Journal of Physics: Conference Series **91** (2007) 012003 doi:10.1088/1742-6596/91/1/012003

Scalar fields: at the threshold of astrophysics

F. S. Guzmán

Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo. Edificio C-3, Cd. Universitaria, A. P. 2-82, 58040 Morelia, Michoacán, México.

E-mail: guzman@ifm.umich.mx

Abstract. In this manuscript the potential existence of self-gravitating complex scalar field configurations is explored. Stable boson stars are presented as potential black hole candidates, and the strengths and weaknesses of such idea are described. On the other hand, Newtonian boson systems are also studied because they are the bricks of the structure within the scalar field dark matter model or the Bose condensate dark matter; the collapse of density fluctuations is described; also the interaction between two structures is shown to allow solitonic behavior, which in turn allows the formation of ripples of dark matter. The processes related to potential observations are also discussed.

1. Introduction

Scalar fields appear in several branches of theoretical physics and topics related to theories of gravity and a complete presentation of results related to astrophysics for all the types of scalar fields would not be practical. That is why in this manuscript we deal only with scalar fields minimally coupled to gravity, that is, the scalar field itself has an identity, and is not a scalar field used within scalar-tensor theories as an artifact to modify the gravitational field. That is, it is assumed here that scalar fields might be of fundamental nature, either obtained from the low energy limit of higher dimensional theories or as a field that represents the mean field approximation of a condensate made of spin-less particles at zero temperature.

One more restriction is that we deal with complex scalar fields. Therefore the Lagrangian density of our system reads:

$$\mathcal{L} = -\frac{R}{16\pi G} + g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + V(|\phi|^2), \qquad (1)$$

which corresponds to a complex scalar field minimally coupled to gravity. Lagrangian (1) enjoys a global $U(1)$ symmetry, which indicates the existence of a conserved scalar charge. An interesting property of this system is that there are solutions for a harmonic time dependent scalar field on a static space-time, the so called boson stars (BSs), which will play a main role in this manuscript. The reason is that BSs have been the most widely studied scalar field objects in astrophysically related grounds.

In the real scalar field case the Lagrangian does not show a global symmetry. However equilibrium configurations called oscillatons have been constructed for the potentials of BSs [1, 2, 3, 4], astrophysical applications have been pointed out in the dark matter problem [5], and the geodesics of such solutions have been also studied in order to establish predictions about their potential existence [4]. However, oscillatons present a very dynamical geometry, which makes it more difficult to study in astrophysical applications involving time scales of the order of the oscillations of the geometry. This is why we do not deal with these solutions in the present manuscript and would be analyzed elsewhere.

2. Boson stars

2.1. Basics of boson stars

The equations obtained from the Lagrangian above are Einstein's equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ with the stress-energy tensor

$$T_{\mu\nu} = \frac{1}{2}[\partial_\mu\phi^*\partial_\nu\phi + \partial_\mu\phi\partial_\nu\phi^*] - \frac{1}{2}g_{\mu\nu}[\phi^{*,\alpha}\phi_{,\alpha} + V(|\phi|^2)]. \quad (2)$$

The equation for the scalar field is the Klein-Gordon equation

$$\left(\Box - \frac{dV}{d|\phi|^2}\right)\phi = 0, \quad (3)$$

where $\Box\phi = \frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}g^{\mu\nu}\partial_\nu\phi]$. Boson stars are characterized by the potential $V = m^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4$, where m is understood as the mass of the boson and λ is the coefficient of a two body self-interaction mean field approximation between bosons.

Boson stars (BSs) are spherically symmetric solutions to the above set of equations under a particular condition: the scalar field has a harmonic time dependence $\phi(r,t) = \phi_0(r)e^{-i\omega t}$, where r is the radial spherical coordinate. This condition implies that the stress energy tensor in (2) is time-independent, which implies through Einstein's equations that the geometry is also time-independent. That is, there is a time-dependent scalar field oscillating upon a time-independent geometry whose source is the scalar field itself. It is possible to construct solutions for boson stars assuming that the metric can be written in Schwarzschild coordinates as

$$ds^2 = -\alpha(r)^2dt^2 + a(r)^2dr^2 + r^2d\Omega^2. \quad (4)$$

The resulting Einstein-Klein-Gordon equations are ordinary on the radial coordinate and define an eigenvalue problem for the frequency ω, provided the conditions of spatial flatness at the origin and asymptotic flatness for large r. BS solutions are constructed by solving the eigenvalue problem for ω, knowing that for each value of the central scalar field $\phi_0(0)$ there is a unique ω. The resulting configurations are presented in Fig. 1a for two different values of λ. When constructing such diagrams it is usually assumed the rescaling of variables given by: $\tilde{\phi}_0 = \sqrt{4\pi G}\phi_0$, $\tilde{r} = mr$, $\tilde{t} = \omega t$, $\tilde{\alpha} = \frac{m}{\omega}\alpha$ and $\Lambda = \frac{2\lambda}{8\pi G m^2}$, which is also the convention used here.

The stability-instability analysis of these solutions has been performed using perturbation theory [6] and full non-linear numerical relativity [7, 8, 9]. Configurations in Fig. 1a are assumed to posses nodeless $\phi_0(r)$ profiles, and solutions with nodes (called excited states) can also be constructed, although they have been shown to be unstable [10] and cannot be considered as astrophysically relevant.

Besides the instability branch that brings a configuration to collapse into a black hole (see caption of Fig. 1a), there is another instability that makes the star to explode. The reason argued is that such configurations have a positive binding energy ($E_B = M - mN$), where N is the conserved charge of (1) and M is the mass function

$M = \frac{r}{2}\left(1 - \frac{1}{a^2}\right)$ [6]. Recently in [9] an exploding BS was shown for a $\Lambda = 0$ configuration and in [11] it has been shown that such behavior is generic also for $\Lambda > 0$.

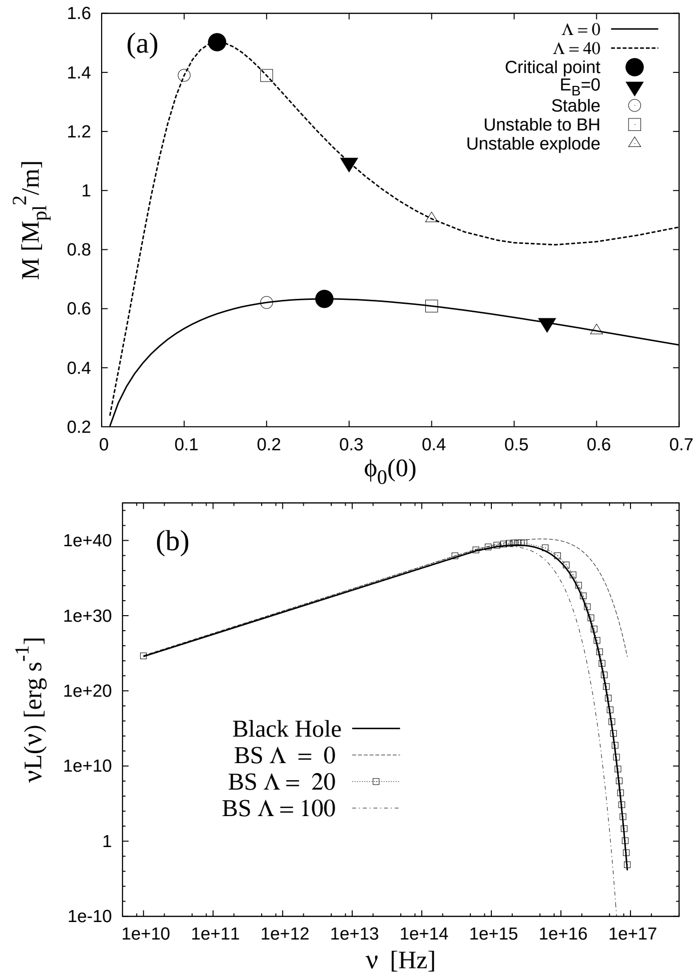

Figure 1. (a) Sequences of equilibrium configurations for two values of Λ are shown as a function of the central value of the scalar field $\phi_0(0)$; each point in the curves corresponds to a solution of the eigenvalue problem and represents a boson star configuration. The filled circles indicate the critical solution that separates the stable from the unstable branch. Those configurations to the left of the maxima, like the white circles, represent stable configurations. The inverted triangles indicate the point at which the binding energy is zero. Those configurations between the filled circles and the inverted triangles (along each sequence) like the white squares, collapse into black holes as a response to a perturbation. Configurations to the right of the inverted triangles, like the white triangles, disperse away. (b) Emission spectra from an accretion disk around a black hole and three BSs with different values of Λ and the same mass $M = 0.633(M_{pl}^2/m)$.

2.2. Boson stars as black hole candidates

BS solutions have been found to match astrophysical parameters that mimic those of some neutron star models, in fact, the self-interaction term of the scalar field ($\Lambda \neq 0$) was originally introduced with this purpose [12]. Such results inspired the

Journal of Physics: Conference Series **91** (2007) 012003 doi:10.1088/1742-6596/91/1/012003

search for observable predictions that could determine the existence of boson stars. Among those proposals, BS were used as black hole candidates (BHC) and some effects were predicted. The method included the use of a simple accretion disk model. The accretion disk model is that of a geometrically thin, optically thick, steady accretion disk. The power per unit area generated by such a disk rotating around a central object is given by [13, 14]:

$$D(r) = \frac{\dot{M}}{4\pi r}\frac{\alpha}{a}\left(-\frac{d\Omega}{dr}\right)\frac{1}{(E-\Omega J)^2}\int_{r_i}^{r}(E-\Omega J)\frac{dJ}{dr}dr, \tag{5}$$

where $\dot{M}$ is the accretion mass rate, r_i is the inner edge of the disk, α and a are the metric functions in (4), E, J, Ω are the energy, angular momentum and angular velocity per unit of mass of a test particle in the accretion disk. For black holes r_i is assumed to be at the ISCO ($r = 6M$) of the hole. Since BSs allow circular orbits in the whole spatial domain it can be considered that $r_i = 0$ for BSs, or any other finite number less than $6M$ without modifying the behavior of the spectrum. Furthermore, by assuming it is possible to define a local temperature we use the Stefan-Boltzmann law so that $D(r) = \sigma T^4$, where $\sigma = 5.67 \times 10^{-5}$ $erg\ s^{-1}\ cm^{-2}\ K^{-4}$ is the Stefan-Boltzmann constant. Now, considering the disk emits as a black body, the dependence of T on the radial coordinate is used to calculate the luminosity $L(\nu)$ of the disk and the flux $F(\nu)$ through the expression for the black body spectral distribution:

$$L(\nu) = 4\pi d^2 F(\nu) = \frac{16\pi h}{c^2}\cos(\zeta)\nu^3\int_{r_i}^{r_f}\frac{rdr}{e^{h\nu/kT}-1}, \tag{6}$$

where d is the distance to the source, r_i and r_f indicate the location of the inner and outer edges of the disk, $h = 6.6256 \times 10^{-27}$ $erg\ s$ is the Planck constant, $k = 1.3805 \times 10^{-16}$ $erg\ K^{-1}$ is the Boltzmann constant and ζ is the disk inclination.

The steps carried out to look at the effects of a boson star acting as a BHC are as follows:

(i) Assume the metric of the space-time is given by (4).

(ii) Define the space-time functions a and α by choosing one of the equilibrium configurations in Fig. 1a and calculate M.

(iii) Define the metric of the equivalent Schwarzschild black hole through $\alpha_{BH} = \sqrt{1-2M/r}$ and $a_{BH} = 1/\alpha_{BH}$.

(iv) Calculate the angular velocity, angular momentum and energy of a test particle for both space-times $\Omega_{BS,BH}$, $L_{BS,BH}$, $E_{BS,BH}$.

(v) Use such quantities to calculate the power emitted in both cases $D_{BS}(r)$ and $D_{BH}(r)$ defined in (5).

(vi) Calculate the temperature of the disk in both cases $T_{BS}(r) = (D_{BS}(r)/\sigma)^{1/4}$ and $T_{BH}(r) = (D_{BH}(r)/\sigma)^{1/4}$.

(vii) Use such temperature to integrate the luminosity $L_{BS}(\nu)$ and $L_{BH}(\nu)$ using (6) for several values of ν. We use $r_f = 50M$ and a wide range of values of r_f can be used that keep the qualitative properties of the spectra.

What we do here is to repeat our algorithm for the same mass of the object but different values of Λ. The result can be found in Fig. 1b, where the spectrum is softened at high frequencies when increasing Λ; in fact we notice that for $\Lambda = 20$ the spectrum could be that of the equivalent black hole. Therefore, it would be difficult to distinguish between a black hole and such boson star assuming the model of accretion disk used here. The configurations whose spectra appear in Fig. 1b, in physical units, are equivalent to consider that the mass of the boson is $m = 1.2 \times 10^{-25}$GeV, which implies a mass $M = 2.8 \times 10^6 M_\odot$ and various values of Λ. In fact, it would be a sort of super massive black hole or boson star [15, 16].

It is usually thought that the more compact a boson star is, the more similar to a BH it should be. However, the bigger the value of Λ, the less compact the boson star [16]. That is the reason why originally BS were proposed as BHC using the case $\Lambda = 0$ in Fig. 1b, which is a stable configuration very near the peak of instability and the most compact; then it was possible to observe that $\nu L(\nu)$ is orders of magnitude bigger for the BS than for the BH around the $10^{16} Hz$ window. This hardening of the spectrum was expected to be a signal of the existence of BSs, however not observed.

The results in Fig. 1b indicate the following set of conclusions: 1) the boson star configuration used in [14] ($\Lambda = 0$ here) shows the hardening of the spectrum at high frequencies and it was expected to be a signal of the existence of BSs. However those observations have not been reported, 2) among the spectra shown in Fig. 1b, the one with self-interaction $\Lambda = 20$ mimics the spectrum of an accretion disk produced by a Schwarschild black hole with the same mass, 3) this idea that an accretion disk model could help to distinguish a BS from a BH is not valid anymore when the self-interaction coefficient gets into the game, 4) apart of the mass of the boson m, it suffices to add the self-interaction parameter Λ to mimic the spectrum produced by a black hole, which provides more freedom to the BS solutions, 5) therefore, boson stars are still able to play the role of BHCs.

2.3. Boson stars as sources of gravitational waves

Another -still less studied- way to determine the existence of boson stars is the observation of gravitational waves. At this point there are various approaches: studies related to perturbation theory and the analysis of the quasi-normal modes of BS [17, 18], studies using full numerical relativity applied to single perturbed boson stars [19] and also for the collision of boson stars [20].

In the case of perturbed single boson stars, it is known that aside of the emission of gravitational waves there is also emission of scalar field matter and the modes show a damping effect on the gravitational wave modes that is not expected in the emission of gravitational radiation from single perturbed perfect fluid stars [17].

3. Scalar field dark matter

3.1. The scalar field dark matter model

The most widely studied dark matter hypothesis consists in assuming that it is made of point-like cold particles that are responsible for the formation of structure in the universe; among the most studied candidates nowadays are the super symmetric particles that would behave as a cold fluid made of particles. However, two problems associated to the point-like nature of dark matter are: i) that the resulting gravitational collapse shows a central density profile that is not flat and ii) it predicts a non-observed amount of small structures. An alternative to ameliorate these two problems consists in assuming that the dark matter is made of an ultra light spin-less particle, the so called Scalar Field Dark Matter Model (SFDM). In the cosmological frame, the analysis of such hypothesis indicated that the mass power spectrum of structures could be controlled through a parameter in the model, that parameter is the mass of the scalar field representing a spin-less particle [21, 22]. Once the mass m of the boson is fixed, the power spectrum suffers a cut-off according to the mass of the smallest structure desired. An interesting assumption in such analysis is that the scalar field potential was a cosh-like potential, that behaved as an exponential at early times and as a free field case (quadratic potential) at late times, whose behavior was that of the usual cold dark matter model and consequently the SFDM enjoys the same advantages at cosmic scale as the standard lambda cold dark matter model.

Because the SFDM requires the existence of a fundamental scalar field for its reliability, it is natural to consider that this scenario fits very well within unification theory scenarios and braneworld models [23]. This by itself is a good enough reason to consider the SFDM as an alternative powerful model. However, once at cosmic scales the model matches with observations, it is necessary to study the predictions of the model at structure scales. In this sense there have been several results indicating that the model is good also at galactic scales and here we briefly summarize such results.

The fully general relativistic version of the model has stationary solutions that explain the flatness of galactic rotation curves assuming the scalar field was real [24]. These solutions were static and difficult to generalize to more interesting cases, which determined that the time-independence of the space-time had to be relaxed and then scalar field dark matter halos were proposed to be gigantic Oscillatons, that is, time dependent fully relativistic real scalar field solutions to the Einstein-Klein-Gordon system of equations [2, 5].

The Newtonian version of the idea is the fluid dark matter made of scalar fields, which was proposed as an alternative galactic dark matter model [25, 26]. These models assume a static gravitational potential and consider the scalar field halos are in dynamical equilibrium.

The newest step is related to the dynamics and formation of such configurations, and devoted mainly to determine the stability and fate of dark matter fluctuations. Assuming the halos are the result of the evolution of an initial fluctuation, there is no way out of writing the set of equations driving the Newtonian version of self-gravitating scalar fields, the Schrödinger-Poisson (SP) system of equations, for fully time dependent wave functions and gravitational potential:

$$\begin{aligned} i\partial_t \Psi &= -\frac{1}{2}\nabla^2\Psi + U\Psi + \Lambda_N|\Psi|^2\Psi, \\ \nabla^2 U &= |\Psi|^2, \end{aligned} \tag{7}$$

which is the low energy-weak field limit of the EKG system of equations, where Ψ is the Newtonian version of the scalar field ϕ above, U is the gravitational potential sourced by the density of probability and Λ_N is again a constant related to the mean field approximation considering a two-particle interaction correction. The SP system is expected to determine the nonlinear evolution at local scales of fluctuations within the SFDM model after the turnaround point. This system of equations also becomes an eigenvalue problem when a harmonic time dependence is assumed $\Psi = e^{i\omega t}\psi(r)$. Such solutions can also be found with zero nodes (ground states) and with many nodes. The only stable ones are those corresponding to ground states, and the process of decay is only thought to be that of the gravitational cooling, conducted by the emission of scalar field [28, 29]. In Fig. 2a, branches of ground state solutions for various values of

Λ_N are shown, and in Fig. 2b we show the attractor behavior of configurations which at initial time assume wave function profiles that do not correspond to ground states. That is, ground state equilibrium configurations appear as the late time fate of quite arbitrary initial density profiles. In the plot, the initial over-density fluctuations are assumed to be spherically symmetric.

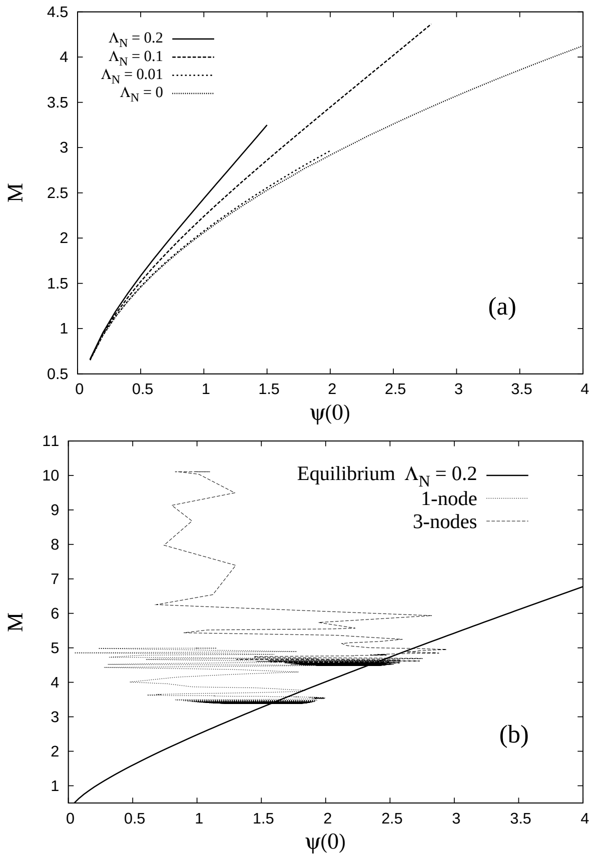

Figure 2. (a) Equilibrium configurations for ground state solutions and various values of Λ_N. The mass is defined by $M = \int |\Psi|^2 d^3x$. Again, the parameter is the central value of the wave function. (b) Attractor behavior for various other configurations that cool down and approach an equilibrium ground state configuration. In fact the two configurations shown, are initially two excited state configurations with one and three nodes. The evolution is calculated by solving the full time dependent SP system of equations (7) using numerical methods (see [26, 27, 28] for details).

An advantage of the SP system versus the EKG is that it is easy to calculate expectation values of observable operators. In particular, it is important to calculate the total energy $E_T = K+W+I$ and the virial relation $2K+W+3I = 0$, where K, W, I are the expectation values of the kinetic, gravitational and self-interaction operators respectively. It has been shown that equilibrium configurations are virialized, and thus arbitrary fluctuations tend toward virialized states.

3.2. Spherical collapse

When the SFDM is assumed to evolve according to the SP system, it is interesting to estimate the virialization time scales depending on the mass of a given initial fluctuation. In this direction relevant results have been found, for instance: it was shown that when the evolution of a structure of galactic mass is followed after the turnaround point, it quickly virializes and tends toward a stationary ground state equilibrium solution of the SP system of equations, whereas one of the mass of a supercluster would still be relaxing at the present time [26]; the condition is that the mass of the boson ($m \sim 10^{-23}$eV) determines the dynamical time scales of the structures [22]. Thus, at the moment the pieces of the model seem to match both, at cosmic and at local scales. Recently, in [28] it was shown that the scalar field gravitational collapse tolerates the introduction of a self-interaction term in the potential, which makes the model to seem quite like a self-gravitating Bose-Condensate at zero temperature. This is so because the Schrödinger equation is the Gross-Pitaevskii equation (the mean field approximation of a Bose Condensate) with a gravitational potential due to the density of probability itself.

3.3. Axial collapse

In [30] it was shown also using numerical evolutions, that spherically symmetric ground state equilibrium solutions of the SP system are stable also against non-spherical perturbations, and moreover, that equilibrium configurations play the role of late-time attractors for initially quite general axisymmetric initial density profiles and not only spherical as shown in [28]. In Fig. 3 we show the evolution of two different initial configurations made of a ground state ψ_{eq} plus an arbitrary contribution $\delta\psi$, where $\delta\psi = Ae^{-x^2/\sigma_x^2 - z^2/\sigma_z^2}$, x, z are the radial and axial cylindrical coordinates respectively, and the sigmas are the widths along the two different coordinates of the perturbation profile $\delta\psi$. The parameters used to generate Fig. 3 are $\sigma_x = 1$, $\sigma_z = 1.5$ for two different amplitudes $A = 0.1, 0.2$. The increase of the mass due to the perturbation is 5% and 9% respectively. Under such conditions, the configuration is not spherical, and instead it shows a considerable integrated difference between the density along z and along x: $S = \int \rho(0,z)dz - \int \rho(x,0)dx$, where $\rho = |\Psi|^2$. Such non-sphericity weighted with the central density is also shown in Fig. 3. The evolution of the two configurations indicates in a M vs ρ plot, that the configurations approach ground state equilibrium configurations (the continuous line in the up-left plot) the set of equilibrium configurations shown in Fig. 2a. This result is the analog of the spherical collapse attractor behavior shown in Fig. 2b. Also shown is the virial relation, where it is evident that the configuration starts oscillating around a virialized state.

3.4. Collision of two structures

Once there is a code that solves the fully time-dependent SP system with axial symmetry, it is natural to investigate the interaction between two structures. It was found that under the condition of positive total energy $E > 0$, the system can show a solitonic behavior. In Fig. 4, one of such cases is presented. Under the opposite condition ($E < 0$) the two blobs collide and form a single structure. When desired, it is possible to add linear momentum along (for example) z to a configuration through the application of the operator $e^{-p_z}\psi_{eq}$ on the wave function of an equilibrium configuration.

The impact of this result might be an explanation for the unusual behavior of dark matter observed in the 1E0657-558 cluster merger [31, 32].

The relaxation process of the collapse is well known to happen through the emission of scalar field, a process called gravitational cooling, and was discovered

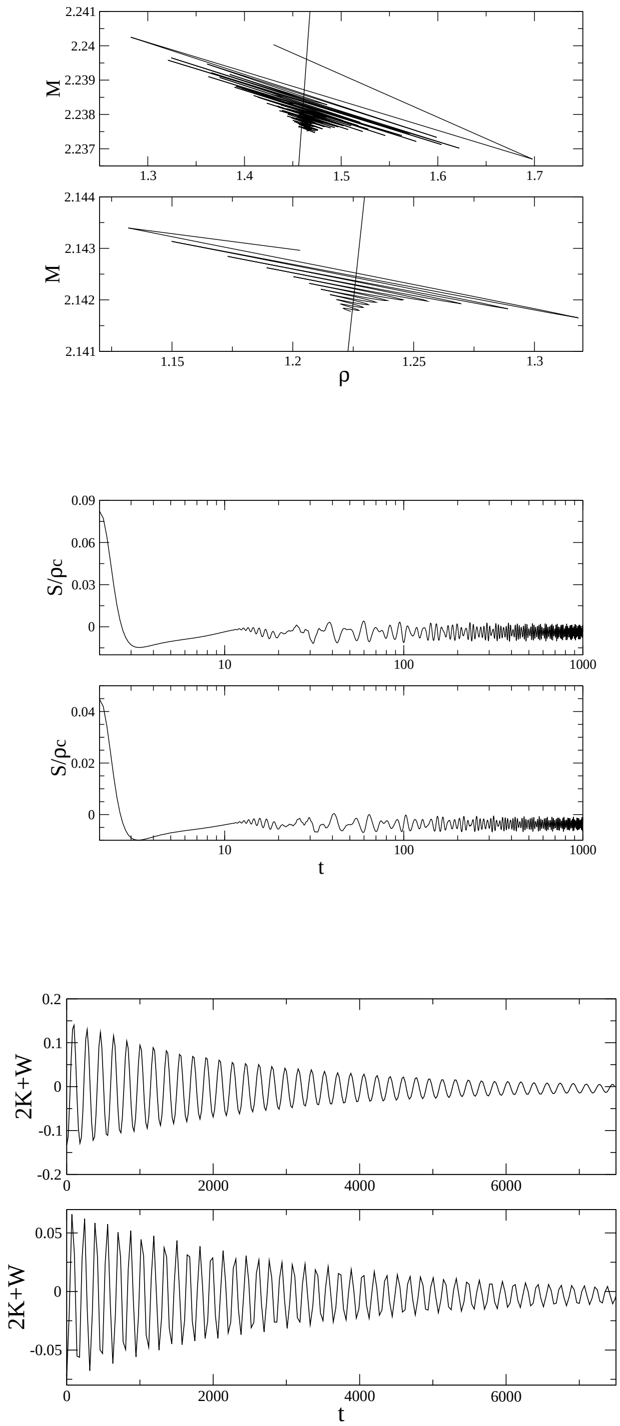

Figure 3. Evolution of two axially symmetric initial data made of an equilibrium configuration plus a non-spherical Gaussian like profile for $\Lambda_N = 0$. Left: we show that the initial axially symmetric configurations evolve toward spherical equilibrium configurations (points in the solid line) through the emission of scalar field. Right: the non-sphericity is shown for both simulations. Bottom: we show the value of the expression $2K + W$; as it oscillates around zero with a decreasing amplitude we conclude that the system approaches a virialized state.

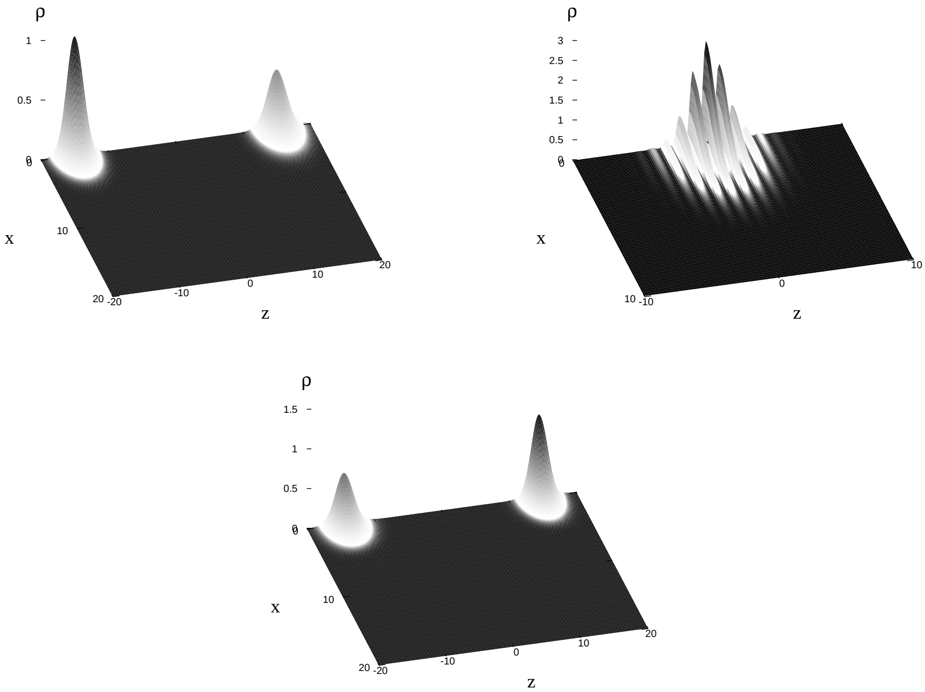

Figure 4. Density $|\Psi|^2$ along the head-on axis of collision at initial, mid and after-collision times. These configurations correspond to $\Lambda_N = 0.2$ and an initial momentum along the head-on axis $p_z = 3.0$; the configurations are initially located in the positions $(x, \pm 15)$. The solitonic behavior is evident and happens under the condition of having an unbounded system with $E_T = K + W + I > 0$, see [33] for further details about this type of behavior.

to happen for spherically symmetric configurations [29, 28]. In the present case of axial symmetry, also scalar field is ejected, however it is not yet understood why the system becomes spherical and what would the process be. It has to be explored the possibility of extracting the amount of energy in gravitational radiation emitted in the process of relaxation.

4. Final comments

The main question about the existence of boson stars is related to the material a BS would be made of. What could the origin of the bosonic matter possibly be and how it is related to the cosmological parameters, are questions that might be answered in the next years. At the present time, it seems tempting to propose definitive observational tests to show either that boson stars exist or not.

On the other hand, the scalar field dark matter seems to pass test after test, and at the moment the simplest way of verifying that galactic halos are made of scalar fields involve the oscillations of equilibrium configurations, which are fingerprints of a particular configuration with a given mass. These oscillations produce particular motion on test particles [4], which for the case of dust and stars in galactic halos made of scalar field should be an indication of the potential existence of galactic scalar field dark matter.

Three main conclusions are in turn: 1) Boson Stars are not discarded as black hole candidates, 2) scalar field dark matter halos should present signals at galactic scales through the peculiar motion of test particles, 3) the apparent solitonic behavior of the Bullet Cluster collision [31, 32] is an indication that dark matter dissipates less momentum than luminous matter, and the condensate presented here shows this property.

What scalar fields in gravity have in common is an important bound on predictability: the mass of the boson fixes the spatial and time scale. The models containing scalar fields consider the mass of the boson and the self-interaction coefficient (or other coefficients of the scalar field potential) as free parameters and therefore one expects that once one of these parameters is fixed by some observations the models will be very restricted and the predictions very clear. As shown here, the only approach to a prediction in the Newtonian case is due to the ultra light nature of the boson mass, which is restricted through the mass power spectrum of structures. In the strong field case (BS case) there are no solid predictions, because there is no preferred scalar field mass suggested by an observation and at the moment one is restricted to study a case where a BS can mimic a black hole without restriction on the parameters of the scalar field potential.

Acknowledgments

This work is partly supported by projects CIC-UMSNH-4.9, PROMEP-UMICH-PTC-121 and PROMEP-UMSNH-CA-22.

References

[1] E. Seidel and W-M. Suen, Phys. Rev. Lett. **66**, 1659 (1991).
[2] M. Alcubierre, R. Becerril, F. S. Guzmán, T. Matos, D. Núñez and L. A. Ureña-López, Class. Quantum Grav. **20**, 2883 (2003). ArXiV: gr-qc/0301105.
[3] L. A. Ureña-López, Class. quantum Grav. **19**, 2617 (2002).
[4] L. A. Ureña-López, T. Matos and R. Becerril, Class. quantum Grav. **19**, 6259 (2002).
[5] M. Alcubierre, F. S. Guzmán, T. Matos, D. Núñez, L. A. Ureña-López and P. Wiederhold, Class. Quantum Grav. **19**, 5017 (2002). ArXiv: gr-qc/0110102.
[6] M. Gleiser, Phys. Rev. **D 38**, 2376 (1988).
[7] E. Seidel and W-M. Suen, Phys. Rev. **D 42**, 384 (1990).
[8] S. H. Hawley and M. W. Choptuik, Phys. Rev. **D 62**, 104024 (2000).
[9] F. S. Guzmán, Phys. Rev. **D 70**, 044033 (2004).
[10] P. Jetzer, Phys. Rep. **220** 4, 163 (1992).
[11] A. Bernal and F. S. Guzmán, Phys. Rev. **D**, submitted.
[12] M. Colpi, S. L. Shapiro and I. Wasserman, Phys. Rev. Lett. **57**, 2485 (1986).
[13] D. Page and K. S Thorne, ApJ **191**, 499 (1974).
[14] D. F. Torres, Nucl. Phys. **B 26**, 377 (2002).
[15] D. F. Torres, S. Capozziello and G. Lambiase, Phys. Rev. **D 62**, 104012 (2000).
[16] F. S. Guzmán, Phys. Rev. **D 73**, 021501 (2006). ArXiV: gr-qc/0512081.
[17] S. Yoshida, Y. Eriguchi and T. Futamase, Phys. Rev. **D 50**, 6235 (1994).
[18] E. Berti and V. Cardoso, Int. J. Mod. Phys. **D 15**, 2209-2216 (2006).
[19] J. Balakrishna, R. Bondarescu, G. Daues, F. S. Guzmán, E. Seidel, Class. Quantum Grav. **23**, 2631-2652 (2006). ArXiV: gr-qc/0602078.
[20] C. Palenzuela, I. Olabarrieta, L. Lehner, S. Liebling, Phys. Rev. **D 75**, 064005 (2007). C. Palenzuela, L. Lehner, S. Liebling, ArXiV:0706.2435
[21] V. Sahni and L. Wang, Phys. Rev. **D 62**, 103517 (2000).
[22] T. Matos and L. A. Ureña-López, Class. quantum Grav. **17**, L75 (2000). T. Matos and L. A. Ureña-López, Phys. Rev. **D 63**, 063506 (2001).
[23] T. Matos, L. A. Ureña-López, M. Alcubierre, R. Becerril, F. S. Guzmán, D. Núñez, 2004, Lect. Notes. Phys. **646**, 401-420. C. G. Boehmer, T. Harko, Class. Quantum Grav., **24**, 3191 (2007).

[24] F. S. Guzmán, T. Matos and H. Villegas-Brena, Astron. Nachr., **320** 3, 97 (1999). F. S. Guzmán and T. Matos, Class. Quantum Grav. **17**, L9 (2000). F. S. Guzmán and T. Matos, Rev. Mex. A. & A. **37**, 63-72 (2001).
[25] A. Arbey, J. Lesgourgues and P. Salati, Phys. Rev. **D 64**, 123528 (2001).A. Arbey, J. Lesgourgues and P. Salati, Phys. Rev. **D 65**, 083514 (2002).A. Arbey, J. Lesgourgues and P. Salati, Phys. Rev. **D 68**, 023511 (2003). A. Arbey, Phys. Rev. **D 74**, 043516 (2006).
[26] F. S. Guzmán and L. A. Ureña-López, Phys. Rev. **D 68**, 024023 (2003). ArXiV: astro-ph/0303440.
[27] F. S. Guzmán and L. A. Ureña-López, Phys. Rev. **D 69**, 124033 (2004). ArXiV: gr-qc/0404014.
[28] F. S. Guzmán and L. A. Ureña-López, ApJ **645**, 814 (2006). ArXiV: astro-ph/0603613.
[29] E. Seidel and W-M. Suen, Phys. Rev. Lett. **72**, 2516 (1994).
[30] A. Bernal and F. S. Guzmán, Phys. Rev. **D 74**, 063504 (2006). ArXiV: astro-ph/0608523.
[31] D. Clowe et al., astro-ph/0608407. Accepted for publication in ApJL.
[32] M. Bradac et al., ApJ **652**, 937-947 (2006). ArXiV: astro-ph/0608408.
[33] A. Bernal and F. S. Guzmán, Phys. Rev. **D 74**, 103002 (2006). ArXiV: astro-ph/0610682.