitons with QCD baryons has been widely accepted by now, but we are still far from complete understanding of the underlying dynamics of the Skyrme model [2]). The theoretical interpretation [3]) of the recent EMC experiment [4]) strongly suggests that the Skyrme solitons are very remote from the naive quark bound-state picture. What picture do we obtain then, if it is at all possible to translate the principle physical content of the Skyrme model into a quark language.

The celebrated Nambu-Jona-Lasinio (NJL) model [5]) has recently re-emerged as a means of establishing a link between quark models and chiral soliton models such as the Skyrme model. A key ingredient is the introduction of the composite fields carrying meson quantum numbers. The implied quark-meson coupling generates the Hartree type mean field for quarks to form a soliton-like bound state. The non-trivial topology of this Hartree potential makes the above soliton solution much resembling to the Skyrmion, although it simultaneously holds the chracteristic of the standard nonrelativistic quark model. The study of the NJL solitons will therefore provide us with valuable informations for reaching deeper understanding of the Skyrme like topological soliton models. The present note is only a brief introduction of such studies.

We start with the chiral symmetric NJL lagrangian [5],

$$\mathcal{L}_{NJL} = \bar{\psi}\, i\gamma^{\mu}\partial_{\mu}\psi + \frac{1}{2} G\,[\,(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma^5 \boldsymbol{\tau}\psi)^2\,], \tag{1}$$

or its equivalent, given as

$$\mathcal{L}'_{NJL} = \bar{\psi}\,[\,i\gamma^{\mu}\partial_{\mu} - g\,(\sigma + i\,\gamma^5 \boldsymbol{\tau}\cdot\boldsymbol{\pi})\,]\,\psi - \frac{g^2}{2G}(\sigma^2 + \boldsymbol{\pi}^2). \tag{2}$$

The effective meson action obtained from this lagrangian is

$$S_{eff}[\sigma,\boldsymbol{\pi}] = -\,i\,N_c \log\det[\,i\gamma^{\mu}\partial_{\mu} - g(\sigma + i\,\gamma^5\,\boldsymbol{\tau}\cdot\boldsymbol{\pi})] - \frac{g^2}{2G}\int d^4x\,(\sigma^2 + \boldsymbol{\pi}^2), \tag{3}$$

where N_c is the number of colors of quarks. In the following, we simply assume that the self-consistent classical solution of the above effective action exists and it satisfies the nonlinear constraint $\sigma^2 + \boldsymbol{\pi}^2 = f_{\pi}^2$. In this case, it is convenient to express the linear combination of σ and $\boldsymbol{\pi}$ in terms of one unitary matrix U as follows:

$$g\,(\,\sigma + i\,\gamma^5\,\boldsymbol{\tau}\cdot\boldsymbol{\pi}\,) \;=\; M\,(\,\frac{1+\gamma^5}{2}\,U + \frac{1-\gamma^5}{2}\,U^{\dagger}\,) \;\equiv\; M\,U^{\gamma^5}. \tag{4}$$

Eq.(3) then reduces to a simple form:

$$S_{eff}[U] = -\,i\,N_c\,\mathrm{Sp}\,\log[\,i\,\gamma^{\mu}\partial_{\mu} - M U^{\gamma^5}\,], \tag{5}$$

which is nothing but the effective action investigated by Diakonov et al. [6] (Here $M = g f_{\pi}$, and $\mathrm{Sp}\,\hat{O} \equiv \int d^4x\; tr_{\gamma}\, tr_f \langle x\,|\,\hat{O}\,|\,x\rangle$.) This expression is formally complete, but of little practical use. In order to obtain actual numerical values for any quantities of physical interest, we must usually resort to some approximation such as the derivative expansion. We can go beyond such a perturbative treatment, by directly evaluating the trace sum of eq.(5) with use of the eigenstates of a time-independent hamiltonian (specified below) as a complete set. This becomes feasible, if we assume a simple time dependence for the unitary matrix U as

$$U(\mathbf{x},t) = A(t)\,U_0(\mathbf{x})\,A^{\dagger}(t). \tag{6}$$

Here $U_0(\mathbf{x})$ stands for the static meson configuration, which is assumed to take the hedgehog form $U_0(\mathbf{x}) = \exp[i\,\boldsymbol{\tau}\cdot\hat{\mathbf{r}}\,F(r)]$ with $F(r) = \pi e^{-r/R}$, and $A(t)$ is a time dependent SU(2) matrix. This just corresponds to the assumption of collective iso-rotation in the Skyrme model [2]. Now, using eq.(6), we can rewrite the operator $D \equiv i\,\gamma^\mu \partial_\mu - M U^{\gamma^5}$ as follows:

$$D = A(t)\,\gamma_0\,(i\,\partial_t - H + \Omega)A^\dagger(t). \tag{7}$$

Here H is the time-independent hamiltonian given as

$$H = \frac{\boldsymbol{\alpha}\cdot\nabla}{i} + M\beta\,(\cos F(r) + i\,\gamma_5\,\boldsymbol{\tau}\cdot\hat{\mathbf{r}}\sin F(r)), \tag{8}$$

and Ω is the collective angular velocity operator defined by

$$\Omega \;\equiv\; i\,A^\dagger\,\dot{A} \;\equiv\; \frac{1}{2}\,i\,\Omega_a\,\tau_a, \tag{9}$$

where $\dot{A} = \frac{d}{dt}A$. Under the above assumption, the effective action can be written as

$$\begin{aligned} S_{eff}[U] = \;& - \;\; i\,N_c\,\mathrm{Sp}\,\log\left[\,i\,\partial_t - H\right] \\ & - \;\; i\,N_c\,\{\,\mathrm{Sp}\,\log\left[\,i\,\partial_t - H + \Omega\right] - \mathrm{Sp}\,\log\left[\,i\,\partial_t - H\right]\}. \end{aligned} \tag{10}$$

Here we have intentionally divided the total action into two parts: the first part has a trivial time dependence and can be related to the static energy of the soliton system [6], while the second part, which depends on Ω, describes the collective iso-rotational energy. In reference to the Skyrme model [2], we assume that the rotational velocity is relatively slow, and the expansion in powers of Ω converges sufficiently fast. The first non-vanishing correction to the static energy comes from the second order term in Ω. After some algebra, we arrive at the following expression for the energy of the quantized soliton with the definite angular momentum $J(= T)$:

$$E \;=\; E_{static} \;+\; \frac{J(J+1)}{2I}\,. \tag{11}$$

Here use has been made of the quantization rule $\Omega_a \;\rightarrow\; \hat{J}_a/I$ ($\hat{J}_a$ is the angular momentum operator).

The discussion above pays little attention to the special role of the valence level (it is the lowest energy eigenstates of the hamiltonian H, which emerges from the positive energy continuum). When the energy of this valence level lies between 0 and M, its contribution must be separately taken into account [6]. Furthermore, the momentum cutoff Λ must be introduced in order to regularize the ultraviolet divergence. This cutoff is determined so as to reproduce the pion kinetic term in the effective meson lagrangian obtained from eq.(5). Adopting the proper-time regularization, this requires to set

$$f_\pi^2 = \frac{N_c M^2}{4\pi^2} \int_{1/\Lambda^2}^{\infty} \frac{dt}{t} e^{-tM^2}. \tag{12}$$

After taking all these into account, we are led to the following formula for the moment of inertia:

$$I = I_0 + I_{v.p.}, \tag{13}$$

where

$$I_0 = \frac{N_c}{2} \sum_{m \neq 0} \frac{< 0 \,|\, \tau_3 \,|\, m >< m \,|\, \tau_3 \,|\, 0 >}{E_m - E_0}, \tag{14}$$

$$I_{v.p.} = \frac{Nc}{8} \sum_{m,n} < n \,|\, \tau_3 \,|\, m >< m \,|\, \tau_3 \,|\, n > \; f(E_m, E_n; \Lambda), \tag{15}$$

with

$$\begin{aligned} f(E_m, E_n; \Lambda) = \; & \frac{\mathrm{sign}(E_m)\mathrm{erfc}(|E_m|/\Lambda) - \mathrm{sign}(E_n)\mathrm{erfc}(|E_n|/\Lambda)}{E_m - E_n} \\ & - \frac{2}{\sqrt{\pi}} \cdot \Lambda \cdot \frac{e^{-E_m^2/\Lambda^2} - e^{-E_n^2/\Lambda^2}}{E_m^2 - E_n^2} \; . \end{aligned} \tag{16}$$

Here $|\, m >$ denotes the eigenstates of the static hamiltonian H with the eigen-energy E_m. In particular, $|\, 0 >$ represents the valence state with the the eigen-energy E_0. To make all the above sums tractable, we introduce the plane-wave basis a la Kahana and Ripka [7]. The momenta of this plane-wave basis are discretized by imposing an appropriate boundary condition at $r = D$ chosen to be sufficiently larger than the soliton size R. The basis is made finite by introducing only those states with the momentum k as $k < k_{max}$. The eigenvalue problem is solved by

Table 1: The soliton size dependence of I.

R(fm)	I_0	$I_{v.p.}$	I(total)
0.4	0.0096	0.0003	0.0099
0.6	0.0043	0.0015	0.0058
0.8	0.0032	0.0039	0.0071
1.0	0.0026	0.0079	0.0105
1.2	-	0.0159	0.0159

diagonalizing the hamiltonian H in the above basis. All the results were checked to remain unchanged with increasing D and k_{max}.

Since we do not have enough space, here we show in table.1 only the final numerical result for I, which was caluculated with eqs.(13) $\sim$ (16), by assuming a simple parametrization $F(r) = \pi e^{-r/R}$ for the soliton profile. We try several choices for R to examine the soliton size dependence of the resultant moment of inertia. One sees that the vacuum quark contribution $I_{v.p.}$ is a rapidly increasing function of R. This feature is quite reasonable, since larger soliton size in our model means stronger chiral background potential, and consequently stronger vacuum polarization. The total moment of inertia I seems to have a minimum around $R \simeq 0.6fm$. (We recall that the static soliton energy evaluated with the same soliton profile has a minimum around the same radius $R \simeq 0.6fm$ [8]).) The value of the moment of inertia at this soliton radius is about $0.0058MeV^{-1}$. It is fairly close to the value $0.005MeV^{-1}$, which is extracted from the observed $N - \Delta$ mass difference with use of the formula $M_\Delta - M_N = \frac{3}{2I}$.

Summarizing our arguments, a semi-classical quantization procedure was carried out, by starting from a schematic form of the mean-field solution of the Nambu-Jona-Lasinio lagrangian. A fundamental quantity appearing in this quantization scheme is the moment of inertia of the soliton system. We have calculated this quantity without recoursing to the derivative expansion, by performing double sum over all the positive and negative energy quark orbitals in a mean potential. A similar analysis can be readily extended to other nucleon observables such as the magnetic moments and the spin expectation value etc. [9,10]. It is hoped that such a study will throw more light on the approximate nature of the fermi-bose correspondence in the $3+1$ dimensional field theoretical models and thereby provide us with valuable information about the utility and the limitation of the Skyrme like topological soliton models.

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Calculating f_π *

TAICHIRO KUGO

Department of Physics, Kyoto University
Kyoto 606, JAPAN

ABSTRACT

A review of the bilocal auxiliary field method is given in the first to emphasize that it gives a systematic approximation scheme consistent with chiral symmetry and reveals an interesting interplay between Schwinger-Dyson equation and Bethe-Salpeter equation. Then applying the method, we examine physical quantities accompanying dynamical chiral symmetry breaking in QCD-like theories. In particular expression for the decay constant f_π is given exactly to the ladder approximation by solving the BS equation. Some results of our numerical calculations for f_π, $\langle\bar{\psi}\psi\rangle$ and $\Sigma(p^2)$ are also reported.

1. Introduction

The spontaneous breakings of chiral symmetries play important roles in various places in particle physics. Namely, the best-known example is QCD theory of strong interaction, in which pion is identified as Nambu-Goldstone boson corresponding to the spontaneous breaking of an approximate $SU(2) \times SU(2)$ chiral symmetry. In the standard electroweak theory of Weinberg-Salam also, some chiral symmetry has to be spontaneously broken, which we may suppose is caused dynamically by a certain strong interaction, e.g., technicolor, 4-fermi interaction and so on. Even in QED in its strong coupling phase, it is known, at least in

⋆ This talk is based on the work in collaboration with K-I. Aoki, M. Bando and H. Nakatani.

the quenched ladder approximation of Scwinger-Dyson equation, that such a dynamical spontaneous breaking takes place. In any case, it is much desirable to understand the dynamical properties concerning spontaneous chiral symmetry breaking more.

The calculable physical quantities relevant here are i) self-energy function $\Sigma(x)$ of fermion, ii) f_π and iii) the vacuum expectation value (VEV) $\langle\bar{\psi}\psi\rangle$. Here f_π and $\langle\bar{\psi}\psi\rangle$ are directly related to the measurable quantities; indeed f_π is the decay constant of NG boson if the chiral symmetry is global, or gives the mass of gauge boson if it is local, and $\langle\bar{\psi}\psi\rangle$ is related to the mass of NG boson for the case of approximate chiral symmetry. On the contrary, $\Sigma(x)$ is not so, although its nonvanishingness equally signals the spontaneous breaking.

I will report in this talk on our recent work of calculating f_π and $\langle\bar{\psi}\psi\rangle$ performed in collaboration with Aoki, Bando and Nakatani.[1] The self-energy function $\Sigma(x)$ has long been calculated by many authors in the quenched ladder approximation in strong coupling QED, and also in QCD in a similar approximation. The decay constant f_π, however, has only been calculated by using a further "approximation", namely Pagels-Stoker's formula,[2] up to now. The Pagel-Stoker formula, which was derived in the so-called dynamical perturbation theory, is indeed a convenient formula since it gives f_π in terms of the knowledge of $\Sigma(x)$ alone. But the nature of this approximation is not necessarily so clear. So we here calculate f_π exactly in the ladder approximation with no additional assumptions.

2. General framework — Auxiliary field method

It is best seen in the *auxiliary bilocal field method* that f_π and $\Sigma(x)$ can be calculated to any common order of approximation, and consistently with the chiral symmetry. We therefore review the auxiliary bilocal field method first.

2.1 LOCAL AUXILIARY FIELD

The history of auxiliary field method itself is very old. It was first introduced by Stratovich and Hubbard[3] in '50s in statistical physics, and was applied in particle physics by Coleman-Jackiw-Politzer and Gross-Neveu[4] in '70s. These authors' method is local field one. So let us see first how it works in the simplest case of local 4-fermi interaction:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{\lambda}{2N}(\bar{\psi}\psi)^2, \tag{2.1}$$

where ψ is an N-plet of Dirac fields $(\psi_1, \psi_2, \cdots, \psi_N)^T$. The generating functional of Green's functions is given in the path-integral expression by

$$Z[\eta, \bar{\eta}] = \int \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp i \int dx(\mathcal{L} + \bar{\eta}\psi + \bar{\psi}\eta). \tag{2.2}$$

We now introduce an auxiliary local field $\chi(x)$,

$$\begin{aligned} 1 &= \int \mathcal{D}\chi \exp i \int dx[-\frac{N}{2\lambda}\chi^2] \\ &= \int \mathcal{D}\chi \exp i \int dx[-\frac{N}{2\lambda}(\chi + \frac{\lambda}{N}\bar{\psi}\psi)^2] \end{aligned} \tag{2.3}$$

and multiply $Z[\eta, \bar{\eta}]$ by this expression of 1. Then we get

$$Z[\eta, \bar{\eta}] = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\chi \exp i \int dx[\bar{\psi}(i\partial\!\!\!/ - m - \chi)\psi - \frac{N}{2\lambda}\chi^2 + \bar{\eta}\psi + \bar{\psi}\eta] \tag{2.4}$$

$$= \int \mathcal{D}\chi \exp iN \left[-\frac{1}{2\lambda}\chi^2 - i\,\mathrm{Tr}\,\mathrm{Ln}(i\partial\!\!\!/ - m - \chi) - \frac{\bar{\eta}}{\sqrt{N}}(i\partial\!\!\!/ - m - \chi)^{-1}\frac{\eta}{\sqrt{N}}\right] \tag{2.5}$$

Note that from (2.4) or (2.3) the equation of motion of χ gives

$$\chi = -\frac{\lambda}{N}\bar{\psi}\psi, \tag{2.6}$$

and hence the auxiliary field χ is essentially a replacement of the composite operator $\bar{\psi}\psi$, Now the exponent of the integrand in the final expression (2.5) gives a quantum action for χ. Since N is factored out as an overall multiplicative factor, the $1/N$ expansion (regarding $\eta/\sqrt{N}$ as of order 1) becomes identical with the "loop" expansion based on this effective action. [This is just like the ordinary loop expansion $=\hbar$ expansion for which $1/\hbar$ is an overall multiplicative factor.]

The effective action to the leading order in $1/N$ is, for the case of vanishing fermion source,

$$S^{\text{"tree"}} = -\frac{1}{2\lambda}\chi^2 - i\,\mathrm{Tr}\,\mathrm{Ln}(i\partial\!\!\!/ - m - \chi), \tag{2.7}$$

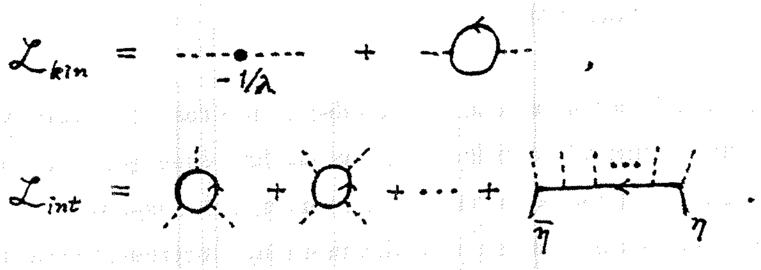
which yields the following effective potential by setting χ x-independent:

$$V^{\text{"tree"}} = \frac{1}{2\lambda}\chi^2 + i\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}\ln(k\!\!\!/ - m - \chi). \tag{2.8}$$

The stationary condition of this determines the VEV $\chi_c = \langle\chi(x)\rangle = -\frac{\lambda}{N}\langle\bar{\psi}\psi\rangle$:

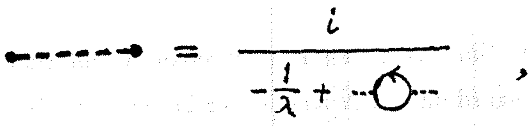
$$\left.\frac{\partial V^{\text{"tree"}}}{\partial\chi}\right|_{\chi=\chi_c} = 0 \longrightarrow \frac{\chi_c}{\lambda} = i\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}\frac{1}{k\!\!\!/ - m - \chi_c}. \tag{2.9}$$

This is nothing but the gap equation or Nambu-Jona-Lasinion's self- consistency equation (for chral symmetry case $m = 0$). The higher order terms of the effective action (or potential) can be calculated using the quantum action in (2.5) which reads diagramatically



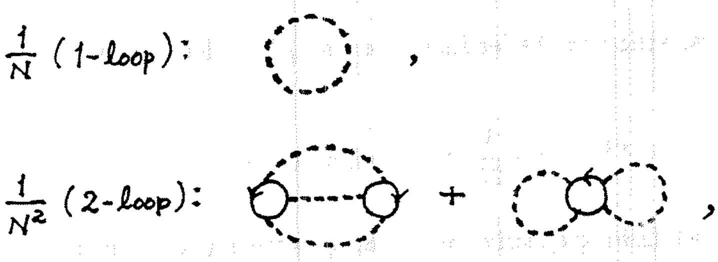
(2.10)

where now the fermion line stands for the propagator $i(k\!\!\!/ - m - \chi_c)^{-1}$ with new mass $m + \chi_c$ determined by (2.9). Thus, using the "dressed" χ-propagator



(2.11)

the higher order terms are given by



(2.12)

and so on.

This auxiliary field method has various merits:

i) It specifies a systematic way of summing Feynman diagrams, with which the ultraviolet behavior becomes often better.

ii) It gives a Lagrangian (action) formalism of bound state(s) χ. Since we have the action $S[\chi]$ or the potential $V(\chi)$, we can discuss the vacuum energy and the stability of solutions of $\partial V/\partial\chi = 0$.

iii) Global symmetries (e.g., chiral symmetry) are kept manifest in the action $S[\chi]$.

2.2 Bilocal auxiliary field

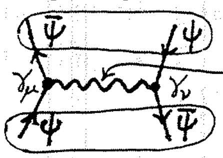
This auxiliary field technique can be extended to the case of Yukawa type interaction by introducing bilocal field $\chi(x,y)$, as was first done by Kleinert and Shrauner.[5] It is in this formulation that the Schwinger-Dyson equation and the Bethe-Salpeter equation come into an intriguing interplay concerning the vacuum stability, as was pointed out first by the present author.[6]

Let us consider the system of QED:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ + eA\!\!\!/)\psi - \frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2\alpha}(\partial A)^2, \tag{2.13}$$

which yields, after integrating out the photon field A_μ

$$\longrightarrow \quad \bar{\psi}i\partial\!\!\!/\psi + \frac{e^2}{2}$$



$$\frac{g^{\mu\nu} - (1-\alpha)p^\mu p^\nu / p^2}{p^2}$$

$$= \bar{\psi}i\partial\!\!\!/\psi + \frac{e^2}{2}\psi(x_1)\bar{\psi}(y_1)K(x_1y_1; x_2y_2)\psi(x_2)\bar{\psi}(y_2), \tag{2.14}$$

that is, effectively, a non-local 4-fermi interaction system.

We now introduce a bilocal auxiliary field $\chi(x, y)$ and add the following Gaussian term to (2.14) so as to cancel the non-local 4-fermi interaction:

$$\begin{aligned} -\frac{1}{2e^2}&[\chi(x_1y_1) - K(x_1y_1; x_1'y_1')\psi(x_1')\bar{\psi}(y_1')] \\ &\times K^{-1}(x_1y_1; x_2y_2)[\chi(x_2y_2) - K(x_2y_2; x_2'y_2')\psi(x_2')\bar{\psi}(y_2')] \\ \equiv -&\frac{1}{2e^2}(\chi - K\psi\bar{\psi})K^{-1}(\chi - K\psi\bar{\psi}). \end{aligned} \tag{2.15}$$

Note here that the auxiliary field $\chi(x, y)$ is a bilocal and bispinor field representing the composite operator

$$\chi_{\alpha\beta}(x, y) = K(xy; zw)\psi_\alpha(z)\bar{\psi}_\beta(w). \tag{2.16}$$

By just the same procedure as before, the generating functional Z of Green's functions is now given by

$$\begin{aligned} Z[\eta, \bar{\eta}] &= \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\chi \exp i \int dx[\bar{\psi}(i\partial\!\!\!/ - \chi)\psi - \frac{1}{2e^2}\mathrm{tr}(\chi K^{-1}\chi) + \bar{\eta}\psi + \bar{\psi}\eta] \\ &= \int \mathcal{D}\chi \exp i[-\frac{1}{2e^2}\mathrm{tr}(\chi K^{-1}\chi) - i\,\mathrm{Tr}\,\mathrm{Ln}(i\partial\!\!\!/ - \chi) - \bar{\eta}(i\partial\!\!\!/ - \chi)^{-1}\eta] \end{aligned} \tag{2.17}$$

Thus, to the "leading order", we get the effective action:

$$S^{\text{"tree"}}[\chi] = -\frac{1}{2e^2}\mathrm{tr}(\chi K^{-1}\chi) - i\,\mathrm{Tr}\,\mathrm{Ln}(i\partial\!\!\!/ - \chi). \tag{2.18}$$

We henceforth work with this action persistently, and wil see the following:[6]

i) The stationary condition $\delta V^{\text{"tree"}}/\delta\chi|_{\chi=\chi_c} = 0$ is just identical with the Schwinger-Dyson (SD) equation for self-energy function $\Sigma(p^2) = \chi_c(p^2)$ of the fermion ψ in the ladder approximation. Thus $\Sigma(p^2)$ is interpreted as a condensation of bilocal field, $\Sigma = \langle\chi\rangle = K\left\langle\psi\bar{\psi}\right\rangle$.

ii) Expanding $S^{\text{"tree"}}[\chi]$ around each possible solution $\langle\chi\rangle = \chi_c$, the condition of diagonalizing the quadratic parts in $\tilde{\chi} = \chi - \chi_c$ reproduces the ladder Beth-Salpeter (BS) equation [with improved fermion propagator $i(i\partial\!\!\!/-\chi)^{-1}$] for $\bar{\psi}\psi$ bound states. Therefore the instability of the vacuum corresponding to the solution $\Sigma = \chi_c$ of SD equation is signaled by the appearance of tachyonic eigenmodes in the corresponding BS equation.

Note also that the chiral symmetry is maintained manifest in $S^{\text{"tree"}}[\chi]$ (or in any higher order approximation in this formalism) and hence that if $\langle\chi\rangle = \chi_c \neq 0$ solution exists in SD equation, then the NG bound state appears in the corresponding BS equation automatically in this approximation.* Now let us see the points i) and ii) in turn explicitly.

First let us switch to the momentum space:

$$\chi(x,y) = \int\frac{d^4q d^4p}{(2\pi)^8}\exp\{-iq\frac{x+y}{2} - ip(x-y)\}\;\chi(p;q),$$
$$K(x_1y_1;x_2y_2) = \int\frac{d^4q d^4p_1 d^4p_2}{(2\pi)^{12}}K_q(p_1;p_2)$$
$$\times\exp\left\{-iq(\frac{x_1+y_2}{2} - \frac{x_2+y_2}{2}) - ip_1(x_1-y_1) - ip_2(x_2-y_2)\right\} \tag{2.19}$$

where p's stand for relative momenta while q for CM momenta. The VEV of

⋆ This fact itself was known to Maskawa[7] independently without the use of auxiliary field method.

$\chi(x,y)$ must be independent of CM coordinate $(x+y)/2$ from translation invariance and hence takes the form

$$\langle \chi(p;q) \rangle = \Sigma(p) \cdot (2\pi)^4 \delta^4(q). \tag{2.20}$$

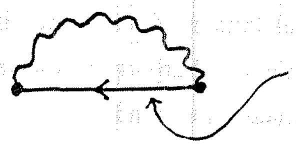
The effective potential is given from the action (2.18) as

$$\begin{aligned} V^{\text{"tree"}}[\Sigma] &= -S^{\text{"tree"}}\left[\chi(p;q) = \Sigma(p)\cdot(2\pi)^4\delta^4(q)\right] \bigg/ \int d^4\left(\frac{x+y}{2}\right) \\ &= \frac{1}{2e^2}\int_{p,k} \mathrm{tr}[\Sigma(p) K^{-1}_{q=0}(p;k)\Sigma(k)] + i\int_p \mathrm{tr}\ln(\not{p} - \Sigma(p)), \end{aligned} \tag{2.21}$$

with abbreviations $\int_p \equiv \int d^4p/(2\pi)^4$ etc. Stationary condition $\delta V^{\text{"tree"}}/\delta\Sigma(p) = 0$ gives

$$\Sigma(p) = ie^2 \int_k K_{q=0}(p;k)\frac{1}{\not{k} - \Sigma(k)}, \tag{2.22}$$

or diagramatically,

$\Sigma(p) =$  $\dfrac{1}{\not{k} - \Sigma(k)}$ (2.23)

This is nothing but the SD equation for the fermion self-energy Σ as announced above! Thus the self-energy Σ is interpreted as a vacuum condensation of the bilocal field χ.

Next we look for the eigenmodes of χ on the vacuum realizing the VEV $\langle\chi\rangle = \Sigma$. Performing a field shift

$$\chi(p;q) \quad \rightarrow \quad \tilde{\chi}(p;q) + \Sigma(p)\cdot(2\pi)^4\delta^4(q) \tag{2.24}$$

in $S^{\text{"tree"}}[\chi]$, we pick up the quadratic parts in $\tilde{\chi}$ (omitting tilde of $\tilde{\chi}$):

$$\begin{aligned} S^{\text{"tree"}}_{\text{quadratic}}[\chi] = \int_q \Big\{ &-\frac{1}{2e^2}\int_{p,k}\mathrm{tr}[\chi(p;-q)K_q^{-1}(p;k)\chi(k;q)] \\ &-\frac{i}{2}\int_p \mathrm{tr}[\chi(p;-q)S_F(p+\frac{q}{2})\chi(p;q)S_F(p-\frac{q}{2})]\Big\} \end{aligned} \tag{2.25}$$

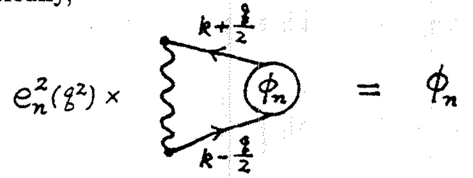
where S_F is the fermion propagator on this vacuum:

$$S_F(p) = i(\not{p} - \Sigma(p))^{-1}. \tag{2.26}$$

For each fixed value of CM momentum q, the quadratic form of χ in (2.25) can be diagonalized by solving the following eigenvalue equation:

$$-i\int_k K_q(p;k)S_F(k+\frac{q}{2})\phi_n(k;q)S_F(k-\frac{q}{2}) = e_n^{-2}(q^2)\phi_n(p;q), \tag{2.27}$$

or diagramatically,



(2.28)

This is just the BS equation as announced above, although it is a bit more improved than the usual ladder BS equation since the fermion line here is not the bare one $i/\not{p}$ but S_F in (2.26) with non-trivial self-energy function $\Sigma(p)$. Owing to the orthonormalization condition

$$-i\int_p \mathrm{tr}[\phi_n(p;-q)S_F(p+\frac{q}{2})\phi_m(p;q)S_F(p-\frac{q}{2})] = \delta_{n,m}, \tag{2.29}$$

the quadratic parts (2.25) of aciton now becomes diagonal:

$$S^{\text{"tree"}}_{\text{quadratic}}[\chi] = \int_q -\frac{1}{2}\sum_n \chi_n(-q)\frac{e_n^2(q^2)-e^2}{e^2}\chi_n(q), \tag{2.30}$$

$$\chi(p;q) = \sum_n \chi_n(q)\phi_n(p;q). \tag{2.31}$$

From this, the 2-point function of the bilocal field χ, for instance, is seen to be given

$$\langle \mathrm{T}\chi(p;-q)\chi(k;q)\rangle = \sum_n ie^2\frac{\phi_n(p;-q)\phi_n(k;q)}{e_n^2(q^2)-e^2} \tag{2.32}$$

which has poles at masses $q^2 = m_n^2$ determined by $e_n^2(m_n^2) = e^2$.

The auxiliary bilocal field method was extensively studied by Morozumi and So[8] for QED case. Another method treating composite operators was also developed by Cornwall-Jackiw-Tomboulis,[9] with which the stability problem was discussed by Fukuda.[10] Comparison of these two methods is done by Haymaker and Matsuki.[11] Related subjects were studied probably by many other authors to whom I apology for not citing them all.

3. Schwinger-Dyson equation for QED and QCD

The Schwinger-Dyson equation (2.23) in Landau gauge ($\alpha = 0$) for QED (with vertex $e\gamma_\mu$) and QCD (with vertex $g\gamma_\mu T^a$) is given, after making Wick rotation and performing the angle integration, as

$$\Sigma(x) = \frac{\lambda(x)}{4x}\int_0^x \frac{y\Sigma(y)dy}{y+\Sigma^2(y)} + \int_x^{\Lambda^2} \frac{\lambda(y)\Sigma(y)dy}{4(y+\Sigma^2(y))} \ , \tag{3.1}$$

where $x = p_{\rm E}^2$ (Euclidean). Now $\Sigma(x)$ is not a bispinor but a scalar function in this gauge. In QED case the coupling λ is a constant $\lambda = 3e^2/4\pi^2$, but in QCD case we are adopting a slightly better approximation than the mere quenched ladder approximation and using the following running coupling constant $\lambda(x)$:

$$\lambda(x) = \frac{3}{4\pi^2}C_2(F)g^2(x) \qquad \Big(\sum_a T^aT^a = C_2(F)1 \Big) \tag{3.2}$$

$$= \lambda_0\left(\theta(t_{\rm IF}-t)1 + \theta(t-t_{\rm IF})\frac{1}{1+At} \right). \quad (t \equiv \ln x/\mu^2) \tag{3.3}$$

This is an approximation devised by Higashijima[12] so as to make the high energy behavior of the solution $\Sigma(x)$ to (3.1) consistent with the leading renormalization group analysis (as we will see shortly). $t_{\rm IF}$ is an infrared cutoff above which $\lambda(x)$ runs according to the leading logarithmic renormalization group but below which $\lambda(x)$ is kept constant to avoid the divergent pole.

Let us define a parameter B in terms of A and λ_0 in (3.3):

$$B \equiv \frac{A}{\lambda_0} = \frac{\beta_0}{12C_2(F)} \,, \tag{3.4}$$

where β_0 is the lowest order coefficient of the β-function of renormalization group equation. B is a renormalization-point independent parameter characterizing the theory. For example, SU(3) color QCD with 3 flavors of quarks has $B = 9/16$. With this parameter B we can deal with various QCD-like theories in a unified way, even including the fixed coupling theories (like QED in this approximation) as a limit $B = 0$. Originally theory is uniquely characterized by the parameter B. But we here treat the infrared cutoff t_{IF} also as an additional free parameter, which is a price we must pay for lacking of our knowledge in strong coupling (or confinement) regime.

The integral equation (3.1) is equivalent to the following differential equation with appropriate boundary conditions:

$$\left[\frac{\Sigma'(x)}{(\lambda(x)/4x)'}\right]' = \frac{x\Sigma(x)}{x+\Sigma^2(x)} \,. \tag{3.5}$$

from which the asymptotic behavior of the solution is easily found as

$$\Sigma(x\to\infty) \sim \frac{1}{x}\left(\ln\frac{x}{\mu^2}\right)^{\frac{1}{4B}-1}, \tag{3.6}$$

This is quite consistent with the operator product expansion result:

$$\Sigma^{\mathrm{OPE}}(x\to\infty) \sim \frac{\lambda(x)}{x}\left(\ln\frac{x}{\mu^2}\right)^{\frac{1}{4B}} \,. \tag{3.7}$$

For the detailed numerical analysis of eq.(3.1), we refer the reader to Ref.[13].

4. Bethe-Salpeter equation and f_π

The Bethe-Salpeter equation (2.28) has a massless solution $e_0^2(q^2=0)=e^2$ automatically whenever $\Sigma \neq 0$ because of chiral symmetry. Let us consider the BS amplitude $\chi \equiv \langle 0|\mathrm{T}\psi\bar{\psi}|\mathrm{Ps}\rangle$ for $J^{PC}=0^{-+}$ massless state Ps (call pion) in the QCD-like theories. The bispinor χ is expanded into the following invariant amplitudes:

$$\begin{aligned}\chi(p+q/2,p-q/2)=&\gamma_5 S(p,q)+\gamma_\mu\gamma_5(P(p,q)(p\cdot q)p^\mu+Q(p,q)q^\mu)\\&+\sigma_{\mu\nu}\gamma_5 T(p,q)(q^\mu p^\nu-p^\mu q^\nu)\ .\end{aligned}\tag{4.1}$$

We also define the same from expansion for the amputated BS amplitude $\hat{\chi}$:

$$\hat{\chi}(p,q)\equiv(\not{p}-\Sigma(p^2))\chi(p,q)(\not{q}-\Sigma(q^2))\ ,\tag{4.2}$$

$$\begin{aligned}\hat{\chi}(p+q/2,p-q/2)=&\gamma_5\hat{S}(p,q)+\gamma_\mu\gamma_5(\hat{P}(p,q)(p\cdot q)p^\mu+\hat{Q}(p,q)q^\mu)\\&+\sigma_{\mu\nu}\gamma_5\hat{T}(p,q)(q^\mu p^\nu-p^\mu q^\nu)\ .\end{aligned}\tag{4.3}$$

[The eigen-functions denoted by ϕ_n in Sect.2 correspond to this amputated BS $\hat{\chi}$.] From the properties under charge conjugation and parity, all the invariant amplitudes $S, P, ..., \hat{Q}, \hat{T}$ defined above are even functions in $(p\cdot q)$. So we write

$$S(p,q)=S(p^2)+O((p\cdot q)^2),\quad etc,\tag{4.4}$$

since $q^2=0$ now. We will need only the first terms $S(p^2), P(p^2), ...,$ to calculate f_π.

The decay constant f_π is defined as usual by

$$\bar{\psi}T^a\gamma_\mu\gamma_5\psi=-f_\pi\partial_\mu\pi^a+\cdots,\tag{4.5}$$

where π^a is renormalized pion asymptotic field. Sandwiching this by $\langle 0|$ and $|\mathrm{Ps}\rangle$, we have

$$f_\pi q_\mu = -\int \frac{d^4p}{i(2\pi)^4} \mathrm{tr}\,[\gamma_\mu\gamma_5\chi(p+q/2, p-q/2)] \quad . \tag{4.6}$$

which by substituting (4.1) immediately leads to

$$f_\pi = \frac{N}{2}\int_0^\infty \frac{x dx}{16\pi^2}(4Q(x) - xP(x)) \quad , \tag{4.7}$$

Here $x = p_{\mathrm{E}}^2$(Euclid) and the factor N is the N of color group SU(N).

In the above, the normalization of the BS amplitude $\chi \equiv \langle 0|\mathrm{T}\psi\bar{\psi}|\mathrm{Ps}\rangle$ was fixed by $\langle \mathrm{Ps}(p)|\mathrm{Ps}(q)\rangle = (2\pi)^3 2p_0\delta^3(p-q)$. With this normalization, the Ward-Takahashi identity

$$-q^\mu\Gamma_{5\mu}(p-q/2, p+q/2) = S_F^{-1}(p-q/2)\gamma_5 + \gamma_5 S_F^{-1}(p+q/2) \quad , \tag{4.8}$$

gives in the limit $q \to 0$

$$f_\pi \hat{S}(x) = 2\Sigma(x) \quad . \tag{4.9}$$

So it is now more convenient to rescale the BS amplitude χ as well as $\hat{\chi}$ by multiplying a factor $f_\pi/2$ so as to take simply $\hat{S}(x) = \Sigma(x)$. Performing this "renormalization", we finally obtain an *exact formula for* f_π:

$$\frac{1}{2}f_\pi^2 = \frac{N}{2}\int_0^\infty \frac{x dx}{16\pi^2}(4Q(x) - xP(x)) \quad , \tag{4.10}$$

Although the formula (4.10) is exact, we need an approximation to obtain the amplitudes $Q(x)$ and $P(x)$. We calculate $Q(x)$ and $P(x)$ using the BS-equation (2.28) for the amputated amplitude $\phi_{\mathrm{Ps}} = \hat{\chi}$, which is exact only to the ladder approximation. We expand it in powers of $(p \cdot q)$, and have to solve it only up to the first order in $(p \cdot q)$ since $Q(x)$ and $P(x)$ are first order terms in (4.1). To the zeroth order in $(p \cdot q)$, it gives after the angle integration is done

$$\hat{S}(x) = \frac{1}{4}\left[\ \lambda(x)\int_0^x dy \frac{y}{x} S(y) + \int_x^{\Lambda^2} dy \lambda(y) S(y)\ \right]. \tag{4.11}$$

On the other hand, the invariant amplitudes of BS amplitue χ and the amputated one $\hat{\chi}$ are related via (4.2) as

$$S(y) = \frac{1}{y + \Sigma^2(y)} \hat{S}(y) \tag{4.12}$$

$$Q(y) = \frac{1}{(y + \Sigma^2(y))^2}\left[(\Sigma^2(y) - y)\hat{Q}(y) + \Sigma(y)\hat{S}(y) + 4y\Sigma(y)\hat{T}(y)\right] \tag{4.13}$$

$$P(y) = \frac{1}{(y + \Sigma^2(y))^2}\left[-2\hat{Q}(y) + (y + \Sigma^2(y))\hat{P}(y) + 2\Sigma'(y)\hat{S}(y) + 4\Sigma(y)\hat{T}(y)\right] \tag{4.14}$$

Thus we see that the BS equation (4.11) with (4.12) substituted becomes of exactly the same form as the previous SD equation (3.1) and hence that $\hat{S}(x) = \Sigma(x)$, as told by WT identity (4.8), actually gives the soluiton to the BS equation (4.11) for the massless NG bound state.

To the first order in $(p \cdot q)$ next, the BS equation gives the following coupled equation:

$$\begin{pmatrix} \hat{Q}(x) \\ \hat{P}(x) \end{pmatrix} = -\lambda \begin{pmatrix} \int_0^x dy \frac{y(2x+y)}{2x^2} + \int_x^{\Lambda^2} dy \frac{3}{2} & \int_0^y dy \frac{-y^2(3x+y)}{12x^2} + \int_x^{\Lambda^2} dy \frac{5x-9y}{12} \\ \int_0^x dy \frac{2y(y-x)}{x^3} & \int_0^x dy \frac{y^2(3x-2y)}{6x^3} + \int_x^{\Lambda^2} dy \frac{1}{6} \end{pmatrix} \begin{pmatrix} Q(y) \\ P(y) \end{pmatrix} \tag{4.15}$$

where we udnerstand the coupling "constant" λ to represent $\lambda(x)$ in the integration region $\int_0^x dy$ and $\lambda(y)$ in $\int_x^{\Lambda^2} dy$, respectively. It is also seen easily that $\hat{T}(x) \equiv 0$ follows from the structure of BS equation. Using (4.13) and (4.14) as well as $\hat{S}(x) = \Sigma(x)$, the equation (4.15) is now an inhomogeneous linear integral equation for $\hat{Q}(x)$ and $\hat{P}(x)$ of the form

$$(1 + K[\lambda, \Sigma]) \begin{pmatrix} \hat{Q} \\ \hat{P} \end{pmatrix} = C[\lambda, \Sigma]\ , \tag{4.16}$$

where the kernel K and the inhomogeneous term C are local functionals of $\Sigma(x)$.

Thus, given a $\Sigma(x)$, we have a unique solution for $\hat{Q}(x)$ and $\hat{P}(x)$, and then we can obtain f_π using the formula (4.10) with (4.13) and (4.14).

If we set $\lambda = 0$ but keeping $\Sigma(x)$ unchanged, the solution to eq.(4.15) is trivial:

$$\hat{Q}(x) \equiv 0 \ , \quad \hat{P}(x) \equiv 0 \ . \tag{4.17}$$

Then the formula (4.10) with (4.13) and (4.14) gives the following f_π:

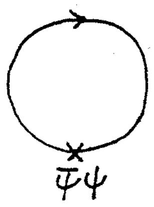
$$f_\pi^2 = \frac{N}{4\pi^2} \int_0^\infty x dx \frac{\Sigma(x)(\Sigma(x) - x\Sigma'(x)/2)}{(x + \Sigma^2(x))^2} \ . \tag{4.18}$$

This formula is nothing but the Pagel-Stoker approximation.[2] Thus we see that the approximation is applicable in the case of weak coupling λ (but strong enough to produce non-zero $\Sigma(x)$). As we will see later, due to the asymptotic freedom in QCD-like theories, the Pagel-Stoker formula gives rather good estimate for f_π.

Before going to the numerical analysis of (4.15), we here discuss the VEV $\langle \bar{\psi}\psi \rangle$ a little. First note that

$$m(\mu) \left\langle (\bar{\psi}\psi)_\mu \right\rangle$$

is a renormalization-point (μ) independent quantity, if $m(\mu)$ is the running mass and $\left\langle (\bar{\psi}\psi)_\mu \right\rangle$ is the composite operator renormalized at μ. Then we can identify the quantity



$$= \frac{1}{16\pi^2} \int_0^{\Lambda^2} dx \frac{x\Sigma(x)}{x + \Sigma^2(x)} \ , \tag{4.19}$$

with the VEV $\left\langle (\bar{\psi}\psi)_\Lambda \right\rangle$ of the renormalized operator at $\mu = \Lambda$. This identification is indeed consistent since $\Sigma(x)$ obtained by Higashijima approximation has the asymptotic behavior as predicted by OPE. Using the leading renormalization group formula for $m(\mu)$, we find

$$\langle(\bar{\psi}\psi)_\mu\rangle = \left[\frac{\lambda(\Lambda)}{\lambda(\mu)}\right]^{\frac{1}{4B}} \cdot \frac{1}{16\pi^2}\int_0^{\Lambda^2} dx \frac{x\Sigma(x)}{x+\Sigma^2(x)}. \tag{4.20}$$

The BS equaiton (4.15) can be solved numerically by discretization. Taking 150 points for $\hat{Q}(x)$ and $\hat{P}(x)$ each, the integral equation (4.15) becomes a 300-dimensional coupled linear equation.

We show some results of our numerical calculations. The ultraviolet cutoff is set to be

$$\ln\left(\Lambda^2/\Lambda_{\mathrm{QCD}}^2\right) = 21 \quad , \tag{4.21}$$

where Λ_{QCD} is the point at which the leading logarithmic running coupling constant diverges. [We set Λ for $B=0$ case to be $\ln(\Lambda^2/\Sigma^2(0)) = 30$.]

A schematic view of the Bethe-Salpeter kernel is in Fig. 1, where we show two cases of $B = 9/16$ (three triplets QCD) and $B = 0$ (non-running coupling constant). The corresponding $\Sigma(x)$, solutions $\hat{Q}(x)$, $\hat{P}(x)$ and the integrands for f_π and for $\langle\bar{\psi}\psi\rangle$, are shown in Fig. 2a and 2b. As for the integrand for f_π, we plot both our ladder exact integrand and the Pagels-Stoker integrand. Both are strongly peaked at near the peak of $|\Sigma'(x)|$.

Changing the infrared cutoff t_{IF}, we get Fig. 3a, where we take the case of $B = 9/16$ (three triplets QCD). Fig. 3b is its zoom-up. Lower t_{IF} drives the infrared coupling constant larger. In any case, we set a renomalization condition, f_π (ladder exact) = 94MeV, which is shown as a thick line of every plots. The reason of taking a specific value of 94MeV is simply for us to easily catch relative scales of various parameters. One sees the following results of our ladder exact calculation.

1. The Pagels-Stoker approximation is rather good. We understand that it is due to the asymptotic freedom of the theory, that is, after the most essential

part of dynamics is taken as a solution of the Schwinger-Dyson equation $\Sigma(x)$, the rest is controlled by the rather 'weak' coupling λ, as is seen in eq. (4.16).

2. The renormalization of $\langle\bar{\psi}\psi\rangle$ works excellently. We get an almost stable value of the renormalized $\langle(\bar{\psi}\psi)_{1GeV}\rangle \equiv \langle\bar{\psi}\psi\rangle_{\mathrm{R}}$, stable against change of the infrared cutoff parameter t_{IF}, while the unrenormalized counter part $\langle(\bar{\psi}\psi)_{\Lambda}\rangle \equiv \langle\bar{\psi}\psi\rangle_{\mathrm{U}}$ depends much on t_{IF}.
3. On the other hand, $\Sigma(0)$ depends quite a lot on t_{IF}. In many articles, $\Sigma(0)$ is used as a mass scale of the spontaneous symmetry breaking. One should note that it does depend on the infrared structure of the running coupling constant, and thus it is not a good measure of physics.
4. As for Λ_{QCD}, it depends on t_{IF}. However, the dependence is negligible for lower t_{IF} region, while in such region, $\Sigma(0)$ diverges.

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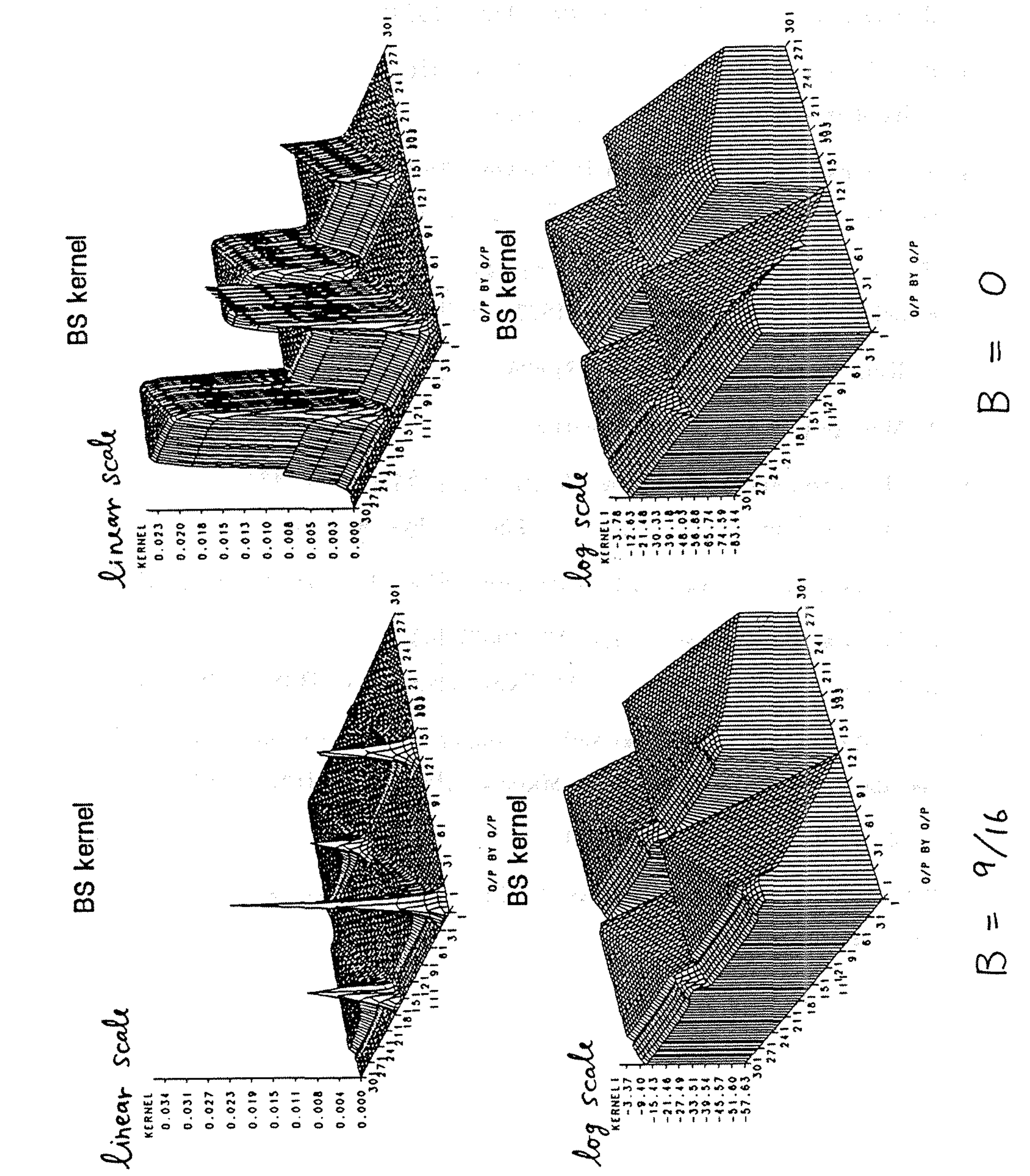

Fig. 1 Schematic View of Bethe Salpeter Kernel

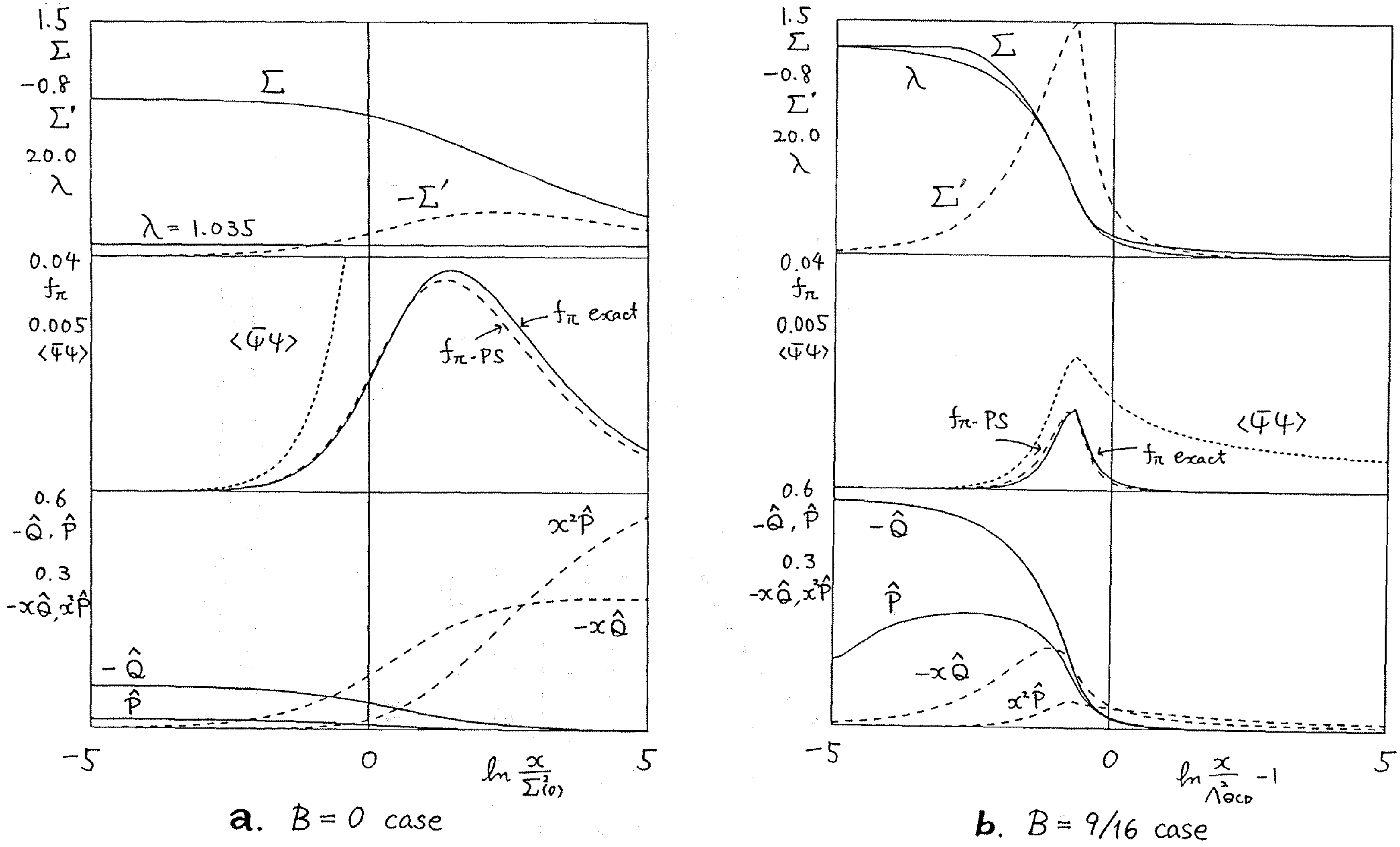

Fig. 2 Solutions of Schwinger-Dyson and Bethe-Salpeter Equations

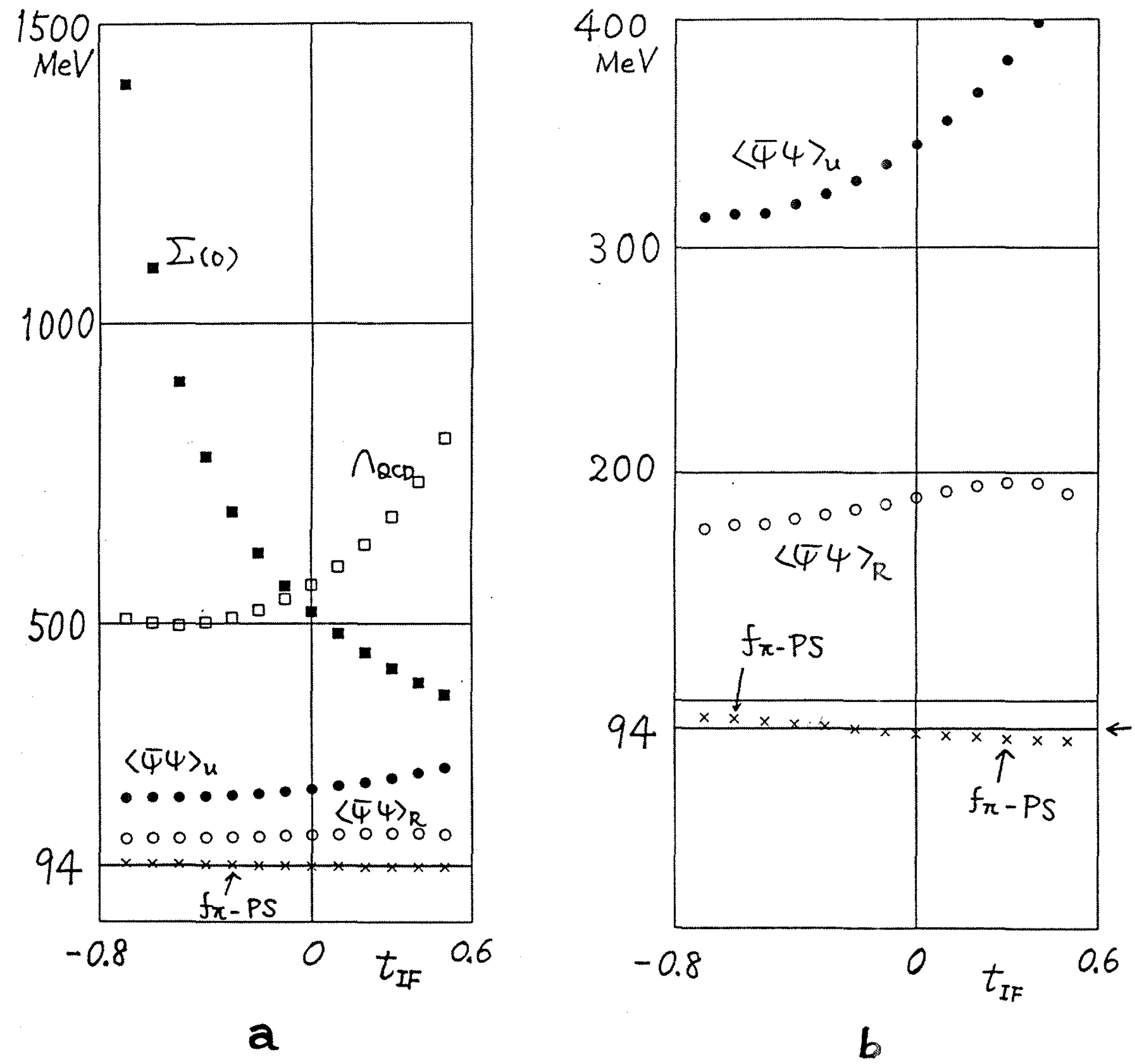

Fig. 3 Value of Physical Quantities
Case of $B = 9/16$(three triplets QCD)

Scalar Bound States in SCQED and Technicolor Model

Misako Suwa

Department of Physics, Niigata University, Niigata 950-21, Japan

Abstract

Using the bilocal auxiliary field method, we have numerically determined the mass spectra of scalar and pseudo-scalar fields as fermion bound states in the framework of the strong coupling QED(SCQED).[12),13)] The consistency of the mass poles with the Miransky's continuum limit is investigated. The renormalizations of the bound states are discussed and the application of SCQED to a technicolor model is also investigated.

On the massless fermion-U(1) gauge field system, it is known that the spontaneous chiral symmetry breaking (χsb) occurs [1),2)] and fermions get a dynamical mass scale ($B_0(0)$) in the strong coupling region. [3),4)] In other words, the scalar bound state of fermion and anti-fermion has a vacuum expectation value. On the other hand, the pseudo-scalar bound states are eaten by W and Z bosons and the bosons become massive.

In the present article, our purposes are the following ; to calculate the mass spectra of the bound states of fermions which appear as the result of the χsb using the method of the bilocal auxiliary fields [8)~10)], to check the consistency of their masses with the Miransky's continuum limit in SCQED, to renormalize the composite fields and to apply our calculations to the technicolor model.[11)]

First, we shall derive the effective action for meson fields (the bound states) form QED action. We start with the well known chiral symmetric action,

$$S_{\mathrm{QED}} = \int d^4x [\bar{\Psi}^a(\gamma_\mu \partial_\mu + ie_0 \gamma_\mu A_\mu)\Psi^a + \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \partial_\mu A_\mu B - \frac{\xi}{2} B^2] \,, \qquad (1)$$

where $\Psi^a(x)$ is the massless fermion field of N_f-flavor interacting with U(1) gauge fields $A_\mu(x)$. By using the method of bilocal auxiliary field developed by Morozumi and So[10)], the effective action S_{eff} for bilocal fields is obtained,

$$S_{eff}[\sigma, \pi, V, A, T] = -\frac{1}{2}\int d^4x d^4y [\sigma^{ab}(x,y) M^{-1}(x-y)\sigma^{ba}(y,x) \\ + \pi^{ab}(x,y)M^{-1}(x-y)\pi^{ba}(y,x)] \\ - \int d^4x d^4y \ \mathrm{tr}\ \ln[\delta(x-y)\delta^{ab}\gamma_\mu\partial_\mu + \sigma^{ab}(x,y) + i\gamma_5\pi^{ab}(x,y)] \ . \quad (2)$$

$M^{-1}(x)$ and $M^{-1}_{\mu\nu}(x)$ are defined as

$$M^{-1} \equiv -\frac{16\pi^2}{3+\xi}\frac{1}{e_0^2}x^2, \quad (3.a)$$

$$M^{-1}_{\mu\nu}(x) \equiv -8\pi^2\frac{1}{e_0^2}\left(x^2\delta_{\mu\nu} + \frac{1-\xi}{\xi}x_\mu x_\nu\right), \quad (3.b)$$

where the vector, axial-vector and tensor terms and the mixing terms with them are ignored for simplicity.

Next, we want to derive the quadratic parts of the effective action for scalar and pseudo-scalar fields in order to obtain their invese propagators. In Landau gauge ($\xi = 0$), the vacuum expectation values of the fields are

$$\begin{aligned} &< 0 \mid \sigma_q^{ab}(P) \mid 0 >= (2\pi)^4\delta^{(4)}(P)B(q^2)\delta_{ab}, \\ &< 0 \mid \pi_q^{ab}(P) \mid 0 >= 0, \end{aligned} \quad (4)$$

where the fields are transformed into the momentum space. P and q are total momentum and relative momentum respectively. The solution of fermion mass scale can be obtained in the strong coupling region, by analyzing the Landau-gauged gap equation of the vacuum expectation value of scalar bound state;

$$-\frac{2\pi}{3\alpha_0}(4q^2\frac{d^2}{(dq^2)^2} + 8\frac{d}{dq^2})B(q^2) + \frac{2(-q^2 + B(q^2)^2)}{(q^2 + B(q^2)^2)^2}B(q^2) = 0, \quad (5)$$

with the infrared and ultraviolet boundary conditions as,

$$\lim_{q\to 0} q^4\frac{dB(q^2)}{dq^2} \to 0, \qquad \lim_{q\to\Lambda}\frac{d(q^2B(q^2))}{dq^2} \to 0. \quad (6)$$

It takes the same form as the ladder Schwinger-Dyson equation for a fermion self-energy.[18] Then only the solution which has no nodes corresponds to the true vacuum expectation value. At the zero momentum point,

$$B_0(0) = \eta\Lambda\exp\left[-\frac{\pi}{\sqrt{\alpha_0/\alpha_c - 1}}\right], \quad (7)$$

where Λ, α_0, α_c, and η are, ultraviolet cut off, bare coupling constant, critical coupling constant, and numerical constant. Since B_0 (0) is proportial to Λ, so if Λ is taken to be infinite, B_0 (0) will diverge. Miransky propsed in his paper that we should take $\alpha_0 \to \alpha_c$

as $\Lambda \to \infty$, $B_0(0)$ remainds finite [5]. This is so called Miransky's continuum limit. Then the scalar and pseudo-scalar quadratic part, $S^{(2)}_{eff}$, of the effective action is

$$S^{(2)}_{eff}[\sigma,\pi] = \int \frac{d^4P}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}[\sigma_q^{ab}(-P)D_\sigma^{-1}(q,P)\sigma_q^{ba}(P) + \pi_q^{ab}(-P)D_\pi^{-1}(q,P)\pi_q^{ba}(P)]. \tag{8}$$

$D_\sigma^{-1}(q,P)$ and $D_\pi^{-1}(q,P)$ correspond to the inverse propagators of the mesons, given by

$$D_\sigma^{-1}(q,P) \equiv -\frac{2\pi}{3\alpha_0}\frac{\partial^2}{\partial q_\mu \partial q_\mu} + \frac{1}{2}\mathrm{Tr}[S_F(-\frac{P_\mu}{2}+q_\mu)S_F(\frac{P_\mu}{2}+q_\mu)], \tag{9.a}$$

$$D_\pi^{-1}(q,P) \equiv -\frac{2\pi}{3\alpha_0}\frac{\partial^2}{\partial q_\mu \partial q_\mu} + \frac{1}{2}\mathrm{Tr}[i\gamma_5 S_F(-\frac{P_\mu}{2}+q_\mu)i\gamma_5 S_F(\frac{P_\mu}{2}+q_\mu)], \tag{9.b}$$

where α_0 $(= e_0^2/4\pi)$ is the bare coupling constant and $S_F(q_\mu)$ means a fermion propagator.

Now, we consider the eigenvalue problems of the inverse propagators eqs.(9.a) and (9.b). They correspond exactly to solving the ladder Bethe-Salpeter equations.[7),18)] Then the scalar mass and pseudo-scalar mass can be estimated by eigenvalues, $\lambda(P)$ and $\mu(P)$, of the Schrödinger-like operators eqs.(9.a) and (9.b),

$$D_\sigma^{-1}(q,P)\psi_\sigma^\lambda(q,P) = \lambda(P)\psi_\sigma^\lambda(q,P), \tag{10.a}$$

$$D_\pi^{-1}(q,P)\psi_\pi^\mu(q,P) = \mu(P)\psi_\pi^\mu(q,P). \tag{10.b}$$

The mass poles of scalar and pseudo-scalar bound states can be calculated numerically as the zero eigenvalue points of them. The wave functions of the bound states, σ and π, are expanded by these eigenfunctions which correspond to the infinite eigenmodes of BS amplitudes, ψ_σ^λ and ψ_π^μ,

$$\sigma_q(P) = \sum_\lambda a_\lambda(P)\psi_\sigma^\lambda(q,P), \tag{11.a}$$

$$\pi_q(P) = \sum_\mu b_\mu(P)\psi_\pi^\mu(q,P). \tag{11.b}$$

Using the method of variation, we get the relation between eigenvalues of those inverse propergators and total momentum P_μ for each set of fixed coupling and cutoff. (Relative momentum q_μ is integrated out.) We show the result for the scalar case in Fig.1 with the gauge coupling constant $C \equiv \alpha_0/\pi$. In order to make the check of our variational method, we also show the result for the pseudo-scalar field in Fig.2, because we know it as a Goldstone boson so its mass pole is expected to be zero. From Fig.1 we can see

$m_\sigma/B_0(0) \sim 1.4$ and is almost independent of α_0, which implies it to be consistent with Miransky's continuum limit.

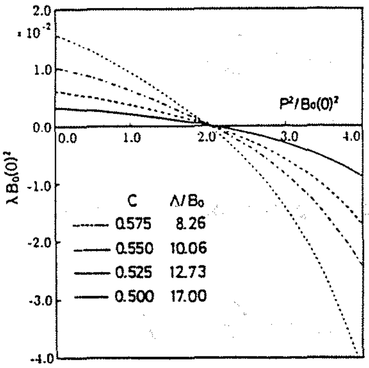

Figure 1. Eigenvalue lines, $\lambda(P)B_0(0)^2$, of a scalar inverse propagatar.

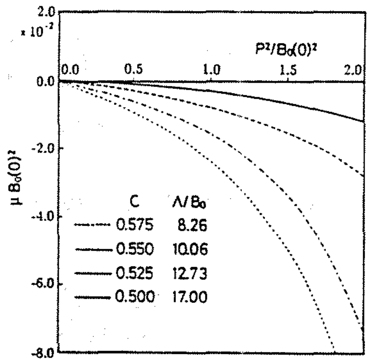

Figure 2. Eigenvalue lines, $\mu(P)B_0(0)^2$, of a pseudo-scalar inverse propagatar.

It can be also seen that the curvature, $\delta^2 V_{eff}(B) \sim D_\sigma^{-1}(0)$, of the effective potential goes to zero as the ultraviolet cutoff tends to infinity (at the same time, the coupling tends to its critical value). The obtained value is slightly smaller than the prediction, ~ 2, by partially conserved dilatational current hypothesis (PCDC).[14] With respect to higher excitation masses,[15] we could not find any physical excitation mass of the scalar state.

One necessity of a renormalization[16] is caused by a fact that the operator $\bar{\Psi}\Psi$ has a ultraviolet linear divergence. So we should conclude field renormalization Z factors for composite fields from our results. In general, they are defined as

$$\sigma_0^{ab} = Z_\sigma^{\frac{1}{2}}\, \sigma_r^{ab}, \tag{12.a}$$

$$\pi_0^{ab} = Z_\pi^{\frac{1}{2}}\, \pi_r^{ab}, \tag{12.b}$$

$$D_{\sigma,0}^{-1}(q,P) = Z_s D_{\sigma,r}^{-1}(q,P), \tag{13.a}$$

$$D_{\pi,0}^{-1}(q,P) = Z_{ps} D_{\pi,r}^{-1}(q,P). \tag{13.b}$$

Our numerical analysis suggests the following Z factors for the bound states;

$$Z_\sigma = Z_s^{-1} = \left(\frac{\Lambda}{B_0(0)}\right)^{2+\epsilon}, \tag{14.a}$$

$$Z_\pi = Z_{ps}^{-1} = \left(\frac{\Lambda}{B_0(0)}\right)^{2+\epsilon'}, \tag{14.b}$$

where ϵ and ϵ' are nearly equal to zero. Powers of Λ in eqs. (14.a) and (14.b) may correspond to twice anomalous dimensions of $\bar{\Psi}\Psi$ and $\bar{\Psi}i\gamma_5\Psi$. Now, we put ϵ and ϵ' zero. The renormalized eigenvalues are shown in Figs.3 and 4. For the scalar case, the good correspondence along one 'renormalized line' is surprising. This fact suggests that the present method could be correct. For the pseudo-scalar case, the correspondence is not so good because the region is so far from the mass pole.

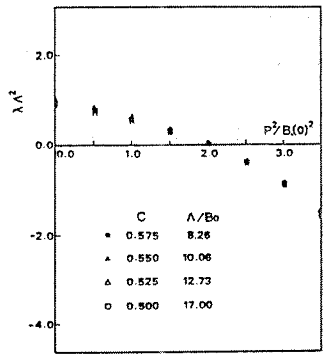

Figure 3. Renormalized eigenvalues for the scalar field, $\lambda(P)\Lambda^2$.

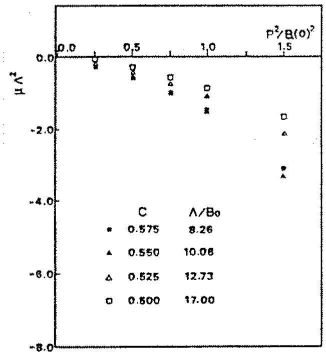

Figure 4. Renormalized eigenvalues for the pseudo-scalar field, $\mu(P)\Lambda^2$.

In the last place, we will apply our results to technicolor theory and estimate the Higgs mass. If we interpret SCQED as U(1) technicolor theory, scalar and pseudo-scalar fields can be regarded as composite Higgs boson(s), h, and Goldstone bosons, Π, as the result of the dynamical χsb of technifermions. To obtain the mass of Higgs particle, we must set a Weinberg-Salam scale at first. For that, we use a weak boson mass scale;

$$M_W^2 = \frac{g_1^2}{4} F_{TC}^2, \tag{15}$$

$$F_{TC}^2 = N_2 F_\pi^2 / N_f, \tag{16}$$

F_π^2 is a bare pion decay constant as numerically given,

$$\begin{aligned} F_\pi^2 &= 4N_f \int \frac{d^4q}{(2\pi)^4} \frac{B_0(q^2)(B_0(q^2) - \frac{q^2}{2}\frac{d}{dq^2}B_0(q^2))}{(q^2 + B_0(q^2)^2)^2}, \\ &= 4.749 \times 10^{-2} N_f B_0(0)^2. \end{aligned} \tag{17}$$

N_2 means the weak doublet number of techni-fermions. From eqs. (15) $\sim$ (17) and using the experimental values of M_W = 81.0 GeV and M_Z = 92.4 GeV, we can obtain F_{TC} $\sim$ 250 GeV . Our SCQED scale ($B_0(0)$) and Higgs mass are determined as

$$B_0(0) = 1.1 \text{ TeV}/\sqrt{N_2}, \tag{18}$$

$$m_h \le 1.6 \text{ TeV}/\sqrt{N_2}, \tag{19}$$

From eq.(19), we expect several hundreds GeV as Higgs mass. It should be noted that, however, the naive one doublet model has anomalies; global $SU(2)$ anomaly and $SU(2)_L - SU(2)_L - U(1)_{SCQED}$ triangle anomaly. So we can conclude $N_2 \ge 2$.[17]

From eq.(19), we expect several hundreds GeV as Higgs mass. It should be noted that, however, the naive one doublet model has anomalies; global $SU(2)$ anomaly and $SU(2)_L - SU(2)_L - U(1)_{SCQED}$ triangle anomaly. So we can conclude $N_2 \geq 2$.[17)]

Finally, we have some questions. (α) Can the mixing effect among scalar-vector-even part of antisymmetric fields and among pseudoscalar-axial vector-odd part of antisymmetric fields be ignored? (β) Is ladder approximation sufficient to predict the mass of a Higgs particle? (γ) Are there any realistic models which can predict N_2? Details of our calculation and further developments involving above points will be reported in separate papers.

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BCS-TYPE RELATION FOR THE COMPOSITE HIGGS BOSON IN GAUGED NAMBU–JONA-LASINIO MODEL*

Susumu Shuto, Masaharu Tanabashi and Koichi Yamawaki **

Department of Physics, Nagoya University
Nagoya 464-01, Japan

Abstract

We study the relation between the dynamical mass of fermion m_f and the mass of the scalar bound state σ ("Higgs boson") m_σ in the gauged Nambu–Jona-Lasinio (NJL) model based on the PCDC hypothesis. In contrast to the result of the pure NJL model $m_\sigma/m_f = 2$, the mass ratio m_σ/m_f does vary according to the gauge coupling λ from 2 ($\lambda = 0$) to 1.2 ($\lambda = \lambda_c \equiv 1/4$). We also find that the mass ratio m_σ/m_f drastically varies near the pure NJL limit, i.e., $m_\sigma/m_f \simeq \sqrt{2}$ for very small (but non-zero) λ. In the top quark condensation scenario for the dynamical electroweak symmetry breaking this implies a mass relation between the Higgs boson and the top quark, $m_H \simeq \sqrt{2}m_t$.

1. Introduction

Dynamical symmetry breaking[1] (DSB) is a familiar phenomenon in many models of quantum field theories. The concept of DSB is also important in modern particle physics to describe the spontaneous symmetry breaking in a natural way. Unfortunately in many models with DSB it is not so easy to determine the parameters of the low energy effective theory because of the non-perturbative nature of the DSB. In this respect, Nambu–Jona-Lasinio (NJL) model[1] is interesting, because we can explicitly calculate the parameters in the dynamical chiral symmetry breaking ($D\chi SB$) phase based on the $1/N$ expansion.

Within $1/N$ leading approximation (or chain approximation) we obtain a simple relation in the mass spectrum of the NJL model (BCS-type relation)[2]:

$$m_\sigma : m_f : m_\pi = 2 : 1 : 0, \tag{1.1}$$

where m_f is the dynamical mass of the fermion and m_σ and m_π mean the masses of scalar bound state σ and pseudoscalar bound state (Nambu-Goldstone (NG) boson) π, respectively.

Unfortunately, the NJL model is not renormalizable in four dimensions. Recently, it was suggested[3] that the NJL model with gauge interaction (gauged NJL model)[4,5,6] becomes renormalizable in the $D\chi SB$ phase even in four dimensions. In fact, explicit calculation of this model based on the gap equation shows that the physical quantities are finite (in the continuum limit) within the ladder approximation[3,7,8,9].

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Important application of this model is a top quark condensation scenario[9,10,11,12,13,14] for the dynamical electroweak symmetry breaking ("top-mode standard model"). In the top-mode standard model, a top quark condensate $\langle \bar{t}t \rangle \neq 0$ triggered by the gauged NJL model (NJL plus QCD) plays a role of the vacuum expectation value of the elementary Higgs boson in the standard model. The model predicted[10] a large top quark mass, typically $m_t \simeq 250\text{GeV}$, and a spinless $\bar{t}t$ bound state (Higgs boson) with a mass $m_H \simeq 2m_t$. The latter relation, $m_H \simeq 2m_t$, [10,11,15] would immediately follow as in the NJL model, Eq.(1.1), if the QCD had only a small effect on the low energy mass spectrum of the composite bosons. On the contrary, if the QCD effect is significant, the BCS-type relation Eq.(1.1) will no longer hold: the prediction $m_H \simeq 2m_t$ must be changed.

Here, we wish to study the effect of gauge interaction on the BCS-type relation in the gauged NJL model. To compute m_σ^2 we regard σ as a "massive dilaton"[4,5,8,16] and employ the Partially Conserved Dilatation Current (PCDC) hypothesis, since it is rather difficult to compute directly the pole position in the $J^{PC} = 0^{++}$ channel of the fermion and antifermion scattering amplitude even in the ladder approximation. In what follows we shall see how this hypothesis actually works in the models in which we can compare the PCDC result with the direct computation of m_σ^2. We then calculate m_σ^2 in the gauged NJL model through the PCDC hypothesis. We find the effect of gauge interaction is rather significant, no matter how small (but non-zero) the gauge coupling may be. In fact, we obtain $m_\sigma \simeq \sqrt{2}m_f$ for the case of a very small gauge coupling.

2. PCDC relation — derivation —

Consider the scalar boson (dilaton) σ which couples to the energy-momentum tensor $\theta_{\mu\nu}$ as

$$\langle 0|\theta_{\mu\nu}|\sigma\rangle = (q^2 g_{\mu\nu} - q_\mu q_\nu)F_\sigma/(D-1), \tag{2.1}$$

where F_σ is the dilaton "decay constant" and D the dimension of space-time. Under the assumption of σ dominance in the channel of θ_μ^μ, we obtain

$$\begin{aligned} i\int d^D x \langle 0|T[\theta_\mu^\mu(x)\theta_\nu^\nu(0)]|0\rangle_c &= \langle 0|\theta_\mu^\mu(0)|\sigma\rangle \frac{1}{m_\sigma^2}\langle\sigma|\theta_\nu^\nu(0)|0\rangle \\ &= m_\sigma^2 F_\sigma^2. \end{aligned} \tag{2.2}$$

Noting the relation of energy-momentum tensor and dilatation current, $D_\mu = x^\nu\theta_{\mu\nu}$, $\partial_\mu D^\mu = \theta_\mu^\mu$, and the Ward-Takahashi identity, we obtain

$$i\int d^D x \langle 0|T[\theta_\mu^\mu(x)\theta_\nu^\nu(0)]|0\rangle_c = \langle 0|i[Q_D, \theta_\nu^\nu(0)]|0\rangle, \tag{2.3}$$

where the dilatation charge Q_D is defined by $Q_D \equiv \int d\vec{x} D_0$. Comparing Eq.(2.2) with Eq.(2.3), we can write the dilaton mass m_σ in terms of F_σ and the vacuum expectation value of the energy-momentum tensor (PCDC relation):

$$m_\sigma^2 = -\frac{\langle 0|i[Q_D, \theta_\nu^\nu(0)]|0\rangle}{F_\sigma^2}. \tag{2.4}$$

When the operator θ^μ_μ obeys a simple scaling $i[Q_D, \theta^\nu_\nu] = d_\theta \theta^\nu_\nu$, the PCDC relation becomes simple:

$$m_\sigma^2 = -Dd_\theta E/F_\sigma^2, \tag{2.5}$$

where the vacuum energy (density) E is defined by $\langle 0|\theta_{\mu\nu}|0\rangle = g_{\mu\nu}E$.

As an illustration for Eq.(2.5), let us consider the φ^4 theory with $D = 4$,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{\lambda}{4!}\left[(\sigma^2 - v^2)^2 - v^4\right]. \tag{2.6}$$

The energy-momentum tensor within the tree approximation is

$$\theta_{\mu\nu} = (\partial_\mu \sigma)(\partial_\nu \sigma) - g_{\mu\nu}\mathcal{L} - \frac{d_\sigma}{2(D-1)}(\partial_\mu\partial_\nu - g_{\mu\nu}\partial^2)\sigma^2, \tag{2.7}$$

where d_σ means the scale dimension of σ. In this case, $\theta^\mu_\mu = (D-4)\frac{\lambda}{4!}\sigma^4 + \frac{\lambda}{3!}v^2\sigma^2 = \frac{\lambda}{3!}v^2\sigma^2$, so that we obtain the scale dimension of θ^μ_μ, $d_\theta = 2$. In the symmetry-broken phase σ develops a vacuum expectation value v, $\sigma = v + \tilde{\sigma}$, $\tilde{\sigma}$ being the dilaton field. Then from the definition of F_σ Eq.(2.1) and E, it is easy to obtain

$$F_\sigma = d_\sigma v = v, \qquad E = -\frac{1}{D \cdot 3!}v^4\lambda, \tag{2.8}$$

where we used $d_\sigma = 1$. PCDC relation Eq.(2.5) reads

$$m_\sigma^2 = -\frac{8E}{F_\sigma^2} = 2\left(\frac{\lambda}{3!}v^2\right). \tag{2.9}$$

Thus the dilaton mass calculated in this way correctly reproduces the mass of $\tilde{\sigma}$ determined from the lagrangian Eq.(2.6).

In more complex systems, the prediction of the dilaton mass in Eq.(2.5) corresponds to the mass term of the dilaton field in the effective theory which contains only the dilaton and has the same scale dimension of θ^μ_μ and dilaton decay constant as those of the original theory. In fact, the PCDC relation Eq.(2.5) can also be derived in terms of the effective theory of dilaton field[17,18].

In the linear σ model of $SU(n_f)_L \times SU(n_f)_R \to SU(n_f)_V$, F_σ can be related to the NG boson decay constant F_π as[16]

$$F_\sigma^2 = \frac{n_f}{2}d_\sigma^2 F_\pi^2. \tag{2.10}$$

where d_σ is the scale dimension of the scalar field in the linear σ model. Then we can rewrite the PCDC relation Eq.(2.5) in terms of F_π;

$$m_\sigma^2 = -2\frac{Dd_\theta E}{n_f d_\sigma^2 F_\pi^2}. \tag{2.11}$$

Eq.(2.11) is our basic formula.

3. BCS-type relation in the renormalizable NJL model with $D < 4$

Let us first check how the formula Eq.(2.11) works in the NJL model in D dimensions ($D < 4$). The model is renormalizable in $1/N$ expansion and we can directly compute m_σ^2 without recourse to the PCDC hypothesis[19].

The lagrangian is

$$\mathcal{L} = \bar{\psi}^j i \not\partial \psi_j + \frac{G}{2N}\left[(\bar{\psi}^j\psi_j)^2 + (\bar{\psi}^j i\gamma_5\vec{\tau}\psi_j)^2\right], \tag{3.1a}$$

where the suffix j runs from 1 to N. Introducing auxiliary fields σ and π, we rewrite Eq.(3.1a) into

$$\mathcal{L} = \bar{\psi}^j i \not\partial \psi_j - \left[\bar{\psi}^j\psi_j\sigma + \bar{\psi}^j i\gamma_5\tau^a\psi_j\pi^a\right] - \frac{N}{2G}\left[\sigma^2 + \pi^2\right]. \tag{3.1b}$$

The effective potential within the $1/N$ leading approximation is given by

$$V(\sigma) = \frac{N}{2}\left[\frac{\sigma^2}{G} - 8\int\frac{d^D p_E}{(2\pi)^D}\ln\left(\frac{p_E^2+\sigma^2}{p_E^2}\right)\right]. \tag{3.2}$$

The stationary condition of the effective potential gives the gap equation,

$$0 = \frac{1}{N}\frac{\partial V}{\partial\sigma} = \frac{\sigma}{G} - 8\int\frac{d^D p_E}{(2\pi)^D}\frac{\sigma}{\sigma^2+p_E^2} = 8\sigma\int\frac{d^D p_E}{(2\pi)^D}\left[\frac{1}{gp_E^2} - \frac{1}{\sigma^2+p_E^2}\right], \tag{3.3}$$

where the dimensionless coupling constant g is defined by $g \equiv 8G\int\frac{d^D p_E}{(2\pi)^D}\frac{1}{p_E^2}$. It is well known that Eq.(3.3) has $D\chi SB$ solution $\sigma \neq 0$ when $g > 1$ ($g = 1$ is a nontrivial ultraviolet fixed point of the bare coupling[19].). The dynamical fermion mass is given by the vacuum expectation value of the auxiliary field, $m_f = \sigma_{sol}$.

Now the vacuum energy in the $D\chi SB$ vacuum can be written in terms of σ_{sol}:

$$\begin{aligned} E =& V(\sigma_{sol}) - V(\sigma = 0) \\ =& -\frac{8N}{(4\pi)^{D/2}D}\Gamma(2-\frac{D}{2})\sigma_{sol}^D. \end{aligned} \tag{3.4}$$

Computing the correlation function for axialvector currents, we find the NG boson decay constant in this model;

$$F_\pi^2 = \frac{4N}{(4\pi)^{D/2}}\Gamma(2-\frac{D}{2})\sigma_{sol}^{D-2}. \tag{3.5}$$

Noting $\theta_\mu^\mu \sim (\bar{\psi}\psi)^2$, $\sigma \sim (\bar{\psi}\psi)$ and $n_f = 2$, we find that the PCDC relation Eq.(2.11) reads

$$m_\sigma^2 = -\frac{d_\theta D\cdot E}{d_\sigma F_\pi^2} = 4\sigma_{sol}^2 = 4m_f^2, \tag{3.6}$$

where $d_\theta = 2(D-1-\gamma_m) = 2$ and $d_\sigma = (D-1-\gamma_m) = 1$, with $\gamma_m(= D-2)$ [19] being the anomalous dimension of the mass operator $(\bar{\psi}\psi)_R$. Note that Eq.(3.6) holds independently of D and perfectly agrees with the direct computation of m_σ^2[19].

Incidentally, effective Yukawa coupling in the low energy effective theory is also calculated (Goldberger-Treiman relation):

$$g_Y^2 = m_f^2/F_\pi^2, \tag{3.7}$$

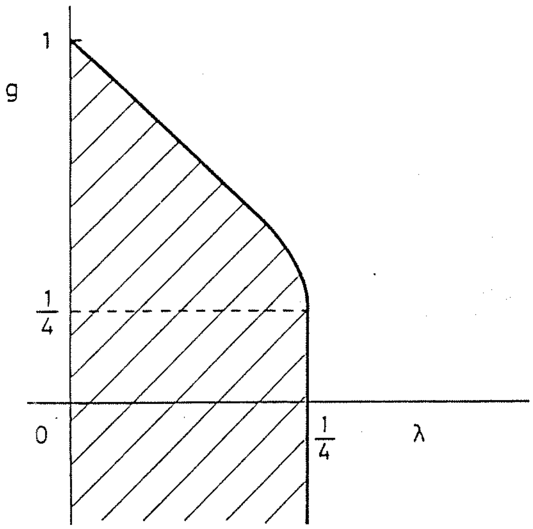

Fig.1.
Critical line in (λ, g) plane is depicted by the solid line, which separates the spontaneously unbroken phase (shaded region) and the broken phase of chiral symmetry.

F_π^2 being given by Eq.(3.5). Note that in the limit $D \longrightarrow 4$ (NJL limit) the NG boson decay constant F_π diverges. Thus in this limit the Yukawa coupling vanishes and we have a trivial (non-interacting) effective theory.

4. BCS-type relation in the gauged NJL model

Having convinced ourselves of validity of the PCDC relation, we now derive a BCS-type relation in the gauged NJL model through the PCDC relation. For simplicity we calculate the case of $U(1)$ gauge theory. The lagrangian is given by

$$\mathcal{L} = \bar{\psi} i \not\partial \psi + \frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \vec{\tau}\psi)^2] + e\bar{\psi}\not{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \text{gauge fixing term.} \tag{4.1}$$

The CJT effective potential[20] is written explicitly[8] in Landau gauge in 2 loop approximation;

$$\begin{aligned} V[\Sigma] = -\frac{1}{4\pi^2}\Bigg\{ &\int_0^{\Lambda^2} dx \cdot x \left[\ln(1+\frac{\Sigma^2(x)}{x}) - \frac{2\Sigma^2(x)}{x+\Sigma^2(x)} \right] \\ &+ \int_0^{\Lambda^2} dx dy \frac{x\Sigma(x)}{x+\Sigma^2(x)} \frac{y\Sigma(y)}{y+\Sigma^2(y)} \left[\frac{\lambda}{x}\theta(x-y) + \frac{\lambda}{y}\theta(y-x) + \frac{g}{\Lambda^2} \right] \Bigg\}, \end{aligned} \tag{4.2}$$

where Σ was defined as $iS^{-1} = \not{p} - \Sigma(-p^2)$ and x,y mean the space-like momentum square, $x \equiv -p^2$, $y \equiv -q^2$. In Eq.(4.2) an ultraviolet cutoff Λ was introduced and we used dimensionless couplings $g \equiv \Lambda^2 G/(2\pi^2)$, $\lambda \equiv 3e^2/(4\pi)^2$.

The stationary condition of the CJT potential Eq.(4.2) yields the gap equation (ladder Schwinger-Dyson equation) which was first studied by Bardeen *et al.*[4]:

$$\Sigma(x) = \frac{g}{\Lambda^2}\int_0^{\Lambda^2} dy \frac{y\Sigma(y)}{y+\Sigma^2(y)} + \lambda \int_0^{\Lambda^2} dy \frac{y\Sigma(y)}{y+\Sigma^2(y)} \left[\frac{1}{x}\theta(x-y) + \frac{1}{y}\theta(y-x) \right]. \tag{4.3}$$

Eq.(4.3) is written into the differential equation and the boundary conditions:

$$\begin{aligned}&\left[x\frac{d^2}{dx^2}+2\frac{d}{dx}+\frac{\lambda}{x+\Sigma(x)^2}\right]\Sigma(x)=0,\\ &\lim_{x\to 0}x^2\frac{d}{dx}\Sigma(x)=0,\qquad \left[1+\left(1+\frac{g}{\lambda}\right)x\frac{d}{dx}\right]\Sigma(x)\Big|_{x=\Lambda^2}=0.\end{aligned}\tag{4.4}$$

If there exists a nontrivial ($D\chi SB$) solution $\Sigma_{sol}\neq 0$, the vacuum energy E for this solution is given by[8]

$$\begin{aligned}E&=V[\Sigma_{sol}]-V[\Sigma\equiv 0]\\ &=-\frac{1}{8\pi^2}\left[\Lambda^4\ln\left(1+\frac{\Sigma^2_{sol}(\Lambda^2)}{\Lambda^2}\right)-\frac{g\Lambda^2}{(\lambda+g)^2}\Sigma^2_{sol}(\Lambda^2)\right].\end{aligned}\tag{4.5}$$

In fact, it was found[5,6] that Eq.(4.4) has a $D\chi SB$ solution when $g>(1+2\gamma')^2/4$ or $\lambda>\lambda_c\equiv 1/4$, where $\gamma'\equiv\frac{1}{2}\sqrt{1-\lambda/\lambda_c}$. (See Fig.1). We scale the bare parameters (λ,g) to get a finite solution Σ in such a way (*à la* Miransky)[21] that they depend on the cutoff Λ. At $\Lambda\to\infty$ the parameters (λ,g) go to on the critical line which separates the χSB phase from the chiral symmetric one[5,6]:

$$\begin{aligned}g&=\frac{(1+2\gamma')^2}{4}\quad\text{for}\quad g>\frac{1}{4},\\ \lambda&=\lambda_c\qquad\text{for}\quad g\leq\frac{1}{4}.\end{aligned}\tag{4.6}$$

Note here that the anomalous dimension is large, $\gamma_m=1+2\gamma'$, in the $D\chi SB$ phase[3].

To solve the gap equation analytically, it is convenient to use the linearized differential equation[8,21] instead of Eq.(4.4): $1/(x+\Sigma^2(x))\to 1/(x+m_f^2)$, with $m_f\equiv\Sigma(m_f^2)$. The solution of this equation is written in terms of the hypergeometric function:

$$\begin{aligned}\Sigma_{sol}(x)&=\xi m_f\cdot\mathrm{F}\!\left(\frac{1}{2}+\gamma',\frac{1}{2}-\gamma',2;-\frac{x}{m_f^2}\right)\\ &\simeq\xi m_f\left[\frac{\Gamma(2\gamma')}{\Gamma(\frac{1}{2}+\gamma')\Gamma(\frac{3}{2}+\gamma')}\left(\frac{x}{m_f^2}\right)^{-\frac{1}{2}+\gamma'}+(\gamma'\leftrightarrow-\gamma')\right],\qquad\text{for } x\gg m_f^2,\end{aligned}\tag{4.7}$$

ξ being a constant ($\simeq 1$[8]).

For calculating m_σ^2 we need to evaluate the vacuum energy E. From the explicit solution Eq.(4.7) the vacuum energy Eq.(4.5) now reads[8]

$$E=-\frac{\xi^2}{8\pi^2}m_f^4\frac{\gamma'\cot(\pi\gamma')}{\pi\left((\frac{1}{2})^2-(\gamma')^2\right)^2},\tag{4.8}$$

in the cutoff $\Lambda\to\infty$ limit (on the critical line). Note also that the vacuum energy becomes finite (at $\lambda>0$).

On the other hand, Pagels-Stoker formula[22,23] for the NG boson decay constant F_π is given by

$$F_\pi^2=\frac{1}{4\pi^2}\int_0^{\Lambda^2}dx\,x\frac{\Sigma^2_{sol}(x)-\frac{1}{4}x\frac{d}{dx}\Sigma^2_{sol}(x)}{(x+\Sigma^2_{sol}(x))^2}.\tag{4.9}$$

This integral converges (for $\lambda > 0$) using the solution of the differential equation. We can evaluate F_π explicitly using the solution of the linearized equation Eq.(4.7):

$$F_\pi^2 \simeq \frac{1}{4\pi^2}\left[\int_0^{m_f^2} dx x\frac{\xi^2 m_f^2}{(x+m_f^2)^2} + \int_{m_f^2}^{\Lambda^2} dx x\frac{\Sigma_{sol}^2(x) - \frac{1}{4}x\frac{d}{dx}\Sigma_{sol}^2(x)}{x^2}\right]$$
$$= \frac{1}{4\pi^2}\xi^2 m_f^2\left\{(\ln 2 - \frac{1}{2}) + \frac{\cot(\pi\gamma')}{\pi\gamma'(1-4\gamma'^2)}\left[(\frac{1}{1-2\gamma'}+\frac{1}{4})e^{2\delta} + (\frac{1}{1+2\gamma'}+\frac{1}{4})e^{-2\delta} - \frac{5}{2}\right]\right\}, \tag{4.10}$$

where

$$\delta \equiv \frac{1}{2}\ln\left(\frac{\Gamma(1+\gamma')\Gamma(\frac{3}{2}-\gamma')}{\Gamma(1-\gamma')\Gamma(\frac{3}{2}+\gamma')}\right) + 2\gamma'\ln 2.$$

Noting $\theta_\mu^\mu \sim (\bar\psi\psi)^2$, we find that $\sigma \sim (\bar\psi\psi)$, and $n_f = 2$, we find that the PCDC relation Eq.(2.11) reads*

$$m_\sigma^2 = -\frac{4\cdot 2(3-\gamma_m)}{(3-\gamma_m)^2}\frac{E}{F_\pi^2}$$
$$= \frac{64\gamma'^2 m_f^2}{(3-\gamma_m)(1-2\gamma')} \tag{4.11}$$
$$\times\left\{\pi\gamma'\frac{1-4\gamma'^2}{\cot(\pi\gamma')}(\ln 2 - \frac{1}{2}) + \left[(\frac{1}{1-2\gamma'}+\frac{1}{4})e^{2\delta} + (\frac{1}{1+2\gamma'}+\frac{1}{4})e^{-2\delta} - \frac{5}{2}\right]\right\}^{-1},$$

Thus we obtained the BCS-type relation Eq.(4.11), which actually depends on the anomalous dimension γ_m (or gauge coupling λ). We also calculate m_σ^2/m_f^2 numerically using the PCDC relation. These results are shown in Fig.2.

Although the agreement of our analytical calculation Eq.(4.11) with the result of the numerical one within the linearized approximation is remarkable, the difference of the result in the linearized approximation and that of the full nonlinear gap equation is most amazing. At sufficiently large cutoff Λ, the numerical result based on the solution of full nonlinear gap equation (Fig.2) strongly suggests

$$m_\sigma \simeq \sqrt{2}m_f, \tag{4.12}$$

if the gauge coupling constant λ is very small (but non-zero).

Important effect of the gauge coupling (however small) can also be seen through the effective Yukawa coupling in the low energy effective theory,

$$g_Y^2 = m_f^2/F_\pi^2, \tag{4.13}$$

which vanishes in the pure NJL limit ($\lambda \to 0$), since the solution Σ_{sol} becomes a constant function and F_π diverges logarithmically in that limit. Thus the inclusion of the gauge coupling makes the theory non-trivial ($g_Y \neq 0$). Numerical result of the gauge coupling dependence of the effective Yukawa coupling is shown in Fig.3.

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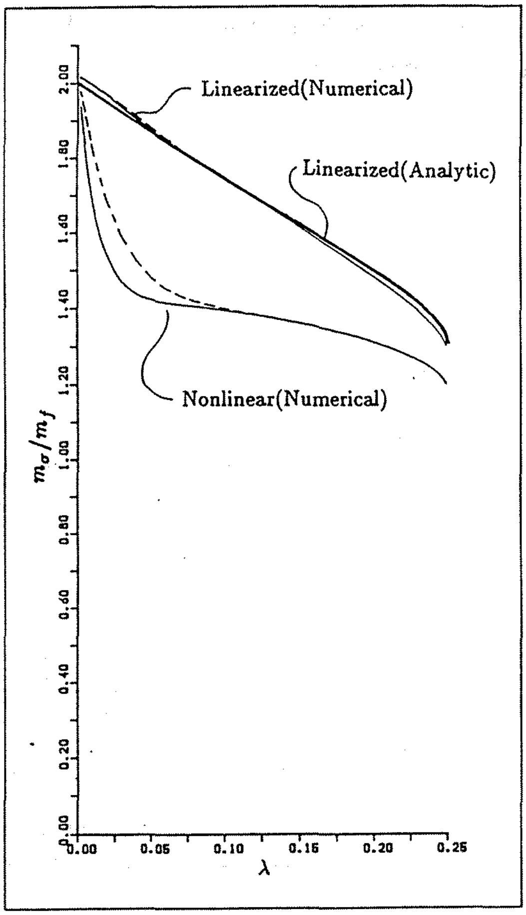
* One obtains the same result for any n_f-flavored model, i.e., $SU(n_f)_L \times SU(n_f)_R \to SU(n_f)_V$ for $n_f > 2$ and $U(1)_L \times U(1)_R \to U(1)_V$ for $n_f = 1$.



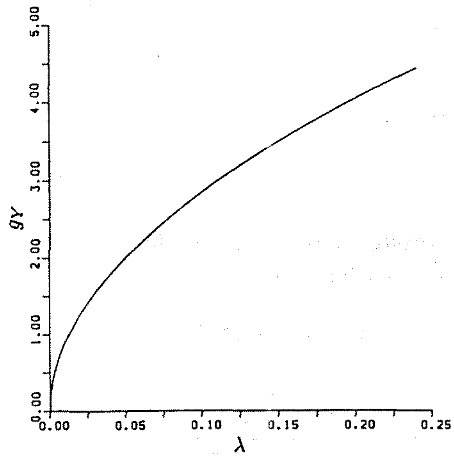


Fig.2.
Dependence of m_σ/m_f on the gauge coupling λ along the critical line. Linearized analytical result is depicted by the bold solid line. Linearized numerical result is depicted by the dashed line ($\Lambda^2/m_f^2 = 10^{10}$) and by the solid line ($\Lambda^2/m_f^2 = 10^{20}$). Full nonlinear numerical result is depicted by the dashed line ($\Lambda^2/m_f^2 = 10^{10}$) and by the solid line ($\Lambda^2/m_f^2 = 10^{20}$).

Fig.3.
Dependence of Yukawa coupling g_Y on the gauge coupling along the critical line.

5. Conclusion

In the $U(1)$ gauged NJL model, we found that the mass ratio m_σ/m_f does vary from 2 ($\lambda = 0$) to 1.2 ($\lambda = \lambda_c = 1/4$) according to the gauge coupling λ. In the pure QED limit ($g = 0$, $\lambda = \lambda_c$), our result ($m_\sigma/m_f \simeq 1.2 < 2$) has the same tendency as that of the lattice Monte Carlo simulation ($m_\sigma/m_f \simeq 1.7 < 2$)[24] and of the analysis through the Nambu-Bethe-Salpeter equation ($m_\sigma/m_f \simeq 1.4 < 2$)[25].

On the other hand, the mass ratio m_σ/m_f drastically varies near the pure NJL limit, i.e.,

$m_\sigma/m_f \simeq \sqrt{2} < 2$, no matter how small (non-zero) the coupling λ may be. If this property of the $U(1)$ gauged NJL model also holds in the QCD-gauged NJL model, the prediction of Higgs boson mass in the top-mode standard model will become

$$m_H \simeq \sqrt{2} m_t, \tag{5.1}$$

in contrast to the previous prediction, $m_H \simeq 2m_t$[10,11,15], ignoring the gauge interaction effect. It is argued[26] that our result Eq.(5.1) can be made consistent with the bootstrap symmetry breaking. This tendency (the Higgs mass decreases with the inclusion of gauge interaction) is also consistent with the renormalization group analysis employed by Bardeen *et al.*[13].

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Dynamical Chiral Symmetry Breaking and Scaling Law in Strong Coupling Unquenched QED*

KEI-ICHI KONDO

Department of Physics, Chiba University,
1-33 Yayoi-cho, Chiba 260, Japan

ABSTRACT

We report the recent developments on the phase structure of the strong coupling QED in the framework of the Schwinger-Dyson equation, taking account of the vacuum polarization effects. The results are compared with those obtained in the quenched planar approximation. The various problems concerning the construction of the continuum QED are discussed in connection with these results.

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§1. Introduction

The study of the critical phenomena in the regularized QED may open a new window to the understanding of the non-asymptotically free (NAF) field theory. As usual, QED is regularized by introducing the cutoffs, and then QED in the euclidean region can be regarded as one of the statistical-mechanical models. Necessary information to take the continuum limit is obtained through the critical behaviors of the model. This viewpoint is extremely powerful, because the tools and the concepts in rigorous and non-rigorous statistical mechanics become available to be able to control the continuum limit. This is demonstrated by the rigorous triviality proof of the $\lambda(\phi^4)_d$ theory,[1][2] and more generally by various achievements of the lattice gauge theories.

The question which we are addressed to is: Can QED be constructed as an interacting field theory? This is a long-standing problem of the constructive field theory,[3] although even at present we must stay in the heuristic level. The most straightforward way to study this issue is to examine the renormalized coupling constant related to the scattering amplitude among fermions, anti-fermions and photons. This is conceptually simple but difficult in actually carrying out. In addition, to study the consistency of the field theory we need some non-perturbative method. An alternative way is to obtain a set of critical exponents and hyperscaling relations in the critical phenomena. For example, in the ferromagnetic model with inverse temperature $J := 1/kT$, which is obtained as the lattice regularized version of scalar field theries, the 4th Ursell (connected 4-point) function U_4, the susceptibility χ and the correlation length ξ are characterized by the critical exponents in the neighborhood (nbd) of the critical point J_c as follows:

$$\bar{U}_4 := \sum_{x_2,x_3,x_4 \in L} U_4(x_1, ..., x_4) \sim (J_c - J)^{-\gamma-2\Delta_4}, \tag{1}$$

$$\chi := \sum_{y \in L} \langle \varphi_x; \varphi_y \rangle \sim (J_c - J)^{-\gamma}, \tag{2}$$

$$\xi^{-1} := m := \lim_{|x| \to \infty} \frac{1}{|x|} \ln \langle \varphi_0; \varphi_x \rangle \sim (\mathrm{J_c} - \mathrm{J})^{\nu}. \tag{3}$$

This implies the following critical behavior for the 4-point renormalized coupling constant (corresponding to the scalar-scalar scattering)

$$g^{(d)} := \frac{\bar{U}_4}{\chi^2 \xi^d} \sim (J_c - J)^{d\nu+\gamma-2\Delta_4}. \tag{4}$$

Then if the lattice model have classical exponents, $\nu = 1/2, \gamma = 1, \Delta_4 = 3/2$ in $d > 4$ dimensions, as predicted by the mean-field (MF) theory, then vanishing of the renormalized coupling constant in the continuum (scaling) limit $J \uparrow J_c$ (the triviality of $\lambda(\phi^4)_d$ theory for $d > 4$) is understood as a result of the violation of the hyperscaling relation:

$$d\nu + \gamma - 2\Delta_4 > 0. \tag{5}$$

On the other hand, in lower dimensions the hyperscaling relation is satisfied, $d\nu + \gamma - 2\Delta_4 = 0$, which implies the non-triviality of the $\lambda(\phi^4)_2$, $\lambda(\phi^4)_3$ theories.

In four-dimensions, the exact MF theory predicts the non-triviality of $\lambda(\phi^4)_4$ theory, while the log-correction to the MF theory implies to the triviality of $\lambda(\phi^4)_4$ theory, as predicted by the renormalization group (RG) analysis. This yields

$$g^{(4)} \sim |\ln(\mathrm{J_c} - \mathrm{J})|^{-1} \downarrow 0(\mathrm{J} \uparrow \mathrm{J_c}). \tag{6}$$

In the broken symmetry phase where the scalar field $\langle\varphi\rangle$ acquires the non-vanishing vacuum expectation value (VEV), $\langle\varphi\rangle \neq 0$, the symmetry $\varphi \longrightarrow -\varphi$ (for 1-component scalar model) is spontaneously broken. In the nbd of the critical point, $\langle\varphi\rangle \sim (J - J_c)^\beta$. If we renormalize the model such that $0 < \langle\varphi_R\rangle < \infty$ to obtain the broken symmetry $\lambda(\phi^4)_d$ theory, then the renormalized 3-, and 4-point coupling constants go to zero in the scaling limit $J \downarrow J_c$, under the requirement of the finiteness of the renormalized two-point function $\langle\varphi_0; \varphi_x\rangle$.[4]

The massless QED in which the bare fermion mass m_0 is set to zero, $m_0 = 0$, possesses the chiral symmetry. Then if the fermion acquires the dynamical mass, m_d, the chiral-symmetry is spontaneously broken. The existence of the strong coupling phase in which the dynamical mass generation takes place and the chiral-symmetry is spontaneously broken above the critical coupling e_c is first demonstrated by Maskawa and Nakajima[5] in the framework of the Schwinger-Dyson (SD) equation, in the quenched planar (ladder) approximation. Recently the scaling law for this phase transition was discovered by Miransky[6] to obey the essential-singularity type:

$$f := \frac{m_d}{\Lambda} \sim \exp[-\pi/\sqrt{e^2/e_c^2 - 1}], \tag{7}$$

which may lead to the non-trivial QED. This result was very suprising to me and I doubt that this must be an artifact of the quenched planar approximation. This is the motivation of my study on the SD equation.

In the chiral-symmetry-breaking (CSB) phase $e > e_c$, the critical exponents are defined for the dynamical fermion mass and the chiral order parameter as follows:

$$m_d \sim \Lambda(e^2 - e_c^2)^{\nu_m}, \tag{8}$$

$$\langle\bar{\psi}\psi\rangle \sim \Lambda^3(e^2 - e_c^2)^{\nu_{ch}}, \tag{9}$$

where $\sim$ should be understood in the sense that

$$\nu_m := -\lim_{e \downarrow e_c} \frac{\ln f}{\ln(e^2 - e_c^2)}, \tag{10}$$

Then $\nu_m < \infty$ implies the power-type scaling, while the essential-singularity type scaling corresponds to $\nu_m = \infty$.

A necessary condition to be able to take the continuum limit is the existence of the 2nd or higher order phase transition point where the correlation length diverges. Existence of such a critical point has been reported by the Monte Carlo simulation.[7] From the Miransky scaling, we may expect that the non-trivial QED could be obtained by taking the continuum limit of the cutoff QED model in the CSB phase. Furthermore the critical point e_c of the "bare" coupling constant was identified with the non-trivial ultraviolet (UV) fixed point. If so, however, we must consider the following questions. How the notorious disaster of the Landau ghost[8] is compatible with the non-triviality of QED? Or, whether the renormalized β function $\beta(e_R^2)$ may have a non-trivial UV fixed point, besides the Gaussioan fixed point resulting from the 1-loop RG equation. Incidentally, the quenched $(QED)_d$ obeys the MF type scaling for $d > 4$.[9] Then $(QED)_d$ is expected to be trivial for $d > 4$.$^{(F1)}$

$(F1)$ Quite recently, Lüscher has obtained in non-compact lattice QED the upper bound on the renormalized coupling constant: $0 \leqslant e_{ren} \leqslant a^{(d-4)}e$ (a:lattice spacing; e:bare coupling), which implies the triviality of $(QED)_d$ for $d > 4$.

In this talk we report the recent results on the SD equation for the fermion propagator in massless QED. The quenched planar approxmation is improved by taking account of the vacuum polarization through the vacuum polarization function at the 1-loop level in the SD equation for the photon propagator.

§2. SD EQUATION

The SD equation for the fermion propagator $S(p)$ is given by

$$S(p)^{-1} = S_0(p)^{-1} + \Sigma(p), \tag{1}$$

with the fermion self-energy part

$$\Sigma(p) := e^2 \int \frac{d^4q}{(2\pi)^4} \gamma_\mu S(q) \Gamma_\nu(q,p) D^{\mu\nu}(q-p) , \tag{2}$$

where $S_0(p) = (\not p + \mathrm{m}_0)^{-1}$ is the bare fermion propagator, $\Gamma_\nu(q,p)$ the vertex function and $D^{\mu\nu}(q-p)$ the photon propagator:

$$D_{\mu\nu}(k) = \frac{1}{k^2} \frac{1}{1+\Pi(k^2)} (\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) + \alpha \frac{k_\mu k_\nu}{k^4} , \tag{3}$$

where the vacuum polarization function $\Pi(k^2)$ is defined as

$$\Pi(k^2) = -\frac{e^2 N}{3k^2} \int \frac{d^4p}{(2\pi)^4} Tr[\gamma_\mu S_0(p) \gamma^\mu S_0(p-k)] . \tag{4}$$

In what follows we consider the Landau gauge $\alpha = 0$, and take the bare vertex approximation, $\Gamma_\mu(q,p) \equiv \gamma^\mu$ (for the arbitrary gauge, see ref.[10]). The vacuum polarization effect is included through the 1-loop vacuum polarization function:

$$\Pi(k^2) = \frac{Ne^2}{12\pi^2} [C + \rho + \ln \frac{\Lambda^2}{\mathrm{k}^2}], \tag{5}$$

where $\rho := \ln \Lambda_{\mathrm{p}}^2/\Lambda^2 := \ln \eta$ and C is a regularization-dependent constant. The SD equation for the fermion propagator is solved in the form of

$$S(p) = [\not p A(p^2) + B(p^2)]^{-1} . \tag{6}$$

Then the SD equation reduces to a pair of integral equations for $A(p^2)$ and $B(p^2)$. Here we introduce the infrared (IR) cutoff ϵ and the ultraviolet (UV) cutoff Λ.

§3. ANALYTICAL RESULTS[11]

3.1 The bifurcation equation In order to consider the scaling law in the nbd of the critical point, it is sufficient to consider the bifurcation solution[12] which obeys the linear equation:

$$B(p) = \frac{3e^2}{16\pi^2} \int_{\epsilon^2}^{\Lambda^2} dq^2 B(q^2) [\frac{d(p^2)}{p^2} \theta(p^2 - q^2) + \frac{d(q^2)}{q^2} \theta(q^2 - p^2)]. \tag{1}$$

where $d(k^2) := 1/[1+\Pi(k^2)]$ and we have adopted the Landau-Abrikosov-Khalatnikov[8] (LAK) approximation:

$$\Pi((p-q)^2) = \Pi(max(p^2, q^2)), \tag{2}$$

which yields $A(p^2) \equiv 1$.

3.2 Asymptotic solution Introducing the new variable $z := z_0 + \ln\Lambda^2/\mathrm{p}^2$ with $z_0 := 3\beta/N + C + \rho$, the above integral equation can be converted to the boundary value problem of the differential equation:

$$\frac{d^2 B(z)}{dz^2} + \left[\frac{2}{z} - \frac{z}{z-1}\right] \frac{dB(z)}{dz} + \sigma \left[\frac{1}{z} - \frac{1}{z^2}\right] B(z) = 0, \sigma := \frac{9}{4N}, \tag{3}$$

with the following boundary conditions,

$$0 = B(z) + \frac{z}{1-z} \frac{dB}{dz}\bigg|_{z=z_0}, \tag{4}$$

$$0 = \frac{dB(z)}{dz}\bigg|_{z=z_\mu := z_0 + \ln\Lambda^2/\mu^2} \tag{5}$$

This is a linear 2nd order differential equation with three singular points, $z = 0, 1, \infty$, of which $z = 0, 1$ are regular singular points and $z = \infty$ is the irregular singular point. Therefore the solution can not be expressed through the known special functions. However Gusynin[13] solved the simplified version of the above equation by restricting to the region $\beta/N \gg 1$, and obtained the critical point $\beta_c = 1.61$ for N = 1, in good agreement with the previous numerical result.[10] Furthermore he has shown that the scaling law is given by the MF type, $f \sim (\beta_c - \beta)^{1/2}$.

We have solved the original differential equation and obtained the two independent asymptotic solutions for large z:

$$B_1(z) = e^z z^{-1-\sigma}\left[1 + \frac{\sigma+\sigma^2}{z} + \ldots + \frac{R_n^{(1)}}{z^n}\right], \tag{6}$$

$$B_2(z) = z^{\sigma}\left[1 + \frac{\sigma-\sigma^2}{z} + \ldots + \frac{R_n^{(2)}}{z^n}\right]. \tag{7}$$

The general solution is given by the linear combination $B(z) = C_1B_1(z)+C_2B_2(z)$. The coefficients C_1, C_2 can be determined from the boundary conditions. The numbers $R_n^{(1)}, R_n^{(2)}$ are calculated up to n = 4, see [11].

<u>3.3 Critical line and scaling law</u> Defining the functions $g(z)$ and $h(z)$ by

$$(1-z)B_1(z) + zB_1'(z) := -e^z z^{-\sigma-n-1}g(z), \tag{8}$$

$$(1-z)B_2(z) + zB_2'(z) := -z^{-\sigma-n}h(z), \tag{9}$$

then they are polynomials in z with deg[g] = n, deg[h] = n+1. Then, using the boundary condition, it is shown that the scaling law for the dynamical fermion mass is given by

$$f \sim [h(z_0)/g(z_0)]^{1/2}, z_0 := \beta/3N. \tag{10}$$

From this we obtain the critical point z_0^c as a zero point of the function $h(z)$ (see Fig.1), and the scaling law is given by the MF type,

$$m_d \sim \Lambda \cdot (N_c(\beta) - N)^{1/2}, \tag{11}$$

since $h'(z_0^c) \neq 0$, which is ascertained by the direct numerical calculations.

<u>3.4 Chiral order parameter</u> It is shown that the chiral order parameter obeys the same scaling as that for the dynamical fermion mass and has the MF critical exponent, since it is shown that for $N \neq 0$

$$\langle\bar{\psi}\psi\rangle \sim \Lambda^2 m_d \sim \Lambda^3 f, \tag{12}$$

which should be compared with the result[15] in the quenched planar approximation; $\langle\bar{\psi}\psi\rangle \sim \Lambda m_d^2 \sim \Lambda^3 f^2$.

3.5 Anomalous dimension The anomalous dimension for the composite operator $\bar{\psi}\psi$ is calculated to result in

$$\gamma_m = 0, \tag{13}$$

in sharp constrast with the quenched planar case,[6] $\gamma_m = 1$.

3.6 The critical fermion number The solution of the SD equation with nonvanishing C, ρ is obtained from that with $C = 0 = \rho$ by shifting the coupling constant as follows:

$$\beta \to \beta - \frac{N}{3}(C + \rho) := \beta', \tag{14}$$

Then this fact tells us that the non-trivial solution exists only in the region

$$N < \frac{3}{C + \rho}\beta. \tag{15}$$

Then the critical flavor number N_c above which there is no chiral symmetry breaking is obtained as the intersection point of $N = (3/C{+}\rho)\beta$ with the critical line obtained when $C = 0 = \rho$, see Fig. 1. The existence of the critical fermion number was first demonstrated by the Monte Carlo simulation.[7]

§4. Numerical study of the SD equation[17]

We have carried out the numerical calcualtions of the original integral equations given in section 2, which invole double integrals. Then our results support the MF scaling, irrespective of the fermion flavor N;

$$m_d \sim \Lambda(\beta_c - \beta)^{1/2}, \langle\bar{\psi}\psi\rangle \sim \Lambda^3(\beta_c - \beta)^{1/2}, \tag{1}$$

except the quenched planar case which corresponds to N = 0 in our scheme. The scaling is unchanged with (Fig.2) or without (Fig.3) the LAK approximation.

§5. Additional 4-fermion interaction[18]

Introducing the additional chiral-invariant 4-fermion interaction

$$\frac{1}{2}G_0[(\bar{\psi}^a\psi^a)^2 - (\bar{\psi}^a\gamma_5\psi^a)^2], (a = 1, ..., N), \tag{1}$$

we consider the SD equation

$$S(p)^{-1} = S_0(p)^{-1} + G_0\langle\bar{\psi}^a\psi^a\rangle + \Sigma(p). \tag{2}$$

This SD equation is solved in the same way as in the pure QED case with the vacuum polarization function given in section 2. The critical line and the scaling law are analytically obtained in the bare parameter space (e^2, g) where $g := NG_0\Lambda^2/4\pi^2$. For the critical line, see Fig.4 and Preprint[18] for the analytic expression. The scaling law for the dynamical fermion mass and the chiral order parameter is given by the MF type, irrespective of the direction approaching the critical line. The "bare" β functions are obtained from the coupling constant flow equation which is obtained by converting the scaling relation; in the CSB phase,

$$\beta(g) = -2(g - g_{cr}), \beta(e^2) = -2(e^2 - e^2_{cr}),$$

while $\beta(g) = 0 = \beta(e^2)$ in the symmetric phase.

§6. Conclusion and Discussion

We have obtained analytically and numerically the non-trivial solutions of the SD equation for the fermion propagator, takeing into account the vacuum polarization effect in massless QED with N fermion flavors. The vacuum polarization effect is included through the vacuum polarization function at the 1-loop level. Within this framework we have shown that in the presence of the vacuum polarization there exists a critical point e_c which separates the weak coupling phase from the strong coupling phase where the chiral symmetry is spontaneously broken.

Based on the asymptotic solution, the critical coupling $e_c(N)$ for each fermion flavor N is obtained, which is in good agreement with the result of the direct numerical calculations of ours for N = 1,2. The scaling law is given by the MF type, irrespective of $N \neq 0$. The critical exponents for the dynamical fermion mass and the chiral order parameter have the same classical MF value, i.e., $\nu = 1/2$.

Additional chiral-invariant 4-fermion interaction is also introduced. We obtained the critical line in the bare parameter space (e^2, g), which connects the pure Nambu-Jona-Lasinio point with the pure QED critical point in the unquenched case. However introduction of the additional 4-fermion interaction does not change the scaling law. So far there is no compelling evidences to support the non-triviality of QED, although it remains to calculate the anomalous dimension on the whole critical line.

Furthere problems to be resolved are:

1) The first Monte Carlo (MC) result of Kogut et al.[16] supports the Miransky scaling in the quenched limit, while the recent result by another group[19] shows the MF scaling even in the quenched limit. Remembering that the quenched planar QED is shown to obey the Miransky scaling, it may be possible that the non-planar effect or the vertex correction may change the scaling, in other words, the essential-singularity type scaling is unstable for the improvement of the approximation.

2) If the 1-loop result presented in this talk is qualiltatively good approximation to the full QED, the original QED coupling vanishes in the continuum limit, since it sufferes from the Landau ghost. However there may exist the induced Yukawa coupling resulting from the spontaneous breakdown of the chiral symmetry which forms the fermion-antifermion (scalar) bound state. Existence of such a state may be judged from the calculation of the pion decay constant f_π. Finite f_π may rescue the strong coupling QED from the naive triviality. The result will be given elsewhere.

3) Confinement or deconfinement of the fermion in the CSB phase is very interesting to go beyond the quenched planar analysis,[21] which is examined by searching for the pole in the fermion propagator.

4) To answer the Landau ghost problem and to obtain the phase structure of QED more completely, we must obtain the vacuum polarization function $\Pi(k^2)$ as a self-consistent solution in the simultaneous SD equation. This is in progress.

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Fig.1

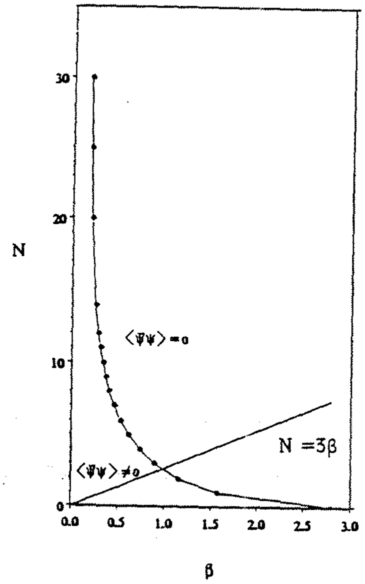


Fig.2

(a) N = 1

(b) N = 2

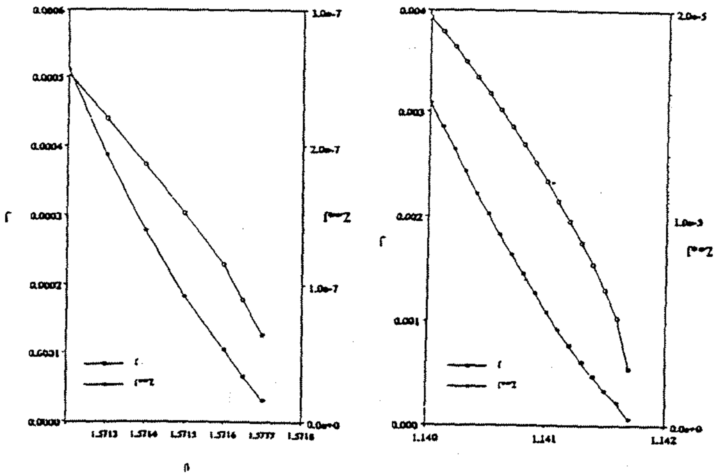


Fig.3

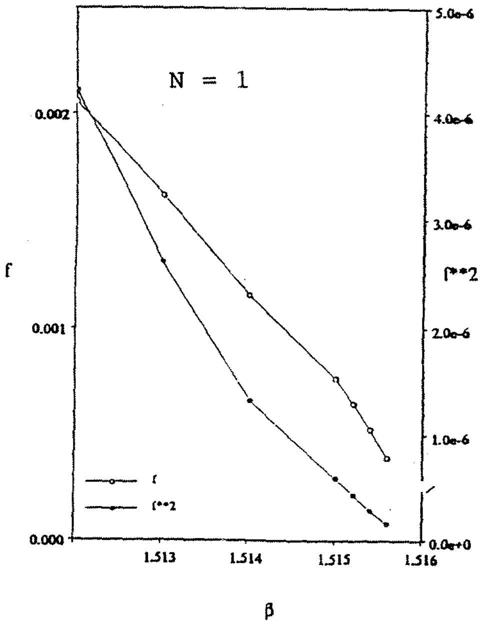
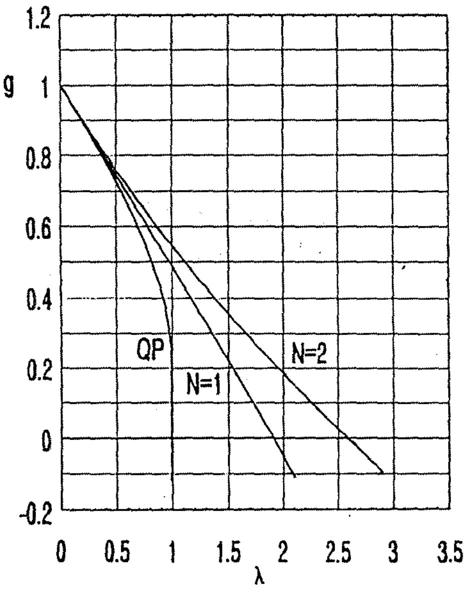


Fig.4



ON THE SOLUTION OF THE SCHWINGER-DYSON EQUATION OF QED

Minoru Hirayama

Toyama University

The Fukuda-Kugo version of the Schwinger-Dyson equation of QED is discussed. Through constructing the Liapunov function, it is analytically proved that Fukuda and Kugo's non-linear differential equation for the fermion propagator possesses chiral-symmetry breaking solutions. The structure of the general solution of their equation is studied by the method developed in the theory of dynamical systems.

1. Introduction

In the course of the analysis of the fermion self-energy of QED, the Schwinger-Dyson equation

$$B(p^2) = \frac{3e^2}{(2\pi)^4}\int d^4q \frac{1}{(p-q)^2} \frac{B(q^2)}{q^2+B^2(q^2)} \qquad (1.1)$$

was discussed by many authors, where $B(p^2)$ is defined by

$$[S_F(p)]^{-1} = \not{p} - B(p^2) \ . \tag{1.2}$$

We are adopting the Landau gauge and the ladder approximation. Fukuda and Kugo[1)] translated (1.3) into a much tractable differential equation

$$g\frac{dg}{du} + 4g = -3u\left(1 + \frac{\lambda}{1+u^2}\right) \ , \tag{1.3}$$

where λ, u and g are given by $\lambda = e^2/(4\pi^2)$, $u(t) = B(p^2)/p$, $g(u) = du(t)/dt$ and $t = \log p$. The boundary conditions on g and u at $t = \pm\infty$ are given by

$$\lim_{t\to\infty} e^{t}(g+3u) = 0, \quad \lim_{t\to-\infty} e^{3t}(g+u) = 0 \ . \tag{1.4}$$

In this talk, with the help of Liapunov's direct method, I present an analytic proof that (1.3) and (1.4) have non-trivial solutions.[2)] I also discuss how the analytic form of solution can be explored.[2)]

2. Liapunov Function of Fukuda-Kugo Equation

Eq.(1.3) is rewritten as

$$\frac{dg}{d\rho} = -\{(4g + \kappa u) + (4g + 3u)u^2\} \ , \tag{2.1}$$

$$\frac{du}{d\rho} = g(1 + u^2) \ , \tag{2.2}$$

where κ and ρ are defined by

$$\kappa = 3(1 + \lambda) > 3 \,, \qquad \frac{d\rho}{dt} = \frac{1}{1+u^2} > 0 \,. \tag{2.3}$$

Apart from mathematical subtleties, it is intuitively clear that any trajectory starting from any point on the (u,g) plane tends to (0,0) as ρ (and so t) goes to infinity if there exists a function L(u,g) satisfying

(i) $L(0,0) = 0$,

(ii) $L(u,g) > 0$ for $(u,g) \neq (0,0)$,

(iii) $\dfrac{dL(u,g)}{d\rho} < 0$ for $(u,g) \neq (0,0)$.

The function L(u,g) is called the Liapunov function at (0,0).[3)]

Making use of (2.1) and (2.2), it can be seen that the function

$$L_1(u,g) = g^2+3u^2+(\kappa-3)\log(u^2+1)+\alpha(ug+2u^2), \; 0 < \alpha < 8, \tag{2.4}$$

satisfies (i), (ii) and (iii) for any value of $\lambda > 0$. It is also seen that the simpler function

$$L_2(u,g) = g^2 + 4gu + (8 + \sqrt{3\kappa})u^2 \quad , \tag{2.5}$$

satisfies (i), (ii) and (iii) for $3 < \kappa < 19 + 8\sqrt{3}$. Thus we understand that the nonlinear differential equation (1.3) with the boundary condition (1.4) possesses non-trivial solutions, which implies that the chiral-symmetry might be broken.

3. General Structure of Solutions of Fukuda-Kugo Equation

Equations (2.1) and (2.2) are expressed in the form

$$\frac{dx}{d\rho} = Ax + f(x) , \tag{3.1}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} g \\ u \end{pmatrix} , \quad A = \begin{pmatrix} -4 & -\kappa \\ 1 & 0 \end{pmatrix} , \tag{3.2}$$

where $f(x)$ consists of terms of third order in x. If we define z by

$$z = Ux , \quad U = \begin{pmatrix} 1 & -\lambda_2 \\ 1 & -\lambda_1 \end{pmatrix} , \tag{3.3}$$

$$\lambda_1 = -2 + \sqrt{1-3\lambda} , \quad \lambda_2 = -2 - \sqrt{1-3\lambda} ,$$

we have

$$\frac{dz}{d\rho} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} z + h(z) , \tag{3.4}$$

where $h(z)$ is given by $h(z) = Uf(U^{-1}z)$ and consists of terms of third order in z. It is known that there exists a unique transformation from z to y[4)]

$$z = y + k(y) \tag{3.5}$$

such that y satisfies the simplest equation

$$\frac{dy}{d\rho} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} y . \tag{3.6}$$

If we write

$$y = z - k(y) = Ux - k(y) = Ux - \psi(x) \ , \tag{3.7}$$

$\psi(x)$ is an infinite power series of x :

$$\psi(x) = \begin{bmatrix} \psi_1(x) \\ \\ \psi_2(x) \end{bmatrix} \ , \tag{3.8}$$

$$\psi_i(x) = \sum_{\ell+m\geq 3} f^{(i)}_{\ell m} \, x_1^{\ell} \, x_2^{m} \ , \quad i = 1, 2 \ . \tag{3.9}$$

Coefficients $\{f^{(i)}_{\ell m}\}$ can be obtained recursively through f(x). Since the general solution of (3.6) is given by

$$y_i = c_i \, e^{\lambda_i \rho} \ , \quad i = 1, 2 \ , \tag{3.10}$$

we get

$$y_1^{\lambda_2} \, y_2^{-\lambda_1} = c \ , \tag{3.11}$$

where c is an arbitrary constant. Now we realize that the general solution of the Fukuda-Kugo equation (1.3) is obtained through solving the equation

$$[g - \lambda_2 u - \psi_1(g,u)]^{\lambda_2} \, [g - \lambda_1 u - \psi_2(g,u)]^{-\lambda_1} = c \tag{3.12}$$

with respect to g. The behaviour of the solution of (3.12) should be classified by the value of the parameter $\sigma = \lambda_2/\lambda_1$.

For $0 < \lambda < \frac{5}{27}$ $(2 < \sigma < 3)$, we recover the analytic form of solution which was originally obtained by Fukuda and Kugo.[1)] For $\frac{5}{27} \leq \lambda < \frac{1}{3}$ $(1 < \sigma \leq 2)$, terms other than those included in the Fukuda-Kugo solution become dominant. For the case of strong coupling $\lambda > \frac{1}{3}$ (σ ; complex) , the solution is given by

$$R\, e^{\frac{2\theta}{\nu}} = \text{const.}\ , \qquad \nu = \sqrt{3\lambda - 1}\ , \tag{3.13}$$

where R and θ are defined by

$$g - \lambda_2 u - \psi_1(g,u) = R\, e^{i\theta}\ . \tag{3.14}$$

The trajectory on the (u,g) plane is a deformed logarithmic spiral.

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Renormalization Group Flow in Lattice QED and Four Fermi Coupling

Masahiro IMACHI

and

Hiroshi YONEYAMA[†)]

Department of Physics, Kyushu University, Fukuoka 812, Japan
and
Department of Physics, Saga University, Saga 840, Japan[†)]

ABSTRACT

Renormalization group flow of the $U(1)$ lattice gauge theory with staggered fermions is studied by the Migdal-Kadanoff renormalization group method. The phase structure is extensively investigated. It is shown that an induced four fermi coupling term becomes relevant in the strong gauge coupling region while it becomes irrelevant in the weak gauge coupling one. The chiral order parameter and the anomalous dimension of the fermion mass operator are calculated.

The Migdal-Kadanoff renormalization group (MKRG) method[1,2] is an approximate but suitable tool to get into an essential feature of the lattice gauge models.[3-8] Such a method may provide us with an important information of the dynamics with the strong four fermi coupling, and is complementary to Monte Carlo calculations which are currently providing interesting results.[9-13] One of the authors (M.I.) has recently studied the theory by incorporating the fermion self-energy to the recursion equation, and found that the four fermi interaction is in fact induced from the original QED in the strong gauge coupling region.[14] In the present paper we make an extensive study of its RG flow and the phase structure.

The main results are as follows. The bare parameter space is divided into two phases, one being the phase where the four fermi coupling is relevant and another where it is irrelevant. Within the former phase, there is a distinction with respect to RG flow between the strong and the weak gauge coupling regions. The chiral order parameter shows a transition separating the two phases. The anomalous dimension γ_m of the fermion mass operator is also calculated by the UT(Unique Trajectory[15]) method[5] in the chiral symmetry unbroken phase. It is found that γ_m is large at the critical line and monotonically decreases as one goes off from the critical line, i.e., γ_m decreases monotonically as the gauge coupling and/or the four fermi coupling C_0 become weak.

Migdal-Kadanoff renormalization group transformation

The recursion equations for the MKRG transformation for $U(1)$ LGT with staggered fermions[16,17] are presented.[14] In general, RG transformations induce couplings which are not in the original bare theory. It is then convenient to write here the most general form of action in the MKRG framework. The lattice action of $U(1)$ gauge group with staggered fermions ψ and $\bar{\psi}$ is given by

$$\begin{aligned} S =& S_g + S_f, \\ S_g =& -2\sum_{plaq}\sum_{q=1}^{\infty}(1-\mathrm{Re}\chi_q(\theta)\,)\beta_q, \\ S_f =& A_0\sum_{n,\mu}\eta_\mu(n)(\varepsilon_+\bar{\psi}(n)U_\mu(n)\psi(n+\mu)+\varepsilon_-\bar{\psi}(n+\mu)U_\mu^\dagger(n)\psi(n)) \\ &- B_0\sum_n\bar{\psi}(n)\psi(n) \\ &- C_0\sum_{n,\mu}\bar{\psi}(n)U_\mu(n)\psi(n+\mu)\ \bar{\psi}(n+\mu)U_\mu^\dagger(n)\psi(n), \end{aligned} \tag{1}$$

where χ_q in S_g denotes the q-irreducible character of a plaquette variable ; $\chi_q(\theta) = \mathrm{Tr}U_q = e^{iq\theta}$ (q =integer, $0 \leq \theta \leq 2\pi$), and β_q is corresponding bare inverse gauge coupling. The fermionic action S_f contains three bare parameters A_0, B_0 and C_0 which represent hopping parameter, mass and four fermi coupling in turn. Positive values of C_0 correspond to an attractive force. ε_+ and ε_- are sign factors ($\varepsilon_+ = -1$ and $\varepsilon_- = +1$), and $\eta_\mu(n) = (-1)^{n_1+n_2+\cdots+n_{\mu-1}}$, where n_i is the i-th coordinate of the site n. The following convention for integrating Grassmann variables ψ and $\bar{\psi}$ is employed:

$$\int d\bar{\psi}d\psi\exp(-p\bar{\psi}\psi+\bar{\psi}\xi+\bar{\zeta}\psi) = p\exp(\frac{1}{p}\bar{\zeta}\xi). \tag{2}$$

A RG transformation consists of two procedures, the decimation and the bond-moving, both for the gauge and fermionic degrees of freedom. In each decimation, the gauge degrees of freedom receive fermion loop corrections, while the fermionic ones contain self-energy corrections.

The recursion equation for the gauge field[3] connecting two scales L and λL is given by

$$F(\lambda L,\theta) = \left[\sum_q \tilde{F}_q(L)^{\lambda^2-1}\tilde{Q}_q(L)\chi_q(\theta)\right]^{\lambda^{D-2}} \tag{3}$$

where $\tilde{F}_q$ is coefficient in the character expansion of the plaquette function $F(L,\theta)$ at scale L,

$$F(L,\theta) = \sum_q \tilde{F}_q(L)\chi_q(\theta), \tag{4}$$

where $F(L,\theta)$ is written in terms of the gauge couplings as

$$F(L,\theta) = F(L,0)\exp\{-2\sum_{q=1}^{\infty}(1-\mathrm{Re}\chi_q(\theta))\beta_q(L)\}. \tag{5}$$

While $\tilde{Q}_q$ is the coefficient in the expansion of $Q(L,\theta)$, which represents the contribution from the innermost plaquette in Fig.1 receiving the fermion loop correction with N_f flavors (vacuum polarization). A RG transformation is thus completed by the bond-moving as is represented by the exponent λ^{D-2} in (3), which is the contribution from the $D-2$ directions perpendicular to the plane on which the plaquette in question is sitting.

The l.h.s. of (3) is also represented by the renormalized couplings $\{\beta_q(\lambda L)\}$ at scale λL in the same manner as (5)

$$F(\lambda L,\theta) = F(\lambda L,0)\exp\{-2\sum_{q=1}^{\infty}(1-\mathrm{Re}\chi_q(\theta))\beta_q(\lambda L)\}. \tag{6}$$

Therefore, by solving (3) and (6) one obtains the recursion equation for the gauge coupling $\{\beta_q(L)\} \rightarrow \{\beta_q(\lambda L)\}$.

Through fermion decimation(Fig.2(a))[14] we obtain fermion parameters $A(\lambda L)$, $B(\lambda L)$ and $C(\lambda L)$ at scale λL from those at L. If we take into account fermion self-energy correction(Fig.2(b)), we have

$$\begin{aligned} A_G(\lambda L) =& A(\lambda L)\left(\frac{\tilde{F}_1}{\tilde{F}_0}\right)^{\lambda}, \\ B_G(\lambda L) =& B(\lambda L), \\ C_G(\lambda L) =& C(\lambda L) - \{1-\left(\frac{\tilde{F}_1}{\tilde{F}_0}\right)^{2\lambda}\}A(\lambda L)\varepsilon_+\varepsilon_-. \end{aligned} \tag{7}$$

The factor $\tilde{F}_1/\tilde{F}_0$ is $\sim\beta$ at strong coupling regions ($\beta \ll 1$) and $\sim(1-1/\beta)$ at weak coupling regions($\beta \gg 1$). Then C_G receives large (small) effect from fermion self energy correction in strong (weak) gauge coupling regions. It may be convenient to define normalized parameters M and G rather than using A, B and C. They are defined by

$$M = B_G/A_G, G = C_G/A_G^2.$$

Renormalization group flow and phase structure[18]

We are now ready to calculate RG flow. Throughout this paper the scale factor λ and the number of staggered fermion N_f are taken to be three and unity, respectively. All the calculations in this section are made for a sufficiently small fixed value of B_0 (=0.05). Its extrapolation to B_0=0 will be discussed in the following section.

Flow of the renormalization group transformations runs in the infinite dimensional parameter space, $(\{\beta_q \;\; ; q = 1 \sim \infty\}, M, G)$. It may then be convenient to project it to various subspaces. In what follows we, in turn, see the one projected to the subspaces of pure gauge (β_1, β_2), gauge and fermion (β_1, G) and pure fermion (G, M).

Flow diagram of gauge coupling β_q is shown in Fig.3. We observe critical point β_{1c} at $2.3 < \beta_{1c} < 2.4$. For $\beta_1 > \beta_{1c}$, trajectories flow to weaker coupling regions, which represents "screening" due to vacuum polarization. For $\beta_1 < \beta_{1c}$, trajectories flow to IR fixed point $\beta_q = 0$ (q = all).

As to the projection onto the subspace, (β_1, G), the RG flow moves as shown in Fig.4. For each trajectory in the figure, starting point corresponds to the bare theory with certain (β_1, C_0). One clearly sees that the two dimensional subspace (β_1, C_0) is divided into two phases in view of the manner of the movement of the G. For small β_1 and all allowed C_0 values, trajectories move up to large G region very quickly. This feature is seen up to the critical point β_{1c}. In the weak gauge coupling region beyond β_{1c}, trajectories move up first but eventually go down to small G for small C_0 values, while for large C_0 values the trajectories move up quickly to the large G region. Namely, in between strong and weak four fermi coupling regions, a critical line runs (see Fig.7).

Keeping the above feature in mind, let us now see the behavior of the trajectories in the fermionic parameter subspace (M, G); For small $\beta_1 (< \beta_{1c})$;

(1) In the very strong coupling region $\beta_1 (\lesssim 1.0)$, a range of bare theories in different β_1 and C_0 values moves on to a scaling trajectory as seen in Fig.5. The functional form of the trajectory reads $G \propto M^2$ for large M and G values.

(2) As β_1 increases beyond 1.0, the flow starts to deviate from such a trajectory, and the slope of the trajectory becomes smaller in the log$G$-log$M$ plot.

For large $\beta_1 (> \beta_{1c})$, the behavior is quite different from the one for small β_1;

(1) For large bare C_0 values, the trajectories move up as shown in Fig.6.

(2) Whereas for small C_0 values, flows move down and converge to a single trajectory, which moves eventually toward $G = 0$.

(3) In between there exists a critical point C_c, at which trajectory moves flat.

(4) The locations of both the critical C_0 value and the convergent trajectory depend on chosen β_1 value. As β_1 increases, the value of C_c monotonically increases as seen

in Fig.7. Such a critical line (actually critical surface in the full parameter space) separates the parameter space into chiral symmetry unbroken(weak four fermi side) and broken(strong four fermi side) phases. Similar critical line is found also in the analysis of Schwinger-Dyson equation of quenched QED.[19]

(5) On the other hand, the value of G of the convergent trajectory at sufficiently large M decreases, as the bare β_1 increases.

In Fig.7 the two phases are labeled by I (chiral symmetry broken) and II (unbroken). Within the chiral symmetry broken phase I , we found a distinct behavior between strong and weak bare gauge couplings. Trajectories for large bare β_1 values move very slowly toward larger β_1 value, while those for small β_1 converge rapidly to the fixed point at $\beta_q = 0$. This appears to suggest that there is a phase boundary between the two regions. We then distinguish the weaker gauge coupling side from the stronger one by naming it the domain III, as indicated in Fig.7. II and III are then connected to the chiral symmetry unbroken and broken phases, respectively , in the Nambu-Jona-Lasinio model.[20]

Chiral order parameter $< -\bar{\psi}\psi >$ is calculated from the partition function Z by

$$\langle\bar{\psi}\psi\rangle_{B_0} = \frac{-1}{N_{sites}}\left[\frac{\partial Z}{\partial B_0}/Z\right]_{B_0}, \tag{8}$$

and by taking linear extrapolation to $B_0 = 0$. N_{site} denotes the total site number, λ^{tD}, with t and D being the number of RG iterations and the space time dimension(=4 in our case), respectively. The result is shown in Fig.8. We observe that $< -\bar{\psi}\psi >$ at strong gauge couplings is much larger than that at weak ones. In the weak gauge coupling region, however, $< -\bar{\psi}\psi >$ is not exactly zero. Subtracting the value ($\approx 5 \times 10^{-5}$) at large β_1, therefore, it can be fitted by an essential singularity form $\alpha\exp(-\gamma/\sqrt{\beta_c - \beta_1})$. The result is insensitive to the assumed value of β_c. For example, a case for β_c=2.3 is shown in the figure.

Anomalous dimension

We will discuss the anomalous dimension of $\bar{\psi}\psi$. Fig.9 shows RG flow for various small values of bare mass B_0 in the symmetric phase (or domain II). One sees that all bare theories (for $C_0 = 0$) with these different values of B_0 converge to single trajectory. Therefore the unique trajectory method applies in order to get the anomalous dimension of $\bar{\psi}\psi$. That is, one sets up a gate on the trajectory , and then count the number of steps t_G of RG transformations necessary to reach the gate from various bare points. The scale at the gate ξ_G and the lattice constant a of a bare point is related by $\log a = -t_G \log\lambda + \log\xi_G$. We found

$$\log B_0 \approx -c(\beta_1)t_G + d(\beta_1), \tag{9}$$

and its slope $c(\beta_1)$ increases as β_1 becomes larger. The $c(\beta_1)$ is calculated to be 0.37, 0.28, 0.19 and 0.14 for β_1=2.5, 3.0, 5.0 and 10.0 in order. This slope gives essentially the anomalous dimension of $\bar{\psi}\psi$ as follows.

The anomalous dimension γ_m is defined by

$$\gamma_m = -\frac{\partial \log m_0(\Lambda)}{\partial \log \Lambda}, \tag{10}$$

where m_0 is a dimensionful bare mass, and Λ denotes an ultraviolet cut-off. In the lattice notation, (10) reads $\gamma_m = \frac{\partial \log B_0(a)}{\partial \log a} - 1$, since $m_0(\Lambda) = B_0(a)/a$ and Λ=$1/a$. γ_m is also represented as

$$\gamma_m = \frac{-1}{\log \lambda} \frac{\partial \log B_0}{\partial t_G} - 1 \tag{11}.$$

For β_1 values in question, (9) leads to

$$\gamma_m \approx c(\beta_1)/\log \lambda - 1. \tag{12}$$

For $\lambda = 3$, γ_m reads $-0.22, -0.41, -0.61$ and -0.71 for β_1=2.5, 3.0, 5.0 and 10.0 in turn. This result seems queer, since it is expected that γ_m is positive and becomes vanishing as β_1 goes to infinity, where the theory becomes free. This is due to the quantitative roughness of the approximation. In the free theory, for example, the mass M ought to change to λM by a scale transformation by λ. However, in the MK framework, or rather generally in approximated RG transformations, M does not transform properly[4] but by λ_{eff} ($\neq \lambda$). Therefore we normalize γ_m in (12) so that $\gamma_m = 0$ is correctly reproduced in the weak gauge coupling limit. Namely, we take λ to be λ_{eff} which is fixed at a large β_1. We choose $\beta_1 = 10.0$ (some other choice, say, $\beta_1 = 15.0$ does not make much difference). The estimated value of λ_{eff} is 1.39. This leads to $\gamma_m = 1.64$, 1.0, 0.36 and 0.0 for $\beta_1 = 2.5$, 3.0, 5.0 and 10.0 (see Fig.10).

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FIGURE CAPTIONS

Fig. 1. Gauge plaquette decimation. The vacuum polarization is contained. Crosses denote the fermion decimations.

Fig. 2. (a)Link function. (b)Fermion self-energy correction.

Fig. 3. Flow diagrams of gauge coupling constants projected onto (β_1, β_2) plane.

Fig. 4. RG flow projected onto β_1-G plane.

Fig. 5. RG flow projected onto M-G plane. Strong gauge coupling case with $\beta_1 = 1.0$ and C_0=0.0(○), 1.0 (□) and 2.0 (△). Intermediately strong gauge coupling case with $\beta_1 =$ 2.0 and C_0=0.0(◇), 1.0(+) and 2.0(×).

Fig. 6. RG flow projected onto M-G plane. Weak gauge coupling case with $\beta_1 = 5.0$ and $C_0 = 0.0$(○), 0.4(□), 0.6(△), 0.9(◇) and 1.0(+).

Fig. 7. Phase diagram in β_1-C_0 plane.

Fig. 8. $< -\bar{\psi}\psi >$ vs. β_1. $C_0 = 0.0$. With fermion self energy correction(○) and without it(□). The former is fitted by $\alpha\exp(-\gamma/\sqrt{\beta_{1c} - \beta_1})$ with $\beta_{1c} = 2.3$, $\alpha = 18.14$, and $\gamma = 12.50$(bold line).

Fig. 9. RG flow projected onto log$M$-log$G$ plane for B_0=0.01(○), 0.025(□), 0.05(△) and 0.1(+). β_1 and C_0 is chosen to be 5.0 and 0.0, respectively.

Fig. 10. γ_m vs. β_1 for $C_0 = 0.0$.

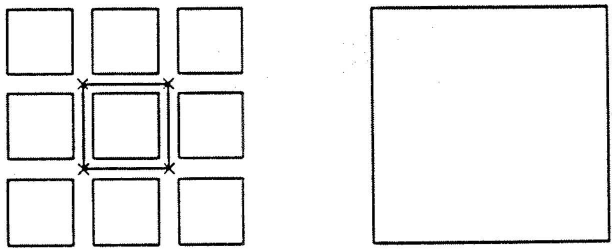


Fig. 1.

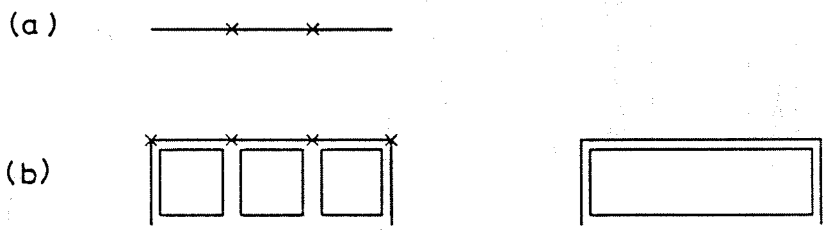
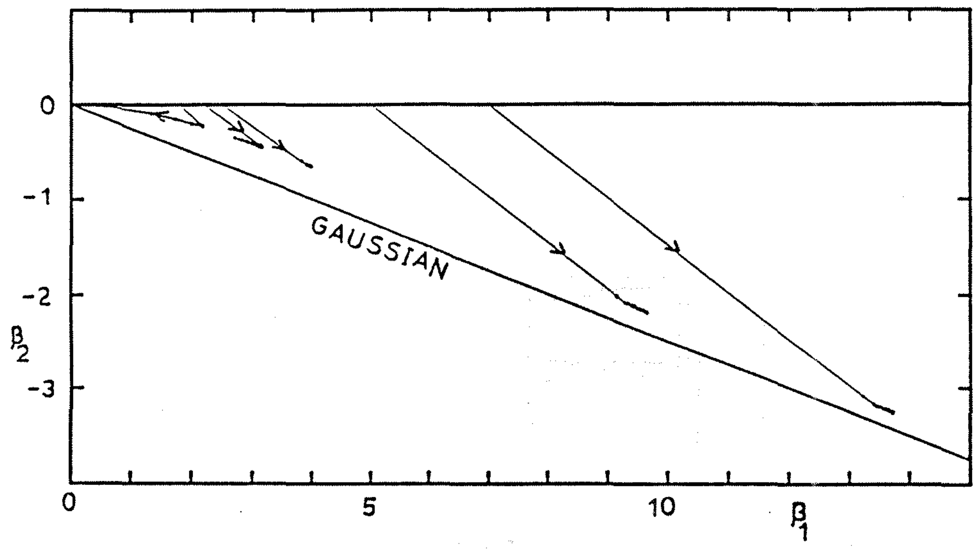


Fig. 2.

Fig. 3



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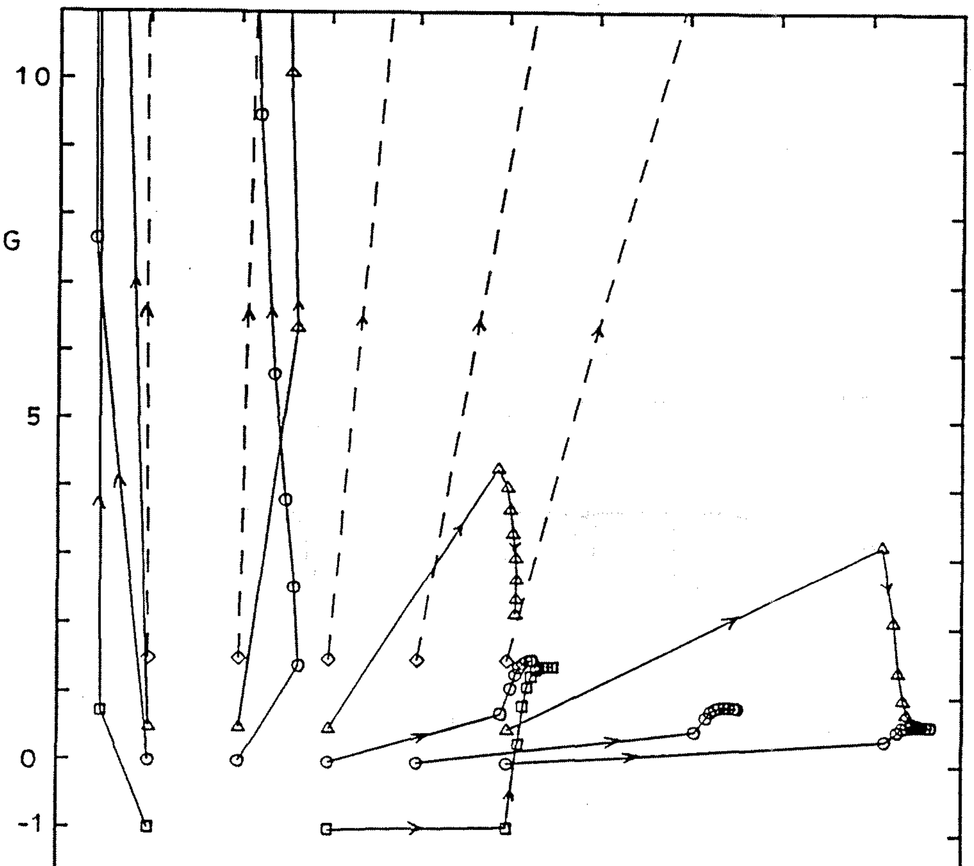

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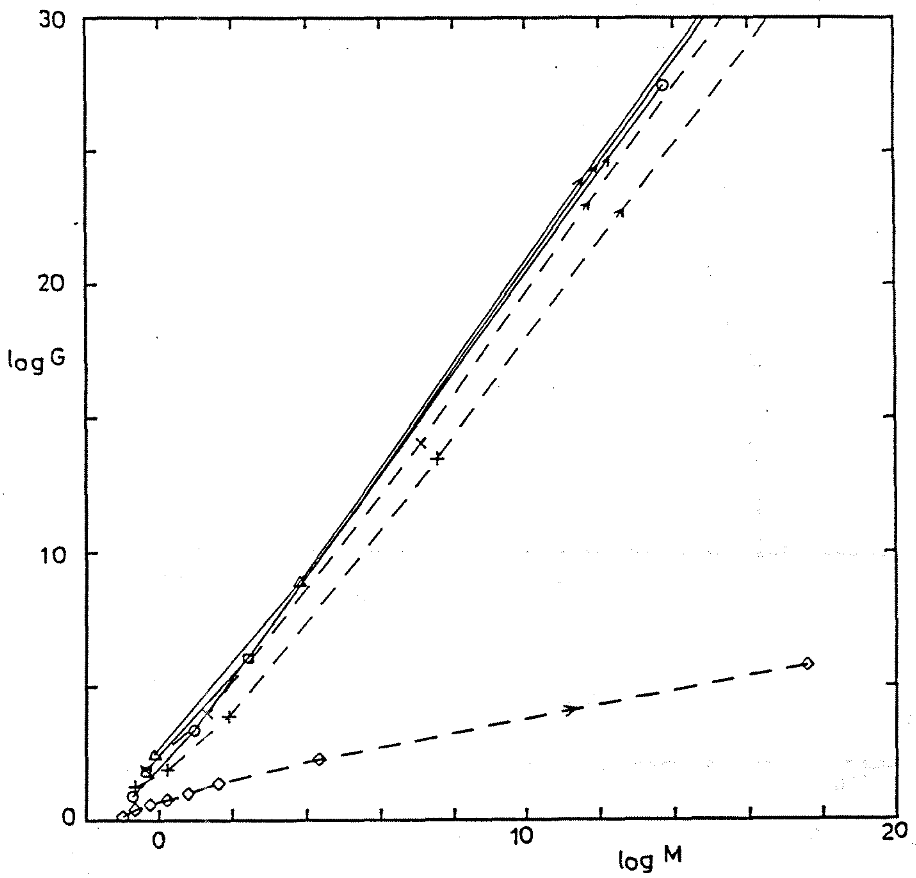

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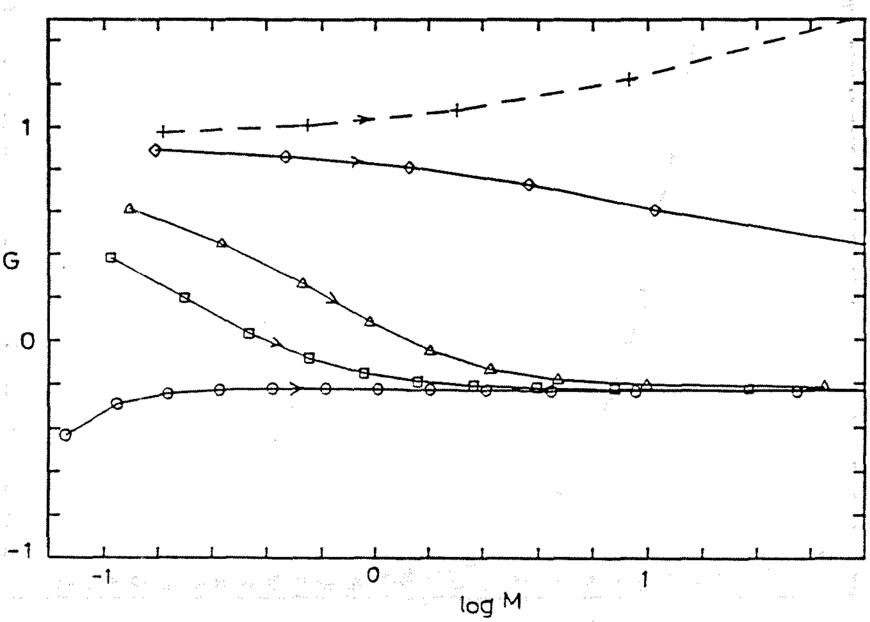


Fig. 7.

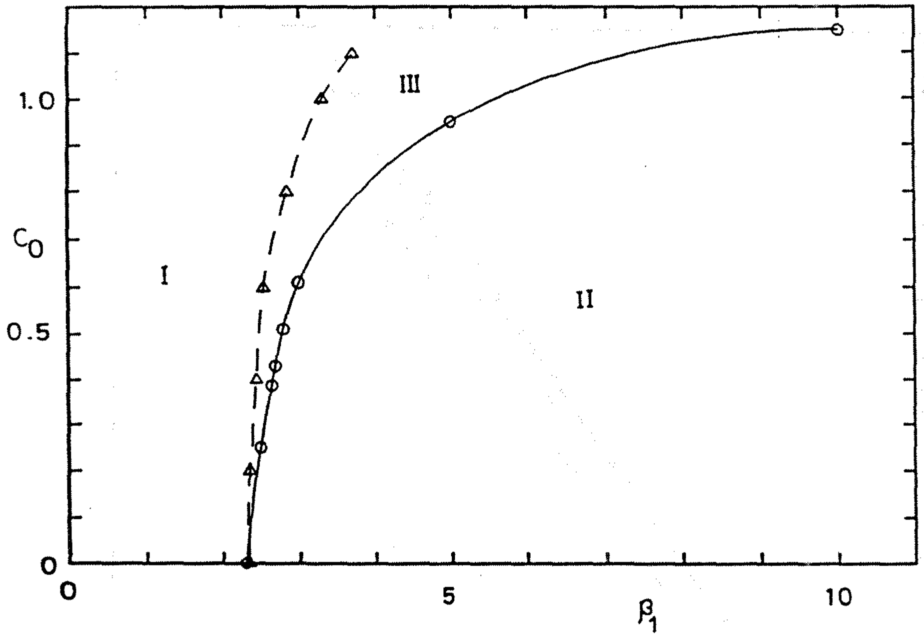

1.0
0.5
0
C_0
I
II
III
0
5
10
β_1

Fig. 8.

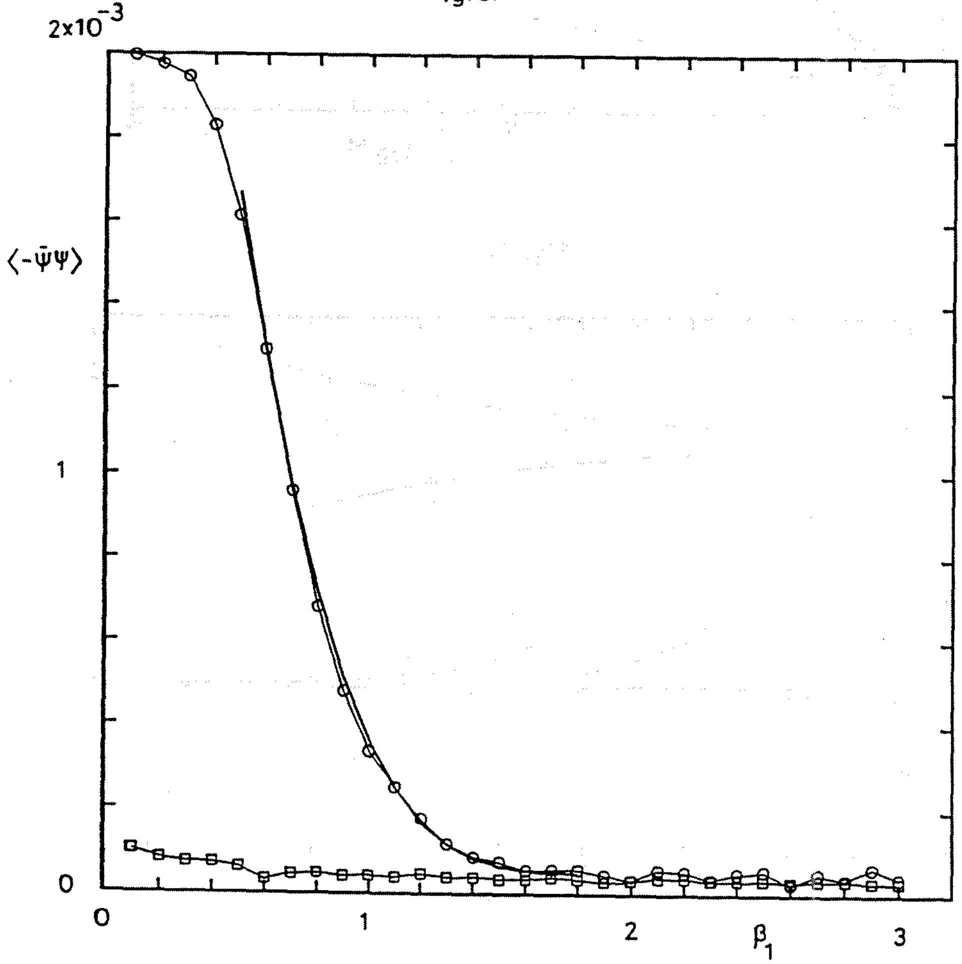

2×10^{-3}
$\langle -\bar{\psi}\psi \rangle$
1
0
0
1
2
3
β_1

Fig. 9 .

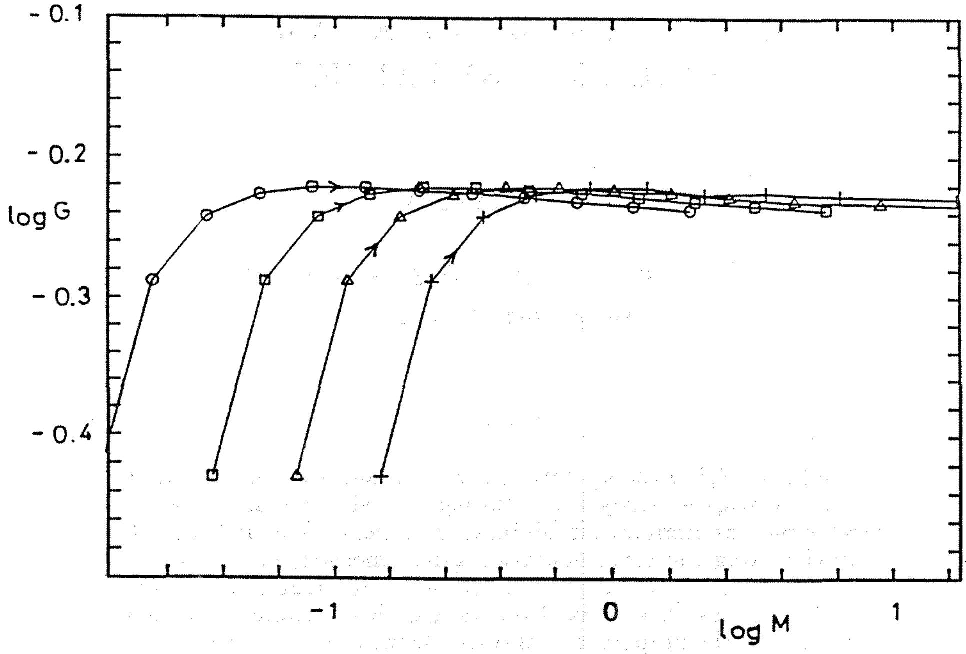
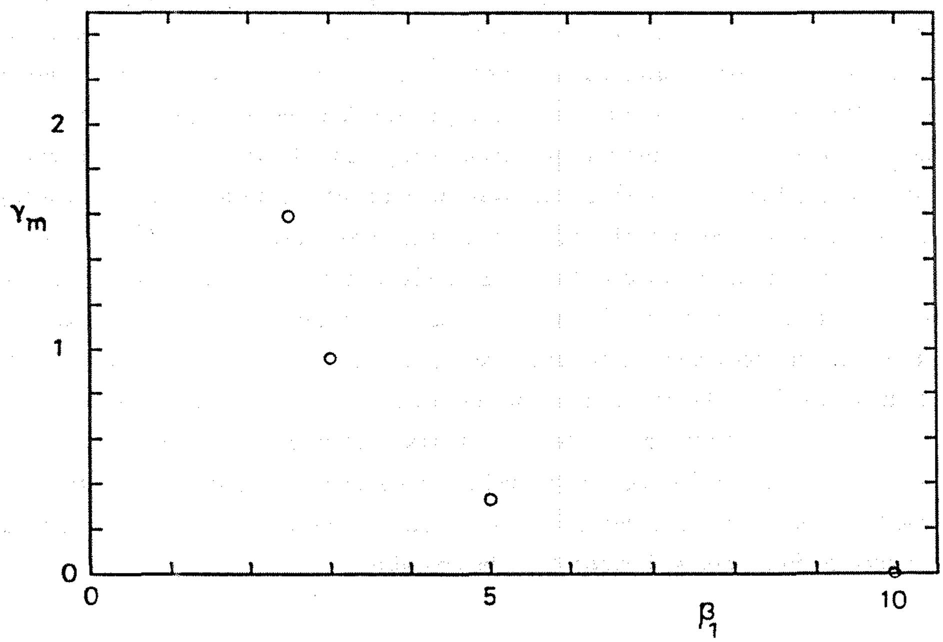


Fig. 10.



CHIRAL PHASE TRANSITION IN THE EFFECTIVE THEORY OF QED PLUS NAMBU-JONA-LASINIO MODEL ON THE LATTICE

YŪKI SUGIYAMA†

Department of Physics, Nagoya University,
Nagoya 464-01, Japan

ABSTRACT

The lattice QED theory with chiral-invariant four-fermion interaction is investigated analytically, through the effective model, which consist of mesons originated in bi-linear fermi fields. We further study this model using the renormalization group method, for taking the continuum limit as critical phenomena on lattice regularized space. We study the existence of fixed points and their scaling properties, and we discuss the properties of theories defined on such fixed points with respect to the triviality.

1. *Introduction.* QED as the perturbatively defined theory leads to the serious field theoretical problem, known as Landau ghost.[1] The possibilities of solving this problem lies on the non-trivial structure in the strong coupling regime. In the viewpoint of the Wilsonian renormalization group (RG) approach[2,3] , the questions are whether non-trivial fixed point(F.P.) , which may be ultra-violet F.P., or another relevant operators really exist. These questions are related to the non-perturbative dynamical effects. For this purpose we choose the lattice QED model with chiral invariant four fermi interactions. We investigate this model analytically, through the effective model. The method is based on the strong coupling expansion and integration of the gauge fields, which derives the local boson system originated in bilinear fermi fields. This method is usually performed to lattice QCD. We further study this model using the renormalization group method, in the viewpoint of the construction of the field theory in the continuum space as critical phenomena on lattice regularized space. We study the existence of trivial and non-trivial fixed points and its scaling properties. Such F.P.s are interpreted in terms of the original lattice QED model and the model including the four fermi interaction and we discuss the properties of theories defined on such fixed points with respect to the triviality.

† Addres after April 1990; Division of Mathematical Science, City College of Mie, Isshinden-Nakano, Tsu 514, Japan

2. *The Effective Model of Lattice QED.* The original model is the 4-dimensional QED with chiral invariant four fermi interaction.

$$
\begin{aligned}
S = & -\frac{1}{2e^2}\sum_{x,\mu>\nu} U_\mu(x)U_\nu(x+\hat{\mu})U^\dagger_\mu(x+\hat{\nu})U^\dagger_\nu(x) \quad + \quad h.c. \\
& -\sum_{x,\mu}\frac{1}{2}[\bar{\psi}(x)\gamma_\mu U_\mu(x)\psi(x+\hat{\mu}) - \bar{\psi}(x+\hat{\mu})\gamma_\mu U^\dagger_\mu(x)\psi(x)] \\
& +m\sum_x \bar{\psi}(x)\psi(x) \\
& +g\sum_x[(\bar{\psi}(x)\psi(x))^2 - (\bar{\psi}(x)\gamma_5\psi(x))^2]
\end{aligned}
\tag{1}
$$

The first term denote the gauge action, which we use the compact U(1) action. $1/e^2$ is the gauge coupling. $\psi(x)$ denotes the naive Dirac fermion with bare mass m. We have kept the naive form of the fermion for chiral invariance in the limit $m = 0$. The last term is the chiral invariant four fermi interaction. If we switch off the gauge interaction the model reduce to the well-known Nambu Jona-Lasinio(NJL) model.[7] We can study lattice QED as well as the gauged NJL model.

Many analytical studies have been made to the gauge systems interacting with fermions on lattice such as QCD or large N model(as the limiting model of QCD) in the similar way.[8,9] The basic idea is as follows. The models are reduced to the system of the elementary excitations on the vacuum of the strong coupling limit, those are several kinds of gauge singlet composite bosons i.e. meson fields. As the gauge coupling becomes weaker, such mesons begin to interact with each other. The similar method is adopted to the QED+NJL model. We survey the process of constructing the effective model. We first make the strong coupling expansion of the partition function, and integrate out the gauge variables. But the exact calculation is so difficult even in the limit of $1/e^2 = 0$. We technically change the dynamical variable $U_\mu(x)$ to the random variable. This change makes the integration of the gauge degrees of freedom easy. As the model is deformed through this process, there is no essential difference whether action is compact or not, with respect to the chiral symmetry breaking. In practice the strong coupling limit, the models are just the same. That produces the sequence of the interaction terms, which consist of bilocal composite fermion fields, such as $\bar{\psi}(x)\gamma_\mu\psi(x+\hat{\mu})$. Second, we perform Fierz rearrangement[8] of all interaction terms, and we get the action rewritten in terms of the local composite fields such as $\bar{\psi}(x)\Gamma\psi(x)$, where Γ denotes the 16-Dirac gamma matrices. We finally rewrite the effective action with local bose fields using the auxiliary field method, such as scalar : $\bar{\psi}(x)\psi(x) \rightarrow \sigma(x)$, pseudoscalar : $i\bar{\psi}(x)\gamma_5\psi(x) \rightarrow \pi(x)$, vector : $\bar{\psi}(x)\gamma_\mu\psi(x) \rightarrow V_\mu(x)$, etc. We note that the process preserve gauge invariance. The obtained effective model is given as follows.

$$
\begin{aligned}
S_{eff} = & -\frac{1}{8}\sum_{x,\mu}\sigma(x)\sigma(x+\hat{\mu}) - g\sum_{x}\sigma(x)^2 - \frac{1}{8}\sum_{x,\mu}\pi(x)\pi(x+\hat{\mu}) - g\sum_{x}\pi(x)^2 \\
& -\frac{1}{8}\sum_{x,\mu,\nu} V_\mu(x)(1-2\delta_{\mu\nu})V_\mu(x+\hat{\nu}) + [A,T] \\
& +\frac{1}{2}\sum_{x}\ln\Big\{\Big[\sum_{\mu}\sigma(x+\hat{\mu}) + 8g\sigma(x) + 4m\Big]^2\Big[\sum_{\mu}\pi(x+\hat{\mu}) + 8g\pi(x)\Big]^2 \\
& \qquad + \sum_{\mu}\Big[\sum_{\nu}(1-2\delta_{\mu\nu})V_\mu(x+\hat{\nu})\Big]^2 + \ldots\Big\} \\
& -(\frac{1}{4})^3\beta V_p[\sigma,\pi] \\
& + o(\beta^2)V_6[\sigma,\pi,] + \text{ higher order terms....} \\
& + \text{other interaction terms}
\end{aligned}
\tag{2}
$$

where

$$
\begin{aligned}
V_p[\sigma,\pi] = \sum_{x,\mu>\nu} \Big\{ & \sigma(x)\sigma(x+\hat{\mu})\sigma(x+\hat{\mu}+\hat{\nu})\sigma(x+\hat{\nu}) \\
& +\pi(x)\pi(x+\hat{\mu})\pi(x+\hat{\mu}+\hat{\nu})\pi(x+\hat{\nu}) \\
& +\sum_{conb.}\varepsilon(conb.)\sigma(x)\sigma(x+\hat{\mu})\pi(x+\hat{\mu}+\hat{\nu})\pi(x+\hat{\nu})\Big\}
\end{aligned}
$$

where A and T are axial vector and tensor fields, respectively. The logarithmic potential is the contribution from the fermionic determinant. This effective action has infinite series of non-local polynomial interactions, those are derived from the plaqutte type gauge interactions, and they are very complicated. But fortunately, from the point of view the RG method, almost all the terms must be irrelevant operators, and the leading contributions to the scaling behavior comes from the lower power polynomial interactions. Besides the contributions of the higher spin variables will not be so important. In the following discussion, only the scalar and the pseudo-scalar fields σ,π will be considered as dynamical variables. Vector fields are replaced by some constant values as the mean fields and others are neglected.* The reduced system of σ and π maintains the chiral invariance within this part. We can study the chiral phase transition of the original QED+NJL model by the effective model.

3. *The Mean Field Study of the Effective Model.* In order to check whether the obtained effective model correctly shows the physical content, we study mean field (MF) action of this model. MF theory assumes that σ and π are uniformly distributed, i.e.,$\sigma(x)$=σ, $\pi(x)$=π.

$$
\begin{aligned}
\frac{S_{MF}}{N} = & -\frac{1}{8}h(\sigma^2+\pi^2) + \frac{1}{4}\gamma(\sigma^2+\pi^2)^2 \\
& +\frac{1}{2}\ln\Big[h^2(\sigma^2+\pi^2)\Big]
\end{aligned}
\tag{3}
$$

* If we use staggered fermions the same effective model can be derived within this approximation.

where

$$h = d + 8g$$
$$\gamma = (\frac{1}{4})^3 2d(d-1)/e^2$$

Set the $\pi = 0$ in the chiral limit $m = 0$, the chiral condensate $< \sigma >$, which corresponds to $< \bar{\psi}\psi >$, is given by the minimum of the resulting effective action. MF action gives us the physical picture of mechanism of the chiral transition in the simple way. The contributions of the anti-ferro spin system induced by Fierz rearrangement of the fermi fields, and the plaquett interaction term induced from the gauge interaction determine the chiral symmetry breaking. The obtained phase diagram are consistent with the results of Schwinger-Dyson approach. The logarithmic potential make the action unbounded and $< \sigma >$ is unstable in the weak coupling region, which causes the difficulty in studying by the RG method. Then, in the effective action vector fields are replaced by some constant values V as the MF and the logarithmic potential is expanded with respect to $1/2d(d-1)V^2$, where d is the spacetime dimension. Again, the MF method is applied this apploximated effective action. The resulting phase diagram is not changed in the qualitative feature, but the stability of the field is improved.

4. *The Renormalization Group Study of the Effective Model.* For the purpose of using the RG method, we rewrite the model in momentum space for small p^2, then any kind of non-local polynomial interaction derived from eq.(2) reduces to the following much simpler form.

$$\begin{aligned} S_{eff} = &-\frac{1}{2}\int dp(p^2+r)(\sigma(p)\sigma(-p)+\pi(p)\pi(-p)) \\ &-\int\prod_{i=1}^{4} dp_i\delta(\sum_{i=1}^{4} p_i)(\sigma(p_1)\sigma(p_2)+\pi(p_1)\pi(p_2))(\sigma(p_3)\sigma(p_4)+\pi(p_3)\pi(p_4)) \\ &\qquad (\beta_1+\beta_2\sum_{i=1}^{4} p_i^2) \\ &- \text{higher power polynomial interactions....} \end{aligned} \tag{4}$$

We remark that interaction kernel has momentum dependence. This shows that the interactions are non-local. The parameters r,β_1 and β_2 are the polynomials of $1/e_c^2$ and g. Here, we take the chiral limit, then all the odd polynomial interaction terms are dropped. The RG equations are derived by diagrammatic expansion[3] with respect to β_1 and β_2. The later investigations are based on the algebraic equations for the coupling parameters of the RG steps as diagrammatically shown. In the case $\beta_2 \geq 0$, the situation is the same as the scalar models, which has only the trivial fixe point; $r^*, \beta_1^*, \beta_2^* = 0$. If we spread the parameter space to the negative region of β_2, the small value of β_2^* at the non-trivial fixed point is indicated by the solution of the RG equations. Choosing $\beta_2 \sim -\tau$ for the expansion parameter, the investigations are performed similar to the well-kown ε-expansion method.[3] The contributions from higher power terms are evaluated systematically, and the results are consistent within the order τ

5. *Fixed Points, Scaling properties and Triviality* For the purpose of studying the scaling properties of the F.P.s, we linearized the RG equations near the fixed points, and calculate the eigenvalues of their transition matrix, which provides the eigenvalues of the scaling operators.

When the parameter space is restricted to the β_2=0 plane, only the trivial F.P.;$r^*, \beta_1^* = 0$ exists, the corresponding F.P. of the original QED+NJL model is $g^* = -(d - v/16)/8$, $e^{*2} = 2 \times 4^2 d(d-1)/v^2$ where $v = d(d-2)^2 V$, which has the eigenvalues are $\lambda_1 = 4, \lambda_2 = 1$,. This shows this F.P. has 1-relevant operator, which corresponds to the mass operator $\sigma^2 + \pi^2$ and this is just the MF value. Besides, there is 1-marginal operator, which is the contribution beyond the linearized RG equations. This contribution is essential in this model. This leads to the logarithmic correction to the MF scaling behavior as long as β_1 is small. According to the investigation of the constructive field theory of scalar models,[10] the scaling properties result in the trivial theory. Here 'triviality' means the renormalized $\sigma - \pi$ 4-body coupling goes to zero in the limit of the cutoff-infinity within the domain of this F.P. The situation is not changed, even if the parameter space is extended to the β_2-axis. Only the 1-irrelevant operator is added.

But in the negative β_2 region, another F.P. seems to exist, that is $r^* = -\frac{40}{3}\tau$, $\beta_1^* = \frac{48}{9}\tau$, $\beta_2^* \sim -\frac{1}{c}\tau$ and the eigenvalues are $\lambda_1 = 4(1 - 4\tau)$, $\lambda_2 = 1 - 48\tau$, $\lambda_3 = *$, where the third eigenvalue is unknown in the framework of this trial analysis. But the qualitative considerations of the RG-equations and the scaling properties of the trivial fixed point imply that the third scaling operator may be relevant. This F.P. has clearly different properties , which should have two relevant operators, so called tri-critical.[11] This F.P. has the possibility leading to the non-trivial theory. In order to understand this F.P. in the original QED+NJL model, another parameter axis is needed, corresponding to some kind of interaction. For example, the chiral invariant eight-fermion interaction can be imposed. Any way non trivial F.P. exist beyond the $e^2 - g$ plane.

4. *Summary and Discussion.* We derive the effective theory for lattice QED. The obtained effective model is the local bosonic system with non-local interactions. This model preserves gauge invariance. Using the RG method we investigate the existence of F.P.s and their critical behaviors for the purpose of constructing the continuum theory associated with their F.P.s. The analysis in this report imply the contribution of the scalar and pseudoscalar part are relevant. When the coupling parameter space is restricted to gauge coupling or gauge and four-fermion couplings, each case imply that theory is well described by gaussian model of non-interacting σ and π fields, which correspond to the gauge singlet

bound states of fermions. The similar picture is proposed by several investigations: Monte Calro simulations[14] and Schwinger-Dyson approach.[12,13] Further, we studied the non-trivial F.P.,which scaling property may save the triviality of the theory. The analysis of this work is preliminary. The contributions from the vector and higher spin variables should be included. Such contributions will make the analysis fully systematic. But the important feature of the scaling behavior don't depend heavily on higher spin variables. The results show the basic property of F.P.s, which is the first step for constructing the consistent non-trivial continuum theory.

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Ultraviolet Fixed Point Structure of Renormalizable Four-Fermion Theory in Less Than Four Dimensions *

Yoshio Kikukawa

Department of Physics, Nagoya University, Nagoya 464-01,Japan

Abstract

We study the renormalization properties of the four-fermion theory in less than four dimensions ($D < 4$) in 1/N expansion scheme. It is shown that β function of the bare coupling has a nontrivial ultraviolet fixed point with a large anomalous dimension ($\gamma_{\bar{\psi}\psi} = D - 2$) in a similar manner to QED and gauged Nambu-Jona-Lasinio (NJL) model in ladder approximation. The anomalous dimension has no discontinuity across the fixed point in sharp contrast to gauged NJL model. The operator product expansion of the fermion mass function is also given.

Introduction

Recently the possibility that QED may have a nontrivial ultraviolet(UV) fixed point has been paid much attention from the viewpoints of "zero charge" problem in QED and raising condensate in technicolor model. Actually such a possibility was pointed out in ladder approximation in which the cutoff Schwinger-Dyson equation for the fermion self-energy possesses a spontaneous-chiral-symmetry-breaking solution for the bare coupling larger than a non-zero value ($\alpha_0 \equiv e_0^2/4\pi > \pi/3 \equiv \alpha_c$). We can make this solution finite by letting α_0 have a cutoff dependence in such a way that $\alpha_0(\Lambda) \rightarrow \alpha_c + 0$ ($\Lambda \rightarrow \infty$), α_c being identified as the critical point with scaling behavior of essential-singularity type. At the critical point, fermion mass operator $\bar{\psi}\psi$ has a large anomalous dimension $\gamma_{\bar{\psi}\psi} = 1$, which is indeed crucial to the technicolor.[2]

This problem was further analyzed in ladder approximation in the two-coupling space of the gauged Nambu-Jona-Lasinio (NJL) model, i.e., QED plus a (possibly "induced") four-fermion interaction whose physical dimension becomes 4(=6-2$\gamma_{\bar{\psi}\psi}$) at the critical point due to a large $\gamma_{\bar{\psi}\psi}$ (=1).[3] Quite recently a critical line of this model was discovered in the whole prameter space of two couplings ($\alpha_0(\Lambda), g_0(\Lambda)$), with $g_0(\Lambda)$ being the dimensionless bare four-fermion coupling.[4][5] The most striking feature of the model is the appearance of an even larger anomalous dimension $\gamma_{\bar{\psi}\psi} = 1 + \sqrt{1 - \alpha_0/\alpha_c}\,(\geq 1)$ at the critical line, which in fact suggests the four-fermion interaction may become a relevant operator and renormalizable, in sharp contrast to the symmetric phase where one obtains a smaller $\gamma_{\bar{\psi}\psi} = 1 - \sqrt{1 - \alpha_0/\alpha_c}\,(< 1)$ and accordingly the four-fermion interaction is irrelevant.[6]

An important application of this dynamical symmetry breaking with a very large $\gamma_{\bar{\psi}\psi}$($\simeq 2$ for $\alpha_0 \simeq 0$) is a "top-mode standard model" in which a top quark condensate is responsible for the electroweak symmetry breaking.[7]

However the existence of a critical point for the bare coupling $\alpha_0(\Lambda)$ does not necessarily imply the UV fixed point for the renormalized one $\alpha_{(\mu)}$ in the continuum theory. In fact the β

* This talk is based on the work done in collaboration with K. Yamawaki.[1]

function was argued to be non-negative, $\beta(\alpha_{(\mu)}) \geq 0$, based on the spectral representation.[8] In ladder approximation, there is no simple way to compute $\beta(\alpha_{(\mu)})$ and/or $\beta(g(\mu))$ through the calculation of vertex Green functions and hence no direct comparison with $\hat{\beta}(\alpha_0(\Lambda))$ and/or $\hat{\beta}(g_0(\Lambda))$ obtained through the gap equation (ladder Schwinger-Dyson equation) for the fermion propagator. Also the above discontinuity of $\gamma_{\bar{\psi}\psi}, 1 \pm \sqrt{1-\alpha_0/\alpha_c}$, across the critical line seems to be rather paradoxical (an artifact of ladder approximation ?), though not obviously in contradiction to the operator product expansion (OPE).[9]

In this talk, we wish to clarify these issues by explicitly calculating $\beta(g)$, $\gamma_{\bar{\psi}\psi}(g)$ and the corresponding "bare" quantities $\hat{\beta}(g_0)$, $\hat{\gamma}_{\bar{\psi}\psi}(g_0)$ of the four-fermion theory in less than four dimensions ($2 < D < 4$) in 1/N expansion; the theory in fact was shown to be renormalizable and was also demonstrated to have a nontrivial UV fixed point for the renormalized coupling, $g_{(\mu)} = g^* \neq 0$, and a large anomalous dimension $\gamma_{\bar{\psi}\psi}(g^*) = D-2$ at the fixed point.[10] We shall show $\hat{\beta}(g_0)$ and $\hat{\gamma}_{\bar{\psi}\psi}(g_0)$ are very similar to $\beta(g)$ and $\gamma_{\bar{\psi}\psi}(g)$, respectively; $\hat{\beta}(g_0)$ possesses a UV fixed point $g_0(\Lambda) = g_c$ in much the same way as the ladder QED and the gauged NJL model, while $\hat{\gamma}_{\bar{\psi}\psi}(g_0)$ becomes large, $\hat{\gamma}_{\bar{\psi}\psi}(g_c) = D-2$, although having no discontinuity across the fixed point in contrast to the gauged NJL model. The discontinuity of $\hat{\gamma}_{\bar{\psi}\psi}(g_0)$ may be traced to the fact that usually in ladder approximation $g_0(\Lambda)$ is not renormalized in the symmetric phase : Taking account of the renormalization of $g_0(\Lambda)$ in our model indeed fill in the gap of $\hat{\gamma}_{\bar{\psi}\psi}$. The large anomalous dimension without discontinuity will be shown to be consistent with the operator product expansion of the fermion mass function, which actually holds in a quite nontrivial fashion.[1]

1/N expansion and Renormalized theory

Let us start with the following four-fermion theory,

$$\mathcal{L}_{4F}(x) = \bar{\psi}^a i\partial\!\!\!/ \psi^a + G_0(\bar{\psi}^a\psi^a)^2/2N,$$

where $\psi^a(x)$ is a four-component Dirac fermion and the suffix runs from one to N. The space-time dimension is less than four. This system has a symmetry under discrete chiral transformation; $\psi^a(x) \to \gamma_5\psi^a(x)$. By introducing an auxiliary field $\sigma(x)$, we rewrite the Lagrangian into

$$\mathcal{L}_\sigma(x) = \bar{\psi}^a i\partial\!\!\!/ \psi^a - M\bar{\psi}^a\psi^a - (N/2G_0)\tilde{\sigma}^2 - \tilde{\sigma}\bar{\psi}^a\psi^a - (NM/G_0)\tilde{\sigma},$$

where the σ field has been shifted to $\tilde{\sigma}$ by a vacuum expectation value $\langle\sigma\rangle = M$ determined through a self-consistent equation, the gap equation, which is derived from the condition that the new variable has no vacuum expectation value.

We now perform 1/N expansion to evaluate Green functions. The fermion propagator and the vertex are of order $O(N^0)$, while the boson propagator and the tadpole are of order $O(1/N)$ and $O(N)$, respectively. The boson propagator, the gap equation and the scalar vertex in 1/N leading order are given by,

$$D_\sigma(p)^{-1} = iN\Big[\frac{1}{G_0} - i\int^\Lambda \frac{d^Dk}{(2\pi)^D}\mathrm{Tr}(\frac{1}{p\!\!\!/ + k\!\!\!/ - M}\frac{1}{k\!\!\!/ - M})\Big],$$

$$\langle\tilde{\sigma}\rangle \propto M(1 - iG_0\int^\Lambda \frac{d^Dk}{(2\pi)^D}\frac{4}{k^2-M^2}) = 0, \tag{1}$$

$$\Gamma_0^{\bar{\psi}\psi}(p,q) = iND_\sigma(p-q)/G_0.$$

The gap equation has two solutions, a symmetric solution $M = 0$ and a spontaneously broken one $M \neq 0$.

This model can be simultaneously renormalized for both solutions as follows. We define the renormalized coupling $G_{(\mu)}(\equiv g(\mu)\mu^{2-D})$;

$$\frac{1}{G(\mu)} \equiv \frac{Z_G}{G_0} \equiv \frac{1}{G_0} - i\int^{\Lambda} \frac{d^Dk}{(2\pi)^D}\mathrm{Tr}(\frac{1}{\not p + \not k}\frac{1}{\not k})\Big|_{p^2=-\mu^2},$$

$$Z_G = 1 - G_0(\frac{\Lambda^{D-2}}{g_c} - \frac{\mu^{D-2}}{g^*}), \tag{2}$$

where

$$g^* = \frac{(4\pi)^{D/2}(D-2)}{[8(D-1)\Gamma(2-D/2)B(D/2,D/2)]}, \quad g_c = (4\pi)^{D/2}\frac{(D-2)}{8}\Gamma(D/2),$$

with $\Gamma(B)$ being the gamma (beta) function. Notice that the renormalization constant Z_G can be defined to be mass-independent (Zero Mass Renormalization Procedure)[11] even for the broken solution $M \neq 0$, and also that $Z_\psi = Z_\sigma = 1$ in 1/N leading order.

The β function of the renormalized coupling, $\beta(g) = \mu\frac{\partial g(\mu)}{\partial\mu}$, is calculated from (2);

$$\beta(g) = (D-2)\frac{g}{g^*}(g^* - g),$$

which is valid in both phases (solid line in Fig.1). It is now evident that g^* is the UV fixed point which separates the symmetric and broken phases of the symmetry. In D=2 we have $g^* = 0$ ($(D-2)/g^* \to 2/\pi$), which is just the asymptotic freedom of Gross-Neveu model.

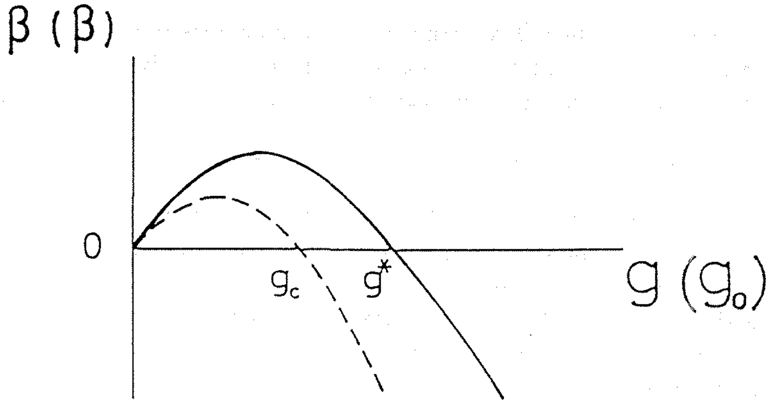

Fig. 1. β functions of renormalized coupling (solid line), and of bare coupling (dashed line).

For the scalar vertex, we can take $Z_{\bar\psi\psi} = Z_G$. Thus the anomalous dimension of the mass operator $\bar\psi^a\psi^a$, $\gamma_{\bar\psi\psi} = \mu\frac{\partial ln Z_{\bar\psi\psi}}{\partial\mu}$, is also obtained from (2),

$$\gamma_{\bar\psi\psi}(g) = (D-2)\frac{g}{g^*}.$$

(solid line in Fig.2). There is no discontinuity at $g(\mu) = g^*$.

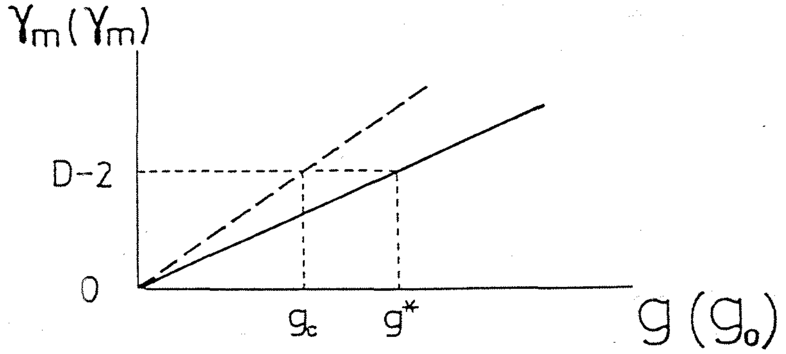

Fig. 2. Anomalous dimensions of mass operator, renormalized one (solid line) and bare one (dashed line).

Bare quantities

Let us now turn to the cutoff dependence of the bare coupling in the present model. From (2) we obtain, for $\Lambda \gg \mu$,

$$\hat{\beta}(g_0) = \Lambda \frac{\partial}{\partial \Lambda} g_0 \Big|_{\mu, G_{(\mu)}-fixed} \simeq (D-2)\frac{g_0}{g_c}(g_c - g_0), \tag{3}$$

where $g_0 \equiv G_0 \Lambda^{D-2}$ (dashed line in Fig.1). Since the gap equation (1) leads to a relation,

$$\frac{\Lambda^{D-2}}{g_c}\frac{g_c - g_0(\Lambda)}{g_0(\Lambda)} = \frac{\mu^{D-2}}{g^*}\frac{g^* - g(\mu)}{g(\mu)},$$

g_c turns out to be the critical point which divides the two phases, corresponding to g^*. On the other hand, from the gap equation (1), we obtain (for $g_0 > g_c$)

$$\hat{\beta}(g_0) = \Lambda \frac{\partial}{\partial \Lambda} g_0 \Big|_{M-fixed} \simeq (D-2)\frac{g_0}{g_c}\left(g_c - \frac{g_0}{(1 + M^2/\Lambda^2)}\right),$$

which is actually the β function widely discussed in the ladder QED and the gauged NJL model. In the limit $M/\Lambda \ll 1$, this reduces to (3). This reflects the fact that we can renormalize simultaneously the gap equation and the boson propagator by the renormalization of the coupling.

The anomalous dimension $\hat{\gamma}_{\bar{\psi}\psi}(g_0(\Lambda)) = -\Lambda \frac{\partial ln Z_{\bar{\psi}\psi}}{\partial \Lambda}$ is also calculated from (2),

$$\hat{\gamma}_{\bar{\psi}\psi}(g_0) \simeq (D-2)\frac{g_0}{g_c},$$

(dashed line in Fig.2). This does not have a discontinuity at $g_0(\Lambda) = g_c$ in contrast to the gauged NJL model.

Note that (3) is valid both in the symmetric and the broken phases. This is contrasted with the ladder QED in which the renormalization of α_0 is performed only through the gap equation for the fermion propagator, which is trivial in the symmetric phase ($\hat{\beta}(\alpha_0) = 0$), but not through that of other Green functions such as fermion-photon vertex and fermion four-point function. The lack of renormalization of the bare coupling in the symmetric phase is also shared by the ladder gauged NJL model ($\hat{\beta}(g_0) = \hat{\beta}(\alpha_0) = 0$ below the critical line).

It may be this non-renormalization of the bare coupling in the symmetric phase that caused the discontinuity of the anomalous dimension, $\gamma_{\bar{\psi}\psi} = 1 \pm \sqrt{1 - \alpha_0/\alpha_c}$, across the critical line in the gauged NJL model. In order to clarify this point in our model, we incorporate a fermion bare mass m_0 into the gap equation (1);

$$M = m_0 + iG_0 \int^{\Lambda} \frac{d^D k}{(2\pi)^D} \frac{4}{k^2 - M^2} = m_0 - G_0 \frac{\langle \bar{\psi}^a \psi^a \rangle_0}{N}, \tag{4}$$

where M is defined as $S_F^{-1}(p) = \not{p} - M$. m_0 is renormalized as $m_0 = Z_{\bar{\psi}\psi} m_R$. In the symmetric phase, (4) itself does not require the renormalization of g_0. Were it not for any renormalization of g_0 through other Green functions than the fermion propagator, one would conclude that $m_0 \sim M(1 - \frac{g_0}{\Lambda^{D-2}} \frac{\Lambda^{D-2}}{g_c}) \sim \Lambda^0$. This is indeed what happened to the ladder gauged NJL model. However, in the case at hand, g_0 is actually renormalized through the boson propagator renormalization (2) in such a way that $1 - g_0(\Lambda)/g_c \sim \Lambda^{2-D}$; namely $m_0 \sim 1/\Lambda^{D-2}$.

Mass function and OPE

We define mass function of fermions, the effective coupling and the effective mass as follows,

$$\Sigma(q; g_R, m_R, \mu) \equiv B(q; g_R, m_R, \mu)/A(q; g_R, m_R, \mu),$$

$$S_R^{-1}(q; g_R, m_R, \mu) = A(q; g_R, m_R, \mu)\not{q} - B(q; g_R, m_R, \mu),$$

and

$$Q\frac{dm(Q)}{dQ} = -\gamma_{\bar{\psi}\psi} m(Q)\,; \quad m(\mu) = m_R,$$

$$Q\frac{dg(Q)}{dQ} = \beta(g)\,; \quad g(\mu) = g_R,$$

$$m(Q) = m_R \;\exp\Big\{-\int_{\mu}^{Q} \gamma_{\bar{\psi}\psi} \frac{dQ'}{Q'}\Big\}.$$

Based on OPE and renormalization group equation, the general formula of the asymptotic behavior of the mass function is;

$$\begin{aligned}\Sigma(q; g_R, m_R, \mu) =& m(Q)\,\mathrm{Tr}\{\Gamma_R^{\bar{\psi}\psi}(q, q; g(Q), 0, Q)\}/4 \\ &+ iQ^2 C_{\bar{\psi}\psi}(q; g(Q), Q)\;\exp\Big\{+\int_{\mu}^{Q} \gamma_{\bar{\psi}\psi} \frac{dQ'}{Q'}\Big\}\;\langle 0|\,\bar{\psi}\psi\,|0\rangle_{(\mu)},\end{aligned} \tag{5}$$

where $-q^2 \equiv Q^2 \gg \mu^2, m_R^2$. The second term in R.H.S. is a dynamical mass associated with the spontaneous chiral symmetry breaking, which was given by Politzer.[12] The first term, a current mass with the explicit chiral symmetry breaking, can be obtained as follows. In the scheme of ZMRP, we straightforwardly have

$$\Gamma_R^{\bar{\psi}\psi}(q; g_R, m_R, \mu) = -\frac{\partial}{\partial m_R} S_R^{-1}(q; g_R, m_R, \mu),$$

from the corresponding formula in terms of bare quantities. There is no singularity in the limit that the renormalized mass goes to zero, so that, we next expand each term in both sides with respect to the renormalized mass m_R. Then,

$$\Gamma_R^{\bar{\psi}\psi}(q, q; g_R, 0, \mu) = B'(q; g_R, 0, \mu) - A'(q; g_R, 0, \mu)\not{q}.$$

The first term in the expansion of B function is a dynamical mass in broken phase, or is equal to zero in symmetric phase because of chiral symmetry. Here we take the explicit breaking term as perturbation to broken phase, so that,

$$\begin{aligned}\Sigma(q;g_R,m_R,\mu) &\simeq m_R\frac{B'(q;g_R,0,\mu)}{A(q;g_R,0,\mu)}+O(m_R{}^2)+(Dynamical\ Mass),\\ &\simeq \frac{m_R}{A(q;g_R,0,\mu)}\mathrm{Tr}\{\Gamma_R^{\bar\psi\psi}(q,q;g_R,0,\mu)\}/4+(Dynamical\ Mass).\end{aligned} \tag{6}$$

Renormalization group equations for $A(q;g_R,0,\mu)$ and $\Gamma_R^{\bar\psi\psi}(q,q;g_R,0,\mu)$ can be solved as follows.

$$A(q;g_R,0,\mu)=\exp\Big\{-\int_\mu^Q 2\gamma_\psi\frac{dQ'}{Q'}\Big\}, \tag{7}$$

$$\Gamma_R^{\bar\psi\psi}(q,q;g_R,0,\mu)=\exp\Big\{-\int_\mu^Q(\gamma_{\bar\psi\psi}+2\gamma_\psi)\frac{dQ'}{Q'}\Big\}\Gamma_R^{\bar\psi\psi}(q,q;g(Q),0,Q), \tag{8}$$

where $-q^2=Q^2$. What we want follows from eq.(6),(7),and (8).

In the case of the four-fermion theory considered, the scalar vertex and the Wilson coefficient function in the zero mass limit are

$$\Gamma_R^{\bar\psi\psi}(q,q;g(Q),0,Q)=iND_\sigma(0;g(Q),0,Q)/G(Q)=\frac{g^*}{g^*-g(Q)},$$

$$C_{\bar\psi\psi}(q;g(Q),0,Q)=\frac{iG(Q)}{NQ^2}.$$

Thus we obtain the asymptotic form of the fermion mass,

$$\begin{aligned}M &\simeq m(Q)\,\frac{g^*}{g^*-g(Q)}\ -\frac{G(Q)}{N}\ \exp\Big\{+\int_\mu^Q\gamma_{\bar\psi\psi}\frac{dQ'}{Q'}\Big\}\ \langle 0|\,\bar\psi\psi\,|0\rangle_{(\mu)},\\ &\simeq m_R\,\frac{g^*}{g^*-g(\mu)}-\frac{G(\mu)}{N}\ \langle 0|\,\bar\psi\psi\,|0\rangle_{(\mu)}.\end{aligned} \tag{9}$$

In the second equality, we use the relation

$$\frac{G(\mu)}{G(Q)}=\frac{g^*-g(\mu)}{g^*-g(Q)}=\exp\Big\{+\int_\mu^Q\gamma_{\bar\psi\psi}\frac{dQ'}{Q'}\Big\},$$

which follows from $Z_G=Z_{\bar\psi\psi}$. Eq.(9) in fact agrees with the gap equation (1), if it is expanded in the renormalized mass, and the subtraction of the operator $\bar\psi\psi$ is considered. The subtraction procedure is as follows,[13]

$$:(\bar\psi\psi):\equiv(\bar\psi\psi)_0-\frac{Nm_0}{G_0}+\frac{Nm_0|\Gamma_0^{\bar\psi\psi}(q,q;g_R,0,\mu)|}{G_0}\equiv Z_{\bar\psi\psi}^{-1}(\bar\psi\psi)_R$$

by means of which $\bar\psi\psi$ can be renormalized by $Z_{\bar\psi\psi}$ defined through the scalar vertex and does not mix with the operator $\mathbf{1}$. Note that the effective mass $m_{(Q)}$ is multiplied by the nontrivial factor $g^*/(g^*-g(Q))$, which precisely compensates rapid damping of $m_{(Q)}\sim Q^{-\gamma_{\bar\psi\psi}(g^*)}$ to yield the first term of (9), a constant mass. This is a remarkable difference from that in QCD-like theories.

Summary

The four-fermion theory in $D < 4$ dimensions is an explicit example of the model possessing a nontrivial UV fixed point in both the bare and the renormalized couplings. It shares many interesting features with QED and gauged NJL model in ladder approximation in four dimensions. The gap in the anomalous dimension across the fixed point in the gauged NJL model may be caused by the non-renormalization of the coupling in the symmetric limit. The large anomalous dimension is consistent with the OPE formula of mass function including the nontrivial coefficient.

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Observable Consequences of the Strong Coupling Phase of QED

Taizo Muta†

Department of Physics,
Hiroshima University, Hiroshima 740 Japan

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PHOTON PAIRING IN QUANTUM ELECTRODYNAMICS

T. Inagaki

Department of Physics, Faculty of Science and Technology,
Keio University, Yokohama 223, Japan

Abstract

In this talk, we discuss photon pairing phenomenon in the strong coupling phase of massive Quantum Electrodynamics (QED) through the analysis of the Cooper equation. Using the well known low energy effective Lagrangian for photon, it is shown that when the coupling constant exceeds some finite value, the normal vacuum of QED becomes unstable with respect to the formation of the photon pair. It is also found that the pairing instability is *enhanced* and the critical coupling has a tendency to become smaller in the presence of the weak constant electric field. This may give a theoretical basis for the anomalous GSI e^+e^- events.

1. Introduction

A number of observations based on the computer simulations using the lattice gauge theory and on the Schwinger-Dyson equation suggest that there exists the strong coupling phase in Quantum Electrodynamics (QED) when the coupling constant $\alpha = (e^2/4\pi)$ becomes larger than some critical value.[1)2)3)] Much effort has been paid to investigate the characteristics of this new phase of QED, because it is hoped that it may give a theoretical basis for explaining the GSI peak[4)] and may resolve the Flavour-Changing Neutral Current (FCNC) problem in the technicolour theory.[1)] The purpose of this talk is to give some discussions on the photon pairing phenomenon in the strong coupling phase of massive QED with the help of the Cooper equation that is well known in the theory of the superconductivity. The talk is based on the paper listed ref.5).

2. The low energy effective Lagrangian and the Hamiltonian for photon

Let us begin with the Euler Heisenberg effective Lagrangian for photon $\mathcal{L}_{eff}$ [6)] ;

$$\mathcal{L}_{eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + a(F^{\mu\nu}F_{\mu\nu})^2 + b(F^{\mu\nu}\tilde{F}_{\mu\nu})^2 \ , \tag{1}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, $\tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ and $a = \frac{4}{7}b = (\alpha^2/90m^4)$. Here $\varepsilon^{\mu\nu\rho\sigma}$ is the completely anti-symmetric tensor with $\varepsilon_{0123} = +1$, A^μ the renormalized photon field, m the electron mass and $\alpha = (e^2/4\pi)$ the renormalized fine structure constant. Here and in the following the indices μ, ν, ... run from 0 to 3 and the indices i, j, ... run from 1 to 3 .

The effective Lagrangian in the presence of the external field A_c^μ can be obtained the deviding the photon field A^μ in (1) into two parts,

$$A^\mu \rightarrow A_c^\mu + A^\mu \ . \tag{2}$$

(We use the same notation A^μ as in (1), since there may be no confusions.) Here we define the A_c^μ by $[\delta S/\delta A_\mu]_{A=A_c} = 0$, where S is the action given by $S = \int d^4x \mathcal{L}_{eff}$. Then we get $\mathcal{L}_{eff}$ as the sum of three terms, i.e. $\mathcal{L}_{eff} = \mathcal{L}_c + \mathcal{L} + \mathcal{L}_G$, where

$$\begin{aligned}
\mathcal{L}_c &= -\frac{1}{4}G^{\mu\nu}G_{\mu\nu} + a(G^{\mu\nu}G_{\mu\nu})^2 + b(G^{\mu\nu}\tilde{G}_{\mu\nu})^2 \ , \\
\mathcal{L} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + a(F^{\mu\nu}F_{\mu\nu})^2 + b(F^{\mu\nu}\tilde{F}_{\mu\nu})^2 \ , \\
\mathcal{L}_G &= 2a(G^{\mu\nu}G_{\mu\nu})(F^{\rho\sigma}F_{\rho\sigma}) + 2b(G^{\mu\nu}\tilde{G}_{\mu\nu})(F^{\rho\sigma}\tilde{F}_{\rho\sigma}) \\
&\quad +4a(G^{\mu\nu}F_{\mu\nu})^2 + 4b(G^{\mu\nu}\tilde{F}_{\mu\nu})^2 \\
&\quad +2a(G^{\mu\nu}F_{\mu\nu})(F^{\rho\sigma}F_{\rho\sigma}) + 2b(G^{\mu\nu}\tilde{F}_{\mu\nu})(F^{\rho\sigma}\tilde{F}_{\rho\sigma}) \ ,
\end{aligned} \tag{3}$$

and $G^{\mu\nu} = \partial^\mu A_c^\nu - \partial^\nu A_c^\mu$, $\tilde{G}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}$. We can neglect $\mathcal{L}_c$ since it has no effect on our problem.

From now on, we choose the Feynman gauge for convenience by adding $\mathcal{L}_{GF} = -\frac{1}{2}(\partial_\mu A^\mu)^2$ to $\mathcal{L}_{eff}$ but our arguments are gauge invariant, of cource.

The canonical momentum Π^μ defined by $\Pi^\mu = (\partial\mathcal{L}'_{eff}/\partial\dot{A}_\mu)$ ($\mathcal{L}'_{eff} \equiv \mathcal{L}_{eff} + \mathcal{L}_{GF}$) is given by $\Pi^0 = -(\partial_\mu A^\mu)$, $\Pi^k = \Pi^{(1)k} + \Pi^{(2)k}$, where

$$\begin{aligned}
\Pi^{(1)k} &= -F^{0k} + 8a(F^{\mu\nu}F_{\mu\nu})F^{0k} + 8b(F^{\mu\nu}\tilde{F}_{\mu\nu})\tilde{F}^{0k} \ , \\
\Pi^{(2)k} &= 8a(G^{\mu\nu}G_{\mu\nu})F^{0k} + 8b(G^{\mu\nu}\tilde{G}_{\mu\nu})\tilde{F}^{0k} + 16a(G^{\mu\nu}F_{\mu\nu})G^{0k} \\
&\quad +16b(G^{\mu\nu}\tilde{F}_{\mu\nu})\tilde{G}^{0k} + 16a(G^{\mu\nu}F_{\mu\nu})F^{0k} + 16b(G^{\mu\nu}\tilde{F}_{\mu\nu})\tilde{F}^{0k} \\
&\quad +8a(F^{\mu\nu}F_{\mu\nu})G^{0k} + 8b(F^{\mu\nu}\tilde{F}_{\mu\nu})\tilde{G}^{0k} \ .
\end{aligned} \tag{4}$$

Therefore, the Hamiltonian density $\mathcal{H}_{eff} = \Pi_\mu\dot{A}^\mu - \mathcal{L}'_{eff} = \mathcal{H}_0 + \mathcal{H}_I + \mathcal{H}_G$ is obtained up to $O(\alpha^2)$ as follows;

$$\begin{aligned}
\mathcal{H}_{eff} &= \mathcal{H}_0 + \mathcal{H}_I + \mathcal{H}_G \ , \\
\mathcal{H}_0 &= -\frac{1}{2}\Pi_\mu\Pi^\mu + \frac{1}{4}(F^{ij}F_{ij}) - \Pi_0(\partial_k A^k) + \Pi_k(\partial^k A_0) \ ,
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}_I =& -a(2\Pi^k\Pi_k - F^{ij}F_{ij})^2 - b(2\varepsilon_{0ijk}\Pi^i F^{jk})^2 \ , \\
\mathcal{H}_G =& -8a(G^{\mu\nu}G_{\mu\nu})(2\Pi^k\Pi_k - F^{ij}F_{ij}) - 8b(G^{\mu\nu}\tilde{G}_{\mu\nu})(2\varepsilon_{0ijk}\Pi^i F^{jk})^2 \\
& -4a(G^{ij}F_{ij} - 2G^{0j}F_{0j})^2 - 4b[\varepsilon_{0ijk}(G^{0i}F^{jk} - \Pi^i G^{jk})]^2 \\
& -4a(2\Pi^k\Pi_k - F^{ij}F_{ij})(G^{ij}F_{ij} - 2G^{0j}F_{0j}) \\
& -4b(2\varepsilon_{0ijk}\Pi^i F^{jk})[\varepsilon_{0ijk}(G^{0i}F^{jk} - \Pi^i G^{jk})] \ . \qquad (5)
\end{aligned}$$

3. The Cooper equation

Now if we introduce $a^{(\Lambda)\dagger}(\mathrm{k})$ as the creation operator for photon with momentum k and helicity Λ , the Cooper state we use in the following is given as

$$|C\rangle = \sum_{\Lambda=R,L} \int \frac{d^3k}{(2\pi)^3 2k_0} f(\mathrm{k})\, a^{(\Lambda)\dagger}(\mathrm{k})\, a^{(\Lambda)\dagger}(-\mathrm{k})\, |0\rangle [2\delta^{(3)}(0)]^{-1/2} \ , \qquad (6)$$

where $k_0 = |\mathrm{k}|$, $\delta^{(3)}(0) = (2\pi)^{-3}\int d^3x = (2\pi)^{-3}V$ (V is the volume of this system.), $|0\rangle$ is the normal vacuum of QED and $f(\mathrm{k})$ the weight function which is determined by the variational principle.

Then, the expectation value of the Hamiltonian $H(=\int d^3x\mathcal{H}_{eff})$ under the constant electric field only $(G_{0i} = E_i = const, G_{ij} = 0)$, can be written as the sum of three terms;

$$\begin{aligned}
\langle C| : H : |C\rangle =& \langle C| : H_0 : |C\rangle + \langle C| : H_I : |C\rangle + \langle C| : H_G : |C\rangle \ , \\
\langle C| : H_0 : |C\rangle =& 2\int d^3k\, k_0\, |f(\mathrm{k})|^2 \ , \\
\langle C| : H_I : |C\rangle =& -\frac{8}{\pi^3}\int d^3k d^3k'\, k_0 k'_0 f^*(\mathrm{k}) f(\mathrm{k}')\{a[4 + (1+\cos\theta)^2] - b(1+\cos\theta)^2\} \ , \\
\langle C| : H_G : |C\rangle =& \int d^3k\, k_0^{-1}\chi(\mathrm{k})|f(\mathrm{k})|^2 \ , \qquad (7)
\end{aligned}$$

where ": ... :" stands for the normal ordering and $H_0 = \int d^3x\mathcal{H}_0$, $H_I = \int d^3x\mathcal{H}_I$, $H_G = \int d^3x\mathcal{H}_G$ and θ is the angle between k and k$'$. If we denote the angle between k and the electric field vector E as φ, the function $\chi(\mathrm{k})$ in (7) can be described as follows,

$$\chi(\mathrm{k}) = -16(a+b)k_0^2|\mathrm{E}|^2\sin^2\varphi \ . \qquad (8)$$

In order to minimize the expectation value of the normal ordered Hamiltonian under the normalization condition $\langle C|C\rangle = \int d^3k|f(\mathrm{k})|^2 = 1$, we take the variation of $\langle C| : H : |C\rangle - \mathcal{E}\langle C|C\rangle$ with respect to $f^*(\mathrm{k})$. Thus we get the Cooper equation;

$$\begin{aligned}
(2k_0 + \frac{\chi(\mathrm{k})}{k_0} - \mathcal{E})f(\mathrm{k}) =& \frac{8}{\pi^3}k_0\int d^3kd^3k'k'_0 f(\mathrm{k}') \\
& \times\{a[4 + (1+\cos\theta)^2] - b(1+\cos\theta)^2\} \ , \qquad (9)
\end{aligned}$$

where $\mathcal{E}$ is the energy of our Cooper state.

In the following discussion, we analyze this Cooper equation in two cases separately; without or with the external electric field.

(i) Photon pairing in the absence of the external field.

In this case, we denote the solution of the Cooper equation as $f_0(\mathbf{k})$ and the energy of the Cooper state as $\mathcal{E}_0$. Then we obtain the following integral equation. We assume here that the solution of this Cooper state depends only on k_0, because the system has the rotational symmetry. Therefore, the Cooper equation takes the form,

$$(2k_0 - \mathcal{E}_0) f_0(k_0) = \frac{2^7}{3\pi^2}(4a - b)k_0 \int_0^\Lambda dk'_0\, k'^3_0 f_0(k'_0) \quad , \tag{10}$$

where Λ is the ultraviolet cut off. Since $b = (7/4)a > 0$, photon-photon interaction in massive QED is attractive in the low energy region.

The solution of (10) has obviously the form $f_0(k_0) \propto k_0(2k_0 - \mathcal{E}_0)^{-1}$ and the energy eigenvalue of the Cooper state is determined by the following equation,

$$1 = \frac{g}{m^4} \int_0^\Lambda \frac{k_0^4 dk_0}{2k_0 - \mathcal{E}_0} \quad , \tag{11}$$

where $g = (16/15)\pi^2\alpha^2$. Equation (11) shows that there exists negative energy eigenvalue state $(\mathcal{E} < 0)$ when

$$g > g_0 \equiv 2m^4 \left[\int_0^\Lambda dk_0 k_0^3\right]^{-1} = 8(m/\Lambda)^4 \quad . \tag{12}$$

This means that when $\alpha > \alpha_c = \sqrt{15/2}\,\pi\,(m/\Lambda)^2$, the normal vacuum of massive QED becomes unstable with respect to the formation of the photon pair (Cooper instability) and the new condensed vacuum is realized after the condensation of these pairs.

The above results agree qualitatively with those obtained in ref.3) using the B-S equation.

(ii) Photon pairing under the electric field.

By assuming that $\mathbf{E}$ is small, we solve (9) perturbatively in $\mathbf{E}$. We expand the energy $\mathcal{E}$ of the Cooper state and the weight function $f(\mathbf{k})$ with respect to $\mathbf{E}$,

$$\begin{aligned} \mathcal{E} &= \mathcal{E}_0 + \mathcal{E}_1|\mathbf{E}|^2 + \dots \quad , \\ f(\mathbf{k}) &= f_0(k_0) + f_1(k_0)(\mathbf{k}\cdot\mathbf{E})^2 + f_2(k_0)|\mathbf{E}|^2 + \dots \quad . \end{aligned} \tag{13}$$

$\mathcal{E}_0$ and $f_0(k_0)$ satisfy (10), of cource. The energy shift $\mathcal{E}_1|\mathbf{E}|^2$ caused by the external electric field is calculated easily and is given by,

$$\mathcal{E}_1|\mathbf{E}|^2 = -\frac{32}{3}(a+b)\left[\int_0^\Lambda \frac{k_0^5 dk_0}{(2k_0-\mathcal{E}_0)^2}\right]\left[\int_0^\Lambda \frac{k_0^4 dk_0}{(2k_0-\mathcal{E}_0)^2}\right]^{-1} < 0 \ . \tag{14}$$

Equation (14) means that if we consider up to $O(|\mathbf{E}|^2)$, the energy eigenvalue of the Cooper state becomes smaller, therefore the critical coupling α_c' for $\mathbf{E} \neq 0$ has the tendency to become smaller than α_c for $\mathbf{E} = 0$. (Fig.1)

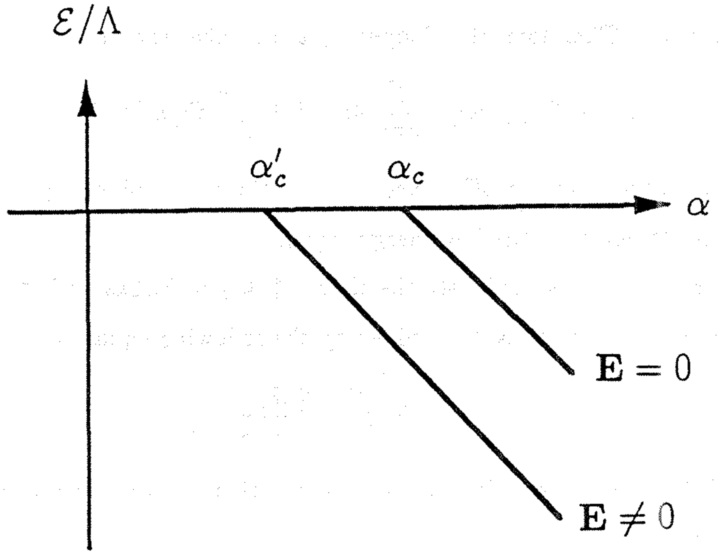

Fig.1 The relation between the energy $\mathcal{E}/\Lambda$ and the fine structure constant α in the absence of the electric field $(\mathbf{E} = 0)$ and under the presence of the electric field $(\mathbf{E} = 0)$.

4. Comments

In this talk, we have seen the Cooper instability of photon can occur in the massive QED. Therefore, the next task, which is of great interest, is to consider the characteristics of the stable condensed vacuum just as in the theory of the superconductivity. Since many observable phenomena in this strong coupling phase would be dependent upon the condenced nature of the vacuum, the formulation has to be established just like the Bogoliubov transformation in the superconductor theory, to discuss the characteristics of this phase.

Especially in connection with the chiral symmetry breaking, it is also very interesting whether the photon pairing phenomenon occurs in *massless* QED because this theory is very unstable in the infrared region. If this phenomenon occurred, what would the critical coupling be?

Although our results bring us a hope for explaining the anomalous GSI event, the photon pairing under the strong electromagnetic field should be studied of cource before applying our conclusions to the real experiment.

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A Search for correlated e^+e^- pairs in the decay of ^{241}Am

T. Asanuma, M. Minowa, T. Tsukamoto, S. Orito, and T. Tsunoda
Department of Physics, Faculty of Science, University of Tokyo, Tokyo 113, Japan

ABSTRACT

Correlated electron-positron pairs are searched for in α-decay of ^{241}Am using a pair of germanium detectors and plastic scintillation counters. A stringent upper limit of 1.5×10^{-9}/α-decay is obtained at the 95% confidence level to the e^+e^- pair of invariant mass above 1.4MeV/c^2, which might come from hypothetical neutral particles produced in the α decay of ^{241}Am.

The final result of this experiment has been published in ref.[1]. Please refer to it for the detail.

Referrence

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A Search for Correlated e^+e^- pairs in the fission process

T. Tsunoda, S. Nakamura, M. Minowa, and S. Orito

Department of Physics
Faculty of Science
University of Tokyo, 113 Japan

abstract

Correlated electron-positron pairs are searched for in fission decay process of ^{252}Cf source. A stringent upper limit of (9-20)$\times 10^{-9}$ /fission(depending on the mass) is obtained at the 95% confidence level to the e^+e^- pair of invariant mass above 40 MeV/c^2, which could come from hypothetical neutral particles produced in the fission process of ^{252}Cf.

Recent observations of peculiar narrow peaks[1] in the spectrum of electrons and positrons emitted from heavy-ion-collisions at Gesellschaft für Schwerionenforschung(GSI) promoted a series of searches for hypothetical neutral particles in e^+e^- collisions[2,3], and e^+-nucleus collisions[4,5,6]. While resonance searches in the e^+e^- interactions all gave essentially negative results, two experiments at Oak Ridge National Lab. by Erb et al.[4] and at Institute for Nuclear Study of the University of Tokyo(INS) by Sakai et al.[5] gave indications of a similar peak as observed at GSI in the energy spectra of final state electrons or positrons emitted in the e^+-U and e^+-Th collisions. We also became aware of a nuclear emulsion study[7] where e^+e^- pairs were reported. In independent analysis[8,9] three event clusters appeared around masses of 1.2, 2.1 and 9.2 MeV/c^2. These results imply that the origin of the narrow spectrum is not a simple neutral particle, but is closely related to the dynamical motions of nuclei in the collision. Peccei et al.[10] suggested that such dynamical motions of nuclei could produce a new phase of strong coupling of QED, and the exotic particle in this new phase might decay into e^+e^- pair. And the data of emulsion study suggest that this particle might have excited states.

A spontaneous fission process (of e.g.^{252}Cf) is very similar to the heavy-ion-collisions in the following points;

1. Heavy nucleus (Total Z=98) breaks up like the heavy-ion-collisions just after the collisions occur.
2. $\Delta Q \cong 200$MeV. Each fission fragment accelerates to high relative velocity ($\beta \approx .08$) in 10^{-21}sec. This time span is similar to the time span of the heavy-ion-collisions.

This process is illustrated in Fig.1

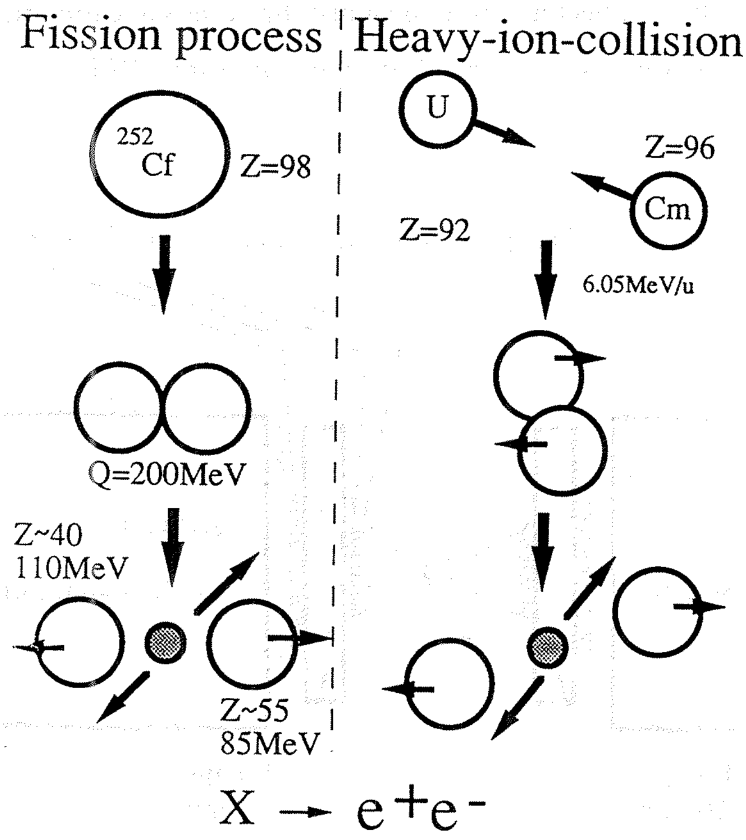

Fig.1 Fission process and heavy-ion-collision

We searched for back-to-back e^+e^- pairs which might be emitted from the decay of the postulated bound state or, more generally, neutral particle (X) during the fission of ^{252}Cf source. Our experimental set up is sketched in Fig.2. Electrons and positrons are detected by four layers of plastic scintillation counters of thickness 1mm, a 10mm thick lucite Čerenkov counter, and a lead glass counter of 4.7 radiation length on each side of the source. Electrons and positrons of 20-100 MeV(total mass equal to 40-200 MeV/c^2) can be detected by lead glass counters. The linearity and the efficiency of these detectors were obtained by a beam calibration at Institute for Nuclear Study of the University of Tokyo(INS).

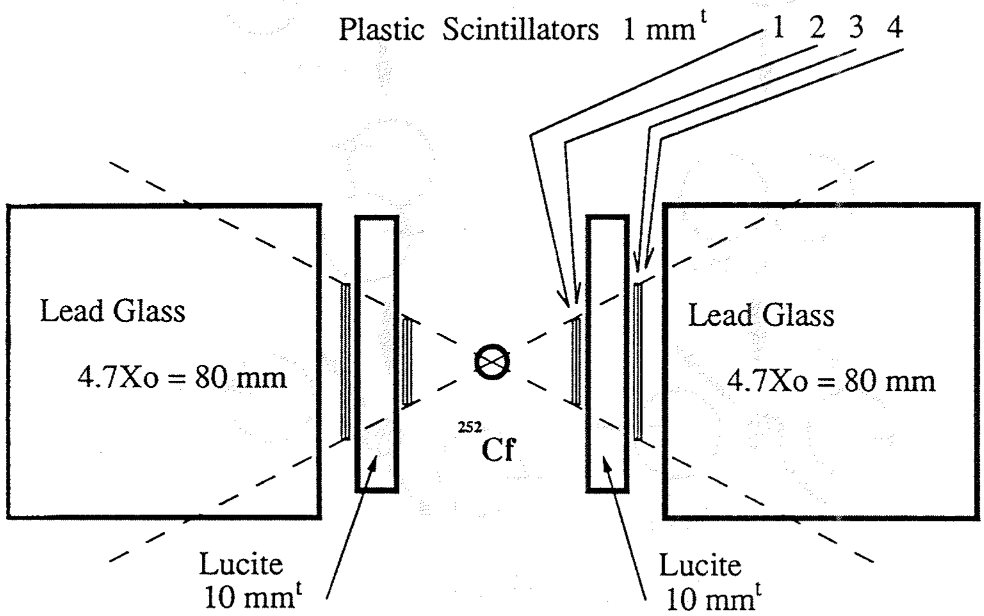

Fig.2 Set up

The data were accumulated for a total of 10^6 sec with a 2μCi ^{252}Cf source. No candidate with E_e>10 MeV was observed. Taking into account the detectors' acceptance, we have obtained a stringent upper limit of (9-20)×10^{-9}/fission (depending on the mass as is shown in Fig.3) at the 95% confidence level to the probability of the production and e^+e^- decay of the hypothetical neutral particle with mass above 40 MeV/c^2.

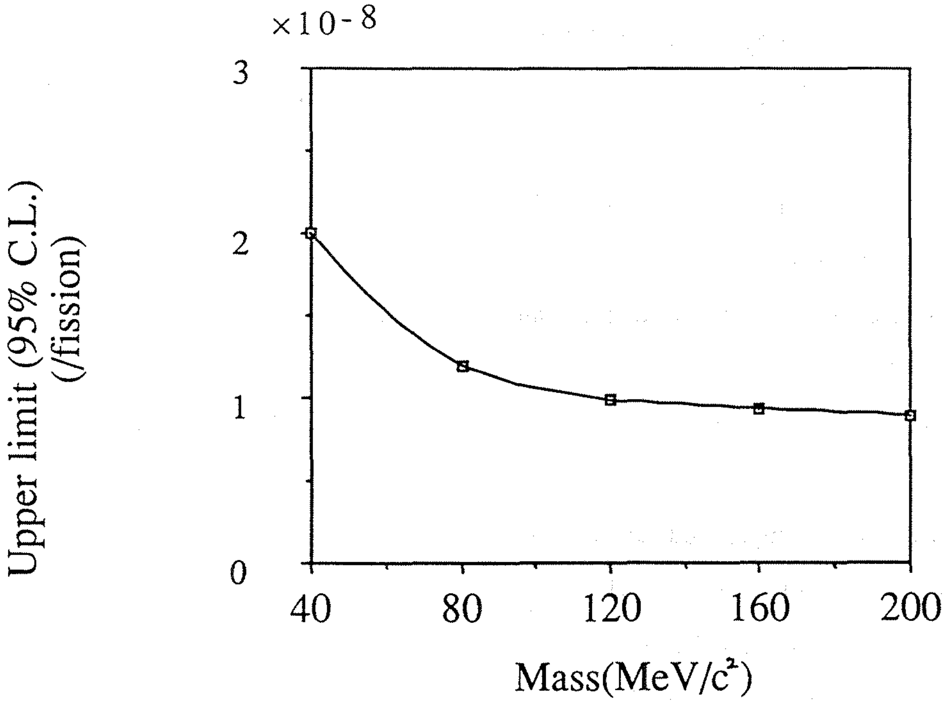

Fig.3 Upper limit to the production and e^+e^- decay of the hypothetical neutral particle

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STRONG COUPLING QED AND GAMMA-RAY BURSTS

Tomoyuki Hanawa

Department of Astrophysics, Nagoya University

Abstract

This paper reviews observations of gamma-ray bursts (GRB) and their models. GRB is the phenomenon that γ-ray emission from a certain direction increases suddenly for a short duration of 48 ms to 1000 s. The GRB sources have not been identified yet with known celestial objects. The theories of γ-ray bursts are still controvertial and one of them is based on the theory of the phase transition of QED under strong electromagnetic fields.

1. Introduction

Gamma-ray burst (GRB) is still a mystery in astrophysics although it has passed more than 15 years since their discovery. No celestial object has been identified with a GRB source. No confident theory has been proposed for the GRB production. GRB may involve several different types of phenomena.

This paper summarizes observations of GRB and describes the constraints derived from the observations briefly. Several models of GRB are introduced with emphasis on the model by Accetta, Caldi and Chodos[1)], where γ-rays are produced by the phase transition of QED in a strongly magnetized neutron star. See the references[2)] listed at the end of this paper for further study on GRB.

2. Observations of GRB

Gamma-ray bursts are detected with γ-ray and x-ray detectors on board orbiting satellites. A typical GRB spectrum is continuous and ranges from several keV to 100 MeV. Absorption and emission features are reported for several GRBs. Absorption lines[3)] are found around $E = 20$ keV and 40 keV and intepreted as cyclotron absorption by strong magnetic fields of several 10^{12} G. A possible emission feature around 400 keV[4)] is thought to be a gravitationally redshifted e^+ e^- annihilation line. Both the features imply that a GRB source has strong gravity and is likely to be a neutron star.

The duration of GRB ranges 48 ms to 1000 s and its rise time from 10 ms to 1 s. Pulsations are found in the time histories of several GRBs. These short timescale variations give a constraint that GRB sources should be compact ($r < c\delta t$). Again a neutron star is a probable GRB source candidate.

The spatial direction of a GRB source is determined by simultaneous observation of GRB with more than three satellites. GRB arrival time depends on the direction of the GRB source and the position of the observing satellite. The celestial position of GRB source is derived from the difference in the arrival time. No optical counterpart has been found in the region specified by the arrival time analysis. GRB sources are distributed isotropically and have no spatial concentrations. This isotropc distribution suggests at leaset three possibilities that GRB sources are located (1) in the disk of our galaxy, (2) in the halo of our galaxy, or (3) in the cosmological distance. The first possibility is the most conservative among the three and the distance to a GRB source is shorter than 1 kpc if we take the first possibility.

3. GRB Models

The plausible hypothesis that GRB sources are strongly magnetized neutron stars populated in the galactic disk, is derived from the observational constraints shown in the previous section. Most of GRB models are based on this hypothesis and called TGNS (Tera Gauss Neutron Star) models. The model by Accetta, Caldi and Chodos[1)] is one of TGNS models.

TGNS models are classified into several groups according to the γ-ray production mechanism. The first generation TGNS models[5)] interprete GRB as the accretion of a solid object (comet) onto a neutron star. In their models γ-rays are produced by the gravitational energy release of the accreting object. Other TGNS models consider thermonuclear runaway on a neutron star[6)], starquake of a spinning neutron star[7)], and ejection of super dense material from a neutron star[8)], as GRB triggering mechanism.

In Accetta, Caldi and Chodos's model[1)] GRB is triggered by starquake. When a spinning neutron star is decelerated by the interaction with the surrounding, the deceleration is non-uniform and the neutron star is stressed. When the stress reaches a critical level, starquake happens and liberates the stress. By the starquake the phase transition from the ordinary phase to a new pahse of QED happens. In the new phase of QED the eigenstates are positronium-like $e^+ e^-$ bound states. Gamma-rays are produced when the new phase decays back to the ordinary phase.

All the GRB models are still speculative. It is a future problem to construct more sophisticated models of GRB.

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FERMION MASS GENERATION IN CHIRAL-SYMMETRIC GAUGE THEORIES

K. Nishijima

Research Institute for Fundamental Physics

Kyoto University, Kyoto 606

abstract

The possibility of generating fermion masses in chiral-symmetric gauge theories has been examined with the help of renormalization group equations.

1. Renormalization Group Equations

In discussing dynamical generation of fermion masses the most useful tool is the renormalization group (RG) method, and we shall briefly recapitulate essential features of this method.

Green's functions in gauge theories with exponentiated mass-insertion[1)] are defined by

$$G^{(n,m)}(x,...,y,...,z,...:K) \\ =<0|T[\psi(x)...\overline{\psi}(y)...\phi_\lambda(z)...\exp\left(iKm\int d^4uS(u)\right)]|0>, \tag{1.1}$$

where the scalar density S is bilinear in the fermion fields and is normalized by

$$<p|S|p>=\overline{u}(p)u(p). \tag{1.2}$$

$u(p)$ and $\overline{u}(p)$ denote the Dirac spinors of the fermion fields corresponding to a single fermion state $|p>$ of momentum p. Also, for a given local operator $A(w)$ we can define Green's functions of the form:

$$A^{(n,m)}(w;x,...,y,...,z,...:K) \\ =<0|T\left[A(w)\psi(x)...\overline{\psi}(y)...\phi_\lambda(z)...\exp(iKm\int d^4uS(u))\right]|0>. \tag{1.3}$$

These Green's functions satisfy homogeneous Callan-Symanzik (CS) equations of the form:[1)]

$$(\mathcal{D} + n\gamma_F + m\gamma_V)G^{(n,m)}(...;g,m,\alpha:K) = 0, \tag{1.4}$$

where n and m denote the numbers of the fermion fields and of the gauge fields in the T-product, respectively, and

$$\mathcal{D} = m\frac{\partial}{\partial m} + \beta\frac{\partial}{\partial g} - 2\alpha\gamma_V\frac{\partial}{\partial\alpha} - \big(1 + (1-\gamma_S)K\big)\frac{\partial}{\partial K}. \tag{1.5}$$

γ_F, γ_V and γ_S denote, respectively, the anomalous dimensions of the operators ψ, ϕ_λ and S.

For practical purposes it is convenient to change the set of parameters from g, m, α and K to g, m, α and m_R, where m_R is called the effective mass and is defined by[2)]

$$m_R = m\big(1 + KB^{-1}(g)\big), \tag{1.6}$$

where $B(g)$ is characterized by the following equation:

$$\beta\frac{dB}{dg} + (1-\gamma_S)B = 1. \tag{1.7}$$

In terms of the new set of parameters the differential operator $\mathcal{D}$ assumes the following form:[3)]

$$\mathcal{D} = m\frac{\partial}{\partial m} + \beta\frac{\partial}{\partial g} - 2\alpha\gamma_V\frac{\partial}{\partial\alpha} - \gamma_\theta m_R\frac{\partial}{\partial m_R}, \tag{1.8}$$

where γ_θ is related to B through

$$1 + \gamma_\theta = B^{-1}. \tag{1.9}$$

2. Chiral Symmetry

In classical gauge theories the concept of chiral symmetry is equivalent to the vanishing of the bare fermion mass m_0. The classical Dirac equation for the fermion

$$\gamma_\mu(\partial_\mu - ig\frac{\lambda^a}{2}\phi^a_\mu)\psi + m_0\psi = 0 \tag{2.1}$$

is invariant under the chiral transformation

$$\psi \to \exp(i\alpha\gamma_5)\cdot\psi, \tag{2.2}$$

provided that the bare mass m_0 vanishes,

$$m_0 = 0. \tag{2.3}$$

The equivalence ceases to be valid in quantum theory, however, because of the divergent character of the theory. Indeed, in quantum field theory we often encounter a tricky relation:

$$0 \times \infty = \text{finite}, \tag{2.4}$$

which is reminiscent of anomalies characteristic of quantum field theory. We shall elucidate on this point in QCD.

Let us decompose the quark propagator as

$$S_F(x) = -(\gamma\cdot\partial)\Delta_1(x) + m\Delta_2(x), \tag{2.5}$$

and let us assume a spectral representation of $\Delta_i(x)$ as

$$\Delta_i(x) = \int d\mu^2\rho_i(\mu^2)\Delta_F(x,\mu^2), \quad (i=1,2) \tag{2.6}$$

where $\Delta_F(x,\mu^2)$ denotes the free propagator for mass μ. Then the physical mass m is related to the bare mass m_0 through[2)]

$$m_0\int d\mu^2\rho_1(\mu^2) = m\int d\mu^2\rho_2(\mu^2), \tag{2.7}$$

provided that $<\phi^a_\mu>=0$. In the Landau gauge the RG analysis leads us to the following relationships[2)] in QCD:

$$\int d\mu^2\rho_1(\mu^2) = Z_2^{-1}(g), \quad \text{finite}, \tag{2.8}$$

$$\int d\mu^2 \rho_2(\mu^2) = 0. \tag{2.9}$$

Then we can deduce on the basis of Eqs.(2.7), (2.8) and (2.9) that

$$m_0/m = 0. \tag{2.10}$$

This result poses a serious doubt on the equivalence between chiral symmetry and the vanishing bare mass. Suppose that the physical mass m is finite, then Eq.(2.10) implies $m_0 = 0$, and consequently the resulting theory should be chiral-symmetric no matter how we choose the physical mass if we should insist on the classical equivalence. This sounds very unlikely, however. We must admit, therefore, that the classical equivalence should be broken by quantum corrections, and we must look for a proper definition of chiral symmetry expressed in terms of renormalized quantities alone.

3. Dynamical Breakdown of Chiral Symmetry

In an attempt to define chiral symmetry we propose to define it by the existence of an axial-vector current X_λ satisfying the following two conditions:

$$\partial_\lambda X_\lambda = 0, \tag{3.1}$$

and the equal-time commutation relations (ETCR):

$$\delta(x_0 - y_0)[X_0(x), \psi(y)] = -\gamma_5 \psi(y) \delta^4(x-y), \tag{3.2a}$$

$$\delta(x_0 - y_0)[X_0(x), \overline{\psi}(y)] = -\overline{\psi}(y) \gamma_5 \delta^4(x-y). \tag{3.2b}$$

In what follows we shall look for the condition for the existence of such a current in QCD. For this purpose we introduce some unrenormalized expressions first.

$$\begin{aligned} A_\lambda^{(0)} &= i\overline{\psi}^{(0)} \gamma_\lambda \gamma_5 \psi^{(0)}, \\ P^{(0)} &= i\overline{\psi}^{(0)} \gamma_5 \psi^{(0)}, \\ S^{(0)} &= \overline{\psi}^{(0)} \psi^{(0)}. \end{aligned} \tag{3.3}$$

Then we have

$$\partial_\lambda X_\lambda^{(0)} = 2m_0 P^{(0)}. \tag{3.4}$$

A flavor-changing current $X_\lambda^{(0)}$ can be identified with $A_\lambda^{(0)}$, but for a flavor-conserving current as (3.3) we have to identify $X_\lambda^{(0)}$ with the following combination:[4]

$$X_\lambda^{(0)} = A_\lambda^{(0)} - C_\lambda^{(0)}, \tag{3.5}$$

where $C_\lambda^{(0)}$ denotes the Chern-Simons term whose explicit form is irrelevant in what follows. The current satisfies the ETCR (3.2) so that we may assume non-renormalization of $X_\lambda^{(0)}$, and it may be identified with the renormalized one:

$$X_\lambda = X_\lambda^{(0)}. \tag{3.6}$$

The scalar density is renormalized as in Eq.(1.2), but the pseudoscalar density will be renormalized by

$$m_0 P^{(0)} = mP, \tag{3.7}$$

in conformity with the renormalization prescription adopted by Adler and Bardeen.[5] Then we have

$$\partial_\lambda X_\lambda = 2mP. \tag{3.8}$$

Here we have assumed that all flavors of quarks carry the same physical mass for simplicity. This is not the only way of renormalizing $P^{(0)}$, and we shall introduce an alternative prescription in what follows.

The unrenormalized currents satisfy the following ETCR:

$$\delta(x_0 - y_0)[X_0(x), S^{(0)}(y)] = 2iP^{(0)}(y)\delta^4(x-y), \tag{3.9}$$

so that its renormalized version is given by

$$\delta(x_0 - y_0)[X_0(x), S(y)] = 2ibP(y)\delta^4(x-y), \tag{3.10}$$

where

$$P^{(0)} = Z_P P, \quad S^{(0)} = Z_S S, \quad b = Z_P Z_S^{-1}. \tag{3.11}$$

Now we introduce $\tilde{P}$ by

$$\tilde{P} = bP, \quad P^{(0)} = Z_S \tilde{P}. \tag{3.12}$$

The RG equation for b in the Landau gauge reads as

$$\Big(\beta \frac{d}{dg} + \gamma_S - \gamma_P\Big) b = 0, \tag{3.13}$$

and the renormalization prescription (3.7) gives[4)]

$$b = 1 - \gamma_P. \tag{3.14}$$

We can easily find that this b is related to B, with the help of Eq.(1.7), through

$$B = b^{-1}, \tag{3.15}$$

and

$$\gamma_\theta = -\gamma_P. \tag{3.16}$$

Combining these relationships we finally arrive at

$$\partial_\lambda X_\lambda = 2mB(g)\tilde{P}. \tag{3.17}$$

A relationship indicating the anomalous character of the theory is illustrated by

$$mB(g) = m_0 Z_S. \tag{3.18}$$

The bare mass m_0 is zero and Z_S is divergent, whereas the *l.h.s.* is finite in general, so that this relationship is an avatar of the anomalous relation (2.4).

Thus, in order for the theory to be chiral-symmetric Eq.(3.1) must be satisfied. This in turn implies

$$mB(g) = 0. \tag{3.19}$$

There are two ways of satisfying Eq.(3.19), namely, either $m = 0$ or $B(g) = 0$. In the former case the fermion is massless and the theory is trivially chiral-symmetric, but in the latter case chiral symmetry is dynamically broken thereby generating the NG boson.

Next, by starting from Eq.(3.8) and the ETCR (3.2) we can derive the Ward-Takahashi identity in an obvious notation:

$$i(p-q)_\lambda \Gamma_{5\lambda}(p,q) = -2m\Gamma_5(p,q) + S_F^{-1}(p)\gamma_5 + \gamma_5 S_F^{-1}(q), \tag{3.20}$$

where p and q are outgoing and incoming momenta, respectively. In the lowest order perturbation theory, $\Gamma_{5\lambda}(p,q), \Gamma_5(p,q)$ and $S_F^{-1}(p)$ reduce to $\gamma_\lambda\gamma_5$, γ_5 and $(ip\cdot\gamma + m)$, respectively.

Let us assume that $m \neq 0$ and $B(g) = 0$, corresponding to Eq.(3.1), so that $\Gamma_5 = 0$ in Eq.(3.20), and then let us take the limit $g \to p$ to find

$$\lim_{q\to p} i(p-q)_\lambda \Gamma_{5\lambda}(p,q) = \{\gamma_5, S_F^{-1}(p)\} \tag{3.21}$$

The non-vanishing of the *r.h.s.* for $m \neq 0$ implies the existence of a massless pole, in the vertex $\Gamma_{5\lambda}(p,q)$, of the form

$$\gamma_5(p-q)_\lambda/(p-q)^2, \tag{3.22}$$

indicating generation of a massless NG boson.

4. The Schwinger-Dyson Equation

When $mB(g) \neq 0$, the theory is *not* chiral-symmetric. First, we shall study the Schwinger-Dyson (SD) equation in this case. Then the massless NG boson is absent, and

Eq.(3.20) reduces, in the limit $q \to p$, to

$$\{\gamma_5, S_F^{-1}(p)\} = 2m\Gamma_5(p,p). \tag{4.1}$$

The vertex function $\Gamma_5(p,p)$ satisfies a Bethe-Salpeter (BS) equation of the following form:

$$\Gamma_5(p,p) = Z_2 Z_P^{-1}\gamma_5 + \int d^4p' K(p,p')\Gamma_5(p',p'), \tag{4.2}$$

where spinor indices have been suppressed. By combining Eqs.(4.1) and (4.2) we also have a BS equation for the *l.h.s* of Eq.(4.1):

$$\{\gamma_5, S_F^{-1}(p)\} = 2m_0 Z_2\gamma_5 + \int d^4p' K(p,p')\{\gamma_5, S_F^{-1}(p')\}. \tag{4.3}$$

Because of the divergent character of the theory we find[2)]

$$Z_P^{-1} = 0, \quad m_0 = 0, \quad Z_2 = \text{finite}, \tag{4.4}$$

and both Eqs.(4.2) and (4.3) reduce to homogeneous ones.

The so-called SD equation is then given by

$$\{\gamma_5, S_F^{-1}(p)\} = \int d^4p' K(p,p')\{\gamma_5, S_F^{-1}(p')\}. \tag{4.5}$$

Although m_0 has been put equal to zero, the system described by this equation is *not* chiral-symmetric. In what follows we shall study the behavior of $\{\gamma_5, S_F^{-1}(p)\}$ for large values of p^2 with the help of the RG equation:

$$(\mathcal{D} + \gamma_P - 2\gamma_F)\Gamma_5(p,p;g,m,m_R) = 0. \tag{4.6}$$

The anomalous dimensions are given in QCD by

$$\begin{aligned}
\beta(g) &= -\frac{b}{2}g^3 + \ldots \qquad &; \; b = \frac{1}{24\pi^2}(33 - 2N_f),\\
\gamma_S(g) &= -cg^2 + \ldots \qquad &; \; c = 1/2\pi^2,\\
\gamma_\theta(g) &= -\gamma_P(g),\\
\gamma_F(g) &\sim O(g^4),
\end{aligned} \tag{4.7}$$

in the Landau gauge. By solving Eq.(4.6) we find the asymptotic form of Γ_5 for large values of p^2 as

$$\Gamma_5(p,p;g,m,m_R) \sim \gamma_5 B(g) Z_2(g) C(g) \Big(\ell n\ \frac{p^2}{m^2}\Big)^{-c/b}, \tag{4.8}$$

and consequently

$$\{\gamma_5, S_F^{-1}(p)\} \sim 2m\gamma_5 B(g) Z_2(g) C(g) \Big(\ell n\ \frac{p^2}{m^2}\Big)^{-c/b}. \tag{4.9}$$

This corresponds to Lane's G_+ solution.[6]

Next we shall study what will become of the NG boson when $mB(g) \neq 0$. The operator $\tilde{P}$ is BRS invariant, and we expect

$$< 0|\tilde{P}(x)\tilde{P}(y)|0 > \neq 0. \tag{4.10}$$

When color confinement is realized, composite hadron states saturate the intermediate states.[7] In particular, we pick out a single particle state $|\pi>$ satisfying

$$< 0|\tilde{P}(x)|\pi > \neq 0. \tag{4.11}$$

Then Eq.(3.17) implies $< 0|X_\lambda(x)|\pi > \neq 0$ and we may put

$$< 0|X_\lambda(x)|\pi > = \frac{2}{M(g)}\partial_\lambda < 0|\tilde{P}(x)|\pi >, \tag{4.12}$$

which defines the proportionality constant $M(g)$. Combination of Eqs.(3.17) and (4.12) yields

$$(\Box - \mu^2) < 0|\tilde{P}(x)|\pi > = 0, \tag{4.13}$$

where

$$\mu^2 = mM(g)B(g). \tag{4.14}$$

Thus, in general, we have a massive pseudoscalar bound state instead of the massless NG boson unless $B(g) = 0$. We shall study the high p^2 behavior of the BS amplitude

$$< 0|T\Big[\psi_\alpha\Big(\frac{x}{2}\Big)\overline{\psi}_\beta\Big(-\frac{x}{2}\Big)\Big]|\pi> . \tag{4.15}$$

For this purpose we introduce the operator product expansion and assume that it is dominated by the pseudoscalar term:

$$T\Big[\psi_\alpha\Big(\frac{x}{2}\Big)\overline{\psi}_\beta\Big(-\frac{x}{2}\Big)\Big] \sim f(x)(\gamma_5)_{\alpha\beta}\tilde{P}(0) + ..., \tag{4.16}$$

so that we have

$$< 0|T\Big[\psi_\alpha\Big(\frac{x}{2}\Big)\overline{\psi}_\beta\Big(-\frac{x}{2}\Big)\Big]|\pi> \sim f(x)(\gamma_5)_{\alpha\beta} < 0|\tilde{P}(0)|\pi> . \tag{4.17}$$

The RG equation for $f(x)$ is given by

$$(\mathcal{D} + 2\gamma_F - \gamma_S)f(x) = 0, \tag{4.18}$$

and the high p^2 behavior of $f(p)$, Fourier transform of $f(x)$, is given by

$$(p^2)^2 f(p) \sim \Big(\ell n\ \frac{p^2}{m^2}\Big)^{c/b} \tag{4.19}$$

The amputated BS amplitude $\tau(p)$ for $p^2 \to \infty$ is related to $f(p)$ through

$$\gamma_5 f(p) \sim S_F(p)\gamma_5\tau(p)S_F(p), \tag{4.20}$$

so that we have for $p^2 \to \infty$ the asymptotic form of $\tau(p)$ as

$$\tau(p) \sim \frac{1}{p^2}\Big(\ell n\ \frac{p^2}{m^2}\Big)^{c/b} \tag{4.21}$$

This corresponds to Lane's G_- solution.[6)]

So far we have assumed $mB(g) \neq 0$ and have obtained massive pseudoscalar bound state, but what will happen when $m \neq 0$ and $B(g) = 0$? Let us assume that $B(g)$ vanishes for $g = g_X$,

$$B(g_X) = 0, \tag{4.22}$$

then Eq.(4.14) indicates that the massive pseudoscalar boson reduces to the massless NG boson. Causality implies

$$\mu^2 = mB(g)M(g) \geq 0. \tag{4.23}$$

Since $B(g)$ changes its sign at $g = g_X$, so does $M(g)$, too, by causality. Namely, we have

$$M(g_X) = 0, \tag{4.24}$$

and hence Eq.(4.12) leads us to

$$< 0|\tilde{P}(x)|\pi >= 0, \quad \text{for} \quad g = g_X. \tag{4.25}$$

In this case we are aware that the *r.h.s.* of Eq.(4.17) vanishes, and Eq.(4.17) must be modified as

$$< 0|T\Big[\psi_\alpha\Big(\frac{x}{2}\Big)\overline{\psi}_\beta\Big(-\frac{x}{2}\Big)\Big]|\pi >\sim h(x)(\gamma_\lambda\gamma_5)_{\alpha\beta}, < 0|X_\lambda(0)|\pi > . \tag{4.26}$$

Non-renormalization of X_λ, expressed by Eq.(3.6), implies

$$\gamma_X = 0, \tag{4.27}$$

and the RG equation for $h(x)$ is given by

$$(\mathcal{D} + 2\gamma_F)h(x) = 0. \tag{4.28}$$

The high p^2 behavior of $h(p)$ is then given by

$$(p^2)^2 h(p) \sim \text{const.} \tag{4.29}$$

The amputated BS amplitude $\sigma(p)$ defined by an equation similar to Eq.(4.20) behaves for large p^2 as

$$\sigma(p) \sim \frac{\text{const}}{p^2}. \tag{4.30}$$

$\sigma(p)$ is proportional to Eq.(3.21) so that we find

$$\{\gamma_5, S_F^{-1}(p)\} \sim \frac{\text{const}}{p^2}, \quad \text{for} \quad g = g_X. \tag{4.31}$$

This is quite distinct from Eq.(4.9).

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STABILITY AT THE ORIGIN IN (2+1)-DIMENSIONAL QED

Takayuki Matsuki

Tsukuba Institute of Science and Technology

1601 Kamitakatsu, Tsuchiura, 300 Japan.

Abstract

Stability at the origin in (2+1)-dimensional QED is studied in the leading order of the $1/N$ expansion with N four-component Dirac fermions. It is found that there are two critical flavor numbers: one is for fermion self-energy and another for wave-function renormalization. It is shown that the effective potential in the direction of fermion wave-function renormalization is always unstable for any flavor number N, which reconfirms that chiral symmetry is broken for any N.

1. Introduction

Quantum Electrodynamics in 2+1 dimensions (QED_3) has been extensively studied by many people[1~7] with a hope that this might be another example having a nontrivial critical behavior as (3+1)-dimensional QED does.

The existence of a critical flavor number in this model was first suggested in Refs. 3) and 4) claiming that the leading order terms in $1/N$ are kept intact. This conclusion seemed to be also supported by the Monte Carlo calculation.[5] On the other hand there appeared a couple of papers in which it was claimed that chiral symmetry is always broken for any N, i.e., there is no critical flavor number at all by taking fermion wave-function renormalization into account properly.[6,7]

In this paper I will show that the latter statement is true by studying stability at the origin in QED_3. That is, a quasi-origin, which is defined to be fermion self-energy $B(p) = 0$ and wave-function renormalization $A(p) = 1$, is always unstable and hence chiral symmetry is broken for any N. An unstable quasi origin is attributable to instability of the effective potential for any N in the direction of fermion wave-function renormalization. There is the same critical flavor number in the direction of fermion self-energy as that in Refs. 3) and 4).

2. Stability at the origin in QED_3

In the following I will apply to QED_3 the effective potential proposed by Cornwall, Jackiw and Tomboulis (CJT)[8] but the results would not change if an alternative effective potential would be used (see, for instance, Ref. 9). Let us consider the case of N fermion flavors in the Landau gauge. Then the CJT effective potential defined as

$$V_{CJT} = -i \int (dp) \mathrm{Tr} \left[\ln S_0^{-1}(p) S(p) - S_0^{-1}(p) S(p) + 1 \right] + \frac{ig^2}{2} \int (dp) \int (dq) \mathrm{Tr} \left[S(p) \gamma^\mu S(q) \Gamma^\nu \right] D_{\mu\nu}(p-q), \tag{1}$$

is calculated to be

$$\begin{aligned} V_{CJT} = & \frac{2N}{\pi^2} \int_0^\infty dp p^2 \frac{A(p)\left(A(p)-1\right)p^2 + B^2(p)}{A^2(p)p^2 + B^2(p)} \\ & - \frac{\tilde{\alpha}}{\pi^4} \int_0^\infty p dp \int_0^\infty q dq \frac{1}{A^2(p)p^2 + B^2(p)} \frac{1}{A^2(q)q^2 + B^2(q)} G(p,q) \\ & \times \left[-4B(p)B(q) \ln \left(\frac{p+q+\tilde{\alpha}}{|p-q|+\tilde{\alpha}} \right) + A(p)A(q)I(p,q) \right], \end{aligned} \tag{2}$$

where

$$S_0(p) = \frac{i}{\not{p}}, \quad S(p) = \frac{i}{A(p)\not{p} - B(p)}, \quad \tilde{\alpha} = \frac{e^2 N}{8}, \tag{3}$$

$$\begin{aligned} I(p,q) = & 2pq - \tilde{\alpha}(p+q-|p-q|) + \tilde{\alpha}^2 \ln \frac{p+q+\tilde{\alpha}}{|p-q|+\tilde{\alpha}} \\ & - \frac{1}{\tilde{\alpha}} \left| p^2 - q^2 \right| (p+q-|p-q|) + \frac{1}{\tilde{\alpha}^2} \left(\ln \frac{p+q+\tilde{\alpha}}{|p-q|+\tilde{\alpha}} - \ln \frac{p+q}{|p-q|} \right), \end{aligned} \tag{4}$$

and the form of the vertex function is assumed to be

$$\Gamma_\mu = \gamma_\mu G(p,q) = \gamma_\mu \left[A(p)\theta(p-q) + A(q)\theta(q-p) \right]. \tag{5}$$

Since spontaneous symmetry breakdown is a phenomenon in a global configuration space, study of infrared behavior of physical quantities in the momentum space is necessary and enough. Henceforth I will use the following approximation which leaves only terms dominant in the infrared region:

$$\ln \frac{p+q+\tilde{\alpha}}{|p-q|+\tilde{\alpha}} = 2\theta(p-q)\left[\frac{q}{\tilde{\alpha}} - \frac{pq}{\tilde{\alpha}^2} + \frac{1}{3}\left(\frac{q}{\tilde{\alpha}}\right)^3 + \frac{p^2 q}{\tilde{\alpha}^3}\right] + 2\theta(q-p)\left[\frac{p}{\tilde{\alpha}} - \frac{pq}{\tilde{\alpha}^2} + \frac{1}{3}\left(\frac{p}{\tilde{\alpha}}\right)^3 + \frac{pq^2}{\tilde{\alpha}^3}\right]. \tag{6}$$

The Schwinger-Dyson equation for QED$_3$ is decomposed into

$$A(p) = 1 + \frac{8}{3\pi^2 N}\frac{1}{p^3}\int_0^\infty \frac{dq}{A(q)q} G(p,q)\left[q^3\theta(p-q) + p^3\theta(q-p)\right] \tag{7}$$

and

$$B(p) = \frac{8}{\pi^2 N}\frac{1}{p}\int_0^\infty \frac{dq}{A^2(q)q} B(q)G(p,q)\left[q\theta(p-q) + p\theta(q-p)\right], \tag{8}$$

which are obtained by functionally differentiating the CJT effective potential, (2), and by retaining only terms of all order in $A(p)$ and linear in $B(p)$. All order terms in $A(p)$ are kept since the vacuum expectation value of $A(p)$ is not known *a priori*. When the effective potential is expanded around the stationary point which is obtained by solving the Schwinger-Dyson equation, one can study stability at the stationary point.

$$V_{CJT}\left[A(p), B(p)\right] = V_{CJT}|_{S.P.} + \frac{1}{2}\int dp \int dq \delta A(p) \left.\frac{\delta^2 V_{CJT}}{\delta A(p)\delta A(q)}\right|_{S.P.} \delta A(q) + \frac{1}{2}\int dp \int dq \delta B(p) \left.\frac{\delta^2 V_{CJT}}{\delta B(p)\delta B(q)}\right|_{S.P.} \delta B(q). \tag{9}$$

There appears no term like $\left.\frac{\delta^2 V_{CJT}}{\delta A(p)\delta B(q)}\right|_{S.P.}$ since it vanishes for the stationary point, $B(p) = 0$. It is a prerequisite for finding the stationary point to numerically solve the Schwinger-Dyson equation. Here I would like to show a qualitative feature of what happens in the vicinity of the stationary point and hence I will substitute the quasi-stationary point for the true one to estimate the second derivatives of the CJT effective potential. The quasi-stationary point adopted in this paper is defined to be

$$A_0(p) = 1, \qquad B_0(p) = 0. \tag{10}$$

Here only $A_0(p)$ deviates from the stationary point in the amount of order of $1/N$ and hence I would expect the results derived from this quasi-stationary point would not significantly differ from the one derived from the true origin. Then the second derivatives are given by

$$\left.\frac{\delta^2 V_{CJT}}{\delta A(p)\delta A(q)}\right|_{Q.S.P.} = -\frac{2N}{\pi^2}\left[p^2\delta(p-q) - \frac{8}{3\pi^2 N}\frac{1}{pq}\left\{q^3\theta(p-q) + p^3\theta(q-p)\right\}\right], \qquad (11)$$

$$\left.\frac{\delta^2 V_{CJT}}{\delta B(p)\delta B(q)}\right|_{Q.S.P.} = \frac{2N}{\pi^2}\left[\delta(p-q) - \frac{8}{\pi^2 N}\frac{1}{pq}\left\{q\theta(p-q) + p\theta(q-p)\right\}\right]. \qquad (12)$$

Therefore the second and the third terms in the expanded effective potential, (9), are given by

$$\frac{1}{2}\int dp \int dq\delta A(p) \left.\frac{\delta^2 V_{CJT}}{\delta A(p)\delta A(q)}\right|_{Q.S.P.} \delta A(q)$$
$$= -\frac{N}{2\pi^3}\int_{-\infty}^{\infty} du\delta\psi_A(-u)\frac{u^2+\omega_A^2}{u^2+\frac{9}{4}}\delta\psi_A(u), \qquad (13)$$

$$\frac{1}{2}\int dp \int dq\delta B(p) \left.\frac{\delta^2 V_{CJT}}{\delta B(p)\delta B(q)}\right|_{Q.S.P.} \delta B(q)$$
$$= \frac{N}{2\pi^3}\int_{-\infty}^{\infty} du\delta\psi_B(-u)\frac{u^2+\omega_B^2}{u^2+\frac{1}{4}}\delta\psi_B(u), \qquad (14)$$

where $t = \ln(p/\tilde{\alpha})$ and

$$\delta\psi_A(u) = \int dte^{-iut}e^{3t/2}\delta A(p), \quad \delta\psi_B(u) = \int dte^{-iut}e^{t/2}\delta B(p), \qquad (15)$$

$$\omega_A^2 = \frac{9}{4} - \frac{8}{\pi^2 N}, \quad \omega_B^2 = \frac{1}{4} - \frac{8}{\pi^2 N}. \qquad (16)$$

3. Discussions

Let us discuss stability at the quasi-stationary point by using the results derived in the former section. When $\omega_A^2 > 0$, the second term in (9) becomes negative definite and when $\omega_A^2 < 0$, it becomes indefinite. In either case, the potential becomes unstable in the A-direction even though it gives one critical flavor number $N_A = 32/9\pi^2$. On the other hand when $\omega_B^2 > 0$, the third term in (9) becomes positive definite and when $\omega_B^2 < 0$, it becomes indefinite, which means the potential becomes stable in the B-direction when

$N > N_B = 32/\pi^2$. This critical flavor number, N_B, is nothing but the one discovered in Refs. 3) and 4). In any case since the A-direction is always unstable, chiral symmetry is broken for any N. These critical flavor numbers are related to the convergence condition for solving the Schwinger-Dyson equation given by (7) and (8). This confirms the results of Ref. 6) in which numerical calculations showed the broken chiral symmetry for any N. Reference 7) also derived the same conclusion as Ref. 6) by analytically solving the approximate Schwinger-Dyson equation.

The accounts in this paper of chiral symmetry breakdown in QED_3 for any N using stability at the origin explain why the Monte Carlo calculation obtained the same conclusion as Refs. 3) and 4). It is because the effects of fermion wave-function renormalization are completely neglected and because the lattice calculation deals with only bare quantities.

I have used the quasi-stationary point to show instability of the effective potential in the direction of fermion wave-function renormalization. The details using the true origin will be published in a separate paper.[10]

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EFFECTIVE ACTION AND S MATRIX
— THE ON-SHELL EXPANSION —

M. Ukita

Department of Physics, Faculty of Science and Technology,
Keio University, Yokohama 223, Japan

abstract

The on-shell expansion of the effective action is discussed from a new standpoint and its close relationship to the S matrix is revealed. The coherent states naturally appear in the course of the argument and play an important role in understanding the equivalence between the on-shell expansion of the effective action and the generating functional of the S matrix elements.

1. Introduction

The effective action(EA) is both an important concept and a useful tool in quantum theory by which we can grasp the physical content of the quantum system by analogy with the classical action. Recently, we have derived in Ref. 1(called I hereafter) the on-shell expansion of the EA and discussed its relation to the S matrix elements. Here we consider the meaning of each step taken in I to understand the on-shell expansion more closely.

In Sec. 2, we review some results of I and consider the meaning of it in Sec. 3. Sec. 4 is devoted to the summary.

2. The On-Shell Expansion of the Effective Action

In this section, we will give some brief summary of the results of I which are connected with the present arguments. Throughout this article, we take the theory of the real scalar field Φ as an example, however, it is possible to extend the investigations here to other systems.

The generating functional $W[J]$ of the connected Green's function $W^{(n)}(x_1,x_2,\ldots,x_n)$ is defined by the functional integral,

$$\exp(iW[J]) \equiv \int D\Phi \exp(i\int d^4x[\mathcal{L}(\Phi)+J(x)\Phi(x)]), \tag{1}$$

where $\mathcal{L}(\Phi)$ is the Lagrangian and $J(x)$ is an external source. $W^{(n)}$ is obtained from $W[J]$ by the functional differentiation as follows:

$$W^{(n)}(x_1,x_2,...,x_n) = \left.\frac{\delta^n W[J]}{\delta J(x_1)\cdots\delta J(x_n)}\right|_{J=0}. \tag{2}$$

The EA $\Gamma[\phi]$ is defined by the Legendre transform of $W[J]$,

$$\Gamma[\phi] \equiv W[J] - \int d^4x J(x)\phi(x), \tag{3}$$

where $\phi(x) \equiv \delta W[J]/\delta J(x)$ and the proper vertex function or the one-particle-irreducible(1PI) Green's function $\Gamma^{(n)}(x_1,x_2,...,x_n)$ is given as

$$\Gamma^{(n)}(x_1,x_2,...,x_n) = \left.\frac{\delta^n \Gamma[\phi]}{\delta\phi(x_1)...\delta\phi(x_n)}\right|_{\phi(x)=\phi_0(x)}, \tag{4}$$

where $\phi_0(x)$ is the vacuum expectation value of the field operator $\hat{\Phi}$ and is determined by the stationary condition of the EA (or the equation of motion of ϕ),

$$\left.\frac{\delta\Gamma[\phi]}{\delta\phi(x)}\right|_{\phi(x)=\phi_0(x)} = 0, \tag{5}$$

which is equivalent to the requirement that the external source vanishes.

We can get a solution of (5) different from $\phi_0(x)$ by the following perturbative calculations.

Expanding $\phi(x)$ into the series $\phi(x) = \phi_0(x) + \sum_{n=1}^{\infty}\Delta\phi^{(n)}(x)$ and substituting this series in (5), instead of $\phi_0(x)$, we get the equation

$$\begin{aligned} &\Gamma^{(2)}(x,x_1)[\Delta\phi^{(1)}(x_1)+\Delta\phi^{(2)}(x_1)+\cdots] \\ &+\frac{1}{2!}\Gamma^{(3)}(x,x_1,x_2)[\Delta\phi^{(1)}(x_1)+\Delta\phi^{(2)}(x_1)+\cdots][\Delta\phi^{(1)}(x_2)+\Delta\phi^{(2)}(x_2)+\cdots]+\cdots \\ &=0, \end{aligned} \tag{7}$$

where the integration with the repeated arguments are understood and the first term is omitted because of (5). We regard $\Delta\phi^{(n)}$ as the quantity of the magnitude $(\Delta\phi^{(1)})^n$, and determine it successively.

The first order equation,

$$\Gamma^{(2)}(x,x_1)\Delta\phi^{(1)}(x_1)=0 \tag{8}$$

is a wave equation, which gives the particle spectrum of the theory, and we call it an on-shell condition. It has been used to discuss the stability of the vacuum in Ref. 2.

As for the higher order equation, it determines $\Delta\phi^{(n)}(x)$ in terms of $\Delta\phi^{(m)}(x)$ with $m<n$ and we reach a simple expression,

$$\Delta\phi^{(n)}(x)=\frac{1}{n!}W^{(2)}(x,x')\widetilde{W}^{(n+1)}(x',x_1,\ldots,x_n)\Delta\phi^{(1)}(x_1)\cdots\Delta\phi^{(1)}(x_n), \tag{9}$$

where $\widetilde{W}^{(n+1)}$ is the connected Green's function whose external legs are amputated by the inverse of the propagator, $[W^{(2)}]^{-1}$. When we evaluate $\Gamma[\phi]$ at this new solution $\phi=\phi_0+\sum\Delta\phi^{(n)}$, we have obtained in I the on-shell expansion.

3. Meanings of the On-Shell Expansion

In the preceding section, we have got the two different functions which satisfy the stationary condition of the EA, namely, ϕ_0 and $\phi_0+\sum\Delta\phi^{(n)}$. We will consider the meanings of them in the following, in order to clarify the relationship between the on-shell expansion of the EA and the S matrix elements.

The former solution $\phi_0(x)$ has been regarded as the vacuum expectation value of the field operator $\hat{\Phi}(x)$, but this interpretation is justified only when the vacuum state is chosen as the boundary states in the definition (1). Usually, we adopt the '$-i\varepsilon$' prescription[3,4] which guarantees the boundary states to be the vacuum. Here we adopt this convention and regard ϕ_0 as the vacuum expectation value and hereafter it is assumed to be a space-time independent constant because of the translational invariance of the vacuum.

As for the latter, $\phi_0+\sum\Delta\phi^{(n)}$, we cannot adopt the '$-i\varepsilon$' prescription, since it will make $\phi_0+\sum\Delta\phi^{(n)}$ and ϕ_0 identical, which implies $\Delta\phi^{(1)}=0$. Therefore, in the following, we will look for the boundary states corresponding to $\Delta\phi^{(1)}\neq0$, which is not the vacuum.

The wave function $\Delta\phi^{(1)}(x)$ determined by the wave equation (8) can be expanded as

$$\Delta\phi^{(1)}(x)=\int d^3k[C^{(-)}(\boldsymbol{k})f_{\boldsymbol{k}}(x)+C^{(+)}(\boldsymbol{k})f^*_{\boldsymbol{k}}(x)], \tag{10}$$

where $f_{\boldsymbol{k}}(x)$ is given by

$$f_{\boldsymbol{k}}(x)=\frac{1}{\sqrt{(2\pi)^3 2k_0}}e^{-ikx}, \tag{11}$$

where $k_0=(\boldsymbol{k}^2+m^2)^{1/2}$ (m is the mass of the particle), $kx=k_0x_0-\boldsymbol{k}\cdot\boldsymbol{x}$, and $C^{(\pm)}(\boldsymbol{k})$ are arbitrary function of the spatial momentum $\boldsymbol{k}$. These are determined by the boundary conditions imposed by the initial or the final states.

Substituting (10) into (9), we integrate over x_i's by the use of the (inverse of the) Lehman–Symanzik–Zimmermann(LSZ) reduction formula. After some calculations, we get

$$\Delta\phi^{(n)}(x)=\sum_{m=0}^{n}\frac{1}{m!(n-m)!}\left[\prod_{i=1}^{m}Z^{-1/2}\int d^3k_iC^{(+)}(\boldsymbol{k}_i)\right]\left[\prod_{j=1}^{n-m}Z^{-1/2}\int d^3p_jC^{(-)}(\boldsymbol{p}_j)\right]$$

$$\times<0|\hat{a}_{\mathrm{out}}(\boldsymbol{k}_1)\cdots\hat{a}_{\mathrm{out}}(\boldsymbol{k}_m)\hat{\Phi}(x)\hat{a}^\dagger_{\mathrm{in}}(\boldsymbol{p}_1)\cdots\hat{a}^\dagger_{\mathrm{in}}(\boldsymbol{p}_{n-m})|0>_{\mathrm{c}}, \tag{12}$$

where the asymptotic fields,

$$\hat{\Phi}_{\mathrm{out(in)}}(x)=\int d^3k\left[\hat{a}_{\mathrm{out(in)}}(\boldsymbol{k})f_{\boldsymbol{k}}(x)+\hat{a}^\dagger_{\mathrm{out(in)}}(\boldsymbol{k})f^*_{\boldsymbol{k}}(x)\right], \tag{13}$$

are defined by the LSZ asymptotic condition with the weak limit

$$\hat{\Phi}(x)\to\sqrt{Z}\hat{\Phi}_{\mathrm{out(in)}}(x) \qquad (x_0\to+(-)\infty)\ . \tag{14}$$

The subscript 'c' in (12) implies that only the connected parts are taken. Then we finally evaluate the sum $\phi_0+\sum\Delta\phi^{(n)}$ with (12) as

$$\phi_0+\sum_{n=1}^{\infty}\Delta\phi^{(n)}=<0|e^{Z^{-1/2}\int d^3kC^{(+)}(\boldsymbol{k})\hat{a}_{\mathrm{out}}(\boldsymbol{k})}\hat{\Phi}(x)e^{Z^{-1/2}\int d^3pC^{(-)}(\boldsymbol{p})\hat{a}^\dagger_{\mathrm{in}}(\boldsymbol{p})}|0>_{\mathrm{c}}$$

$$\equiv<C^{(+)}|\hat{\Phi}(x)|C^{(-)}>_{\mathrm{c}}\ . \tag{15}$$

From (15), we find that the solution $\phi_0+\sum\Delta\phi^{(n)}(x)$ is the expectation value of the field operator $\hat{\Phi}(x)$ with respect to the *coherent states* $|C^{(\pm)}>$ characterized by the functions $C^{(\pm)}(\boldsymbol{k})$.

In terms of these coherent states, the physical meaning of the on-shell expansion can be made clear as follows. We use these coherent states as the boundary states in the definition of $W[J]$ as

$$\exp(iW[J])=<C^{(+)}|\exp(i\int d^4x J(x)\hat{\Phi}(x))|C^{(-)}>, \tag{16}$$

which of course gives the relation

$$\phi_0+\sum_{n=1}^{\infty}\Delta\phi^{(n)}(x)=\left.\frac{\delta W[J]}{\delta J(x)}\right|_{J=0}. \tag{17}$$

The Legendre transformation from this generating functional provides the EA defined through the coherent states. On the other hand, the generating functional of the S matrix elements is given by the amplitude between the coherent states as $S[C^{(+)},C^{(-)}]\equiv<C^{(+)}|C^{(-)}>$. As a result, the EA evaluated at $\phi(x)=\phi_0+\sum\Delta\phi^{(n)}(x)$ is related to the generating functional of the connected S matrix elements as seen from the equation

$$\begin{aligned} i\Gamma[\phi_0+\textstyle\sum\Delta\phi^{(n)}]&=iW[0]=<C^{(+)}|C^{(-)}>_c \\ &=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{1}{m!}\frac{1}{n!}\left[\prod_{i=1}^{m}Z^{-1/2}\int d^3k C^{(+)}(\boldsymbol{k}_i)\right]\left[\prod_{j=1}^{n}Z^{-1/2}\int d^3p C^{(-)}(\boldsymbol{p}_j)\right] \\ &\quad\times<0|\hat{a}_{\rm out}(\boldsymbol{k}_1)\cdots\hat{a}_{\rm out}(\boldsymbol{k}_m)\hat{a}^{\dagger}_{\rm in}(\boldsymbol{p}_1)\cdots\hat{a}^{\dagger}_{\rm in}(\boldsymbol{p}_n)|0>_c. \end{aligned} \tag{18}$$

Of course, we can obtain all the S matrix elements by the functional differentiation $i\Gamma[\phi_0+\sum\Delta\phi^{(n)}]$ with respect to $C^{(\pm)}(\boldsymbol{k})$ as

$$\left.\frac{\delta^m}{\delta C^{(+)}(\boldsymbol{k}_1)\cdots\delta C^{(+)}(\boldsymbol{k}_m)}\frac{\delta^n}{\delta C^{(-)}(\boldsymbol{p}_1)\cdots\delta C^{(-)}(\boldsymbol{p}_n)}i\Gamma[\phi_0+\textstyle\sum\Delta\phi^{(n)}]\right|_{C^{(\pm)}=0}$$

$$=<0|\hat{a}_{\rm out}(\boldsymbol{k}_1)\cdots\hat{a}_{\rm out}(\boldsymbol{k}_m)\hat{a}^{\dagger}_{\rm in}(\boldsymbol{p}_1)\cdots\hat{a}^{\dagger}_{\rm in}(\boldsymbol{p}_n)|0>_c$$

$$=<0|\hat{a}_{\rm in}(\boldsymbol{k}_1)\cdots\hat{a}_{\rm in}(\boldsymbol{k}_m)\hat{S}\hat{a}^\dagger_{\rm in}(\boldsymbol{p}_1)\cdots\hat{a}^\dagger_{\rm in}(\boldsymbol{p}_n)|0>_{\rm c}. \tag{19}$$

We have now accomplished our purpose: to understand the meaning of the on-shell expansion of the EA and to clarify its relation to the S matrix. The details of the above arguments will soon appear.[5)]

4. Summary

We have found that the on-shell expansion of the EA is directly related to the generating functional of the connected S matrix elements. This shows a new aspect of the EA, that is, it provides all the S matrix elements in addition to the vacuum expectation value of the field operator, the spectrum of the particle excited above this vacuum. In other words, all the observables are calculable through the on-shell expansion.

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INVERSION METHOD AND ITS APPLICATIONS

R. Fukuda

Department of Physics, Faculty of Science and Technology,
Keio University, Yokohama, 223 Japan

Abstract:

The inversion method, which is the generalization of the method of the free energy, and the effective potential (or the action), is explained and applied to several problems. These include strong coupling *QED* and the confining parameter in *QCD*.

Introduction In various fields of physics, we encounter the situation where the ground state which is realized in nature is not that of the naive perturbative one. It is realized after the "condensation" of the objects which behave as normal particles in the perturbative phase. The attractive interaction between normal particles is the driving force of the condensation. The usual way to study the phenomenon is to introduce the free energy and find its minimum. For the field theory of zero temperature, the effective potential or the action plays the role. Let us recall the way how they are defined.

First the Lagrangian L of the system is changed into $L_J = L + JO$ where J is the artificial source and the operator O is chosen to break the symmetry of L so that the order parameter $<O> \equiv \phi$ calculated by L_J is non-zero even in the perturbation theory. Calculate the vacuum action functional $W[J]$ in the theory governed by L_J and the effective action $\Gamma[\phi]$ is defined, through the Legendre transformation, as

$$\Gamma[\phi] = W[J] - J\partial W/\partial J, \qquad \partial W/\partial J = \phi. \tag{1}$$

The relation $\phi = \partial W/\partial J$ is inverted to express J as the function of ϕ. The stationarity condition $\partial\Gamma/\partial\phi = -J = 0$ assures the recovery of the original theory at this point.

Inversion Method We can generalize the above procedure to the case where ϕ is not written as the expectation value of some operator. Change L to L_J where $L_{J=0}=L$, which is the only requirements for L_J. It need not be $L+JO$. Then the order parameter ϕ is calculated in perturbative series;

$$\phi = \sum_{n=0}^{\infty} (g^2)^n h_n(J) \tag{2}$$

where g is the coupling strength and $h_n(J)$ is calculable diagrammatically. The modified Lagrangian L_J is so chosen as to get the non-zero series (2). Now we invert (2) to get

$$J = \sum_{m=0}^{\infty} (g^2)^m f_m(\phi). \tag{3}$$

The co-efficient function $f_m(\phi)$ for $m \leq N$ are obtainable from $h_n(J)$ with $n \leq N$. The solution to $J=0$ is looked for in the inverted form (3). We can get the non-perturbative solution, if it exists at all, by this method besides the trivial solution $\phi=0$.

Illustration The ladder Schwinger-Dyson equation is derived by this method as an example. Take the Lagrangian L_{QED} of the Quantum Electrodynamics (QED) and consider the electron self-energy function $S_F(p)$. Here $g=e$, the charge of the electron , and the source term is introduced by changing the action of QED by adding $\int d^4pJ(p)\overline{\psi}(-p)\psi(p)$ where ψ is the electron field. The original series (2) up to e^2 is

$$S_F(p) = S_0^J(p) - e^2 S_0^J(p)\int d^4q[\gamma_\mu S_0^J(p+q)\gamma_\nu D_0^{\mu\nu}(q)]S_0^J(p), \tag{4}$$

where $D_0^{\mu\nu}$ is the photon propagator and $(S_0^J)^{-1}(p) = S_0^{-1}(p)-iJ(p)$ is the inverse of the free electron propagator in the presence of J. The lowest inversion ($e^2=0$) gives us $J(p) = iS_F^{-1}(p)-iS_0^{-1}(p)$ so that the inverted series up to e^2 is

$$J(p) = iS_F^{-1}(p) - iS_0^{-1}(p) - ie^2\int d^4q\gamma_\mu S_F(p+q)\gamma_\nu D^{\mu\nu}(q), \tag{5}$$

which is just the ladder Schwinger-Dyson equation if we set $J=0$.

Strong Coupling QED[2)] Consider the gauge invariant order parameter $\phi=<\overline{\psi}(x)\psi(x)>$ and the term $J\int d^4x\overline{\psi}(x)\psi(x)$ is added to the action of QED. The vacuum action function $W[J]$ is calculated. It has the form

$$W[J] = -iTr\ln(S_0^J)^{-1} + (i/2)Tr\ln D_0^{-1} - i\sum_{n=1}^{\infty}(e^2)^n W_J^{(n)}, \tag{6}$$

where $(S_0^J)^{-1} = p+J$ and $W_J^{(n)}$ is the vacuum graphs of the order $(e^2)^n$ in the presence of J. We calculate ϕ by the formula $\phi = \partial W/\partial J/\Omega$ where Ω is the space-time volume. The term up to e^2 is calculated below. It involves the two loop diagram having one photon propagator and is evaluated by opening the photon propagator, which is nothing but the vacuum polarization graph with mass insertion of the electron. This produces the gauge invariant results. The original series is obtained in this way which is given for small J as;

$$\phi = (J/4\pi^2)[\Lambda_f^2+(3\alpha/2\pi)\{1-(J^2/\Lambda_p^2)(\ln J^2/\Lambda_p^2)^2\}] + O(J^3\ln J) \tag{7}$$

where $\Lambda_f(\Lambda_p)$ is the electron (photon) momentum cut-off. The inverted series is

$$J = (4\pi^2\phi/\Lambda_f^2)\{1-\frac{\alpha}{\alpha_c}+24\pi^3\alpha(\phi^2/\Lambda_f^6)(\ln\frac{16\pi^2\phi^2}{\Lambda_f^4\Lambda_p^2})^2\}, \tag{8}$$

where $\alpha=e^2/4\pi$ and $\alpha_c=2\pi/3\eta$, $\eta=\Lambda_p/\Lambda_f$. This is noting but the negative of the derivative of the effective potential. Equation (8) has the same form, except for the $(\ln\phi^2)^2$ term, as the Landau theory of the phase transition. We conclude; for $\alpha>\alpha_c$ the chiral condensation ($\phi\neq0$) is realized. Our theory predicts the mean field type behavior near $\alpha=\alpha_c$; $\phi\sim(\alpha-\alpha_c)^{1/2}/\ln(\alpha-\alpha_c)$.

QCD and string tension For Quantum Chromodynamics (QCD), the expected non- perturbative solution behaves near $g=0$ as $\phi^{1/\delta}\sim\mu\exp(1/2b_0g^2)$, where all the quantities are the renormalized ones and μ is the subtraction point. The index $\delta>0$ is the dimension of ϕ in mass unit and $\beta(g)=b_0g^3+b_1g^5+\cdots$. We change the variable from ϕ to $t\equiv g^2\ln\phi^{1/\delta}/\mu$ which is of the order unity near the solution. The source J is assumed to have the dimension of mass and consider the following inverted series by extracting the factor $\phi^{1/\delta}$. This factor is always present since $\phi=0$ is one of the solution to $J=0$;

$$J = \phi^{1/\delta}\sum_{n=0}^{\infty}(g^2)^n f_n(t)\equiv\phi^{1/\delta}f(t,g^2). \tag{9}$$

Before calculating $f_n(t)$ explicitly, the renormalization group equation tells us much about the form of $f_n(t)$.

In order to see this, we choose J in such a way that J is independent of the subtraction parameter μ: $\mu\frac{dJ}{d\mu}=0$. This can always be done by multiplying a suitable factor in front of J. By noting that when applied to the right hand side of (9),

$$\mu\frac{d}{d\mu} = \mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}+\mu\frac{d\phi^{1/\delta}}{d\mu}\frac{\partial}{\partial\phi^{1/\delta}},$$

and by requiring that the each coefficient of $(g^2)^n$ vanishes, we get the set of ordinary differential equations for $f_n(t)$. The first member of this set is

$$(2b_0t-1)f_0{}'(t)+\frac{\gamma_1}{\delta}f_0(t) = 0, \tag{10}$$

where $f_0{}'\equiv\frac{df_0}{dt}$ and

$$\frac{1}{\phi}\mu\frac{d\phi}{d\mu} = \gamma(g) = \gamma_1g^2+\gamma_2g^4+\cdots. \tag{11}$$

The solution is

$$f_0(t) = C\left(t-\frac{1}{2b_0}\right)^{-\frac{\gamma_1}{2\delta b_0}} \tag{12}$$

where C is the integration constant. Since $b_0<0$, $\delta>0$, the non-trivial solution to $J=0$ is present if $\gamma_1>0$.

The sign $\gamma_1>0$ has the physical meaning. For the two quark field $\phi\sim qq$ or $\overline{q}q$ for instance, the first term of the anomalous dimension is calculated by the one gluon exchange diagram between two quarks or antiquark which determines whether the force acting between two fermions is attractive or repulsive. The correspondence is indeed[1)]

$\gamma_1>0 \leftrightarrow$ ***attractive***,

$\gamma_1<0 \leftrightarrow$ ***repulsive***,

therefore our conclusion is that ϕ condenses as long as we have the attractive force between two particles. The condition $\gamma_1>0$ for the condensation of ϕ can be used as a generalized criterion in the case where ϕ is not written as the product of two fields.

Now we know that the correct non-perturbative value of ϕ is

$$\phi^{1/\delta} = \mu\exp\int^g \frac{1-\gamma(x)/\delta}{\beta(x)}dx. \tag{13}$$

Thus the variable t has the expansion

$$t = g^2\ln\frac{\phi^{1/\delta}}{\mu} = \frac{1}{2b_0}-\frac{1}{2b_0}\{\frac{\gamma_1}{\delta}+\frac{b_1}{b_0}\}g^2\ln g^{-2}$$

$$+\ Cg^2+dg^4+\cdots, \tag{14}$$

where C, d etc. are some constants. The solution (12) to the lowest truncation reproduces the first term of the expansion (14). In order to discuss the higher truncation systematically and most conveniently, we define $\overline{f}(t,g^2)$ as

$$\overline{f}(t,g) = K(g)^{-1}f(t,g)^{1/\eta}, \tag{15}$$

where

$$K(g) \equiv \lim_{g_0\to 0} g_0^2\exp-\int_{g_0}^{g}\frac{dx\gamma(x)}{\delta\eta\beta(x)} \tag{16}$$

$$= g^2\{1+(\frac{\gamma_2}{\gamma_1}-\frac{b_1}{b_0})g^2+\cdots\}, \tag{17}$$

$$\eta = -\frac{\gamma_1}{2\delta b_0}. \tag{18}$$

Remember that $K(g)$ has the Taylor expansion about g=0. The merit of using $\mathcal{F}(t,g)$ is that it satisfies a simple equation;

$$(\mu\frac{\partial}{\partial\mu}+\hat{\beta}(g)\frac{\partial}{\partial g})\mathcal{F}(t,g) = 0, \tag{19}$$

where ϕ is fixed in this equation and

$$\hat{\beta}(g) = \frac{\beta(g)}{1-\gamma(g)/\delta} = b_0g^3+\hat{b}_1g^5+\cdots, \tag{20}$$

$$\hat{b}_1 = b_1 + \frac{\gamma_1 b_0}{\delta}. \tag{21}$$

The function $\mathcal{F}$ has the expansion

$$\mathcal{F}(t,g^2) = \frac{W_{-1}(t)}{g^2}+W_0(t)+g^2W_1(t)+.... \tag{22}$$

and $W_l(t)$ $(l\geq-1)$ satisfies

$$(1-2b_0t)W_l'-2b_0lW_l-2\hat{b}_1[tW_{l-1}'+(l-1)W_{l-1}] = 0, \tag{23}$$

where $W_{-2}\equiv 0$. The solution up to W_0 is given as

$$\mathcal{F}(t,g^2) = \frac{C_{-1}}{g^2}(t-\frac{1}{2b_0}) - \frac{C_{-1}\hat{b}_1}{2b_0^2}\ln(1-2b_0t) + C_0 \tag{24}$$

with C_{-1} and C_0 being the integration constants which are calculated through the series (9) diagrammatically. The zero of (24) is slightly modified compared with the lowest value $\frac{1}{2b_0}$, and it behaves for small g^2 as

$$t = \frac{1}{2b_0} - \frac{\hat{b}_1}{2b_0^2}g^2\ln g^{-2} + 0(g^2\ln\ln g^{-2}) \tag{25}$$

so that we have recovered the second term of the correct expansion (14) but the term $O(g^2\ln\ln g^{-2})$ should be $O(g^2)$. The discrepancy is of course due to the truncation (24) and we have for $\phi^{1/\delta}$, the solution to $\overline{J}=0$, the scale non-invariant behavior under the variation of μ

$$\phi^{1/\delta}(g^2)^{\eta} \propto (\ln g^{-2})^{\frac{\hat{b}_1}{2b_0^2}}. \tag{26}$$

The right hand side should be constant for the correct solution since the left hand side is the scale invariant quantity.

We can improve the situation by taking into more terms of W_l. This can most conveniently be done by deriving the partial differential equation for $K(t,g^2)\equiv\dfrac{-2b_0}{C_{-1}}\times(W_0+g^2W_1+g^4W_2+\cdots)$. By multiplying $(g^2)^l$ to (23) and summing up from $l=-1$, we get

$$[\{(1+e^2)s-e^2\}\frac{\partial}{\partial s} + (1+e^2)e^2\frac{\partial}{\partial e^2}]\widetilde{K}(s,e^2) = 1 \tag{27}$$

where $e^2=g^2\dfrac{\hat{b}_1}{b_0}$, $\widetilde{K}(s,e^2)=\dfrac{b_0}{\hat{b}_1}K(t,g^2)$ and $s=1-2b_0t$. We sum up the term $\dfrac{(\ln s)^m}{s^l}$ with $1\leq m\leq l$ appearing in W_l which are the most singular terms in W_l. This can be accomplished by looking for the solution to (27) with $1+e^2$ replaced by unity. So that we solve

$$\{(s-e^2)\frac{\partial}{\partial s} + e^2\frac{\partial}{\partial e^2}\}\widetilde{K}_0(s,e^2) = 1 \tag{28}$$

and get the solution in the implicit form,

$$\widetilde{K}_0 = \ln(s+e^2\widetilde{K}_0) \tag{29}$$

$$= \ln(s+e^2\ln(s+e^2\ln(s+e^2\ln\cdots))). \tag{30}$$

The lowest solution $\widetilde{K}_0=\ln s$ reproduces (24). The next truncation gives $\widetilde{K}_0=\ln(s+e^2\ln s)$ which leads to the solution

$$t = \frac{1}{2b_0} - \frac{\hat{b}_1}{2b_0^2}g^2\ln g^{-2} + O(g^2\ln\ln\ln g^{-2}) \tag{31}$$

and for $\phi^{1/\delta}$

$$\phi^{1/\delta}(g^2)^\eta \propto (\ln\ln g^{-2})^{\frac{\hat{b}_1}{2b_0^2}}. \tag{32}$$

The situation is improved but still it is not scale invariant.

In order to obtain the scale invariant formula for the solution ϕ, we have to use the exact solution $\widetilde{K}_0$ given by (29). Therefore we solve (29) and the equation

$$\frac{s}{g^2} + \frac{\hat{b}_1}{b_0}\tilde{K}_0 + \widetilde{C}_0 = 0 \tag{33}$$

simultaneously where $\widetilde{C}_0=-2b_0C_0/C_{-1}$. By inserting $s=\exp\widetilde{K}_0-e^2\widetilde{K}_0$ obtained from (29) into (33), $\widetilde{K}_0$ is given by

$$\widetilde{K}_0 = \ln(-g^2\widetilde{C}_0) \tag{34}$$

which leads to

$$\begin{aligned} s &= 1-2b_0t=1-2b_0g^2\ln\frac{\phi^{1/\delta}}{\mu} \\ &= -g^2\widetilde{C}_0 - \frac{\hat{b}_1}{b_0}\ln(-g^2\widetilde{C}_0) \end{aligned} \tag{35}$$

so that

$$\phi^{1/\delta} = \mu\exp\{\frac{1}{2b_0g^2}+\frac{\widetilde{C}_0}{2b_0}+\frac{\hat{b}_1}{2b_0^2}\ln(-g^2\widetilde{C}_0)\}. \tag{36}$$

The scale parameter μ is eliminated using the QCD scale parameter Λ_{QCD};

$$\Lambda_{QCD} = \mu\exp\{\frac{1}{2b_0g^2} - \frac{b_1}{2b_0^2}\ln(\frac{-b_0g^2}{1+\frac{b_1}{b_0}g^2})\} \tag{37}$$

which is the expression assuming $\beta(g)=b_0g^3+b_1g^5$. From (36) and (37) we finally get, by taking the limit $g\rightarrow 0$,

$$\phi^{1/\delta}(g^2)^\eta = D\Lambda_{QCD}, \tag{38}$$

where D is the numerical constant given by

$$D = \exp\{\frac{\tilde{C}_0}{2b_0}+\frac{b_1}{2b_0^2}\ln(\frac{\tilde{C}_0}{b_0})+\frac{\gamma_1}{2\delta b_0}\ln(-\tilde{C}_0)\} \tag{39}$$

$$= \exp\{\frac{\tilde{C}_0}{2b_0}+\frac{\hat{b}_1}{2b_0^2}\ln(\frac{\tilde{C}_0}{b_0})+\frac{\gamma_1}{2\delta b_0}\ln(-b_0)\}. \tag{39'}$$

The above two expressions are equivalent. These formulas suggest that the solution exists as long as $\tilde{C}_0<0$. The numerical evaluation of $\tilde{C}_0$ for the case of

$$\phi=<\bar{\psi}\psi>, \quad \frac{1}{g^2}\times(\text{string tension}), \qquad <G^2_{\mu\nu}>$$

in QCD is under way but the sign of $\tilde{C}_0$ for these quantities is indeed negative.

Finally the formula for the energy density, more exactly the difference of the energy density ΔE between the normal and the condensed vacuum, is given below. ΔE has no anomalous dimension so it needs a separate discussion. The source should be introduced in such a way that the energy is lowered, i.e. $\Delta E<0$. Let us write $\Delta E=-aJ^4(a>0)$ then the inverted series has the form, with $\varepsilon\equiv-\Delta E/a$ and $t=g^2\ln\frac{\varepsilon^{1/4}}{\mu}$,

$$J = \varepsilon^{1/4}\{1+g^2f_1(t)+g^4f_2(t)+\cdots\}. \tag{40}$$

Define $W(t,g^2)$ by

$$W(t,g^2) = \{g^2f_1(t)+g^4f_2(t)+\cdots\}^{-1}$$
$$\equiv \frac{W_{-1}(t)}{g^2}+W_0(t)+g^2W_1(t)+\cdots, \tag{41}$$

then by the same arguments as before, we have up to W_0

$$W(t,g^2) = \frac{\bar{C}_{-1}}{g^2}(t-\frac{1}{2b_0}) - \frac{\bar{C}_{-1}b_1}{2b_0^2}\ln(1-2b_0t)+\bar{C}_0.$$

The scale invariant formula for $\varepsilon^{1/4}$ is now

$$\varepsilon^{1/4} = \overline{D}\Lambda_{QCD},$$

$$\overline{D} = \exp[-\frac{1+\overline{C}_0}{\overline{C}_{-1}} + \frac{b_1}{2b_0^2}\ln\{\frac{2(1+\overline{C}_0)}{-\overline{C}_{-1}}\}].$$

For the real solution to exist $(1+\overline{C}_0)/\overline{C}_{-1}<0$ should be satisfied. The calculation of $\overline{C}_0$ requires two loop vacuum diagrams under the presence of the suitable source which is not yet carried out.

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EFFECTIVE ACTION AND THE ENERGY LEVELS OF HYDROGEN-, HELIUM-, AND LITHIUM-LIKE ATOMS

M. Komachiya

Department of Physics, Faculty of Science and Technology,
Keio University, Yokohama 223, Japan

Abstract

A systematic derivation of the energy eigenvalue equations for one-, two-, and three-electron atoms is presented in terms of the effective action. By using this method, one can naturally include the field theoretical corrections into the wave equations.

1. Introduction

In this talk we present an application of the generalized on-shell condition,[1),2)] which is obtained by the second derivative of the effective action, and we want to start with a quick review of this formalism. The talk is based on the work with R.Fukuda.[3)]

For simplicity, let us consider the scalar field $\Phi(x)$ and the Lagrangian density $L(\Phi)$ of a system. The generating functional $W[J]$ of the connected Green's function is introduced as

$$\exp(iW[J]) = \int [d\Phi] \exp[\, i\int_{-\infty}^{+\infty} d^4x \,(\, L(\Phi) + J(x)\Phi(x)\,)\,], \tag{1}$$

and the effective action $\Gamma[\phi]$ is defined by the Legendre transformation,

$$\Gamma[\phi] \equiv W[J] - \int d^4x \; J(x)\phi(x), \tag{2}$$

$$\phi(x) \equiv \delta W[J]/\delta J(x). \tag{3}$$

The stationary condition,

$$\delta\Gamma[\phi]/\delta\phi(x) = -J(x) \equiv 0, \tag{4}$$

determines the ground state expectation value of $\Phi(x)$, $<\Phi(x)>_{J=0} \equiv \phi^{(0)}(x)$. We then look for another solution of (4) in the form of $\phi(x)= \phi^{(0)}(x)+\Delta\phi(x)$ and, by assuming $\Delta\phi(x)$ is small, we find the following eigenvalue equation for $\Delta\phi(y)$ (the generalized on-shell condition),

$$\int d^4y \, [\, \frac{\delta^2\Gamma[\phi]}{\delta\phi(x)\delta\phi(y)} \,]_0 \, \Delta\phi(y) = 0, \tag{5}$$

where $[\,]_0$ denotes the value of $[\,]$ evaluated at $\phi(x)=\phi^{(0)}(x)$.

If we take the space-time translational-invariant case, the zero of the kernel in (5) coincides with the pole of the Green's function by the relation,

$$\int d^4y \, [\, \frac{\delta^2\Gamma[\phi]}{\delta\phi(x)\delta\phi(y)} \,]_0 \, [\, \frac{\delta^2 W[J]}{\delta J(y)\delta J(z)} \,]_{J=0} = -\,\delta^4(x-z). \tag{6}$$

So, eq.(5) determines the particle spectrum or the mode. In the same way, if we study the case where the time-independent external field exists (e.g. the nuclear Coulomb field), eq.(5) is expected to determine the energy eigenvalue and its eigenfunction ($\propto\Delta\phi(x)$) of the excited level. We utilize this fact and study the systematical derivation of the equations that determine the energy levels of hydrogen-, helium-, and lithium-like atoms.

2. One-Electron Atoms

We consider QED under the external field and use the Lagrangian density of the form,

$$L_J \equiv -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(\, i\partial\!\!\!/ - m - eA\!\!\!/ \,)\psi - j_\mu A^\mu + J^A_\mu A^\mu + J_\psi \psi + J_{\overline{\psi}} \overline{\psi}, \tag{7}$$

where $j(x) \equiv (Z|e|\delta^3(x) , 0, 0, 0)$ is the source of the nuclear Coulomb field with the atomic number Z. The last three source terms are used as probes. If we want to discuss the finite nuclear size and/or the nuclear magnetic moment, they can be included as the modification of j_μ. Here we notice that the term $J_\psi \psi$ (or $J_{\overline{\psi}} \overline{\psi}$) is necessary for the investigation of the one fermion-number channel, while the term $J^A_\mu A^\mu$ is used for convenience.

The effective action Γ can be obtained with the help of the Legendre transformation formula given by De Dominicis and Martin [4] but with a small modification. For the Grassmann variables, we employ the following definitions of Γ and its functional derivatives,[5]

$$\Gamma[\phi_i] \equiv W[J_i] - J_i\phi_i \qquad (\phi_i \equiv \frac{\overrightarrow{\delta}}{\delta J_i} W[J_i]), \tag{8}$$

$$\overleftarrow{\delta}\Gamma[\phi_i]/\delta\phi_i = -J_i. \tag{9}$$

The expression of Γ is then summarized as follows,

$$\Gamma[<A>,<\psi>,<\overline{\psi}>] \equiv W[J^A, J_\psi, J_{\overline{\psi}}] - J^A \frac{\delta W}{\delta J^A} - J_\psi \frac{\overrightarrow{\delta} W}{\delta J_\psi} - J_{\overline{\psi}} \frac{\overrightarrow{\delta} W}{\delta J_{\overline{\psi}}}$$

$$= \frac{1}{2}<A^\mu>iD_0^{-1}{}_{\mu\nu}<A^\nu> + <\overline{\psi}_i>iS_0^{-1}{}_{ij}<\psi_j> - j_\mu<A^\mu> - i\kappa^{(1)}, \tag{10}$$

where D_0 (S_0) is the bare photon (electron) propagator and $\kappa^{(1)}$ denotes the sum of the one-particle irreducible (1-PI) vacuum diagrams. Graphically $<\psi>$, $<\overline{\psi}>$, and $<A^\mu>$ are expressed by the broken lines which directly connect to the vertices (Fig.1).

As the solution of the stationary condition of Γ, we can choose $<\psi>=<\overline{\psi}>=0$ and, after substituting them, the on-shell condition for $\Delta<\psi>$ is obtained in the following form,

$$(S_0^{-1}{}_{ij} + [\frac{\overleftarrow{\delta}}{\delta\langle\psi_j\rangle} (\frac{\overleftarrow{\delta}\,\kappa^{(1)}}{\delta\langle\overline{\psi}_i\rangle})]_0)\ \Delta\langle\psi_j\rangle = 0. \tag{11}$$

This is the equation that determines the full energy levels of the hydrogen-like atoms. For example, if we concentrate ourselves on the tree diagram of $\kappa^{(1)}$, we get $\langle A^\mu\rangle = -iD_0^{\mu\nu}j_\nu \equiv A_c^\mu$ as the stationary solution and eq.(11) becomes,

$$[\ i\partial\!\!\!/_x - m + \gamma_0 \frac{Ze^2}{4\pi|\boldsymbol{x}|}\ ]\ \Delta\langle\psi\rangle_x = 0. \tag{12}$$

This is nothing but the Dirac equation under the Coulomb potential. In the same way, if we choose the diagrams shown in Fig.1, the lowest-order radiative corrections are properly taken into account and we get the modified Dirac equation from (11) as

$$[\ i\partial\!\!\!/ - m - \Sigma - eA_c^\mu \{\ \gamma_\mu + \overline{\omega}_{\mu\rho}D_0^{\rho\sigma}\gamma_\sigma + \Lambda_\mu\ \}\ ]\ \Delta\langle\psi\rangle = 0, \tag{13}$$

where $\overline{\omega}_{\mu\rho}$, $-i\Sigma$, and $-ie\Lambda_\mu$ denote the lowest-order contribution of the vacuum polarization, the electron self-energy, and the vertex correction, respectively. In this way, we can systematically include the quantum field theoretical corrections into the relativistic wave equation.

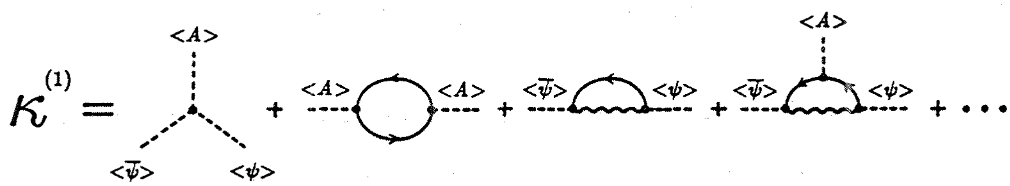

Fig.1 Some of the diagrams included in $\kappa^{(1)}$, which contribute to the modified Dirac equation (13). The wavy line is the photon propagator. The solid line with an arrow denotes the electron one.

3. Two- and Three-Electron Atoms

Next we consider the helium- and lithium-like atoms. In the case of the helium-like atoms, we start with the Lagrangian density (7) plus new source term $(1/2!)K(a,b)\Phi(a)\Phi(b)$ where $\Phi \equiv [\ \psi, \overline{\psi}, A\ ]$ and a,b denote the species of the fields as well as the other degrees of freedom. (Summations over the repeated indices are implied.) In the same way, for the lithium-like atoms, we further use the source term $(1/3!)M(a,b,c)\Phi(a)\Phi(b)\Phi(c)$. Each source is to be antisymmetrized for Grassmann components and to be symmetrized for the others. We notice that the sources $K(\psi,\psi)$ $(,K(\overline{\psi},\overline{\psi}))$ and $M(\psi,\psi,\psi)$ $(,M(\overline{\psi},\overline{\psi},\overline{\psi}))$ are necessary for the investigations of two and three fermion-number channels. Other sources are employed in order to use the (modified) De Dominicis-Martin rules. For the new arguments of Γ, we introduce the notations $<ab>$ and $<abc>$. They are defined as the connected part of $2!\overleftarrow{\delta}W/\delta K(a,b)$ and $3!\overleftarrow{\delta}W/\delta M(a,b,c)$, respectively.

As the stationary solutions, $<\psi>$, $<\overline{\psi}>$, $<\psi\psi>$, $<\overline{\psi}\,\overline{\psi}>$, $<\psi\psi\psi>$, $<\overline{\psi}\,\overline{\psi}\,\overline{\psi}>$, and other variables which couple to them can be set equal to zero. Then we get the on-shell conditions in the form of the Nambu-Bethe-Salpeter type wave equations. The result is summarized for helium-like atoms,

$$\frac{1}{2!}\,S_{jk}^{-1}\,S_{il}^{-1}\Delta<\psi_k\psi_l> = [\,\frac{\delta}{\delta<\psi_k\psi_l>}\,(\frac{\delta\,\kappa^{(2)}}{\delta<\overline{\psi}_i\overline{\psi}_j>})\,]_0\,\Delta<\psi_k\psi_l>, \tag{14}$$

and for lithium-like atoms,

$$\frac{1}{3!}\,S_{ii'}^{-1}\,S_{jj'}^{-1}\,S_{kk'}^{-1}\,\Delta<\psi_{i'}\psi_{j'}\psi_{k'}>$$

$$= [\,\frac{\overleftarrow{\delta}}{\delta<\psi_{i'}\psi_{j'}\psi_{k'}>}\,(\frac{\overleftarrow{\delta}\,\kappa^{(3)}}{\delta<\overline{\psi}_i\overline{\psi}_j\overline{\psi}_k>})\,]_0\,\Delta<\psi_{i'}\psi_{j'}\psi_{k'}>, \tag{15}$$

where S_{ij} denotes the full fermion propagator (*i.e.* the stationary solution of $<\psi_i\overline{\psi}_j>$). In (14), $\kappa^{(2)}$ represents the sum of the one- and two-particle irreducible (1,2-PI) vacuum diagrams constructed out of $<a>$, $<ab>$ (propagator), and the original QED vertex. Similarly, $\kappa^{(3)}$ in (15) denotes the sum of the one-, two-, and three-particle irreducible (1,2,3-PI) vacuum graphs made up of $<a>$, $<ab>$, and $<abc>$. Graphically, each $<abc>$ is represented by the full vertex with the full propagators.[5] The term "three-particle irreducible (3-PI)" usually means the graphs which cannot be disconnected by cutting any three internal lines. But even when the graph is disconnected by this process, if one (and only one) of the disconnected part is the full vertex itself, we also call it the 3-PI graph by the conventions adopted in Ref.4.

4. Comments

Nuclear recoil corrections can be included into our formalism by considering the on-shell condition for $\Delta<\psi_N\, \psi \cdots \psi>$, where ψ_N denotes the nucleon field operator and ψ is that of the electron.

Our method is also available for the non-relativistic models. For example, we can derive the Schrödinger equation under the external potential instead of the Dirac equation (12). The extensions for other cases are straightforward.

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FERMION MASS GENERATION IN QED$_3$*

Yūichi Hoshino[a], Toyoki Matsuyama[b†] and Chikage Yoshida-Habe[c]

a) Kushiro National College of Technology, Otanoshike, Kushiro 084, Japan

b) Research Institute for Fundamental Physics, Kyoto University, Kyoto 606, Japan

c) Department of Physics, Hokkaido University, Sapporo 060, Japan

Abstract

We investigate the spontaneous generation of fermion mass in QED$_3$ with two component Dirac fermions by using the Schwinger-Dyson technique. In the ladder approximation, we show that the single two-component fermion becomes massive, thus the parity symmetry is broken dynamically. In the case with many flavor, an effect of vacuum polarization on the mass generation is studied in 1/N approximation.

1. Introduction

(2+1)-dimensional quantum electrodynamics (QED$_3$) has attracted widespread attention. The theory has highly non-trivial structures which are not seen in theories in even space-time dimensions. One of the typical aspects of the (2+1)-dimensional gauge theories is that the Chern-Simons term is allowed in the Lagrangian and the gauge field becomes (topologically) massive without violating the gauge invariance.[1)] From the practical aspects, QED$_3$ or the variants of it are expected to be important as effective theories of solid-state-physical phenomena in 2-dimensional space, e.g., the quantum Hall effect, the high-T$_c$ superconductivity and so on. Keeping these status in mind, we proceed the investigation of a dynamical symmetry breaking in QED$_3$.

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Previously Appelquist et al.[2] investigated QED_3 with four-component fermions by analyzing the Schwinger-Dyson (S-D) equation. The vacuum polarization effect is included in the 1/N approximation (N is the number of fermion flavor). The integral kernel is linearized in introducing a cut off and the integral S-D equation is rewritten to a differential equation. Further they set the wave function renormalization to unit. Then they discovered a critical number of the flavor ($N_c = \frac{32}{\pi^2}$). Thus the chiral symmetry is broken below N_c and unbroken above N_c. On the other hand, Dagotto et al.[3] also found the critical value $N_c \cong 3.5$ in the lattice calculation.

These works inspire the more detailed investigations of the dynamics of QED_3.[4] In the analyses of Appelquist et al.[2] , two drastic approximations have been done as mentioned above. The first is the linearization of the integral kernel. An infrared behavior of the kernel is crucial to the dynamical mass generation. The linearization procedure might disturb the infrared behavior. The second is to neglect the effect of the wave function renormalization. When the effect of the vacuum polarization is included, the wave function renormalization cannot be put to unit even in the Landau gauge.

In this paper, we reanalyze the S-D equation for QED_3 without using the above two approximations. We do not linearize the integral kernel and include the effect of the wave function renormalization. We solve the coupled integral equation directly without introducing any cut off. At first, the ladder S-D equation for single two-component Dirac fermion is solved in the Landau gauge. We find a non-trivial solution so that single two-component Dirac fermion becomes massive dynamically. Next we analyze the case with N two-component fermions including the effect of the vacuum polarization in 1/N approximation. The dynamically generated mass is estimated.

2. Lowest ladder analysis

In this section, we study the lowest ladder S-D equation for single two-component fermion in the Landau gauge. The lowest ladder S-D equation is given as

$$B(p) = \frac{e^2}{4\pi^2}\frac{1}{p}\int_0^\infty dk \frac{B(k)k}{A(k)^2k^2 + B(k)^2}\ell n\Big(\frac{p+k}{p-k}\Big)^2\Big(1+\frac{1}{2}\eta\Big), \tag{1a}$$

$$A(p) = 1 + \eta\frac{e^2}{4\pi^2}\frac{1}{p^2}\int_0^\infty dk \frac{k^2A(k)}{A(k)^2k^2 + B(k)^2}\Big(1 - \frac{p^2+k^2}{2pk}\Big)\ell n\Big(\frac{p+k}{p-k}\Big)^2 \tag{1b}$$

where the fermion propagator S' has been defined by $S'^{-1}(p) = A(p^2)p - B(p^2) = p - i\Sigma(p)$. η is the covariant gauge fixing parameter. We choose the Landau gauge $\eta = 0$. Then eq. (1b) gives us $A = 1$ and eq. (1a) becomes

$$B(p) = \frac{e^2}{4\pi^2}\frac{1}{p}\int_0^\infty dk \frac{B(k)k}{A(k)^2k^2 + B(k)^2}\ell n\Big(\frac{p+k}{p-k}\Big)^2. \tag{2}$$

The kernel has the singularity at $p = k$ and it seems that eq.(2) does not have a solution except the trivial solution. But the integral formula $\int_0^\infty dx \frac{x}{1+x^2}\ell n\Big(\frac{1+x}{1-x}\Big)^2 = \frac{\pi^2}{2}$ strongly suggests that there exists a non-trivial solution. We solve eq.(2) by using a numerical method. Inputting an initial trial function, we seek a convergent solution by an iteration method.

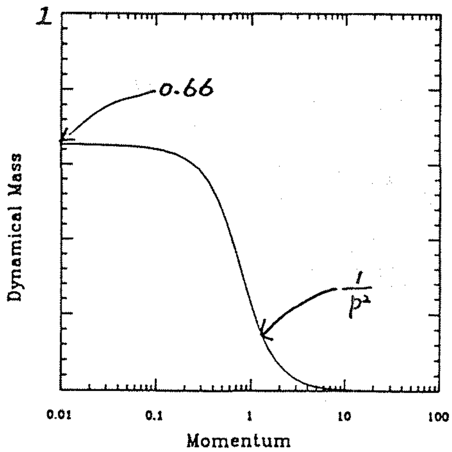

Figure 1. The non-trivial solution in lowest ladder approximation in the $\frac{e^2}{2\pi} = 1$ unit

The results are as follows. We find a non-trivial solution (Fig 1.). The generated mass is $\Sigma(0) = \frac{e^2}{\pi^2}$. Then the effective potential[5] has the value $V_{eff} = -0.033(\frac{e^2}{2\pi})^3$ so that the non-trivial solution is stable. The vacuum expectation value $< \overline{\psi}\psi >$ is $1.04(\frac{e^2}{2\pi})^2$. Thus we have discovered that the parity-violating effective mass of single two-component fermion is generated dynamically in the lowest ladder approximation.

3. 1/N Analysis

We include the vacuum polarization effect in 1/N approximation in this section. As was known, the vacuum polarization of a two-component fermion contains a parity-violating part as the parity anomaly.[6] The affect of the part on the mass generation is a very important issue that should be investigated. The topological consideration suggests a fruitful structure of the dynamics in QED_3.[7] But in this paper, we ignore the part only for the simplicity as was done in the previous analysis. The analysis of the case including the parity-violating part is now progress and will appear in future.

The S-D equation including the effect of the vacuum polarization in $1/N$ approximation is

$$\begin{aligned} A(p) =& 1 - \frac{1}{N}\frac{e^2}{2\pi}\frac{1}{4\pi}\int_0^\infty dk \frac{k}{p^3}\frac{A(k)}{A(k)^2k^2+B(k)^2}\Big\{2pk + c(x-y) + c^2\ell n\Big(\frac{y+c}{x+c}\Big) \\ &+ \frac{1}{c}xy(x-y) + \frac{1}{c^2}(xy)^2\ell n\Big(\frac{y(x+c)}{x(y+c)}\Big) + \eta(2pk + (p^2+k^2)\ell n\Big(\frac{x}{y}\Big))\Big\}, \quad (3a) \\ B(p) =& \frac{1}{N}\frac{e^2}{2\pi}\frac{1}{2\pi}\int_0^\infty dk \frac{kB(k)}{A(k)^2k^2+B(k)^2}\Big\{2\ell n\Big(\frac{y+c}{x+c}\Big) + \eta\ell n(\frac{y}{x})\Big\}, \quad (3b) \end{aligned}$$

where we have defined variables as $c = \frac{e^2}{16}N$, $x = |p-k|$, $y = p+k$. Here we should notice that A does not equal to unit even in the Landau gauge. So we solve the coupled integral equations in the Landau gauge $\eta = 0$. The method of solving the equations is the same as the lowest ladder case. We have used the numerical iteration method.

As the result, we have found a flavor dependence of the generated mass(Fig. 2). We can see that the effective mass decreases as the flavor number increases. This is due to the screening effect by the vacuum polarization.

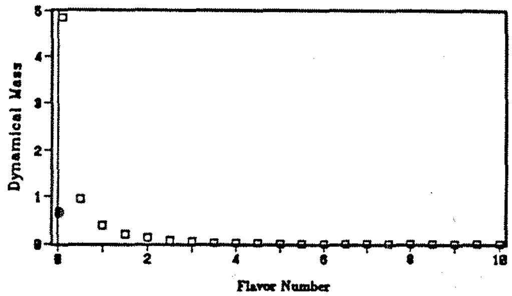

Figure 2. The flavor dependence of the generated mass in the $\frac{e^2}{2\pi} = 1$ unit: The dot indicates the value of the lowest ladder case.

The most important problem that should be answered is whether there exists the critical flavor number or not. Within the precision of the numerical calculation at present, it is difficult to give a definite conclusion. Now we proceed to improve the precision.

4. Conclusion and discussion

We have investigated the dynamical fermion mass generation in QED_3 by solving S-D equation. In the lowest ladder approximation, we have found the non-trivial solution. Thus the single two-component fermion becomes massive dynamically breaking the parity symmetry. Further we have included the vacuum polarization effect in the $1/N$ approximation. We have estimated the effective fermion mass under the screening effect by the vacuum polarization.

The important point of our analyses is that we do not linearize the integral kernel and include the effect of wave function renormalization. And also, we have done the analyses in the two-component formalism. The symmetry to which we concentrate is the parity symmetry, besides the four-component formalism treats the chiral symmetry.

The crucial problems are still remained. The first is whether the critical number of the flavor can be found in our approach. The second is how the parity anomaly affects the dynamical mass generation. The studies on these subjects are proceeding.

More details of this work and a development after this workshop are appeared in Refs. 8 and 9.

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The Critical Behavior of QED in any dimension

HAJIME NAKATANI

Department of Physics, Nagoya University,
Nagoya 464-01, Japan

ABSTRACT

We consider the critical behavior of QED in $3 \sim 6$ dimensional space-time. We obtain the chiral-symmetry-breaking solutions of the Schwinger-Dyson equation in d-dimensional QED in the quenched ladder approximation and show that for $d > 4$ the sclaing law is the mean-field type. We also study QED_3 beyond the quenched ladder approximation and show that the scaling law is dependent of the value of the infrared cutoff.

In this report, we consider the critical behavior of the d-dimensional Quantum Electromagnenic Dynamics (QED_d).[1~3] Especially we study QED_d where d is not equal to 4* . In QED_4, it is well-known that in the quenched ladder approximation, the scaling behavior is the singurality-type or so called "Miransky scaling".[4] Is this scaling common irrespective of the space-time dimensions? Is this scaling also correct even if there is vacuum polarization included? Here we try to answer these questions in the framework of the Schwinger-Dyson equation.

* For QED_4, see Kondo's reprot in this proceedings.

First, we consier the scaling law of the dynamical mass and the coupling in QED_d in the quenched ladder approximation, that is, the vertex $\Gamma_\mu(p,k)$ is bare and the vacuum polarization function $\Pi(k)$ is zero;

$$\Gamma_\mu(p,k) \; = \; \gamma_\mu \qquad \text{and} \qquad \Pi(k) \; = \; 0 \; . \tag{1}$$

We write the fermion propagator $S(p)$ as

$$S(p)^{-1} = A(p^2)\, \not p + B(p^2) \; . \tag{2}$$

Then the Schwinger-Dyson equation for $A(p^2)$ and $B(p^2)$ is written by

$$A(x) = 1 + g_d \int_\epsilon^{\Lambda^2} dy \frac{y^{(d-2)/2} A(y)}{yA^2(y) + B^2(y)} L_d(x,y) \tag{3}$$

$$B(x) = g_d \int_\epsilon^{\Lambda^2} dy \frac{y^{(d-2)/2} A(y)}{yA^2(y) + B^2(y)} K_d(x,y) \tag{4}$$

where $x := p^2, y := k^2$, ϵ is the infrared cutoff, Λ the ultraviolet cutoff and

$$g_d^2 := \frac{e^2}{2^d \pi^{(d+1)/2} \Gamma(\frac{d-1}{2})} \; . \tag{5}$$

$L_d(x,y)$ and $K_d(x,y)$ are kernels. As proved in ref.[2], in quenched planar approximationan and in the Landau gauge $L_d(x,y)$ is simply zero so that the wave renormalization function $A(x) = 1$. Note that this is consisitent with the Ward-Takahashi identity. $K_d(x,y)$'s in $3 \sim 6$ dimensions are the following,

$$K_3(x,y) = \frac{2}{\sqrt{xy}} \ln \frac{\sqrt{x} + \sqrt{y}}{|\sqrt{x} - \sqrt{y}|} \; , \tag{6}$$

$$K_4(x,y) = \frac{3\pi}{x + y + |x - y|} \; , \tag{7}$$

$$K_5(x,y) = \frac{2(x+y)}{xy} - \frac{(x-y)^2}{(xy)^{3/2}} \ln \frac{\sqrt{x} + \sqrt{y}}{|\sqrt{x} - \sqrt{y}|} \; , \tag{8}$$

$$K_6(x,y) = \frac{5\pi}{8}\left[\frac{3x-y}{x^2}\theta(x-y) + \theta(y-x)\right] . \tag{9}$$

We solve numerically the above SD equation with kernels (6) $\sim$ (9) to obtain the scaling laws against the dimensionless coupling β_d,

$$\beta_d := \frac{\Lambda^{4-d}}{(d-1)g_d^2} . \tag{10}$$

The dynamical mass can be writen using β_d as

$$m = \Lambda f(\beta_d) , \tag{11}$$

where $f(\beta_d)$ is defined as the scaling function.

The numerical results show the following scaling functions,

$$f_3(\beta_d) \propto \frac{1}{\beta_d} , \tag{12}$$

$$f_4(\beta_d) \propto \exp\left(-\frac{\pi}{\sqrt{\beta_d/\beta_d^c - 1}}\right) , \tag{13}$$

$$f_5(\beta_d) \propto \sqrt{\beta_d^c - \beta_d} , \tag{14}$$

$$f_6(\beta_d) \propto \sqrt{\beta_d^c - \beta_d} . \tag{15}$$

where β_d^c is the critical coupling. In QED_4, the scaling law is singularity type. On the other hand, in QED_3 there is no phase transition, that means that only symmery-breaking phase survives. This is also confirmed by the analytical solution in the bifurcation method. And in QED_d($d > 4$) the scaling law is the mean-field type. Although we have no analytical proof, in any higher dimensions than 4 the scaling would be the mean-field type. So these results imply that the singularity-type scaling is rather spacial one in QED, and to confirm the scaling type we should study more details in QED beyond the quenched ladder approximation.

As one of examples beyond the quenched ladder approximation, we consider the vacuum polarization effect in QED_3.

QED_3 is an interesting model which is superrenormalizable and have a similarity with QED_4. And also it seems to be related to the recent study for high T_c superconductor, the quark confinement and so on. Furthermore, the calculability of the angular integration in SD equation without any approximation makes easy to analyze the flavor dependence of the model.

So far many pepole has been discussing this model, in the framework of the Schwinger-Dyson(SD) equation combined with the $1/N$ expansion for the vacuum polarization.[5~8] There are two claims about the question, whether or not there exists the critical point of the fermion number N_c in QED_3. Appelquist, Nash and Wijewardhana[6] (ANW) pointed out that there exists the finite critical point, $N_c = 32/\pi^2$, using Appelquist et al.'s assumption[5] that the wave function renormalization would be negligible in the large N limit. Matsuki et al.[9] also obtained the same result from the viewpoint of the effective potential. The existence of the critical point is also supported by the Monte Carlo(MC) calculation by Dagotto et al.[10] On the other hand, Pennington and Webb[7] (PW) and Atkinson, Jhonson and Pennington[8] (AJP) claimed that if one takes into account the $1/N$ correction to the wave-function renormalization, the critical point N_c in the infinite cutoff limit goes away to infinity against ANW's result. This means that only the symmetry-breaking-phase survives in QED_3.

Generally, in QED the wave-function renormalization is unavoidable if the vacuum polarization in photon propagator is included. This is in sharp contrast with the quenched planar QED in the Landau gauge.[2] In fact, the one-loop correction to the photon propagator leads to the non-trivial wave-function renormalization even in the Landau gauge. Therefore Appelquist et al.'s assumption is not justified *a priori*, if the effect of the fermion loop is included.

We solve the SD gap equation in QED_3 combined with the $1/N$ expansion for the vacuum polarization without using the Appelquist et al.'s assumption.[5,6] Actual calculation have been done with the approximately equivalent differntial equation.[3] We consider the leading correction in the $1/N$ expansion, i.e. the one-loop correction in the photon propagator for massless fermion,[11]

$$\Pi(p) \; = \; \frac{\tilde{\alpha}}{p} \,, \tag{16}$$

where

$$\tilde{\alpha} := \frac{e^2 N}{8} \,. \tag{17}$$

The SD equation for the fermion propagator in Landau gauge is written by

$$\begin{aligned} A(p) \; = \; & 1 - \frac{\tilde{\alpha}}{\pi^2 N} \frac{1}{p^3} \int_{\epsilon}^{\infty} dk \frac{k A(k) G(p^2, k^2)}{k^2 A(k)^2 + B(k)^2} \\ & \times \left[\tilde{\alpha}^2 \ln \frac{p + k + \tilde{\alpha}}{|p - k| + \tilde{\alpha}} - \tilde{\alpha}(p + k - |p - k|) + 2pk \right. \\ & - \frac{1}{\tilde{\alpha}} |p^2 - k^2| (p + k - |p - k|) \\ & \left. - \frac{1}{\tilde{\alpha}^2} (p^2 - k^2)^2 \{ \ln \frac{p + k + \tilde{\alpha}}{|p - k| + \tilde{\alpha}} - \ln \frac{p + k}{|p - k|} \} \right] \,, \end{aligned} \tag{18}$$

$$B(p) \; = \; \frac{4\tilde{\alpha}}{\pi^2 N} \frac{1}{p} \int_{\epsilon}^{\infty} dk \frac{k B(k) G(p^2, k^2)}{k^2 A(k)^2 + B(k)^2} \ln \frac{p + k + \tilde{\alpha}}{|p - k| + \tilde{\alpha}} \,. \tag{19}$$

We have paid special attention to the critical value N_c of the fermion flavor and the scaling law in the neighborhood of N_c. We showed that the scaling behavior of the dynamical mass is restricted by the inequality and discussed the relation between the scaling law and the "generalized vertex ansatz",

$$\Gamma_\mu(p, k) = \gamma_\mu A(k)^n \,. \tag{20}$$

Our numerical results show that the scaling law depends on the infrared cutoff. Actually in the limit of the infrared cutoff $\epsilon \to 0$, we have three types of the scaling law depending on the vertex ansatz, i.e. the exponential type, the essential-singularity type and the power-law type,

$$f(N) \propto \exp(-CN) \,, \qquad \text{for } n < 2 \,, \tag{21}$$

$$f(N) \propto \exp(-2\pi/\sqrt{N_c/N - 1}) , \qquad \text{for } n = 2 , \tag{22}$$

$$f(N) \propto (N_c - N)^{\lambda} \qquad \text{for } n > 2 . \tag{23}$$

We can give an explanation on this difference based on the concept of the effective coupling. The result of the exponential type would be the physical one and agrees with the previous result obtained by PW and AJP. Recent works by Atkinson, Johnson and Maris[14] proposed the $n = 2$ case as the physical one from the analysis of the anomalous dimensions.

On the other hand, in the presence of the finite infrared cutoff, the scaling obeys the mean-field type independent of the vertex ansatz,

$$f(N) \propto (N_c - N)^{1/2} , \tag{24}$$

and N_c has a finite value which depends on the infrared cutoff. According to MC results, there exists a finite critical value for fermion flavor. It is, however, still an open question what the scaling type really is in QED_3. It should be remarked that infrared cutoff introduced in our framework may correspond to the lattice size in MC simulation, while the ultraviolet cutoff corresponds to the lattice spacing. It appears that our framework provides us with a possibility, which enables us to explain apparently conflicting results based on the SD equation[7,8] and the MC simulation.[10] It is quite interesting that resent MC simulation by DESY group[13] is fit with the analysis of the mena-field method.

Our investigation have been restiricted in the Landau gauge and quenched ladder apporximation or atmost including the one-loop correction in the vacuum polarigation. Beyond these restriction we plan to perform the numerical calculation of the SD equation beyond one-loop corrction[12] to the vacuum plarization including the improvement of the vertex.[15]

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1989 WORKSHOP ON

DYNAMICAL SYMMETRY BREAKING

Nagoya

21–23 December, 1989

Program

Dec. 21 (Thu.)

11:00–12:05 Chair T. Maskawa (RIFP)

11:00–11:05 *Opening:* Y. Ohnuki (Nagoya)
11:05–12:05 Y. Nambu (Chicago/RIFP)
Model Building Based on Bootstrap Symmetry Breaking

12:05–13:30 lunch

13:30–15:10 Chair T. Maskawa (RIFP)

13:30–14:20 K. Yamawaki (Nagoya)
Fun with Large Anomalous Dimension in Dynamical Symmetry Breaking
14:20–14:45 M. Tanabashi (Nagoya)
Dynamical Symmetry Breaking due to Strong Coupling Yukawa Interaction
14:45–15:10 I. Watanabe (Hiroshima)
Color-sextet Condensation

15:10–15:40 Coffee Break

15:40–18:10 Chair T. Kugo (Kyoto)

15:40–16:30 J. Kubo (Kanazawa)
Is There Any Relation between Dynamical Symmetry Breaking and Reduction of Coupling Constants ?
16:30–16:55 N. Maekawa (Kyoto)
Compositeness Condition in Renormalization Group Equation
16:55–17:20 M. Yasuè (INS)
Extra Weak Bosons Implied by Complementarity in a Confining Gauge Theory
17:20–17:45 K. Akama (Saitama Medical College)
Dynamics of the Nambu-Jona-Lasinio Type for Subquarks
17:45–18:10 M. Wakamatsu (Osaka)
Baryon Physics Based on the Nambu–Jona-Lasinio Model

18:30–20:30 Banquet

Dec. 22 (Fri.)

9:00–12:10		Chair R. Fukuda (Keio)
9:00– 9:50	T. Kugo (Kyoto) *Calculating* f_π	
9:50–10:15	M. Suwa (Niigata) *Scalar Bound States in SCQED and Technicolor Model*	
10:15–10:40	S. Shuto (Nagoya) *BCS-type Relation for the Composite Higgs Boson in the Gauged Nambu-Jona-Lasinio Model*	
10:40–10:55	Coffee Break	
10:55–11:45	K.-I. Kondo (Chiba) *Dynamical Chiral Symmetry Breaking and Scaling Law in Strong Coupling Unquenched QED*	
11:45–12:10	M. Hirayama (Toyama) *On the Solution of the Schwinger-Dyson Equation of QED*	

12:10–13:30 lunch

13:30–15:10		Chair K.-I. Kondo (Chiba)
13:30–14:20	M. Imachi (Kyushu) *Renormalization Group Flow in Lattice QED and Four Fermi Coupling*	
14:20–14:45	Y. Sugiyama (Nagoya) *Chiral Phase Transition in the Effective Theory of QED plus Nambu-Jona-Lasinio Model on the Lattice*	
14:45–15:10	Y. Kikukawa (Nagoya) *Ultraviolet Fixed Point Structure of Renormalizable Four-Fermion Theory in Less Than Four Dimensions*	
15:10–15:40	Coffee Break	
15:40–17:50		Chair Z. Maki (RIFP)
15:40–16:30	T. Muta (Hiroshima) *Observable Consequences of the Strong Coupling Phase of QED*	
16:30–16:55	T. Inagaki (Keio) *Photon Pairing in Quantum Electrodynamics*	
16:55–17:10	T. Asanuma (Tokyo) *A Search for Correlated* e^+e^- *Pairs in the Decay of* ^{241}Am	
17:10–17:25	T. Tsunoda (Tokyo) *A Search for Correlated* e^+e^- *Pairs in the Fission Process*	
17:25–17:50	T. Hanawa (Nagoya) *Strong Coupling QED and Gamma-ray Bursts*	

Dec. 23 (Sat.)

9:00–10:40 Chair K. Higashijima (KEK)

9:00– 9:50 K. Nishijima (RIFP)
Fermion Mass Generation in Chiral-symmetric Gauge Theories

9:50–10:15 T. Matsuki (Tsukuba Institute of Science and Technology)
Stability at the Origin in (2+1)-dimensional QED

10:15–10:40 M. Ukita (Keio)
Effective Action and S Matrix — the On-shell Expansion —

10:40–10:55 Coffee Break

10:55–12:10 Chair T. Muta (Hiroshima)

10:55–11:45 R. Fukuda (Keio)
Inversion Method and its Applications

11:45–12:10 M. Komachiya (Keio)
Effective Action and the Energy Levels of Hydrogen-, Helium-, and Lithium-like Atoms

12:10–13:30 lunch

13:30–14:20 Chair K. Yamawaki (Nagoya)

13:30–13:55 Y. Hoshino (Kushiro National College of Technology)
Fermion Mass Generation in QED_3

13:55–14:20 H. Nakatani (Nagoya)
Critical Behavior of QED in Any Dimension

LIST OF PARTICIPANTS

Akama, K.	Saitama Medical Univ.
Aoyama, S.	Univ. of Padova - Italy
Aramaki, M.	Nagoya Univ.
Asanuma, T.	Univ. of Tokyo
Endo, K.	Hiroshima Univ.
Fujigaki, M.	Aichi Medical Univ.
Fukuda, R.	Keio Univ.
Fusaoka, H.	Aichi Medical Univ.
Hanawa, T.	Nagoya Univ.
Harada, M.	Nagoya Univ.
Hayakawa, Y.	Nagoya Univ.
Hirayama, M.	Toyama Univ.
Hoshino, Y.	Kushiro Nat. Coll. of Tech.
Hosono, S.	Nagoya Univ.
Imachi, M.	Kyushu Univ.
Inagaki, T.	Keio Univ.
Kan, H.	Nihon Univ.
Kanki, T.	Kinki Univ.
Kikukawa, Y.	Nagoya Univ.
Kitakado, S.	Toyata Inst. of Tech.
Kitamura, I.	Chubu Univ.
Kitazawa, N.	Nagoya Univ.
Komachiya, M.	Keio Univ.
Kondo, K.-I.	Chiba Univ.
Kubo, J.	Kanazawa Univ.
Kugo, T.	Kyoto Univ.
Maekawa, N.	Kyoto Univ.
Maki, Z.	RIFP, Kyoto Univ.
Maskawa, T.	RIFP, Kyoto Univ.
Matsuki, T.	Tukuba Inst. Sci. Tech.
Matsuoka, T.	Nagoya Univ.
Matsushima, T.	Nagoya Univ.
Matsuyama, T.	RIPF, Kyoto Univ.
Mishima, T.	Nagoya Univ.
Murai, T.	Nagoya Univ.
Muta, T.	Hiroshima Univ.
Nakatani, H.	Nagoya Univ.
Nakatsu, T.	Univ. of Tokyo
Nambu, Y.	Univ. of Chicago - U.S.A.
Nishijima, K.	RIFP, Kyoto Univ.
Nishitani, T.	Kikuchi College of Optometry
Ogawa, S.	Nagaya Univ.
Ohnuki, Y.	Nagoya Univ.
Ono, T.	Kanazawa Univ.
Sawada, S.	Nagoya Univ.
Seki, R.	INS, Univ. of Tokyo
Shibata, A.	Nagoya Univ.
Shiokawa, K.	Kanazawa Univ.
Shiozaki, K.	Nagoya Inst. of Tech.
Shuto, S.	Nagoya Univ.
So, H.	Niigata Univ.
Suematsu, D.	Kanazawa Univ.

Sugiyama, Y.	Nagoya Univ.
Suwa, M.	Niigata Univ.
Suzuki, T.B.	Nagoya Univ.
Tanabashi, M.	Nagoya Univ.
Teshima, T.	Chubu Univ.
Tsujimaru, S.	Nagoya Univ.
Tsunoda, T.	Univ. of Tokyo
Ukita, S.	Keio Univ.
Wakamatsu, M.	Osaka Univ.
Watanabe, S.	Nagoya Univ.
Watanabe, I.	Hiroshima Univ.
Yamawaki, K.	Nagoya Univ.
Yasue, M.	INS, Univ. of Tokyo
Yasuno, M.	Nagoya Univ.