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WEAK-INTERACTION THEORY AND NEUTRAL CURRENTS*

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I. EFFECTIVE LAGRANGIANS FOR CHARGED AND NEUTRAL CURRENT PROCESSES

After over thirty years of research on the structure of the interaction responsible for β-decay, we have reached the point where an impressively simple and successful phenomenological theory well represents a wealth of data on decays of nucleons, nuclei, and strange particles, as well as data on neutrino interactions with energies from 1 to 50 GeV. All this can be summarized in an effective Lagrangian density, to be used in lowest order, as follows

$$\mathscr{L}_{\mathrm{eff}} = \frac{G}{\sqrt{2}} \mathscr{J}_{\lambda}(x) \mathscr{J}^{\lambda}(x)^{\dagger} \qquad (1.1)$$

Here

$$\begin{aligned} \mathscr{J}_{\lambda}(x) = {} & \bar{e}\gamma_{\lambda}(1-\gamma_5)\,\nu_e + \bar{\mu}\gamma_{\lambda}(1-\gamma_5)\,\nu_{\mu} \\ & + \sum_{i=1}^{3} \left(\bar{d}_i \cos\theta_c + \bar{s}_i \sin\theta_c\right)\gamma_{\lambda}(1-\gamma_5)\,u_i \qquad (1.2) \\ & \left\{ + \sum_{i=1}^{3} \left(\bar{s}_i \cos\theta_c - \bar{d}_i \sin\theta_c\right)\gamma_{\lambda}(1-\gamma_5)\,c_i \right\}^{??} + ?? \end{aligned}$$

$G = 1.02 \cdot 10^{-5}\, M_p^{-2}$ is the Fermi constant. θ_c is the Cabibbo mixing angle ($\sin^2\theta_c \cong 0.055$), while u_i, d_i, s_i are the fields of fractionally charged colored quarks. The last term in curly brackets is the conjectured (but not fully established) charm current involving a fourth quark c_i, about which we shall elaborate later.

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Again and again this model, as described by Eq. (1.1), has successfully survived experimental tests. This is not to say it is beyond challenge. It is well beyond the scope of these lectures to review the status of the phenomenological theory. However, before leaving it, we do wish to mention recent developments which bear on the question of its validity:

1. In a recent LBL experiment,[1] the polarization of the muon emitted in $K_L \rightarrow \mu\pi\nu_\mu$ has been measured, and agrees with the phenomenological theory (supplemented with the PCAC hypothesis). This mitigates a long-standing and vexing discrepancy existing from earlier measurements.[2]

2. Recent experiments on β-transitions in light nuclei[3] have indicated a need for additional anomalous-moment coupling and/or second-class currents[4] of opposite G-parity from that given in Eq. (1.2) in the effective Lagrangian. Such currents are difficult to incorporate in the theoretical structures we discuss in these lectures. It is clearly of great importance to clarify this issue.

3. How to interpret the success of the $\Delta I = 1/2$ rule in $\Delta S=1$ nonleptonic decays remains an ambiguous issue. There is still room for fresh ideas and unconventional interpretations in this area.

4. While there is some experimental evidence for existence of "diagonal" charged-current couplings (e.g., $\bar{\nu}_e\gamma_\lambda(1-\gamma_5)e \quad \bar{e}\gamma^\lambda(1-\gamma_5)\nu_e$ or $\sum_{i,j=1}^{3} \bar{u}_i\gamma_\lambda(1-\gamma_5)d_i \quad \bar{d}_j\gamma^\lambda(1-\gamma_5)u_j$), it is more qualitative than quantitative. A strict current-current structure as written in Eq. (1.1) is far from experimentally established.

5. The algebraic properties of the currents, i.e., their commutation relations among each other (e.g., $SU(3) \otimes SU(3)$ or chiral $SU(6) \otimes SU(6)$) suggest the identification of quark fields made in Eq. (1.1). However it may be a little too strong a conclusion to infer the presence of quark-fields from those algebraic properties alone. In particular there is no evidence from the weak interaction phenomenology for the color degree of freedom.

With the discovery of neutral-current phenomena a few years ago and their apparent connection with weak interactions in general, we now face a situation similar to that encountered in the early days ($t \lesssim 1957$) of β-decay: we now must determine the nature of the effective Lagrangian governing neutral-current phenomena. The processes which have been observed include

$$\begin{aligned}
&\nu_\mu + N \rightarrow \nu_\mu + \text{hadrons} \\
&\bar{\nu}_\mu + N \rightarrow \bar{\nu}_\mu + \text{hadrons} \\
&\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^- \\
&\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-
\end{aligned} \tag{1.3}$$

A fairly general candidate effective Lagrangian for these reactions is

$$\begin{aligned}
\mathscr{L}_{\text{eff}} = \frac{G}{\sqrt{2}}\, \bar{\nu}_\mu \gamma^\lambda (1-\gamma_5)\nu_\mu \Big[& \epsilon_L(e)\, \bar{e}\gamma_\lambda(1-\gamma_5)e \\
& + \epsilon_R(e)\, \bar{e}\gamma_\lambda(1+\gamma_5)e \\
& + (e \longleftrightarrow \mu) \\
& + \epsilon_L(u) \mathop{}_{i=1}^{3} \bar{u}_i\gamma_\lambda(1-\gamma_5)u_i \\
& + \epsilon_R(u) \mathop{}_{i=1}^{3} \bar{u}_i\gamma_\lambda(1+\gamma_5)u_i \\
& + (u \longleftrightarrow d) \\
& + (u \longleftrightarrow s) \\
& + (u \longleftrightarrow c) \\
& + \ldots \Big] \\
+ \frac{G}{\sqrt{2}} \Big[& \epsilon_{LL}(e,u)\, \bar{e}\gamma^\lambda(1-\gamma_5)e \sum_{i=1}^{3} \bar{u}_i\gamma_\lambda(1-\gamma_5)u_i \\
& + \epsilon_{RL}(e,u)\, \bar{e}\gamma^\lambda(1+\gamma_5)e \sum_{i=1}^{3} \bar{u}_i\gamma_\lambda(1-\gamma_5)u_i \\
& + \ldots \Big]
\end{aligned} \tag{1.4}$$

The most vital assumption made here is that the neutrino emitted in ν-induced neutral current processes is the same type as the incident neutrino. This comment is meant to include helicity: at present there is no evidence at all that a right-handed ν_μ (or left-handed $\bar{\nu}_\mu$) exists as a physical particle. If neutral currents proceed according to

$$\nu_{\mu L} + N \rightarrow \nu_{\mu R} + \text{hadrons} \tag{1.5}$$

this would imply existence of a heretofore unknown degree of freedom, and concomitant scalar, pseudoscalar, or tensor neutral-current couplings.[5] There does exist some evidence against pure scalar or pseudoscalar couplings.[6] In any case we here assume they are absent.

Also possible are the reactions

$$\begin{aligned} \nu_\mu + N &\rightarrow L^0 + \text{hadrons} \\ \bar{\nu}_\mu + e^- &\rightarrow \bar{L}^0 + e^- \end{aligned} \tag{1.6}$$

with L^0 some new neutral lepton. Existence of $\bar{\nu}_\mu$-e neutral current processes at GeV energies argues that the mass of L^0 must be small,[7] under 50 MeV (because $\sqrt{s} < 100$ MeV).

There are many coupling constants in Eq. (1.4) to be determined. But this is the basic problem. There are two main routes that may be followed in trying to solve it. They in fact were identified in 1957 for charged-current processes, after the discovery of parity violation provided the impetus needed to straighten out the mess present in that field at that time. One option was simply to measure many, many processes directly and reduce the couplings to a unique form purely from experiment. This program has not been fulfilled to this day, despite its supremely logical nature. (For example, even for muon-decay the couplings have not been uniquely determined.[8]) The other approach was to guess the answer. The masters[9, 10] spake: "Let there be V-A."

And there was V-A. A simple working hypothesis, tested over and over again, has by now become a phenomenological theory.

It is too early to know which path will be the most successful with regard to neutral currents, although we do have a good working hypothesis. Again the masters[10, 11] spake: "Let there be $SU(2) \otimes U(1)$." And thus far the $SU(2) \otimes U(1)$ gauge theory has done very well. In these lectures we shall travel some distance down both paths—sometimes the straightforward, cautious phenomenological path, and sometimes the hypothetical path of the specific $SU(2) \otimes U(1)$ gauge theory. In a situation (as at present) of relatively little data and relatively many phenomenological parameters, the latter path is the most powerful and useful (as long as it survives). However whenever possible the former path is to be preferred: it is safer.[12] I was surprised to find how far it is possible to proceed down the phenomenological path. In particular it may turn out that elastic $\nu_\mu p$ and $\bar{\nu}_\mu p$ scattering will play a pivotal role in disentangling the various couplings.

But before embarking upon specifics, we again warn the reader that already in Eq. (1.4) there contain untested implicit assumptions, which if wrong would greatly influence our present conception of neutral-current phenomena. These include:

1. The identity of the outgoing neutrino, a question we have already addressed.

2. Nonexistence of <u>off-diagonal</u> neutral current reactions, e.g., charm changing neutral current processes, such as $\nu_\mu + u_i \rightarrow \nu_\mu + c_i$. While the absence of $\Delta S=1$ neutral currents and the motivation for charm (GIM mechanism, discussed in Section II) suggests their absence, this is not an inevitability and should be tested.

3. Correctness of the 4-fermion nonderivative coupling structure of Eq. (1.4). This can be tested by observation of the E_ν and Q^2 dependence of neutral current processes.

4. Use of the color-singlet fractionally charged quark structure. One might be able to build a Pati-Salam-like scheme,[13] with broken color degrees of freedom, which might look quite different from Eq. (1.4).

II. HIGHER ORDERS OF THE WEAK INTERACTION

A. High Energy Cutoffs. The charged-current effective Lagrangian rivals the Lagrangian of quantum electrodynamics in predictive power. Given a weak process, short—usually straightforward—calculations (supplemented with the PCAC hypothesis) leads to results in good agreement with experiment. However, it has been known for a long time[14] that such a phenomenological theory is at best only a provisional description valid at low energy. As soon as one considers higher orders in the Fermi coupling, or equivalently weak processes at energies $\sqrt{s} \gtrsim G^{-1/2} \sim 300$ GeV, the phenomenological theory does not make sense. This can be seen best by looking at pure leptonic scattering processes,[15] for example, $e^- + \nu_\mu \rightarrow \nu_e + \mu^-$. Neglecting lepton mass, one helicity-amplitude evidently controls this process. And the total helicity is zero because e^- and ν_μ are both left-handed and have equal and opposite momenta. Because the point Fermi interaction allows no orbital angular momentum the cross section is nonvanishing only in the J=0 partial wave. Compute the invariant amplitude $\mathcal{M}$ in the forward direction $(\vec{p}_e = \vec{p}_{\nu_e} = -\vec{p}_\mu = -\vec{p}_{\nu_\mu})$ and in the center-of-mass frame:

$$\begin{aligned} \mathcal{M} &= \frac{G}{\sqrt{2}} \left(\bar{u}_{\nu_e} \gamma_\lambda (1-\gamma_5) u_e\right) \left(\bar{u}_\mu \gamma^\lambda (1-\gamma_5) u_{\nu_\mu}\right) \\ &= \frac{G}{\sqrt{2}} \left(\frac{2(p_e)_\lambda}{E_e}\right) \cdot \left(\frac{2(p_\mu)^\lambda}{E_e}\right) \\ &= \frac{4G}{\sqrt{2}} \frac{p_e \cdot p_\mu}{E_e E_\mu} = \frac{8G}{\sqrt{2}} \end{aligned} \qquad (2.1)$$

(We normalize $u^\dagger u = 1$, not to m/E in order to avoid awkwardness in handling massless fermion states.) This amplitude should (but does not) fall with energy in order not to violate unitarity ($\mathcal{M} \lesssim \text{const}/s$). The problem is better seen by computing the differential cross section (from an unpolarized incident electron):

$$d\sigma = \underset{\substack{\uparrow \\ \text{flux factor} = 2c}}{\left(\frac{1}{2}\right)} \underset{\substack{\uparrow \\ e^- \text{ spin average}}}{\left(\frac{1}{2}\right)} |\mathcal{M}|^2 \frac{d^3p_{\nu_e}}{(2\pi)^3} \frac{d^3p_\mu}{(2\pi)^3} (2\pi)^4 \delta^4\left(p_e + p_{\nu_\mu} - p_{\nu_e} - p_\mu\right)$$

$$= |\mathcal{M}|^2 \frac{s\,d\Omega}{128\pi^2} \tag{2.2}$$

or

$$\frac{d\sigma}{d\Omega} = \frac{G^2 s}{4\pi^2} \tag{2.3}$$

and

$$\sigma_{tot} = \frac{G^2 s}{\pi} \tag{2.4}$$

Thus instead of $\sigma_{tot} \sim s^{-1}$, we have $\sigma_{tot} \sim s^{+1}$. Something must happen before 300 GeV in the center-of-mass. We now discuss possible fixes for this situation:

1. Simple unitarization: If rescattering corrections are included in a manner parallel to what might be done in strong interactions (Bethe-Salpeter equation, K-matrix approximation, N/D method, etc.), the amplitude would be damped at energies $\sqrt{s} \sim G^{-1/2} \sim 300$ GeV. This puts an effective cutoff $\Lambda \sim 300$ GeV. By dimensional analysis, higher order effects would occur as

$$\mathcal{M} \sim G + \mathcal{O}(G^2\Lambda^2) + \ldots = G\left[1 + \mathcal{O}(G\Lambda^2) + \ldots\right] \tag{2.5}$$

which is as large as the lower order effects. Just this much leads to some qualitative predictions (or at least expectations):

a) Neutral currents (as generated by second order loops $\nu_\mu e^- \to \mu^- \nu_e \to \nu_\mu e^-$) should be of the same order of magnitude as charged currents.

b) There is no *a priori* need to incorporate electromagnetic currents into the scheme.

c) The neutral currents induced by known charged currents acting in higher orders would be expected to be pure V-A.

d) The effective Lagrangian describing the lepton-lepton scattering matrix would contain derivative coupling terms, e.g.,

$$\mathscr{L}^{(2)} \sim G^2 \left[\bar{\ell}\gamma_\lambda \frac{\partial}{\partial x^\sigma} (1-\gamma_5)\ell'\right]\left[\bar{\ell}''\gamma^\lambda \frac{\partial}{\partial x_\sigma} (1-\gamma_5)\ell'''\right] \tag{2.6}$$

which would become significant at very high energies.[16]

e) Similar neutral current and derivative coupling terms would be expected to be generated in the effective Lagrangian for lepton-hadron interactions, provided the short distance behavior of such interactions parallels that of lepton-lepton interactions (as evidenced by the existence of scaling behavior in deep-inelastic processes). This would, if followed straightforwardly, imply a term[16]

$$\mathscr{L}' \sim G\left[\left(\bar{d} \cos \theta_c + \bar{s} \sin \theta_c\right)\gamma_\lambda(1-\gamma_5)\left(d \cos \theta_c + s \sin \theta_c\right)\right] \bar{u}\gamma^\lambda(1-\gamma_5)u \tag{2.7}$$

leading to $\Delta S=1$ neutral current processes such as $K_L \to \mu^+\mu^-$, $K^+ \to \pi^+\mu^+\mu^-$, etc.

2. Simple charged intermediate boson exchange; strong W-W couplings. With an intermediate W exchanged between the lepton pairs (Fig. 1a) the amplitude in Eq. (2.1) is multiplied by a factor $m_W^2\left(m_W^2+t\right)^{-1}$. This factor, in addition to damping the amplitude, distributes it into many partial waves so as to postpone any unitarity crisis to unobservably (i.e., exponentially) high energy. However, without any other restriction it turns out that pair-production processes (Fig. 1b) such as $e^+e^- \to W^+W^-$, with at least one W having helicity zero (longitudinal polarization) in the cms, again violates unitarity in the J=1 wave at $\sqrt{s} \gtrsim G^{-1/2}$. (We shall see this in detail later on.) The simplest panacea (which does not reduce to the previous case) involves supposing that W-W scattering becomes strong[17] but that W couplings to leptons and quarks remain weak

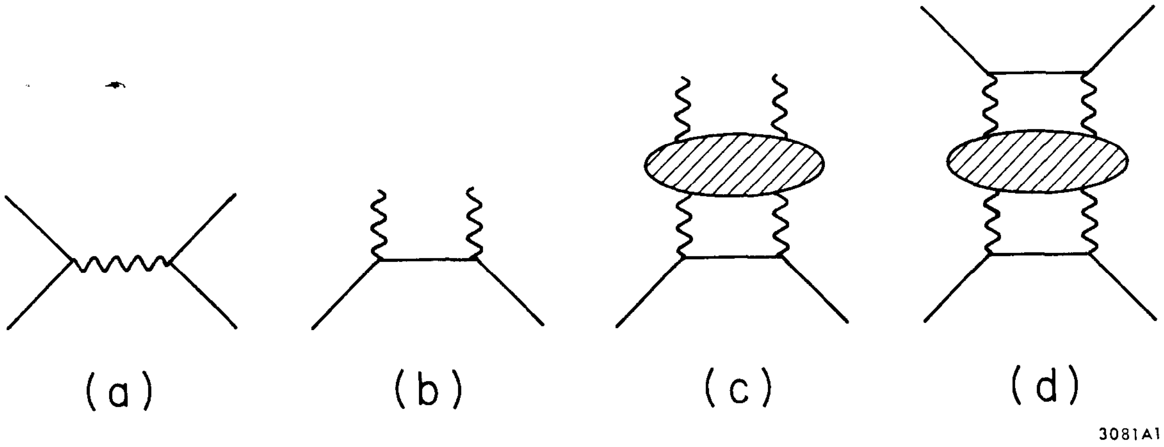

Fig. 1. (a) Weak lepton-lepton scattering via intermediate-boson exchange. (b) Pair production of intermediate bosons by leptons. (c) Intermediate boson rescattering. (d) Lepton-lepton scattering in higher order. The divergence in the loop integration (or in the dispersion integral over $\sigma(\ell\bar{\ell} \to W\bar{W})$) is damped out by the strong W-W rescattering.

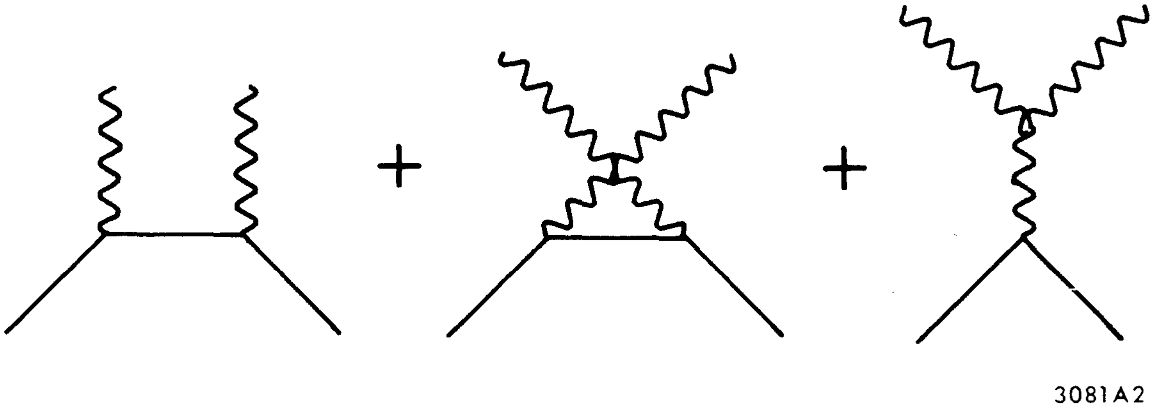

Fig. 2. Amplitudes for $e^+e^- \to W^+W^-$ in a gauge theory. Couplings are arranged so that the sum of these diagrams have a smooth high-energy behavior.

(Fig. 1c). If Λ is the energy for which W-W scattering phase-shifts become of order unity, this will generate higher-order neutral current effects according to Fig. 1d:

$$\mathcal{M}^{(2)} \sim G(G\Lambda^2) \tag{2.8}$$

just as in the previous case. (We again will see this in some detail later on.) With Λ unknown, it is hard to estimate the size of such induced neutral current effects.[18] However it is a safe bet that $\Lambda \gtrsim m_W$. Note also that in this option there could be many W-W resonances of various spins, the W's could like on Regge trajectories, there could be a strong interaction W bootstrap, etc.

3. Simple W-exchange, but cancellations between diagrams to keep the high-energy amplitude growth small (Gauge theories). Under suitable circumstances, it is possible to arrange the couplings of the set of intermediate bosons to fermions (quarks and leptons) and to each other such as to effect cancellations between the individual diagrams[19] shown in Fig. 2. A necessary and sufficient condition that this occur is that the couplings be those of some nonabelian gauge theory. These gauge theories are an elegant starting point for a weak (or for that matter, strong) interaction theory. They are generalizations of quantum electrodynamics introduced long ago by Yang and Mills[20] and designed to handle systems with internal degrees of freedom. It is not our intention to describe this basic starting point in detail. A feeling for it can perhaps be ascertained by the following parallelism:

	Quantum Electrodynamics	Nonabelian Gauge Theories
a)	Electromagnetic potential A_μ	Gauge potentials $B_\mu^{(a)}$, one for each generator T^a in the group algebra: $\left[T^a, T^b\right] = \sum_c f^{abc}\, T^c$

b)	Electromagnetic coupling e	Gauge coupling g
c)	Electromagnetic field $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$	Gauge fields $G^{(a)}_{\mu\nu} = \partial_\mu B^{(a)}_\nu - \partial_\nu B^{(a)}_\mu + g \sum_{bc} f^{abc} B^{(b)}_\mu B^{(c)}_\nu$
d)	Free Lagrangian density $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ + gauge terms	$-\frac{1}{4} \sum_a G^{(a)}_{\mu\nu} G^{(a)\mu\nu}$ + gauge terms
e)	Gauge invariant substitution $i \frac{\partial}{\partial x^\mu} \to i \frac{\partial}{\partial x^\mu} - eA_\mu$	$i \frac{\partial}{\partial x^\mu} \to i \frac{\partial}{\partial x^\mu} - g \sum_a t_a B^{(a)}_\mu$ where the t_a are an appropriate matrix representation (of the generators T_a) for the multiplet on which the derivative operator acts.
f)	Symmetry under local gauge transformations $\psi(x) \to e^{i\Lambda(x)} \psi(x)$ $eA_\mu(x) \to eA_\mu(x) + \frac{\partial \Lambda}{\partial x^\mu}$	$\psi_r(x) \to \left[e^{it_a \Lambda^a(x)}\right]_{rs} \psi_s(x) \equiv U_{rs}(x) \psi_s(x)$ $gt^a \cdot B^{(a)}_\mu \to U^{-1} gt^{(a)} \cdot B^{(a)}_\mu U - U^{-1} \frac{\partial U}{\partial x^\mu}$

One salient feature to notice is that gauge potentials couple trilinearly (and quartically) to each other as a consequence of the nonlinearity in the definition (c) of $G_{\mu\nu}$. That is, unlike electrodynamics, the gauge fields themselves carry quantum numbers and therefore must couple to themselves (just as the gravitational field carries energy and momentum and therefore couples to itself). This allows for couplings as needed in Fig. 2c. However, we shall go no further in this direction, it being sufficient for our purposes to point out the distinguished pedigree of this class of theory.

More significant perhaps is the inevitable unification of weak and electromagnetic interaction coming from this approach. The amplitude in Fig. 2c for electromagnetic production of W^+W^- by e^+e^-, taken by itself, violates unitarity. It must be cancelled by diagrams involving the weak couplings g. Furthermore, invoking only the charged-current weak coupling of e^- to ν_e is not enough: it is present for only negative-helicity e^-; the electromagnetic amplitude for annihilation of a positive-helicity e^- with the (negative-helicity) e^+ into W^+W^- is unaffected. There are two basic options open at this point. The first is to introduce at least one neutral lepton E^o (or, more extravagantly, a doubly charged $E^{\pm\pm}$) coupled (at least in part) via a V+A interaction to e^- (and W^+). The other basic option is to introduce at least one neutral intermediate Z^o, <u>with at least some V+A coupling</u>, to the e^-e^+ system. We repeat: the gauge-theory solution requires

a) either a neutral heavy lepton E^o (with coupling not pure V-A)

b) or a doubly charged $E^{\pm\pm}$ (with coupling not pure V-A)

c) or a neutral intermediate boson Z^o (with coupling to e^+e^- not pure (V-A)

d) or some combination of the above (including more than one of any kind)

e) and the coupling constant g of W's and Z's to fermions of the same order of magnitude as the electromagnetic coupling e.

Again, as in the previous scheme, neutral currents are generated. Indeed they typically appear in lowest order, although it is possible to concoct,[21] with the help of heavy leptons, a scheme with no extra Z^o's. (However, such a scheme is artificial and even in trouble with experiment.[22]) All this will be treated in detail in Section III.[23]

B. The Motivation for Charm. The charm concept was introduced originally[24, 25] both on aesthetic considerations (lepton-hadron parallelism) as well as on an attempt to get closer to the 3 in the then newly-established eightfold way: SU(3). It attained real operational significance only in 1970 when Glashow, Iliopoulos, and Maiani[26] showed its usefulness in alleviating the problems of strangeness-changing neutral currents and higher orders of weak interactions.

We saw that all options treated above require at some level the presence of a neutral-current coupling and that this in turn suggested strongly the existence of semileptonic $|\Delta S=1|$ neutral current couplings as in Eq. (2.7). In addition, given leptonic neutral currents parametrized phenomenologically as

$$\mathscr{L}_{\text{eff}} = \frac{G}{\sqrt{2}} (G\Lambda^2) \left[\bar{e}\gamma_\lambda(1-\gamma_5)e\right] \left[\bar{e}\gamma^\lambda(1-\gamma_5)e\right] \tag{2.9}$$

we would a priori expect similar nonleptonic terms of the same order of magnitude

$$\mathscr{L}_{\text{eff}} \sim \frac{G}{\sqrt{2}} (G\Lambda^2) \left[\bar{d}_c\gamma_\lambda(1-\gamma_5)d_c\right] \left[\bar{d}_c\gamma^\lambda(1-\gamma_5)d_c\right] \tag{2.10}$$

with

$$d_c = d \cos \theta_c + s \sin \theta_c \tag{2.11}$$

Eq. (2.10) contains a $\Delta S=2$ piece

$$\mathscr{L}^{\Delta S=2}_{\text{eff}} = \frac{2G}{\sqrt{2}} (G\Lambda^2) \cos^2 \theta_c \sin^2 \theta_c \left[\bar{d}\gamma_\lambda(1-\gamma_5)s\right] \left[\bar{d}\gamma^\lambda(1-\gamma_5)s\right] + \text{h.c.} \tag{2.12}$$

which has matrix elements between K and $\bar{K}$ and contributes to the K_L-K_S mass difference. Estimates of this[27, 28, 29] lead to the limit

$$\Lambda \lesssim 4 \text{ GeV} \tag{2.13}$$

a remarkably low value. This is where charm enters. Given a fourth charmed quark c coupled with a V-A coupling to the combination

$$s_c = s \cos \theta_c - d \sin \theta_c \tag{2.14}$$

as well as a modest amount of permutation symmetry in their couplings, it is possible to show that anywhere[30] one has a neutral-current coupling

$$\bar{d}_c \Gamma_\lambda d_c \tag{2.15}$$

one should generalize it to

$$\begin{aligned} &\bar{d}_c \Gamma_\lambda d_c + \bar{s}_c \Gamma_\lambda s_c \\ &\equiv \bar{d}\Gamma_\lambda d + \bar{s}\Gamma_\lambda s \end{aligned} \tag{2.16}$$

That is, the Cabibbo strangeness mixing can be rotated away, and with it $\Delta S \neq 0$ neutral-current effects. This is only true in the limit of vanishing quark masses. In order to remain compatible with the K_L-K_S mass difference limit, the charmed-quark mass necessarily had to be less than a few GeV.

This simple and elegant solution to the problem is the GIM mechanism. We emphasize that it is more general than and logically independent of the gauge-theory option of high energy behavior for weak interactions. It was in fact constructed before the flourishing in 1972 of the renormalizable gauge theories.[31]

C. The Higgs Sector. The introduction of gauge-theory couplings is necessary but not sufficient for curing the singular high energy behavior of weak scattering amplitudes. Production of three intermediate bosons in lepton-lepton collisions (Fig. 3) again causes difficulty.[32] Without any cancellations, the cross section would behave as

$$g^6 \frac{s^2}{m_W^6} \sim G(Gs)^2 \tag{2.17}$$

Cancellations do occur after introduction of the quartic coupling between W's in Fig. 3c but they are not complete: there is a residual piece to the J=1 amplitude which gives rise to a cross section of order

$$g^6 \frac{s}{m_W^2} \sim G^2 m_W^2 (Gs) \tag{2.18}$$

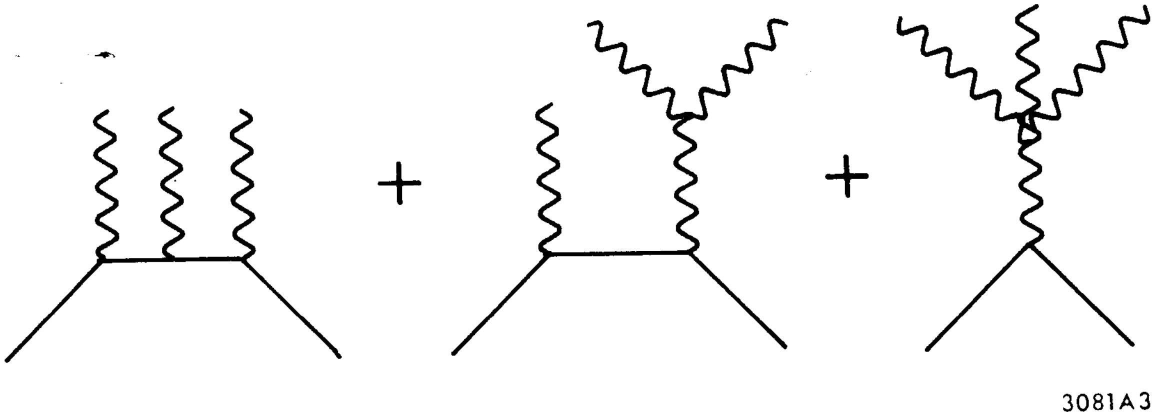

Fig. 3. Typical amplitudes for the production of three gauge bosons in e^+e^- collisions.

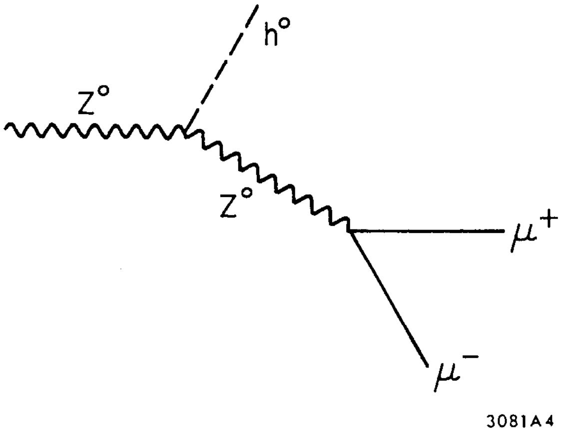

Fig. 4. Diagram for the decay $Z^0 \to h^0\mu^+\mu^-$, where h^0 is a neutral J=0 Higgs-boson.

To fix up this situation without reverting to a previous case involving large phase-shifts, it is necessary to introduce J=0 particles coupled principally to the gauge-bosons W. Such particles, the so-called Higgs particles,[33, 34] also play a role in the more fundamental field-theoretical approach. Just as in quantum electrodynamics, the pristine nonabelian gauge bosons are massless. To generate mass, the J=0 Higgs fields ϕ_r are introduced into the Lagrangian in such a way as to undergo spontaneous breakdown, i.e., the ground state contains a superfluid condensate of some of these J=0 bosons such that $<0\,|\phi_r(x)|\,0>\neq 0$. This generates a mass term for gauge bosons. There is a solid-state analogy in the Ginsburg-Landau model[35] for superconductivity:

Gauge Theory	Ginzburg-Landau
Higgs particle Lagrangian density	Free energy density
$\mathcal{L}=\frac{1}{2}\left\vert\left(i\partial_\mu - g t_a B_\mu^{(a)}\right)\phi\right\vert^2 - V(\phi) + \dots$	$\mathcal{F}=\phi^+ \frac{(\vec{p}-e\vec{A})^2}{2m}\phi + V(\phi) + \dots$
After spontaneous breakdown	
$\mathcal{L}\rightarrow\frac{g^2}{2}\left\vert t_a B_\mu^{(a)} <\phi>\right\vert^2$	$\mathcal{F}\rightarrow\frac{e^2}{2m}\left\vert<\phi>\right\vert^2\vec{A}^2 + \dots$
Boson mass term	Meissner effect
$\mu^2 \sim g^2 \left\vert<\phi>\right\vert^2$	$\mathcal{F}=\frac{1}{2m}e^2<\phi>^2+\frac{1}{2}(E^2+B^2)$ $\Rightarrow \vec{A}(x)=\vec{\epsilon}\,e^{-\lambda x}$
	London penetration depth
	$\lambda\sim\left[\frac{e^2\left\vert<\phi>\right\vert^2}{m}\right]^{1/2}$ = "effective photon mass"

Again we digress. The main point is that the exchange of a J=0 boson (coupled in the way appropriate to the spontaneously broken gauge theory) does restore acceptable high energy behavior to this order. Studies to all orders[23] have shown that this continues to be true, with one qualification. Diagrams containing a fermion loop with three external boson lines may, because of the superficial linear divergence, produce additional problems: this is the triangle anomaly.[36] Discussion of this subtle issue is also beyond the scope of these lectures;[37] it suffices to say that the problem can only be disposed of by again appealing to algebraic cancellation.[38] The trouble in such loop diagrams is independent of fermion mass, and the amplitude, summed over all left-handed fermions which can be inserted in the loop, is multiplied by a factor

$$\mathrm{Tr}\ t_a \left\{ t_b, t_c \right\}$$

where the fermion coupling to the gauge-boson in question is

$$\mathscr{L}' = g\,\bar{\psi}\gamma^{\lambda}(1-\gamma_5)\ t_a\psi B_{\lambda}^{(a)}$$

(Without loss of generality we include only left-handed fermions (and ipso facto right-handed antifermions) in the loop. All fermion degrees of freedom can be included just by redefinitions using CP transformations.) The condition

$$\mathrm{Tr}\ t_a \left\{ t_b, t_c \right\} = 0 \qquad (2.19)$$

then removes any problems with triangle diagrams, and renders the theory a weak-coupling theory at all practical energies; i.e., it is renormalizable.[39]

Renormalizability (including the above condition, Eq. (2.19)) is widely used as a criterion for a successful gauge theory. If consistently followed, we would be forced to abandon general relativity.[40] A much more persuasive reason for consideration of gauge theories is the underlying gauge principle, which links the gauge theories to electrodynamics at a much deeper level.

However, it is not clear what level we reach when we enter this gauge theory scheme of high energy behavior. We have seen that the sector (of Hilbert space) containing the J=0 Higgs particles appears to be unavoidable. The renormalizability requirement, as we saw, demands them. And if renormalizability is abandoned then some other set of particles (fermions in option 1 in Section IIA above, bosons in option 2) interact strongly at some energy. They may be expected to dynamically generate new J=0 resonances. In any model I know, something like the Higgs sector seems to be present, and on a mass scale $\lesssim$ 300 GeV. The full theory, including the Higgs sector, can be a weak-coupling theory provided the masses of Higgs particles are small compared to 300 GeV, perhaps $\lesssim m_W$ or even less. (This seems to be preferred by Weinberg.[41]) However, this option leads to ugly-duckling, cumbersome Lagrangians with a host of rather ad hoc degrees of freedom. Thus, I believe even the gauge theories do not really solve the problem of high energy behavior in a self-contained way; they instead move it into a more inaccessible region, the Higgs sector.

Is there hope of finding real Higgs bosons experimentally? It appears to be very difficult.[42] The coupling of Higgs bosons to fermions is proportional to fermion mass: the Yukawa coupling to a fermion of mass m_f is typically

$$\mathscr{H}' \sim e \frac{m_f}{m_W} \bar{\psi} \psi \phi \qquad (2.20)$$

The Higgs bosons are coupled more substantially to gauge bosons. There is a quadratic coupling similar to the $A^2\phi^2$ seagull in scalar electrodynamics. In addition there is a trilinear $B_\mu B^\mu \phi$ coupling proportional to $e\, m_W$. One hope for entering the Higgs sector is to resonantly produce some neutral Z^o in e^+e^- collisions, and look for the Higgs boson h in the decay $Z^o \rightarrow he^+e^-$ or $Z^o \rightarrow h\mu^+\mu^-$ (Fig. 4). There are other ideas as well,[43] but they are also futuristic and difficult. More thought on these general issues might well be fruitful.

III. THE SU(2)⊗U(1) MODEL

A. Introductory Generalities. We now turn to construction of the simplest and most successful gauge theory model. It consists of no extra fermions (other than a charmed quark), one charged intermediate boson $W^{\pm}$, and one (massive) neutral boson Z^{o}. As a prelude to its construction we consider the general case in order to exhibit how the gauge theory algebra arises. Let

$$L_i = \begin{pmatrix} e^- \\ \nu_e \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \qquad (3.1)$$

be a column vector for the fermions in some internal symmetry space and the coupling to a set of gauge bosons $B^{(a)}_{\mu}$ given by

$$\mathcal{H}' = \left[L_f^{\dagger}\, g^{(a)}_{fi}\, L_i\right] \bar{u}_f \gamma^{\lambda}(1-\gamma_5)\, u_i B^{(a)}_{\lambda} \quad . \qquad (3.2)$$

Similarly, for the coupling of gauge bosons a, b, c with polarizations $\epsilon^{(a)}$, $\epsilon^{(b)}$, $\epsilon^{(c)}$ to each other, write[44] (with the momentum conventions in Fig. 5)

$$\mathcal{H}' = f^{ab,\,c}\left[\epsilon^{(a)}\cdot\epsilon^{(b)}\right]\left[\epsilon^{(c)}\cdot(p_a-p_b)\right]$$

$$+ \text{ cyclic permutations of } a, b, c \qquad (3.3)$$

Hence

$$f^{ab,\,c} = -f^{ba,\,c} \qquad (3.4)$$

as follows from Bose symmetry. With these conventions we may write down amplitudes (in the limit of vanishing fermion mass) for the processes $\ell\bar{\ell} \to WW$ corresponding to Fig. 2 and again to Fig. 6 where momentum labels and indices have been carefully included. The amplitude for Fig. 6a is

$$\mathcal{M}^{(1)} = \bar{u}_f \not{\epsilon}^{(a)} \frac{(\not{p}_i+\not{k}^{(b)})}{(p_i+k^{(b)})^2} \not{\epsilon}^{(b)} \left(\frac{1-\gamma_5}{2}\right) u_i \left[L_f^{\dagger}\, g^{(a)}_{fn} g^{(b)}_{ni}\, L_i\right] \qquad (3.5)$$

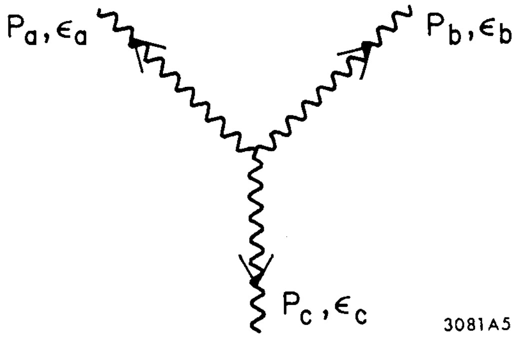

Fig. 5. Trilinear coupling of gauge bosons to each other.

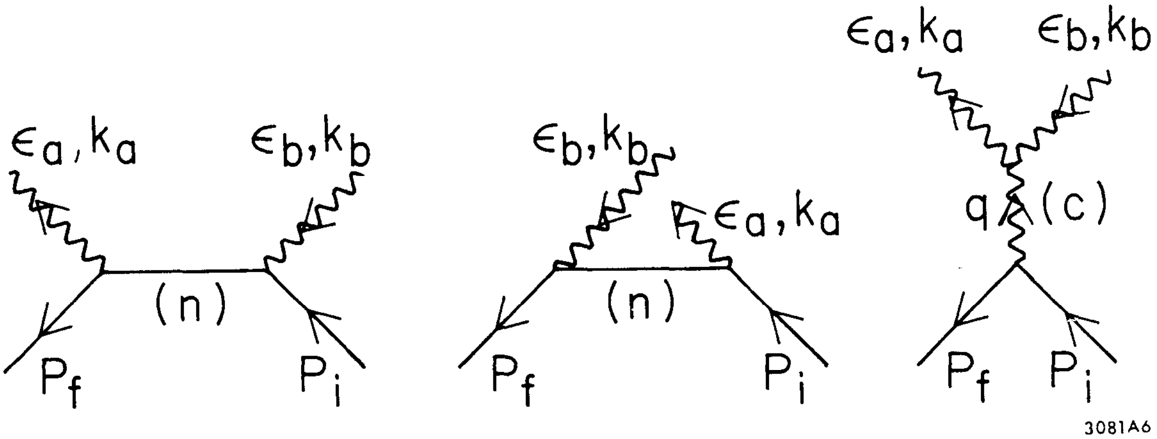

Fig. 6. General amplitudes for the process $\ell\bar{\ell} \rightarrow WW$.

Similarly for Fig. 6b, the amplitude is

$$\mathscr{M}^{(2)} = \bar{u}_f \not{\epsilon}^{(b)} \frac{(\not{p}_f - \not{k}^{(b)})}{(p_f - k^{(b)})^2} \not{\epsilon}^{(a)} \left(\frac{1-\gamma_5}{2}\right) u_i \left[L_f^\dagger g_{fn}^{(b)} g_{ni}^{(a)} L_i\right] \tag{3.6}$$

For the intermediate propagator in Fig. 6c we use

$$D_{\mu\nu} = \frac{\left(-g_{\mu\nu} + m^{-2} q_\mu q_\nu\right)}{q^2 - m^2} \tag{3.7}$$

Because of current-conservation at the bottom vertex, the $q^\mu q^\nu$ term can be dropped. This leaves

$$\mathscr{M}^{(3)} = \frac{-\bar{u}_f \gamma^\lambda \left(\frac{1-\gamma_5}{2}\right) u_i \left[L_f^\dagger g_{fi}^{(c)} L_i\right]}{q^2 - m_c^2} \begin{cases} f^{ab,c}\, \epsilon^{(a)} \cdot \epsilon^{(b)} (k^{(a)} + k^{(b)})_\lambda \\ + f^{bc,a}\, \epsilon_\lambda^{(b)}\, \epsilon^{(a)} \cdot (-k^{(b)} + q) \\ + f^{ca,b}\, \epsilon_\lambda^{(a)}\, \epsilon^{(b)} \cdot (-q - k^{(a)}) \end{cases} \tag{3.8}$$

with

$$q = p_i - p_f = k^{(a)} - k^{(b)} \tag{3.9}$$

Why do these amplitudes misbehave at high energy, when they differ from the form of the amplitudes for $e^+e^- \rightarrow \gamma\gamma$ (which does not) only by algebraic factors? The crucial difference is the form of the polarization vector ϵ_μ for a massive <u>longitudinally polarized</u> gauge boson. If the boson has four-momentum $k_\lambda = (k_0, 0, 0, |\vec{k}|)$, and mass m, then the polarization vector is

$$\epsilon^\lambda = \frac{1}{m}(|\vec{k}|, 0, 0, -k_0) \tag{3.10}$$

in order that $\epsilon \cdot k = 0$ and $\epsilon^2 = -1$. Thus ϵ^λ is $O(k/m)$, not unity at high energy and, without special cancellation, leads to the singular behavior. Actually the singular part of ϵ^λ is proportional to k^λ:

$$\begin{aligned} \epsilon^\lambda &= \frac{k^\lambda}{m} + \frac{1}{m}(|\vec{k}| - k_0, 0, 0, k_0 - |\vec{k}|) \\ &= \frac{k^\lambda}{m} - \frac{m}{|\vec{k}| + k_0}(1, 0, 0, -1) = \frac{k^\lambda}{m} + O\left(\frac{m}{k}\right) \end{aligned} \tag{3.11}$$

It therefore suffices that $\mathscr{M}$ vanish upon setting <u>one</u> of the boson polarization vectors equal to k_λ/m. Let that vector be $\epsilon^{(b)}$. We get for the first two terms

$$\begin{aligned} \mathscr{M}^{(1)} &\to \frac{1}{m}\,\bar{u}_f \not{\epsilon}^{(a)} \left(\frac{1-\gamma_5}{2}\right) u_i \left[L_f^\dagger g_{fn}^{(a)} g_{ni}^{(b)} L_i\right] \\ \mathscr{M}^{(2)} &\to \frac{-1}{m}\,\bar{u}_f \not{\epsilon}^{(a)} \left(\frac{1-\gamma_5}{2}\right) u_i \left[L_f^\dagger g_{fn}^{(b)} g_{ni}^{(a)} L_i\right] \end{aligned} \qquad (3.12)$$

A little more algebra, using $\epsilon^{(r)}\cdot k^{(r)}=0$ and momentum conservation yields

$$\mathscr{M}^{(3)} \to \frac{-1}{m}\,\frac{L_f^\dagger g_{fi}^{(c)} L_i}{\left(m_a^2+m_b^2-2k_a\cdot k_b-m_c^2\right)} \left\{ \begin{aligned} & 2\epsilon^{(a)}\cdot k^{(b)}\,\bar{u}_f \not{k}^{(b)} \left(\frac{1-\gamma_5}{2}\right) u_i \left(-f^{bc,a}+f^{ab,c}\right) \\ & +f^{ca,b}\left(-2k^{(a)}\cdot k^{(b)}+m_b^2\right) \bar{u}_f \not{\epsilon}^{(a)} \left(\frac{1-\gamma_5}{2}\right) u_i \end{aligned} \right.$$

(3.13)

This leads to two conditions. The first is

$$f^{bc,a} = f^{ab,c} = -f^{ba,c} \qquad (3.14)$$

That is, we can remove the comma: f^{abc} must be <u>fully antisymmetric</u>. Then as $k^{(a)}\cdot k^{(b)} \to \infty$

$$\mathscr{M}^{(3)} \to \frac{-1}{m}\,\bar{u}_f \not{\epsilon}^{(c)} \left(\frac{1-\gamma_5}{2}\right) u_i\, f^{cab} \left[L_f^\dagger g_{fi}^{(c)} L_i\right] \qquad (3.15)$$

Putting together Eqs. (3.12) and (3.15) and demanding the cancellation gives

$$L_f \left\{ g_{fn}^{(a)} g_{ni}^{(b)} - g_{fn}^{(b)} g_{ni}^{(a)} - f^{abc} g_{fi}^{(c)} \right\} L_i = 0 \qquad (3.16)$$

or in matrix language

$$\left[g^{(a)}, g^{(b)}\right] = f^{abc} g^{(c)} \qquad (3.17)$$

This is the defining relation for a Lie algebra, and the gauge-coupling constants (up to a factor) are the <u>structure constants</u> for the algebra.

The condition Eq. (3.14), when applied to the coupling of γ to any charged W, turns into a restriction on its magnetic moment[45]: the gyromagnetic ratio

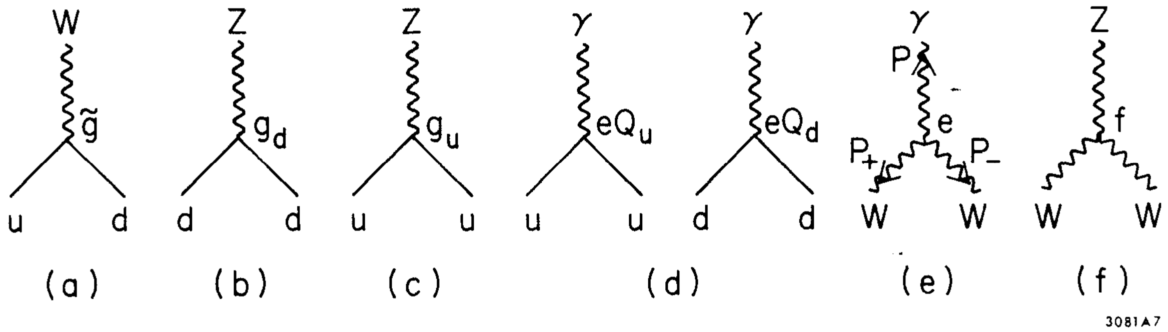

Fig. 7. Yukawa couplings of W to fermions and each other in the $SU(2) \otimes U(1)$ model.

must be 2 (just like the electron gyromagnetic ratio in the absence of radiative corrections).

B. The SU(2) ⊗ U(1) Model in Detail. We now apply this technique to generate by construction all the couplings needed for the Weinberg-Salam SU(2) ⊗ U(1) model. We assume the only gauge bosons are $W^{\pm}$, Z^{0} (and photon), and consider $W^{\pm}$ couplings to u and d quarks. We shall endow these quarks with charge $Q_u=2/3$ and $Q_d=-1/3$ but keep the notation general in order to be able to apply the results to leptons as well. Again we neglect fermion masses, a good approximation at high energies. For the Yukawa couplings illustrated in Fig. 7, we write for the vertices

$$V_{(a)} = \tilde{g}\, \bar{d} \not{\epsilon} \left(\frac{1-\gamma_5}{2}\right) u \qquad W \longleftrightarrow u\bar{d}$$

$$V_{(b)} = g_d\, \bar{d} \not{\epsilon} \left(\frac{1-\gamma_5}{2}\right) d \qquad Z^{o} \longleftrightarrow d\bar{d}$$

$$V_{(c)} = g_u\, \bar{u} \not{\epsilon} \left(\frac{1-\gamma_5}{2}\right) u \qquad Z^{o} \longleftrightarrow u\bar{u}$$

$$\begin{aligned} V_{(d)} &= eQ_u \left\{ \bar{u}\not{\epsilon} \left(\frac{1-\gamma_5}{2}\right) u + \bar{u}\not{\epsilon}\left(\frac{1+\gamma_5}{2}\right) u \right\} \qquad \gamma \longleftrightarrow u\bar{u} \qquad (3.18) \\ &+ eQ_d \left\{ \bar{d}\not{\epsilon} \left(\frac{1-\gamma_5}{2}\right) d + \bar{d}\not{\epsilon}\left(\frac{1+\gamma_5}{2}\right) d \right\} \qquad \gamma \longleftrightarrow d\bar{d} \end{aligned}$$

$$V_{(e)} = e\left[(\epsilon^{-}\cdot\epsilon^{+})\, \epsilon\cdot(p_{-}-p_{+}) + (\epsilon^{+}\cdot\epsilon)\ \epsilon^{-}\cdot(p_{+}-p) + (\epsilon\cdot\epsilon^{-})\ \epsilon^{+}\cdot(p-p_{-})\right]$$

$$V_{(f)} = f\left[(\epsilon^{-}\cdot\epsilon^{+})\ \epsilon\cdot(p_{-}-p_{+}) + \epsilon^{+}\cdot\epsilon\ \ \epsilon^{-}\cdot(p_{+}-p) + (\epsilon\circ\epsilon^{-})\ \epsilon^{+}\cdot(p-p_{-})\right]$$

Now we can again write out the diagrams as we did in the general case. We consider only the left-handed helicity amplitudes first; the others can be handled independently later. The overall factor depending on the spinors, polarization vectors, γ-matrices, and the like should be evident and we will not write it explicitly. Instead we consider, for each diagram in Fig. 8 (note how many of them have potential physical significance!), the relationship among the coupling

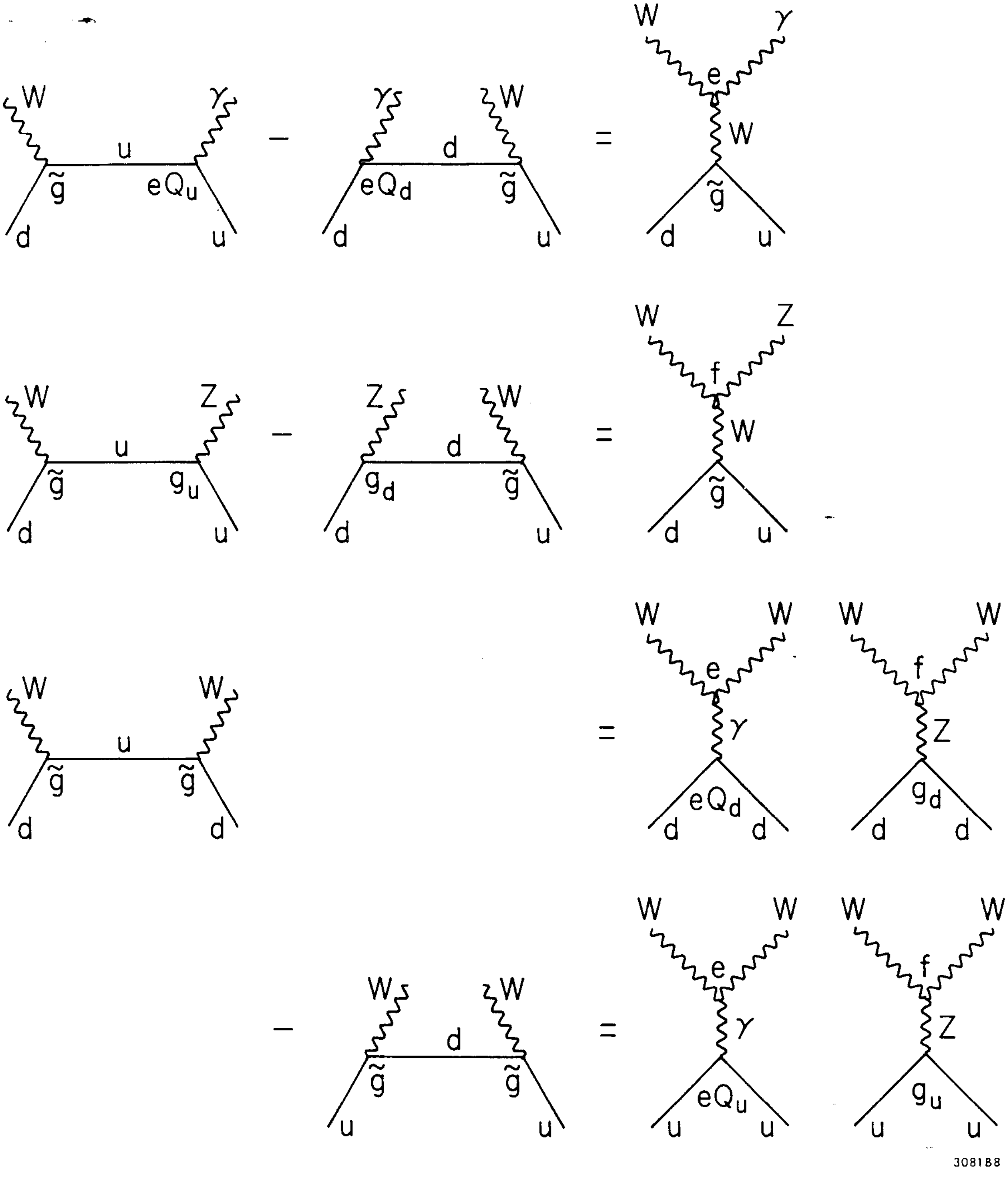

Fig. 8. Diagrams for fermion-antifermion annihilation into two gauge-bosons in the SU(2) ⊗ U(1) model.

constants, as expressed in an excessively succinct way by Eq. (3.17). They are

$$\begin{aligned} \tilde{g}(eQ_u) - (eQ_d)\tilde{g} &= e\tilde{g} \\ \tilde{g}\, g_u - g_d \tilde{g} &= f\tilde{g} \\ \tilde{g}^2 - 0 &= - \left[e^2 Q_d + f g_d\right] \\ 0 - \tilde{g}^2 &= - \left[e^2 Q_u + f g_u\right] \end{aligned} \tag{3.19}$$

Subtracting the last two equations yields the relation

$$2\tilde{g}^2 = e^2 + f^2 = g^2 \tag{3.20}$$

g is the intrinsic SU(2) gauge coupling constant connecting W^+ and W^- to W_0, which is a mixture of A and Z. Indeed letting

$$\begin{aligned} e &= g \sin \theta_W \\ f &= g \cos \theta_W \end{aligned} \tag{3.21}$$

we see that

$$W_0 = A \sin \theta_W + Z \cos \theta_W \tag{3.22}$$

couples with full strength to $W^{\pm}$, while

$$B = A \cos \theta_W - Z \sin \theta_W \tag{3.23}$$

decouples from $W^{\pm}$. The angle θ_W is the Weinberg angle, the most important parameter of the model.

Solving for the couplings gives

$$\begin{aligned} g_u &= \frac{g^2}{2f} - \frac{e^2}{f} Q_u = \frac{g}{\cos \theta_W} \left[\frac{1}{2} - Q_u \sin^2 \theta_W\right] \\ g_d &= - \frac{g^2}{2f} - \frac{e^2}{f} Q_d = \frac{g}{\cos \theta_W} \left[- \frac{1}{2} - Q_d \sin^2 \theta_W\right] \end{aligned} \tag{3.24}$$

We may now repeat these calculations for the right-handed amplitudes. In this case there is no coupling of W to the $(u, d)_R$ doublet. If there is no other heavy quark coupled to u or d and W, with right-handed coupling, then all the terms on the left-hand sides of Eqs. (3.19) vanish, and the solutions for the appropriate right-handed couplings g_u^R and g_d^R of u and d to Z are evidently

$$\begin{aligned} g_u^R &= -\frac{e^2}{f} Q_u = \frac{g}{\cos\theta_W}\left[0 - Q_u \sin^2\theta_W\right] \\ g_d^R &= -\frac{e^2}{f} Q_d = \frac{g}{\cos\theta_W}\left[0 - Q_d \sin^2\theta_W\right] \end{aligned} \qquad (3.25)$$

This has the same structure as Eq. (3.24). While we refrain from a proof, it should come as no surprise that for <u>any</u> constituent its coupling to the Z is likewise

$$g_q = \frac{e}{\sin\theta_W \cos\theta_W}\left[T_3 - Q\sin^2\theta_W\right]_q \qquad (3.26)$$

where T_3 is the third component of the <u>weak</u> isospin of the constituent q. The conventional assignment of weak isospin is

$$\underline{\text{Doublets}} \quad \begin{pmatrix} T_3 = \frac{1}{2} \\ T_3 = -\frac{1}{2} \end{pmatrix} : \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d_c \end{pmatrix}_L \quad \begin{pmatrix} c \\ s_c \end{pmatrix}_L \qquad (3.27)$$

$$\underline{\text{Singlets}} \quad e_R^- \; \mu_R^- \qquad u_R \qquad d_R \qquad s_R \qquad c_R \qquad (3.28)$$

The singlet assignments should, however, be considered tentative. Heavy leptons[46,47] or heavy quarks with new flavors might exist and provide "weak-isospin" partners for the purported singlets. It is one of the fascinating features of the SU(2) ⊗ U(1) model that <u>within that model</u> the measurement of neutral-current processes is able to provide information regarding unseen, heavy fermion degrees of freedom which possess charged-current couplings to the observed fermion degrees of freedom.

With all the couplings determined in terms of the Weinberg angle, the mass of the $W^{\pm}$ can be estimated. Look, for example at muon decay:

$$\mathscr{L}_{\mathrm{eff}} = -\frac{G}{\sqrt{2}} \left[\bar{\nu}_\mu \gamma_\lambda (1-\gamma_5) \mu\right] \left[\bar{e}\gamma^\lambda (1-\gamma_5) \nu_e\right]$$

$$\cong -\frac{\tilde{g}^2}{m_W^2} \left[\bar{\nu}_\mu \gamma_\lambda \left(\frac{1-\gamma_5}{2}\right) \mu\right] \left[\bar{e}\gamma^\lambda \left(\frac{1-\gamma_5}{2}\right) \nu_e\right] \tag{3.29}$$

Therefore, using Eqs. (3.20) and (3.21),

$$\frac{g^2}{m_W^2} = \frac{4G}{\sqrt{2}} = \frac{g^2}{2m_W^2} = \frac{e^2}{2m_W^2 \sin^2 \theta_W} \tag{3.30}$$

or

$$m_W^2 = \left(\frac{2\pi\alpha\sqrt{2}}{G}\right) \cdot \frac{1}{4\sin^2 \theta_W} = \left(\frac{74.6 \text{ GeV}}{2\sin \theta_W}\right)^2 \tag{3.31}$$

The mass of the Z^0, in general, is not determined without further assumption. A conservative approach is to consider the model a two-parameter theory and fit data to it in that manner. However, what is commonly done is to assume a mass formula relating the Z-mass to the W-mass (and θ_W). This mass relation occurs in the simplest model of spontaneous breakdown involving a minimum number of Higgs bosons.[10] It can also be obtained by the following general picture[48]: In the beginning we suppose that we have an SU(2) triplet (W^+, W^-, W^0) degenerate in mass and a singlet B^0 with possibly different mass. Then an additional term mixing B_0 and W^0 is included. The crucial point is that in the absence of the B-W mixing the $W^{\pm}$ and W^0 are assumed to be degenerate. This means the mass-term in the effective hamiltonian must be

$$\mathscr{H}' = m_W^2 \left(W^+W^- + \frac{1}{2} W_0^2\right) + \frac{1}{2} \mu_1^2 B^2 + \mu_2^2 BW_0 \tag{3.32}$$

Into this we may insert Eqs. (3.22) and (3.23) expressing B_0 and W^0 in terms of A and Z. We then demand the coefficient of A^2 as well as the term mixing A with Z be zero, because A and Z are mass eigenstates and the photon is massless.

This gives two equations, which can be solved for μ_1^2 and μ_2^2. Given μ_1^2 and μ_2^2, one may then further solve for m_Z. One obtains, after a short amount of algebra

$$m_Z = \frac{m_W}{\cos\theta_W} \tag{3.33}$$

It is evident that this is the right answer, because the resultant H'_{mass} is

$$\begin{aligned}\mathscr{H}'_{\text{mass}} &= m_W^2 W^+W^- + \frac{1}{2} m_Z^2 Z_0^2 \\ &= m_W^2 \left[W^+W^- + \frac{1}{2}\cdot\frac{1}{\cos^2\theta_W}\left(\cos\theta_W W_0 - \sin\theta_W B\right)^2\right]\end{aligned} \tag{3.34}$$

which clearly satisfies (uniquely) the hypotheses we made.

Knowledge of the mass of the Z allows us to reconstruct the fōrm of the neutral-current effective Lagrangian. From Eq. (3.26) we have (dropping an overall minus sign)

$$\mathscr{L}_{\text{eff}} = \frac{e^2}{m_Z^2 \sin^2\theta_W \cos^2\theta_W}\left[\bar\psi_1\gamma_\lambda\left(\frac{1\pm\gamma_5}{2}\right)\psi_1\right]\left[\bar\psi_2\gamma^\lambda\left(\frac{1\pm\gamma_5}{2}\right)\psi_2\right]\left(T_3\text{-}Q\sin^2\theta_W\right)_1 \left(T_3\text{-}Q\sin^2\theta_W\right)_2 \tag{3.35}$$

But using Eq. (3.33) for the Z-mass and Eq. (3.30) relating the W-mass to the Fermi coupling G gives the neat result

$$\mathscr{L}_{\text{eff}} = \frac{G}{\sqrt{2}}\,\bar\psi_1\gamma_\lambda(1\pm\gamma_5)\,\psi_1\cdot\bar\psi_2\gamma^\lambda\left(\frac{1\pm\gamma_5}{2}\right)\psi_2\cdot 2\left(T_3\text{-}Q\sin^2\theta_W\right)_1\left(T_3\text{-}Q\sin^2\theta_W\right)_2 \tag{3.36}$$

The neutral current structure is the <u>same</u> as the charged-current one, with only the extra coefficient

$$\epsilon = 2\left(T_3\text{-}Q\sin^2\theta_W\right)_1\left(T_3\text{-}Q\sin^2\theta_W\right)_2 \tag{3.37}$$

to normalize the strength for any given choice of fermions and their helicities. For ν-induced neutral currents (the only processes for which there in fact exists positive evidence!), we have

$$2\left(T_3 - Q\sin^2\theta_W\right)_{\nu_\mu \text{ or } \nu_e} = 1 \qquad (3.38)$$

and the amplitude strength relative to the charged current strength is simply

$$\boxed{\epsilon = T_3 - Q\sin^2\theta_W} \qquad (3.39)$$

If one only remembers this one result, that is sufficient to reconstruct predictions for ν-induced processes in the $SU(2) \otimes U(1)$ model.

With an additional assumption, even the final parameter in this model, θ_W, can also be estimated theoretically, as shown by Georgi, Quinn, and Weinberg.[49] The assumption is that the $SU(2) \otimes U(1)$ model is only a small portion of the full weak-interaction theory, and that there is a simple group G within which $SU(2) \otimes U(1)$ resides as a subgroup. Such a situation is natural[50] in a truly unified weak-electromagnetic theory, for then there is only one independent coupling constant (instead of the two coupling constants of $SU(2) \otimes U(1)$).

The main consequence of this embedding of $SU(2) \otimes U(1)$ into G is a large proliferation of the gauge-bosons in the model; one assumes[50] that all the remaining bosons have masses large compared with the $W^\pm$ and Z^0 and contribute negligibly to present-day phenomenology. This is not an especially disagreeable possibility; there exists a hierarchy of masses in the fermion sector ($m_e \ll m_\mu \ll m_U$; $m_{u,d} \ll m_s \ll m_c$) which is not at all understood. A similar such hierarchy in the intermediate-boson sector may bear some relationship. But in any case, assuming such an embedding, the result of Georgi, Quinn, and Weinberg is the following: Let $\mathscr{R}$ be the basis of some fermion representation of the group G; i.e., $\mathscr{R}$ is some multiplet of fermions appropriate to the group G. It therefore can be decomposed according to the $SU(2) \otimes U(1)$ subgroup. Then the GQW result is

$$\sin^2 \theta_W = \frac{\sum_{i \text{ in} \mathscr{R}} T_{3i}^2}{\sum_{i \text{ in} \mathscr{R}} Q_i^2} \tag{3.40}$$

where the sum goes over all chiral (two-component) degrees of freedom in the multiplet $\mathscr{R}$, T_{3i} is third component of weak isospin of the ith fermion, and Q_i is its charge.

The proof of Eq. (3.40) is short. We note first that W^o and B, gauge bosons appropriate to $SU(2) \otimes U(1)$, are also gauge-bosons of G. The coupling of W_0 to fermion i is proportional to T_{3i}. The coupling of B is a weak isosinglet; call its coupling to the ith fermion T_{0i}. Because W_0 and B are both gauge particles for the group G, we must have, for any representation of G

$$\sum_{i \text{ in} \mathscr{R}} T_{3i}^2 = \sum_{i \text{ in} \mathscr{R}} T_{0i}^2 \tag{3.41}$$

because a symmetry operation U of the group G can transform W_0 into B; U however transforms $\mathscr{R}$ into itself. (Another way to see this is to look at the W_0 and B propagators; the vacuum polarization insertions (in the symmetry limit) in Fig. 9 must be the same. But they are proportional to the quantities in Eq. (3.41).)

Completion of the proof is now simple algebra. The photon is a gauge particle for G; furthermore it lies in the $SU(2) \otimes U(1)$ subgroup (by construction; see the previous sections). Therefore the charge Q_i must be a linear combination of T_{3i} and T_{0i}; indeed from Eqs. (3.22) and (3.23), it is clear that

$$A^\lambda = W_0^\lambda \sin \theta_W + B^\lambda \cos \theta_W \tag{3.42}$$

and necessarily

$$Q_i = (\text{const}) \left[T_{3i} \sin \theta_W + T_{0i} \cos \theta_W \right] \tag{3.43}$$

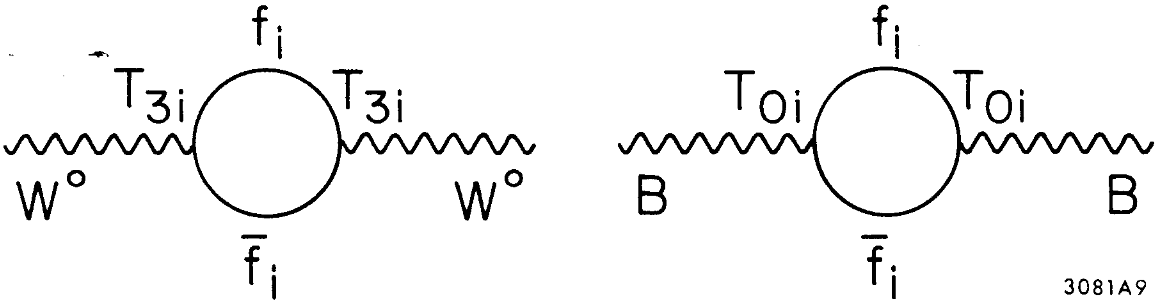

Fig. 9. Vacuum polarization insertions in gauge boson W_0 and B propagators. Note that they are proportional to T_{3i}^2 and T_{0i}^2 respectively.

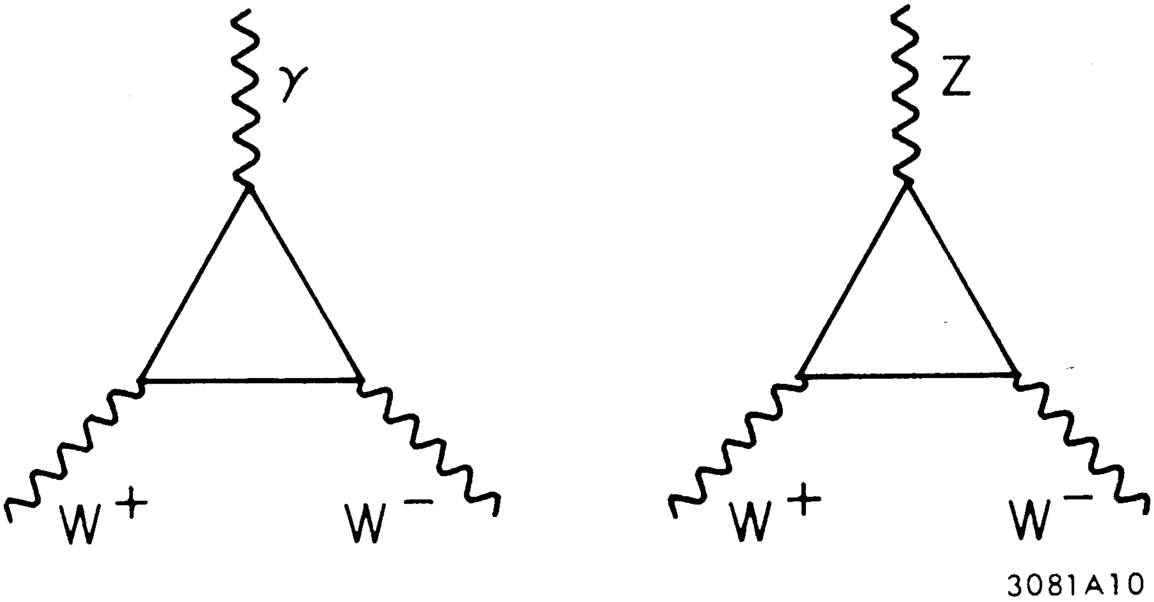

Fig. 10. Triangle diagrams in SU(2) ⊗ U(1) model.

Finally, in order to have $\Delta Q_i = \pm 1$ for members of weak isospin multiplets, we need the constant to be $(\sin\theta_W)^{-1}$; i.e.,

$$Q_i = T_{3i} + \cot\theta_W\, T_{0i} \tag{3.44}$$

Now we square Eq. (3.44), and sum over members i in the multiplet $\mathscr{R}$ of G. The cross term vanishes

$$\sum_i T_{3i}\, T_{0i} = 0 \tag{3.45}$$

when summed over any weak-isospin submultiplet. We obtain

$$\sum_{i\ \mathrm{in}\ \mathscr{R}} Q_i^2 = \sum_{i\ \mathrm{in}\ \mathscr{R}} T_{3i}^2 + \cot^2\theta_W \sum_{i\ \mathrm{in}\ \mathscr{R}} T_{0i}^2 \tag{3.46}$$

and upon utilizing Eq. (3.41), we obtain the main result, Eq. (3.40).

It is fun to estimate $\sin^2\theta_W$ using the known fermions. We take 3 or 4 examples:

1. <u>All leptons form a basis $\mathscr{R}$ for a representation of the group G.</u> Then

$$\sin^2\theta_W = \frac{\left(\frac{1}{2}\right)^2_{e_L^-} + \left(\frac{1}{2}\right)^2_{\mu_L^-} + \left(\frac{1}{2}\right)^2_{\nu_\mu} + \left(\frac{1}{2}\right)^2_{\nu_e} + ?}{(1)_{e_L^-} + (1)_{e_R^-} + (1)_{\mu_L^-} + (1)_{\mu_R^-} + ?} = \frac{1+?}{4+?} = .25 + ? \tag{3.47}$$

where the ? signifies the contribution of unknown degrees of freedom we have omitted.

2. <u>All quarks separately form a basis for a representation $\mathscr{R}$ of G.</u> Then

$$\sin^2\theta_W = \frac{3\cdot\left\{\left(\frac{1}{2}\right)^2_{u_L} + \left(\frac{1}{2}\right)^2_{d_L} + \left(\frac{1}{2}\right)^2_{s_L} + \left(\frac{1}{2}\right)^2_{c_L} + ?\right\}}{3\cdot 2\cdot\left\{\left(\frac{2}{3}\right)^2_{u} + \left(\frac{1}{3}\right)^2_{d} + \left(\frac{1}{3}\right)^2_{s} + \left(\frac{2}{3}\right)^2_{c} + ?\right\}} = \frac{3+?}{\frac{20}{3}+?} = .45 + ? \tag{3.48}$$

Consistency demands that these two values be <u>the same</u>. This requires either proliferation of fermion degrees of freedom or else combining both lepton and hadron degrees of freedom in <u>all</u> representations of G. Elegant examples of this concept have been given by Georgi and Glashow,[51] and by Gursey and Sikivie.[52,53] Combination of all fermion degrees of freedom—quarks and leptons—leads to the estimate

$$\sin^2\theta_W = \frac{\left(\frac{1}{2}\right)^2\{4 + 3.4 + ?\}}{2\cdot\left\{2 + 3\cdot\frac{10}{9} + ?\right\}} = \frac{4 + ?}{\frac{32}{3} + ?} = .375 + ? \tag{3.49}$$

Such fully unified models require intermediate bosons of fractional charge and baryon number (leptoquarks) and in at least many cases tend to destabilize the proton via reactions such as $q+q \rightarrow \bar{q}+\ell$. This requires such leptoquark gauge-bosons to be extremely massive, typically with mass $\gtrsim 10^{15}$ GeV.

A panacea which is considerably less grand in scope is to increase the leptonic estimate of $\sin^2\theta_W$ by presuming the existence of massive neutral heavy leptons E^0 and M^0 coupled via right-handed currents to e^- and μ^- respectively. This leads to the estimate

$$\sin^2\theta_W = \frac{\frac{1}{4}\cdot(8 + ?)}{2\cdot(2 + ?)} = .50 + ? \tag{3.50}$$

which is more in line with the estimate from the quarks. However both values, as we shall see, are in rather marginal agreement with the data, which prefers a value of $\sin^2\theta_W$ somewhat smaller. Nevertheless, all these estimates are not all that far from the data, and may be taken as another argument in favor of gauge theories, and in particular the $SU(2)\otimes U(1)$ model, as a correct description of weak-interaction phenomena.

Before closing this section, we should mention the question of triangle anomalies in the $SU(2)\otimes U(1)$ model. The only triangle diagrams involve $W^+W^-Z^0$ and $W^+W^-\gamma$. We see that upon summing over all fermions, the condition for cancellation of the $W^+W^-\gamma$ triangle-anomaly is, from Eq. (2.19)

$$\sum_{\text{left-handed fermions } i} \langle i|\{T^+, T^-\}Q|i\rangle = 0 \tag{3.51}$$

There is no new information that comes from the W^+W^-Z triangle couplings, because automatically

$$\sum_i \langle i | \left\{T_+, T_-\right\} T_3 | i \rangle = 0 \tag{3.52}$$

With the assignment of the standard model, Eq. (2.19), this means we should have

$$\sum_{\substack{\text{(all fermions i in}\\ \text{left-handed weak}\\ \text{doublets)}}} Q_i = 0 = \underset{e}{-1} + \underset{\nu_e}{0} - \underset{\mu}{1} + \underset{\nu_\mu}{0} + 3\left(\underset{u}{\frac{2}{3}} - \underset{d}{\frac{1}{3}} - \underset{s}{\frac{1}{3}} + \underset{c}{\frac{2}{3}}\right) \tag{3.53}$$

which actually works! Such a condition is widely considered a boon, and a positive indication of the relevance of SU(2) ⊗ U(1), charm, and nothing else as a correct model of nature. However, the only basis for demanding anomaly-cancellation is renormalizability, and in a rather high order of perturbation theory at that. As mentioned before this may not be a safe criterion to demand of a physical theory.

IV. INTERMEDIATE BOSONS IN GENERAL

The apparent current-current structure of charged-current weak interactions—and perhaps of neutral-current reactions as well—strongly suggests the intermediate boson hypothesis, even in the absence of some underlying gauge theory. However, the hypothesis of an underlying gauge theory is more predictive. We saw that in the SU(2) ⊗ U(1) model the masses of $W^\pm$ and Z^0 are determined rather well, m_W typically in the range 50-75 GeV, and m_{Z^0} in the range 75-85 GeV.

It turns out that such estimates can be generalized beyond the SU(2) ⊗ U(1) model.[54] The main assumption needed for such a generalization is, as in the preceding discussion, that the underlying gauge group G be simple, i.e., have only one coupling constant. Another is that the low-energy effective weak

Lagrangian is of current-current form and built from tricolored fractionally charged quarks. With a few more relatively innocuous assumptions, it is possible to construct upper and lower bounds for the masses of W and upper bounds for the mass of the Z. Here the W and Z are defined as the least massive charged and neutral gauge-bosons in the theory (apart from the photon) which couple the known fermions. Before stating these bounds define

$\mathscr{R}$ = basis for some fermion representation of G, as before.

$R_0 = \sum_{i \text{ in} \mathscr{R}} Q_i^2$ (as in the usual definition of R for $e^+e^- \to$ hadrons) (4-component fermions summed).

M = number of independent terms in $\mathscr{J}_\lambda^\dagger \mathscr{J}^\lambda$ coupling in the effective charged-current Lagrangian.

B_f = branching ratio of W into fermion pair f (assuming $m_W >> m_f$ for all f in $\mathscr{R}$).

Then the result is

$$\text{I:} \quad 75 \text{ GeV} \sqrt{B_{e\bar{\nu}_e} R_0} \leq m_W \leq 75 \text{ GeV} \sqrt{\frac{R_0}{M}} \tag{4.2}$$

A less restrictive result, which can be obtained without assuming the current-current structure of the effective Lagrangian, but only that the effective Lagrangian for semileptonic processes is of the usual form, is

$$\text{II:} \quad m_W \leq 75 \text{ GeV} \left(\frac{4}{3}\right)^{1/4} \sqrt{\frac{R_0}{M'}} \tag{4.3}$$

where

$$M' = \begin{cases} \geq 3 & \text{if only u, d, s quarks are accepted as members of } \mathscr{L}_{\text{eff}}. \\ \geq 6 & \text{if u, d, s, c quarks, along with the GIM coupling, are accepted.} \end{cases} \tag{4.4}$$

Examples of what these formulae mean are given in Table I for various choices of the representation $\mathscr{R}$.

Table I: General Bounds on m_W (in GeV).

Representation R	Lower Bound I	Upper Bound I	Upper Bound II (fewer assumptions)
$e^-, \nu_e, \mu^-, \nu_\mu, \ldots$?	$\geq 75 \pm$?	$\leq 75 \pm$?	--
$u_i, d_i, s_i, \ldots$? (color)	$\geq 61 \pm$?	$\leq 61 \pm$?	--
$u_i, d_i, s_i, c_i, \ldots$? (GIM, color)	$\geq 56 \pm$?	$\leq 56 \pm$?	--
$e^-, \nu_e, \mu^-, \nu_\mu, \ldots$? $u_i, d_i, s_i, \ldots$? (color)	$\geq 67 \pm$?	$\leq 67 \pm$?	$\leq 68 \pm$?
$e^-, \nu_e, \mu^-, \nu_\mu, \ldots$? $u_i, d_i, s_i, c_i, \ldots$? (GIM, color)	$\geq 61 \pm$?	$\leq 61 \pm$?	$\leq 71 \pm$?
$e^-, \nu_e, \mu^-, \nu_\mu, E^o, M^o, \ldots$? (E^o, M^o, coupled to e^-, μ^- with right-handed currents)	$\geq 53 \pm$?	$\leq 53 \pm$?	--

The method for deriving these bounds is to first embed G in an SU(N) group (with N = numbers of 2-component fermion degrees of freedom in the multiplet $\mathscr{R}$), and then to use straightforward Schwartz inequality methods. One sees that the results are very similar (not surprisingly) to those obtained within the SU(2) $\otimes$ U(1) scheme.

The bounds on the Z^o mass, regrettably, are less restrictive. Defining

$$R = \left.\frac{\sigma_{tot}(\nu N \to \nu \text{ hadrons})}{\sigma_{tot}(\nu N \to \mu^- \text{ hadrons})}\right|_{E_\nu \text{ large}} , \left(\begin{matrix}\text{but below new-}\\ \text{flavor thresholds}\end{matrix}\right)$$

$$\bar{R} = \left.\frac{\sigma_{tot}(\bar{\nu} N \to \bar{\nu} \text{ hadrons})}{\sigma_{tot}(\bar{\nu} N \to \mu^+ \text{ hadrons})}\right|_{E_\nu \text{ large}} \qquad (4.5)$$

$$R_{cc} = \left.\frac{\sigma_{tot}(\bar{\nu} N \to \mu^+ \text{ hadrons})}{\sigma_{tot}(\nu N \to \mu^- \text{ hadrons})}\right|_{E_\nu \text{ large}}$$

we find, under the previous assumptions

$$m_{Z^o} \leq (75 \text{ GeV}) R_0^{1/2} \left[\frac{2\left(1-R_{cc}^2\right)}{M'\left(R-\bar{R}R_{cc}^2\right)}\right]^{1/4} \qquad (4.6)$$

This turns out to be a rather poor restriction in comparison to what is obtained within the SU(2) ⊗ U(1) model. Competitive bounds are obtained with the strong assumption that only one Z^o mediates the observed neutral-current phenomena. Then

$$\text{IV:} \qquad m_{Z^o} \leq (75 \text{ GeV}) R_0^{1/2} \left[\frac{(1+R_{cc})}{4M'(R+R_{cc}\bar{R})}\right]^{1/4} \qquad (4.7)$$

If in addition ν_μ-ν_e universality is presumed, one can gain an additional improvement of a factor $\sqrt[4]{2}$:

$$\text{V:} \qquad m_{Z^o} \leq (75 \text{ GeV}) R_0^{1/2} \left[\frac{1+R_{cc}}{8M'(R+R_{cc}\bar{R})}\right]^{1/4} \qquad (4.8)$$

Finally, with all assumptions but the last one, a direct estimate (via the technique used to obtain lower bounds for m_W) gives

$$\text{VI:} \qquad m_Z \approx (75 \text{ GeV}) R_0^{1/2} \left[\frac{B_{\nu_\mu\bar{\nu}_\mu} B_{had}(1+R_{cc})}{M'(R+R_{cc}\bar{R})}\right]^{1/4} \qquad (4.9)$$

We summarize these bounds and estimates in Table II.

Table II: Bounds and Estimates of m_{Z^0} (in GeV).

Representation	Upper Bound III (general)	Upper Bound IV (only one Z^0 for observed phenomena)	Upper Bound IV (as in IV, plus ν_μ-ν_e universality)	Estimate VI[a] (as in IV)
$e^-, \nu_e, \mu^-, \nu_\mu, \ldots$ $u_i, d_i, s_i, \ldots$ (color)	$\leq 189 \pm ?$	$\leq 107 \pm ?$	$\leq 90 \pm ?$	--
$e^-, \nu_e, \mu^-, \nu_\mu, \ldots$ $u_i, d_i, s_i, c_i, \ldots$ (GIM, color)	$\leq 184 \pm ?$	$\leq 104 \pm ?$	$\leq 88 \pm ?$	$69 \pm ?$

[a] We use $B_{\nu_\mu \bar{\nu}_\mu} = 8\%$, $B_{had} = 75\%$, in accord with the simplest of guesses using only statistical weights.

Can the W and Z actually be produced in any foreseeable future? There is considerable reason to believe that, if these estimates are correct, the answer is yes. In hadron-hadron collisions, the Drell-Yan mechanism[55,56] appears thus far to provide a good description of the electromagnetic production of massive dilepton pairs. This mechanism is a parton model process in which pointlike quarks and antiquarks annihilate into the lepton pairs. It is beyond the scope of these lectures (however, cf. the lectures of D. Hitlin; these proceedings) to describe this. Suffice it to say that, if the process $q+\bar{q} \rightarrow \ell^+ + \ell^-$ can be observed, so also can the resonant processes $q+\bar{q} \rightarrow W$ or $q+\bar{q} \rightarrow Z^0$. One only needs the width for $W \rightarrow q+\bar{q}$ or $Z \rightarrow q+\bar{q}$ to calculate the cross sections,[57] which turn out to be in the range $10^{-33} - 10^{-35}$ cm^2 and accessible to future pp storage rings.

It is a straightforward, model independent, calculation to estimate the width of the W as a function of its mass. The simplest of Feynman-diagram calculations gives

$$\Gamma(W^- \to e^- \bar{\nu}_e) = \frac{G m_W^3}{6\pi\sqrt{2}} \tag{4.10}$$

with a branching ratio (assuming u, d, s, c, color, and GIM charm) of

$$B_{e\bar{\nu}_e}^{-1} = \underset{e\bar{\nu}_e}{1} + \underset{(\mu\bar{\nu}_\mu)}{1} + \underset{(u\bar{d})}{3} + \underset{(c\bar{s})}{3} + ? \geq 8 \tag{4.11}$$

or

$$B_{e\bar{\nu}_e} \leq 12.5\% \tag{4.12}$$

Staying within the SU(2) ⊗ U(1) scheme and taking the experimental (as well as theoretical!) limits on $\sin^2 \theta_W$, to be discussed in Section V,

$$.25 \lesssim \sin^2 \theta_W \lesssim .50 \tag{4.13}$$

leads to

$$\begin{aligned} m_W &= 64 \pm 11 \text{ GeV} \\ m_Z &= 80 \pm 6 \text{ GeV} \end{aligned} \tag{4.14}$$

and

$$\Gamma(W^- \to e^- \bar{\nu}_e) \sim 115 \text{ MeV} \tag{4.15}$$

Thus

$$\Gamma(W \to \text{all}) \gtrsim 900 \text{ MeV} \tag{4.16}$$

The W is rather broad!

For the Z we estimate, in the SU(2) ⊗ U(1) scheme

$$\frac{\Gamma(Z \to \nu_\mu \bar{\nu}_\mu)}{\Gamma(W^- \to e^- \bar{\nu}_e)} = \left(\frac{m_Z}{m_W}\right) \frac{\left(\frac{1}{2} \cdot \frac{e}{\sin\theta_W \cos\theta_W}\right)^2}{\left(\frac{e}{\sqrt{2}\sin\theta_W}\right)^2} = \frac{1}{2}\left(\frac{m_Z}{m_W}\right)^3 \tag{4.17}$$

The ratio (m_Z/m_W) comes from dimensional analysis; evidently

$$\Gamma = m \times (\text{factors } 2, \pi, \text{ etc. from phase-space}) \times (\text{coupling constants}) \qquad (4.18)$$

The coupling constant of Z to $\nu_\mu \bar{\nu}_\mu$ is the second factor in the numerator, obtained from Eq. (3.26), while the W coupling to $e\bar{\nu}_e$ in the denominator is found in Eq. (3.30). Finally we have used the mass formula relating m_Z and m_W; Eq. (3.33).

This yields the estimate (using $m_W = 65$ GeV, $m_Z = 80$ GeV)

$$\Gamma(Z \to \nu_\mu \bar{\nu}_\mu) \sim 110 \text{ MeV} \qquad (4.19)$$

Then, remembering that Z-couplings are proportional to $(T_3 - Q\sin^2\theta_W)$, all other partial widths are immediately written down:

$$\frac{\Gamma(Z \to \mu^+\mu^-)}{\Gamma(Z \to \nu_\mu\bar{\nu}_\mu)} = \frac{\left(-\frac{1}{2} + \sin^2\theta_W\right)^2_{LH} + \left(\sin^2\theta_W\right)^2_{RH}}{\left(\frac{1}{2}\right)^2} = 1\text{-}4\sin^2\theta_W + 8\sin^4\theta_W \qquad (4.20)$$

$$\frac{\Gamma(Z \to u\bar{u})}{\Gamma(Z \to \nu_\mu\bar{\nu}_\mu)} = \frac{3\left\{\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)_{LH} + \left(\frac{2}{3}\sin^2\theta_W\right)^2_{RH}\right\}}{\left(\frac{1}{2}\right)^2} = 3\text{-}8\sin^2\theta_W + \frac{32}{3}\sin^4\theta_W \qquad (4.21)$$

(Notice the factor 3 for color; it <u>must</u> be there if one includes it in describing R in $e^+e^- \to$ hadrons.) Finally

$$\frac{\Gamma(Z \to d\bar{d})}{\Gamma(Z \to \nu_\mu\bar{\nu}_\mu)} = \frac{3\left\{\left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right)^2 + \frac{1}{9}\sin^4\theta_W\right\}}{\left(\frac{1}{2}\right)^2} = 3\text{-}4\sin^2\theta_W + \frac{8}{3}\sin^4\theta_W \qquad (4.22)$$

For $\sin^2\theta_W = 0.36$, one calculates

$$\frac{\Gamma(Z \to \mu^+\mu^-)}{\Gamma(Z \to \nu_\mu\bar{\nu}_\mu)} = 0.6 \qquad \frac{\Gamma(Z \to u\bar{u})}{\Gamma(Z \to \nu_\mu\bar{\nu}_\mu)} = 1.5 \qquad \frac{\Gamma(Z \to d\bar{d})}{\Gamma(Z \to \nu_\mu\bar{\nu}_\mu)} = 1.9 \qquad (4.23)$$

and therefore, using GIM, color, and charm:

$$\frac{\Gamma_{\text{tot}}}{\Gamma(Z \to \nu_\mu \bar{\nu}_\mu)} \geq \underset{(\nu_e)}{1} + \underset{(\nu_\mu)}{1} + \underset{(e)}{0.6} + \underset{(\mu)}{0.6} + \underset{(u)}{1.5} + \underset{(c)}{1.5} + \underset{(d)}{1.9} + \underset{(s)}{1.9} = 10.0 \quad (4.24)$$

or

$$\Gamma_{\text{tot}}^{Z^o} \approx 1.1 \text{ GeV}$$

$$B_{Z \to e^+e^-} \lesssim 0.06 \quad (4.25)$$

In addition to W and Z production in strong interactions, resonant production of Z^o in e^+e^- colliding beams would clearly be an extraordinary powerful and clean way[58] of studying all objects of moderate mass coupled to Z^o. To get an idea of what is involved, we need only look at the peak cross section, as follows from the Breit-Wigner formula:

$$\sigma_{BW}(e^+e^- \to Z^o) = \frac{12\pi\, \Gamma(Z \to e^+e^-)\, \Gamma(Z \to f)}{\left(s - m_Z^2\right)^2 + m_Z^2\, \Gamma_{\text{tot}}^2} \quad (4.26)$$

At the resonance peak, this means

$$\sigma_{\text{peak}}(e^+e^- \to Z^o) = \frac{12\pi}{m_Z^2} B_{e^+e^-} B_f \quad (4.27)$$

Notice, for $m_Z \sim 80$ GeV and $\Gamma_Z \gtrsim 1$ GeV, the finite machine-resolution ($\ll 1\%$) does not lower significantly the peak cross section (unlike the case of resonant production of the ψ). Taking the SU(2) $\otimes$ U(1) estimates

$$B_{e^+e^-} \sim 6\%$$

$$B_f \sim 1 - B_{\nu_e \bar{\nu}_e} - B_{\nu_\mu \bar{\nu}_\mu} \approx 80\% \quad (4.28)$$

$$m_Z \sim 80 \text{ GeV}$$

along with a luminosity $\mathscr{L} \sim 10^{32}$ cm^{-2} sec^{-1} (a typical futuristic estimate for such rings) this gives

$$\text{Rate of Z-production} \sim 10 \text{ Z's/sec} \quad . \quad (4.29)$$

With such a yield of pure Z's, one should think hard about rare decay-modes of the Z^o. One that comes to mind is the decay $Z^o \to W^+e^-\bar{\nu}_e$. However, with SU(2)⊗U(1) parameters, a crude estimate gives a discouraging number $(B(We\bar{\nu}_e) \lesssim 10^{-6})$. More interesting is the decay $Z^o \to h^o\mu^+\mu^-$, with h_0 the Higgs boson in the simple SU(2)⊗U(1) model. A straightforward calculation (slightly beyond the scope of these lectures), optimistically assuming that a single Higgs boson is responsible for the Z mass, gives

$$\frac{1}{\Gamma(Z \to \mu\mu)}\frac{d\Gamma}{dx} = \frac{\alpha}{4\ \sin^2\theta_W\ \cos^2\theta_W}\ \frac{\left[1-x+\frac{x^2}{12}+\frac{2}{3}\frac{m_h^2}{m_Z^2}\right]\left(x^2-\frac{4m_h^2}{m_Z^2}\right)^{1/2}}{\left(x-\frac{m_h^2}{m_Z^2}\right)^2} \tag{4.30}$$

where

$$x = \frac{2E_{higgs}}{m_Z} \tag{4.31}$$

and the kinematic limits are

$$\frac{2m_h}{m_Z} \leq x \leq 1+\frac{m_h^2}{m_Z^2} \tag{4.32}$$

Rough numerical integration provides the yield shown in Fig. 11. We see that for $m_h \lesssim 40$ GeV, the branching ratio relative to μ pairs $B(Z \to h^o\mu^+\mu^-)/B(Z^o \to \mu^+\mu^-)$, is $\gtrsim 3 \times 10^{-5}$. Recalling that a 6% $\mu^+\mu^-$ branching ratio still means ~ 0.6 $Z^o \to \mu^+\mu^-$ events/second, this leaves a tolerable production of Higgs bosons. The signature evidently is very good; one looks at a peak in the mass recoiling against an energetic acoplanar dilepton pair. We must, however, point out that this estimate, as is any estimate which directly involves the Higgs sector, is very unreliable: the theoretical status is very poorly understood.[42] Indeed there is no certainty that $m_h \lesssim 40$ GeV; Higgs bosons could be ten times more massive.[59] And there could well be several.

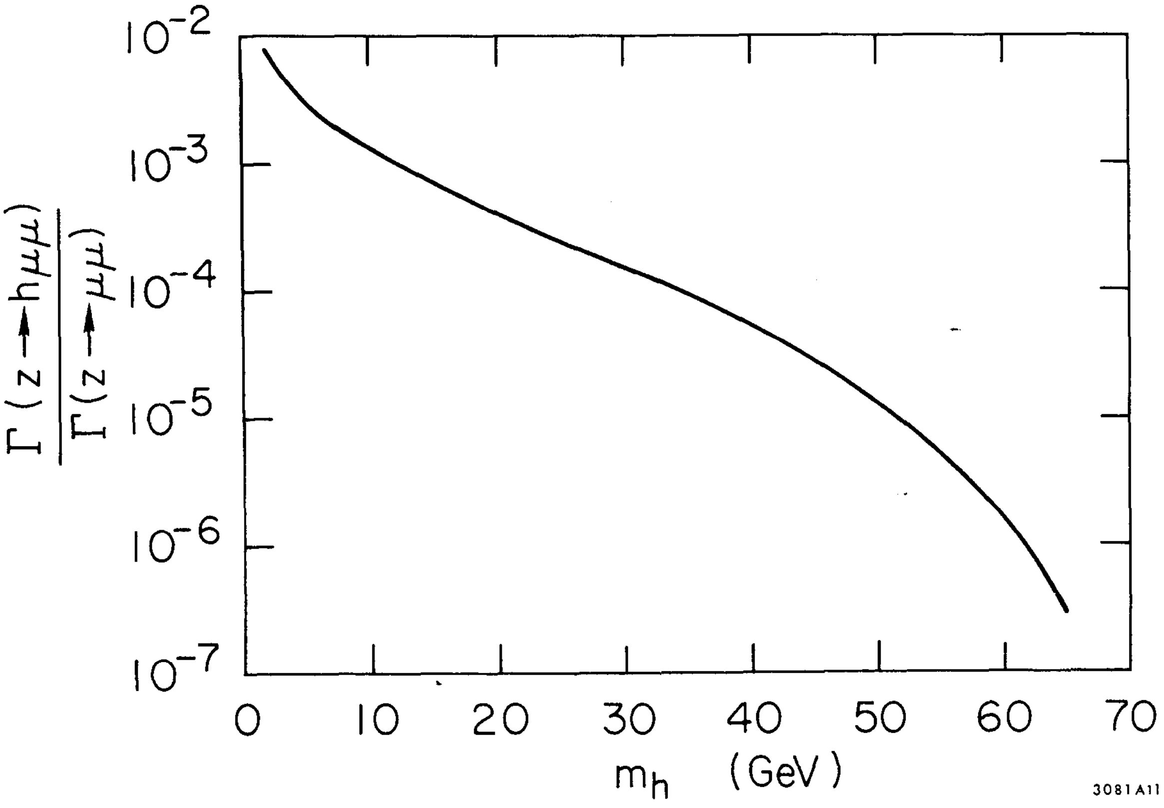

Fig. 11. Estimated branching ratio of $Z \rightarrow h^o\mu^+\mu^-$ relative to $Z^o \rightarrow \mu^+\mu^-$. We have taken $\sin^2\theta_W = 1/3$.

On the other hand, if $m_h < \frac{1}{2} m_Z$, the decay $Z^o \rightarrow h^o h^o$ has a very large branching ratio, nearly competitive with $Z^o \rightarrow \mu^+ \mu^-$. Observability of this process depends upon the dominant decay modes of h^o, an uncertain matter indeed.

V. SOME NEUTRAL CURRENT PHENOMENOLOGY

Thus far, we have only laid the groundwork for describing the observations on charged and neutral-current phenomena. We now discuss some of the relevant experimental information. Regrettably, there is not the time to discuss even all the neutral-current data which bears on the issues. We sacrifice any extensive discussion of resonance and/or single pion production, i.e., reactions such as

$$\begin{aligned} \nu p &\rightarrow \nu \Delta^+ \\ \nu n &\rightarrow \nu p \pi^- \\ \nu p &\rightarrow \nu p \pi^+ \pi^- \end{aligned} \qquad (5.1)$$

This is not to be interpreted that these reactions are not of interest; on the contrary it should be possible to learn a great deal, especially isospin-structure of the neutral current couplings, by such studies. An excellent and most authoritative introduction to the subject can be found in the lectures of Adler[60] at the 1975 Hawaii Summer School. We shall instead concentrate on what might be learned from existing deep-inelastic data, from the new data on elastic νp and $\bar{\nu} p$ scattering, and from the experiments on parity-violation in atomic physics which are now in progress.

We shall approach the phenomenology from two points of view:

1. Can we determine the neutral-current couplings (as defined in the effective Lagrangian in Eq. (1.4)) in a model independent way?

2. Is the SU(2) ⊗ U(1) gauge theory model (or variants thereof) in accord with the data??

A. Leptonic Neutral Currents. There are three channels for which there exists some data:

$$\begin{aligned} &\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^- \\ &\nu_\mu e^- \to \nu_\mu e^- \\ &\bar{\nu}_e e^- \to \bar{\nu}_e e^- \end{aligned} \tag{5.2}$$

The first two have been studied in the CERN PS neutrino beam, both in the heavy liquid bubble chamber Gargamelle,[61] and in a counter experiment (Aachen-Padua) behind it.[62] The Gargamelle data consist of three $\bar{\nu}_\mu e^-$ events and no $\nu_\mu e^-$ events above background, from which come the estimates

$$\begin{aligned} \sigma_{\bar{\nu}_\mu e^-} &= \left(.11 {}^{+.21}_{-.09}\right) \times 10^{-41}\ E_\nu\ \mathrm{cm}^2\ \mathrm{GeV}^{-1} \\ \sigma_{\nu_\mu e^-} &< .26 \times 10^{-41}\ E_\nu\ \mathrm{cm}^2\ \mathrm{GeV}^{-1}\ (90\%\ \mathrm{conf.}) \end{aligned} \tag{5.3}$$

The Aachen-Padua group, on the other hand, find a considerably larger yield, and quote[63]

$$\begin{aligned} \sigma_{\bar{\nu}_\mu e^-} &= (.54 \pm .17) \times 10^{-41}\ E_\nu\ \mathrm{cm}^2\ \mathrm{GeV}^{-1} \\ \sigma_{\nu_\mu e^-} &= (.24 \pm .12) \times 10^{-41}\ E_\nu\ \mathrm{cm}^2\ \mathrm{GeV}^{-1} \end{aligned} \tag{5.4}$$

The theoretical cross section for $\nu_\mu e^-$ scattering is easily derived from the effective Lagrangian written down in Section I; Eq. (1.4). It is (at high energy)

$$\sigma_{\nu_\mu e^-} = \frac{2G^2 m_e E_\nu}{\pi} \left\{ |\epsilon_L(e)|^2 + \frac{1}{3} |\epsilon_R(e)|^2 \right\} \tag{5.5}$$

while

$$\sigma_{\bar{\nu}_\mu e^-} = \frac{2G^2 m_e E_\nu}{\pi} \left\{ \frac{1}{3} |\epsilon_L(e)|^2 + |\epsilon_R(e)|^2 \right\} \tag{5.6}$$

Putting in the numbers gives the estimates

$$|\epsilon_L(e)|^2 + \frac{1}{3}|\epsilon_R(e)|^2 = \begin{cases} \leq .15 \text{ (90\% conf.) GGM} \\ .31 \pm .10 \text{ AP} \end{cases}$$

$$|\epsilon_R(e)|^2 + \frac{1}{3}|\epsilon_L(e)|^2 = \begin{cases} .06^{+.12}_{-.05} \text{ GGM} \\ .14 \pm .07 \text{ AP} \end{cases} \qquad (5.7)$$

The SU(2) ⊗ U(1) model, from the rule in Eq. (3.39) that (for neutrino-induced reactions) $\epsilon = T_3 - Q \sin^2 \theta_W$, gives

$$\epsilon_L(e) = -\frac{1}{2} + \sin^2 \theta_W$$
$$\epsilon_R(e) = \sin^2 \theta_W \qquad (5.8)$$

provided e_R^- is a weak singlet; if it is a weak doublet (i.e., e_R^- is coupled via $W^\pm$ to a heavy lepton E^o) evidently $\epsilon_R(e) = \epsilon_L(e)$. The present limits, as quoted by Gargamelle, give rise to the allowed regions in Fig. 12. Clearly these measurements are not yet good enough to draw much of any quantitative conclusions. However knowledge of the very existence of the neutral current reaction $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$ is, of course, of very great importance.

Data on the reaction $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$ at low energy (using reactor $\bar{\nu}_e$'s of a few MeV energy) has recently been reported.[64] The ordinary diagonal charged-current coupling is expected to contribute here as well as any additional neutral current coupling. Adding this to Eq. (1.4), along with a Fierz transformation, gives

$$\mathscr{L}_{\text{eff}} = \frac{G}{\sqrt{2}} \bar{\nu}_e \gamma_\lambda (1-\gamma_5) \nu_e \left[\begin{array}{l} (1+ \epsilon_L(e)) \, \bar{e} \gamma^\lambda (1-\gamma_5) e \\ + \epsilon_R(e) \, \bar{e} \gamma^\lambda (1+\gamma_5) e \end{array} \right] \qquad (5.9)$$

with again a differential cross section of a form as in Eq. (5.6); when $E >> m_e$

$$\sigma_{\bar{\nu}_e e^-} = \frac{2G^2 m_e E}{\pi} \left\{ |\epsilon_R(e)|^2 + \frac{1}{3}|1 + \epsilon_L(e)|^2 \right\} \qquad (5.10)$$

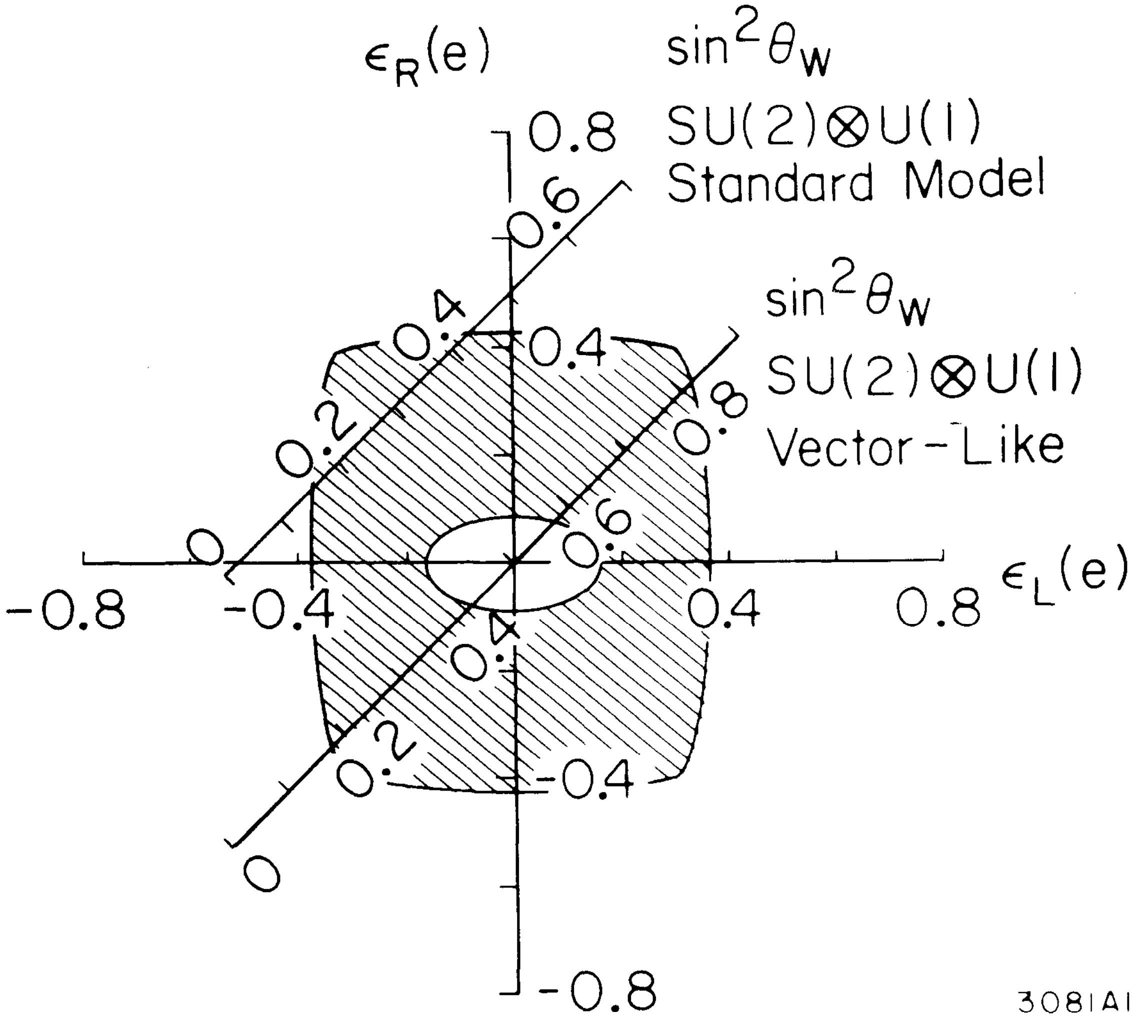

Fig. 12. Allowed values of the coupling constants $\epsilon_L(e)$ and $\epsilon_R(e)$ governing the processes $\nu_\mu e^- \rightarrow \nu_\mu e^-$ and $\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$. We have used the limits as quoted by Gargamelle.

Because the detection efficiency is dependent upon positron energy, the one-standard deviation limits on the coupling constants are actually quoted. These are reproduced in Fig. 13, along with the $SU(2) \otimes U(1)$ predictions, for both the weak-singlet and weak-doublet options for e_R^-. Consistency exists for both options over a considerable range of $\sin^2 \theta_W$, as well as for the pure charged-current coupling corresponding to $\epsilon_L(e) = \epsilon_R(e) = 0$.

In general, it is clear that, while the existence of these processes appear to be established and to have a reasonable magnitude, it is difficult to draw decisive quantitative conclusions.

B. Deep-Inelastic Neutral Currents. There exist three measurements of the processes $\nu N \rightarrow \nu$ hadrons and $\bar{\nu} N \rightarrow \bar{\nu}$ hadrons at relatively large energies,[65,66,67] where the scaling concepts used in charged-current deep-inelastic scattering are found to be of use. In interpreting these measurements, we shall make a few simplifying assumptions (which should not introduce errors of much more than 10% or so). We shall first of all adopt the naive quark-parton model description of the cross section. This is probably not too terrible an assumption, inasmuch as this description reproduces charged-current data, and much of the neutral-current phenomena are isospin (and SU(3) rotations) of the charged current phenomena. Secondly we shall neglect any contribution of strange quark-partons (or strange antipartons), as well as any new currents involving charmed quarks or other new flavors. The neglect of new charged currents, as well as neutral currents, is especially dangerous when considering $\bar{\nu}$-induced processes. The experimentalists measure the ratio of the $\bar{\nu} N$ neutral current cross section to the $\bar{\nu} N$ charged-current cross section. If there is anomalous behavior in the $\bar{\nu} N$ charged-current process (cf., the lectures of S. Wojcicki, these Proceedings), this will reflect itself in the numbers quoted for the neutral-current measurements. However most of the neutral-current data is for

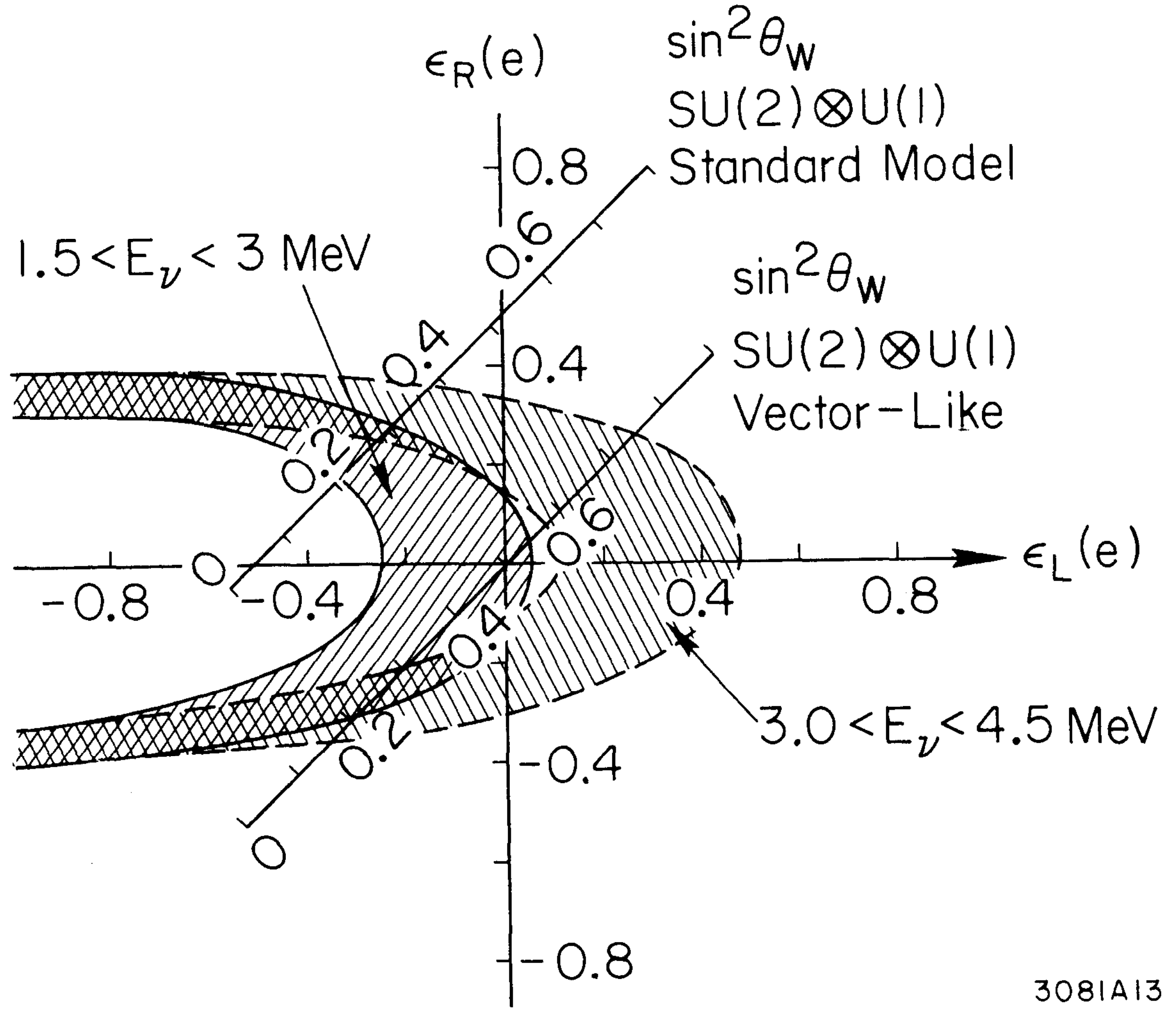

Fig. 13. Allowed values of the coupling constants $\epsilon_L(e)$ and $\epsilon_R(e)$ as determined from $\bar{\nu}_e e^-$ scattering.

$E_{\bar{\nu}} \lesssim 50$ GeV, while the anomalous behavior is probably only large for $E_{\bar{\nu}} \gtrsim 50$ GeV.

We now turn to the expressions for the cross sections for charged and neutral current processes. Using the simple parton model calculations,[68] one obtains for the charged-current process $\nu p \to \mu^-$ hadrons

$$\sigma_{cc}^{\nu p} = \frac{2G^2ME_\nu}{\pi}\left\{\int_0^1 dx\left[d(x) + \frac{1}{3}\bar{u}(x)\right]\right\} \tag{5.11}$$

where

$$\frac{d(x)}{x} = \frac{dN_d}{dx} = \text{number of down quarks in an energetic proton possessing fraction x (in dx) of the proton total momentum} \tag{5.12}$$

with similar definitions for u(x), $\bar{u}(x)$, and $\bar{d}(x)$. The experiments we shall discuss are on complex nuclei, and using charge symmetry, i.e.,

$$\begin{aligned} u(x)_{\text{proton}} &= d(x)_{\text{neutron}} \\ \bar{u}(x)_{\text{proton}} &= \bar{d}(x)_{\text{neutron}} \end{aligned} \tag{5.12}$$

we write for the cross section per nucleon N

$$\sigma_{cc}^{\nu N} = \frac{2G^2ME_\nu}{\pi}\int_0^1 dx\left[q(x) + \frac{1}{3}\bar{q}(x)\right] \tag{5.13}$$

with

$$\begin{aligned} q(x) &= \frac{1}{2}\left[u(x) + d(x)\right] \\ \bar{q}(x) &= \frac{1}{2}\left[\bar{u}(x) + \bar{d}(x)\right] \end{aligned} \tag{5.14}$$

Likewise

$$\sigma_{cc}^{\bar{\nu}N} = \frac{2G^2ME_\nu}{\pi}\int_0^1 dx\left[\frac{1}{3}q(x) + \bar{q}(x)\right] \tag{5.15}$$

Using the effective Lagrangian, Eq. (1.4) we can immediately write down the corresponding neutrino cross section on a proton:

$$\sigma_{NC}^{\nu p} = \frac{2G^2ME_\nu}{\pi} \left\{ |\epsilon_L(u)|^2 \int_0^1 dx \left[u(x) + \frac{1}{3}\bar{u}(x)\right] + |\epsilon_R(u)|^2 \int_0^1 dx \left[\frac{1}{3} u(x) + \bar{u}(x)\right] \right.$$
$$\left. + |\epsilon_L(d)|^2 \int_0^1 dx \left[d(x) + \frac{1}{3}\bar{d}(x)\right] + |\epsilon_R(d)|^2 \int_0^1 dx \left[\frac{1}{3} d(x) + \bar{d}(x)\right] \right\} \tag{5.16}$$

On an average nucleon N, this is

$$\sigma_{NC}^{\nu N} = \frac{2G^2ME_\nu}{\pi} \left\{ \left[|\epsilon_L(u)|^2 + |\epsilon_L(d)|^2\right] \int_0^1 dx \left[q(x) + \frac{1}{3}\bar{q}(x)\right] \right.$$
$$\left. + \left[|\epsilon_R(u)|^2 + |\epsilon_R(d)|^2\right] \int_0^1 dx \left[\frac{1}{3} q(x) + \bar{q}(x)\right] \right\} \tag{5.17}$$

and for antineutrinos

$$\sigma_{NC}^{\bar{\nu} N} = \frac{2G^2ME_\nu}{\pi} \left\{ \left[|\epsilon_L(u)|^2 + |\epsilon_L(d)|^2\right] \int_0^1 dx \left[\frac{1}{3} q(x) + \bar{q}(x)\right] \right.$$
$$\left. + \left[|\epsilon_R(u)|^2 + |\epsilon_R(d)|^2\right] \int_0^1 dx \left[q(x) + \frac{1}{3}\bar{q}(x)\right] \right\} \tag{5.18}$$

Define again the ratios R, $\bar{R}$, R_{cc} already introduced in Eq. (4.5). The experimental values quoted for these quantities are

	Gargamelle	HPWF	CITF
$R = \frac{\sigma_{Nc}^{\nu N}}{\sigma_{cc}^{\bar{\nu} N}} =$	.25 ± .04	.29 ± .04	.24 ± .04
$\bar{R} = \frac{\sigma_{Nc}^{\bar{\nu} N}}{\sigma_{cc}^{\bar{\nu} N}} =$	.39 ± .06	.39 ± .10	.35 ± .11
$R_{cc} = \frac{\sigma_{cc}^{\bar{\nu} N}}{\sigma_{cc}^{\nu N}} =$	.38 ± .02		

(5.19)

Inasmuch as

$$R_{cc} = \frac{\int_0^1 dx\left[\frac{1}{3}\, q(x) + \bar{q}(x)\right]}{\int_0^1 dx\left[q(x) + \frac{1}{3}\,\bar{q}(x)\right]} \tag{5.20}$$

we find from Eqs. (5.17) and (5.18) that

$$\begin{aligned} R &= |\epsilon_L(u)|^2 + |\epsilon_L(d)|^2 + R_{cc}\left\{|\epsilon_R(u)|^2 + |\epsilon_R(d)|^2\right\} \\ \bar{R} &= |\epsilon_L(u)|^2 + |\epsilon_L(d)|^2 + R_{cc}^{-1}\left\{|\epsilon_R(u)|^2 + |\epsilon_R(d)|^2\right\} \end{aligned} \tag{5.21}$$

These can be solved for the basic couplings

$$\begin{aligned} |\epsilon_L(u)|^2 + |\epsilon_L(d)|^2 &= \frac{R_{cc}^{-1}R - R_{cc}\bar{R}}{R_{cc}^{-1} - R_{cc}} \\ |\epsilon_R(u)|^2 + |\epsilon_R(d)|^2 &= \frac{\bar{R} - R}{R_{cc}^{-1} - R_{cc}} \end{aligned} \tag{5.22}$$

Putting in the numbers gives

$$\begin{aligned} |\epsilon_L|^2 &\equiv |\epsilon_L(u)|^2 + |\epsilon_L(d)|^2 = .26 \pm .04 \\ |\epsilon_R|^2 &\equiv |\epsilon_R(u)|^2 + |\epsilon_R(d)|^2 = .06 \pm .05 \end{aligned} \tag{5.23}$$

The error-assignments here are not to be taken seriously, especially for the value of $|\epsilon_R|^2$. It would be best for the experimental groups to directly quote their estimate of these quantities. Especially important is to firmly establish that $|\epsilon_R(u)|^2 + |\epsilon_R(d)|^2 \neq 0$. It is my understanding that in fact there is evidence both from Gargamelle and CITF that, at a level approaching two standard deviations, a pure left-handed (V-A) neutral-current coupling is ruled out. In the case of the CITF experiment, this is exhibited in Fig. 14. The quantities g_L^2 and g_R^2 in that plot are defined by the expression

$$d\sigma_{Nc}^{\nu N}/dy = \left[g_L + g_R(1-y)^2\right] \frac{G^2ME}{\pi} \tag{5.24}$$

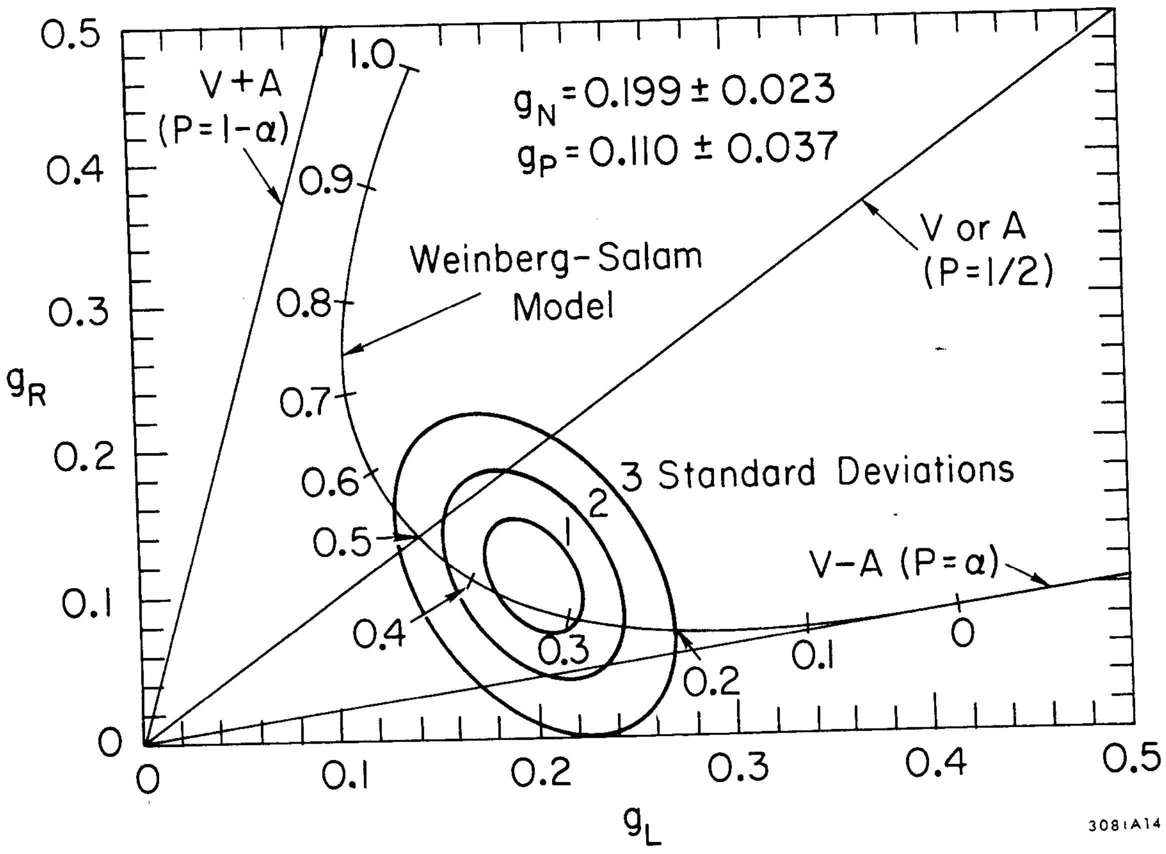

Fig. 14. Range of left and right helicity coupling allowed by the CITF neutral-current data. See text for definitions of g_L and g_R.

with

$$y = \frac{E_{\text{hadron}}}{E_\nu} = \frac{\nu}{E_\nu} \qquad 0 \leq y \leq 1 \tag{5.25}$$

Hence

$$g_L \sim |\epsilon_L|^2 + \frac{\int_0^1 dx\, \bar{q}(x)}{\int_0^1 dx\, q(x)} |\epsilon_R|^2$$

$$g_R \sim |\epsilon_R|^2 + \frac{\int_0^1 dx\, \bar{q}(x)}{\int_0^1 dx\, q(x)} |\epsilon_L|^2 \tag{5.26}$$

The antiquark content is best determined by studies of the charged-current y-distributions, and is conventionally parametrized by the quantity

$$B = \frac{\int_0^1 dx\, [q(x) - \bar{q}(x)]}{\int_0^1 dx\, [q(x) + \bar{q}(x)]} \tag{5.27}$$

Estimates of B, in the appropriate energy range, vary from ~0.7 to ~0.95, giving an antiquark content varying from

$$.16 \gtrsim \frac{\int_0^1 \bar{q}(x)\, dx}{\int_0^1 q(x)\, dx} \gtrsim .05 \tag{5.28}$$

(See the lectures of S. Wojcicki, these Proceedings; in particular Fig. 28, for more details.)

What is theoretically expected for ϵ_L and ϵ_R? In the SU(2)⊗U(1) model, the values are easily calculated from Eq. (3.39), which states that $\epsilon(q) = \left(T_3 - Q\sin^2\theta_W\right)_q$. Evidently u_L and d_L form a weak doublet (we here ignore effects of Cabibbo mixing); however one must decide upon the assignments of u_R and d_R. Without introducing new quarks of charge greater than unity, this leaves four basic variants to consider[69]:

1. u_R and d_R weak singlets: this is the standard model.

2. d_R is singlet and $(u\ b)_R$ a doublet; b is a heavy "bottom" quark of charge -1/3. This model (with $m_b \sim 4$ GeV) gives a good account of the anomalous behavior of $\bar{\nu}$-induced charged current processes at high energy.[70] It has been discussed by several authors,[70,71] including Gursey and Sikivie[72] who embed it into a fully unified theory based on the exceptional group E7. We shall see that this model is consistent with most neutral-current data.

3. u_R a singlet and $(t\ d)_R$ a doublet involving a new heavy "top" quark. This model fares poorly when compared with experiment, as we shall see.

4. $(u\ b)_R$ and $(t\ d)_R$ both weak doublets. This leads to a parity-conserving vector neutral current,[73] which is in considerable disagreement with experiment. In particular the HPWF group has tested this hypothesis against their data[74] and claim it is ruled out by at least 3 standard deviations. Models based upon this scheme have been quite popular.

The coupling constants for the schemes are given below. In all variants

$$\epsilon_L(u) = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W \tag{5.29}$$

$$\epsilon_L(d) = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \tag{5.30}$$

For the right-handed currents we have

$$\begin{array}{lll}
\text{Standard model} & \epsilon_R(u) = -\frac{2}{3}\sin^2\theta_W & \epsilon_R(d) = \frac{1}{3}\sin^2\theta_W \\
(u\ \ b)_R & \epsilon_R(u) = \frac{1}{2}-\frac{2}{3}\sin^2\theta_W & \epsilon_R(d) = \frac{1}{3}\sin^2\theta_W \\
(t\ \ d)_R & \epsilon_R(u) = -\frac{2}{3}\sin^2\theta_W & \epsilon_R(d) = -\frac{1}{2}+\frac{1}{3}\sin^2\theta_W \\
\text{Vector-like} & \epsilon_R(u) = \frac{1}{2}-\frac{2}{3}\sin^2\theta_W & \epsilon_R(d) = -\frac{1}{2}+\frac{1}{3}\sin^2\theta_W
\end{array} \tag{5.31}$$

These are plotted in Fig. 16. We see that the vector model and $(t\ \ d)_R$ models do not agree well with the data. It should also be kept in mind that we have assumed the mass formula, Eq. (3.33) relating m_W and m_Z. Changing m_Z by a factor 2 changes $|\epsilon_L|^2$ and $|\epsilon_R|^2$ by a factor 16. We see therefore that the standard model not only agrees remarkably well with the data, but that no other choice of m_Z than that given by the mass-formula would give agreement. This is also true of the vector-like, and to some extent, of the $(t\ \ d)_R$ models but <u>not</u> of the $(u\ \ b)_R$ model which could accommodate a change in m_Z, provided it were accompanied by a change in $\sin^2\theta_W$.

As an additional complication, it is also possible to introduce mixing of u_R and d_R with additional heavy quarks which have a different weak isospin. This will evidently allow predictions which interpolate between the cases we have discussed.

C. Elastic Neutrino-Proton Scattering. Two experiments[75, 76] have recently been performed at Brookhaven in which the elastic scattering processes $\nu_\mu p \rightarrow \nu_\mu p$ and $\bar{\nu}_\mu p \rightarrow \bar{\nu}_\mu p$ have been observed. These experiments are especially useful in determining the isospin-structure of the neutral-current couplings. The formalism for describing this process as well as a comparison with data has been discussed in several recent theoretical papers.[77, 78, 79] Usually this is done in the context of models. Here we shall first approach it from the general point of view of finding additional restrictions on the couplings $\epsilon_{L(R)}(u)$ and

$\epsilon_{L(R)}(d)$. We write the effective Lagrangian in terms of vector and axial coupling:

$$\mathscr{L}_{\text{eff}} = \frac{G}{\sqrt{2}} \begin{bmatrix} \left\{\epsilon_L(u) + \epsilon_R(u)\right\} \bar{u}\gamma_\lambda u + \left\{\epsilon_R(u) - \epsilon_L(u)\right\} \bar{u}\gamma_5\gamma_\lambda u \\ + \left\{\epsilon_L(d) + \epsilon_R(d)\right\} \bar{d}\gamma_\lambda d + \left\{\epsilon_R(d) - \epsilon_L(d)\right\} \bar{d}\gamma_5\gamma_\lambda d \end{bmatrix} \bar{\nu}_\mu \gamma^\lambda (1-\gamma_5)\, \nu_\mu \qquad (5.32)$$

We again ignore all contributions of the strange quark current, even for this elastic process. When matrix-elements of $\mathscr{L}_{\text{eff}}$ are taken between proton states, we shall have to deal with amplitudes $<p\,|\bar{u}\gamma^\lambda u\,|p>$ and $<p\,|\bar{d}\gamma^\lambda d\,|p>$, which in turn are related to the electromagnetic form factors of the proton and neutron. Specifically[80]

$$<p\,|\tfrac{2}{3}\bar{u}\gamma_\lambda u - \tfrac{1}{3}\bar{d}\gamma_\lambda d\,|p> = \bar{u}(p')\left[\gamma_\lambda F_{1p}(q^2) - \frac{\kappa_p}{4M}[\gamma_\lambda, \not{q}]F_{2p}(q^2)\right] u(p) \qquad (5.33)$$

and from charge symmetry

$$<p\,|\tfrac{2}{3}\bar{d}\gamma_\lambda d - \tfrac{1}{3}\bar{u}\gamma_\lambda u\,|p> = \bar{u}(p')\left[\gamma_\lambda F_{1n}(q^2) - \frac{\kappa_n}{4M}[\gamma_\lambda, \not{q}]F_{2n}(q^2)\right] u(p) \qquad (5.34)$$

with $p'=p+q$ and $\kappa_p=1.79$, $\kappa_n=1.91$ the anomalous magnetic moments. The Dirac form factors are normalized to unity (or zero) at $q^2=0$. For the axial form factors, we know the I=1 portion from β-decay and from analyses of the quasi-elastic processes $\nu_\mu + n \to \mu^- + p$:

$$<p\,|\bar{u}\gamma_5\gamma_\lambda u - \bar{d}\gamma_5\gamma_\lambda d\,|p> = 1.24\,\bar{u}(p')\gamma_5\gamma_\lambda u(p)\, F_A(q^2)^{I=1} \qquad (5.35)$$

Note there is only one form factor; there can be no induced pseudoscalar piece that contributes (because of the conserved ν-currents).

For the I=0 axial form factor, there is considerable uncertainty.[81] In this discussion we shall use what we consider a "best guess". We choose the ratio of isoscalar and isovector axial matrix elements to be the same as the ratio of the isoscalar and isovector magnetic contributions (proportional to the total moments of proton and neutron, i.e., to $G_M \sim F_1 + \kappa F_2$). The reasoning is that this choice is in accord with static SU(6), the naive quark model, and the improvements given by the Melosh transformation.[82] This is equivalent to the statement that the F/D ratio for axial form factors (as measured in hyperon β-decay processes) is the same as the F/D ratio for magnetic moments. The theoretical justification, such as it is, is that in the static limit, both the axial current and the magnetic moment operator reduces to $\vec{\sigma}\lambda_i$ (with λ_i the SU(3) matrices); hence all matrix elements are proportional.

There is some additional experimental justification for this choice. An old sum rule[83] relates the (intrinsic) asymmetry A in deep-inelastic scattering of longitudinally polarized electrons by longitudinally polarized protons to a matrix element of I=0 and I=1 axial-vector currents between protons at $q^2=0$:

$$\begin{aligned}\bar{u}\gamma_5\gamma_\lambda u \int_1^\infty \frac{d\omega}{\omega}\left[\nu W_2(\omega)\right]\cdot A &= <p\,|\tfrac{4}{9}\bar{u}\gamma_5\gamma_\lambda u + \tfrac{1}{9}\bar{d}\gamma_5\gamma_\lambda d\,|p> \\ &= \frac{1}{6}\,(1.24)\,\bar{u}\gamma_5\gamma_\lambda u + \frac{5}{18} <p\,|\bar{u}\gamma_5\gamma_\lambda u + \bar{d}\gamma_5\gamma_\lambda d\,|p>\end{aligned} \tag{5.36}$$

With the SU(6) prediction of +0.6 for the isoscalar-to-isovector mixture, it turns out that the isoscalar contribution should approximately equal the isovector (with the same sign). The weighted asymmetry should be approximately zero for the neutron[84] while for the proton

$$\int_0^1 \frac{d\omega}{\omega}\cdot\nu W_2\cdot A \cong \frac{1}{3}\left|\frac{g_A}{g_V}\right| \sim \frac{1}{3}\,(1.24) \tag{5.37}$$

Recent data[85] show a large asymmetry of the correct sign, and my own rough estimate of the sum gives a value between ~0.5 and ~1.0 of the right-hand side of Eq. (5.37). This would imply a value of the isoscalar form factor (at $q^2=0$) somewhere between zero and the SU(6) prediction which we use. In any case what we do use for the isoscalar axial form factor is explicitly

$$<p\,|\bar{u}\gamma_5\gamma_\lambda u + \bar{d}\gamma_5\gamma_\lambda d\,|p> = (1.24)\cdot\frac{3(1+\kappa_p+\kappa_n)}{(1+\kappa_p-\kappa_n)}\,\bar{u}\gamma_5\gamma_\lambda u\,F_A(q^2) \tag{5.38}$$

Here we blindly assume the same q^2 dependence for I=0 and I=1 axial form factors. The justification this time (a not very good one) is only that this works for the magnetic form factors reasonably well.

At this point, one has all the ingredients for estimating the cross sections. The matrix elements needed are given in Eqs. (5.33), (5.34), (5.35) and (5.38) and can be inserted into the matrix element of $\mathscr{L}_{\text{eff}}$ in Eq. (5.32), taken between proton states. The resultant amplitude is then squared, spin-sums taken, and phase-space calculated. As in elastic electron-proton scattering, there is great advantage in utilizing the Sachs form factors[80]

$$G_E = F_1 + \frac{\kappa Q^2}{4M^2} F_2 \qquad G_M = \frac{F_1 + \kappa F_2}{1+\kappa} \tag{5.39}$$

and, in accord with experiment,[86] approximating them with the dipole form

$$\begin{aligned} G_{Ep} = G_{Mp} = G_{Mn} \cong \left(1 + \frac{Q^2}{m_V^2}\right)^{-2} \\ m_V^2 = .71 \text{ GeV}^2 \end{aligned} \tag{5.40}$$

(Note the Drell definition $Q^2 = -q^2 > 0$ is used here.)

Likewise the axial form factor is conventionally written

$$G_A(q^2) = F_A(q^2) = \left(1 + \frac{Q^2}{m_A^2}\right)^{-2} \tag{5.41}$$

with typically $m_A^2 \sim 0.9 \text{ GeV}^2$ from experiment although values from 0.7 to 1.2 GeV^2 might be entertained.[87] When all this is put together one finds the differential cross section:

$$\begin{aligned}\frac{d\sigma_p^{\nu,\bar{\nu}}}{dQ^2} = \frac{G^2}{2\pi}\Bigg[&\left(\epsilon_V^1+3\epsilon_V^0\right)^2\left(1+\frac{Q^2}{m_V^2}\right)^{-4}\left[(1+\tau)^{-1}\left(1-\tau\frac{M}{E}\right)^2-\tau\frac{M^2}{E^2}\right]\\ &+\left(4.7\,\epsilon_V^1+2.6\epsilon_V^0\right)^2\left(1+\frac{Q^2}{m_V^2}\right)^{-4}\left[\frac{\tau}{1+\tau}\left(1-\tau\frac{M}{E}\right)^2+\frac{\tau^2M^2}{E^2}\right]\\ &+(1.24)^2\left(\epsilon_A^1+0.55\,\epsilon_A^0\right)^2\left(1+\frac{Q^2}{m_A^2}\right)^{-4}\left[\left(1-\tau\frac{M}{E}\right)^2+\tau(1+\tau)\frac{M^2}{E^2}\right]\\ &\pm(1.24)\left(4.7\epsilon_V^1+2.6\epsilon_V^0\right)\left(\epsilon_A^1+.55\,\epsilon_A^0\right)\left(1+\frac{Q^2}{m_V^2}\right)^{-2}\left(1+\frac{Q^2}{m_A^2}\right)^{-2}4\tau\frac{M}{E}\left(1-\tau\frac{M}{E}\right)\Bigg]\end{aligned} \tag{5.42}$$

with

$$\tau = Q^2/4M^2 \tag{5.43}$$

and, in a hopefully obvious notation,

$$\begin{aligned}\epsilon_V^1 &= \frac{1}{2}\left[\epsilon_L(u)+\epsilon_R(u)-\epsilon_L(d)-\epsilon_R(d)\right]\\ \epsilon_V^0 &= \frac{1}{2}\left[\epsilon_L(u)+\epsilon_R(u)+\epsilon_L(d)+\epsilon_R(d)\right]\\ \epsilon_A^1 &= \frac{1}{2}\left[\epsilon_L(u)-\epsilon_R(u)-\epsilon_L(d)+\epsilon_R(d)\right]\\ \epsilon_A^0 &= \frac{1}{2}\left[\epsilon_L(u)-\epsilon_R(u)+\epsilon_L(d)-\epsilon_R(d)\right]\end{aligned} \tag{5.44}$$

The various contributions should be fairly recognizable. The first line is the contribution proportional to the electric coupling G_E^2, the second line to the magnetic coupling G_M^2, and the third to G_A^2. The only V-A interference occurs between the (spin-dependent) magnetic and axial form factors. To go from νp to $\bar{\nu}$p scattering just involves changing the sign of the V-A interference term. To go from νp to νn elastic scattering, it is only necessary to change the sign of ϵ_V^1 and ϵ_A^1. If one wishes to test the dependence of this cross section on the magnitude of the isoscalar axial-vector form factor, one simply multiplies ϵ_A^0 by an appropriate scale factor η (with, probably, $0 \lesssim \eta \lesssim 1$ according to the rough indications from polarized electroproduction). In order to get an idea of the sensitivity of the BNL data to the four ϵ-parameters, we first average over the neutrino spectrum

$$\langle \frac{M}{E_\nu} \rangle \approx .5 \qquad \langle \frac{M^2}{E_\nu^2} \rangle \approx .4 \tag{5.45}$$

and then average the kinematical factors dependent on τ over the range $0.3\ \mathrm{GeV}^2 < Q^2 < 0.9\ \mathrm{GeV}^2$, using $m_V^2 = m_A^2 = 0.71\ \mathrm{GeV}^2$ for simplicity. This gives $<\tau> \cong 0.13$ and a cross section dependence

$$\sigma \sim 0.8\gamma_E^2 + 2\gamma_M^2 + 1.4\gamma_A^2 \pm 1.4\gamma_M\gamma_A$$

with

$$\begin{aligned} \gamma_E &= \epsilon_V^1 + 3\epsilon_V^0 \\ \gamma_M &= \epsilon_V^1 + .55\epsilon_V^0 \\ \gamma_A &= \epsilon_A^1 + .55\epsilon_A^0 \end{aligned} \tag{5.46}$$

The experimental piece of information we shall use is the small ratio of $\sigma_{\bar{\nu}p}$ to $\sigma_{\nu p}$ reported by the HPW experiment:

$$\frac{\sigma_{\bar{\nu}p}}{\sigma_{\nu p}} = 0.4 \pm 0.2 \tag{5.47}$$

This requires a large interference term; in particular

$$\frac{2.8\gamma_M\gamma_A}{0.8\gamma_E^2 + 2\gamma_M^2 + 1.4\gamma_A^2 + 1.4\gamma_M\gamma_A} \gtrsim 0.6 \pm 0.2 \tag{5.48}$$

The ratio above is maximized when $\gamma_M \approx \gamma_A$; $\gamma_E = 0$, giving the value ~ 0.58. Large excursions away from this value are not tolerable, and without belaboring the point here with numerical examples, the result is that

i) $|\gamma_M - \gamma_A|$ cannot be too large

ii) $\gamma_M+\gamma_A$ is large and is rather well determined from σ_{tot}

iii) γ_E cannot dominate the total cross section.

These results are not sensitive to the choice of m_A^2. Upon noticing that (from Eqs. (5.46) and (5.44))

$$\begin{aligned} \gamma_M+\gamma_A &= 1.55\ \epsilon_L(u) - 0.45\epsilon_L(d) = 1.55\left[\epsilon_L(u) - 0.3\epsilon_L(d)\right] \\ \gamma_M-\gamma_A &= 1.55\ \epsilon_R(u) - 0.45\epsilon_R(d) = 1.55\left[\epsilon_R(u) - 0.3\epsilon_R(d)\right] \end{aligned} \tag{5.49}$$

we see that it is simply $\epsilon_L(u) - 0.3\epsilon_L(d)$ which is required to be as large as possible in order to maximize the interference term.

We may visualize this better by looking at ϵ_L space and ϵ_R space as shown in Fig. 15. The regions allowed (at the one-standard deviation level) by the deep-inelastic data are shown shaded, along with predictions of various SU(2) ⊗ U(1) models. A crude estimate of the limits on $\epsilon_L(u) - 0.3\epsilon_L(d)$ from the line of arguments given above is shown as the shaded band on the right. However, it is clearly better to simply search all values of ϵ's in the allowed regions and ask how acceptable the fit to the elastic scattering data is. A somewhat cursory search reveals that the only region in ϵ_L space in which there is a large ratio of νp to $\bar{\nu}$p scattering is the lower right quadrant $\epsilon_L(u) > 0$, $\epsilon_L(d) < 0$. There is a little restriction on $\epsilon_R(u)$ and $\epsilon_R(d)$, in general, although there is considerable correlation between allowed values of $\vec{\epsilon}_R$ with the precise value of $\vec{\epsilon}_L$. However these lectures are an inappropriate place to perform a serious analysis of the allowed values of the couplings. Suffice it to say that the values of $\epsilon_L(u)$ and $\epsilon_L(d)$ are constrained to lie fairly near that predicted by SU(2) ⊗ U(1) models, and that more accurate data is needed to meaningfully restrict further the right-handed couplings.

Several serious analyses have been performed to see how the data compares with popular SU(2) ⊗ U(1) models. We exhibit selections from the work of Albright, Quigg, Schrock, and Smith,[78] which compares the observed differential

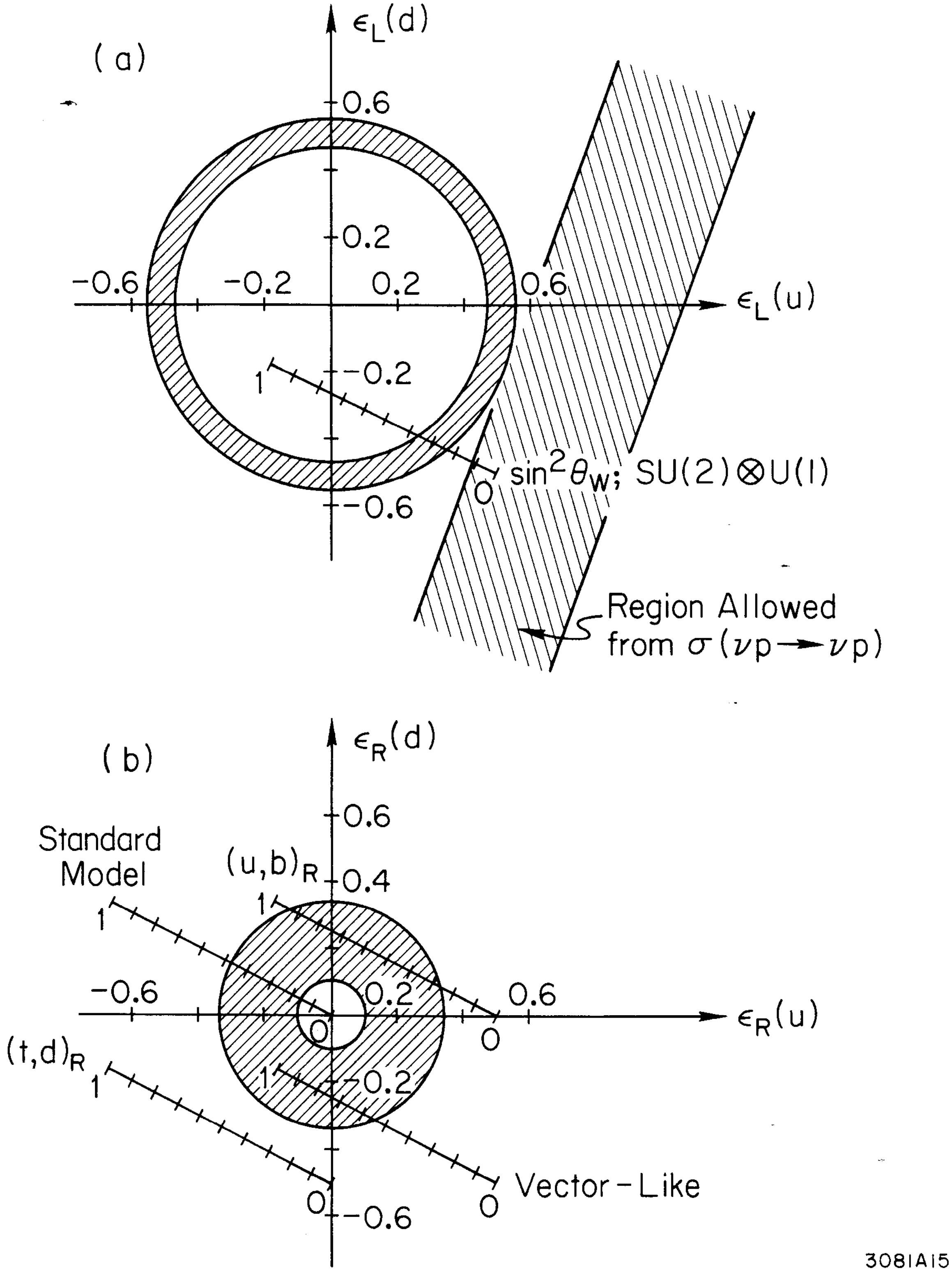

Fig. 15. Allowed regions (a) in $\epsilon_L(u)$-$\epsilon_L(d)$ space, and (b) in $\epsilon_R(u)$-$\epsilon_R(d)$ space from the deep-inelastic neutral-current data. The shaded panel on the right is the allowed region (at ~1 standard deviation) coming from the νp and $\bar{\nu} p$ elastic scattering measurements. Predictions of various $SU(2) \otimes U(1)$ models are given as a function of $\sin^2 \theta_W$.

cross sections of HPW[76] with their calculations. These are shown in Figs. 16-18. Evidently vector-like theories (for which there is no interference term and therefore $d\sigma_{\bar{\nu}p}/dQ^2 = d\sigma_{\nu p}/dQ^2$) have difficulty with experiment. Even the dependence on Q^2 is poor. The standard $SU(2) \otimes U(1)$ model, as well as the $(u, b)_R$ variant, are acceptable fits to the data, given the uncertainties in choice of $G_A(Q^2)$ and the limited statistics of the $\bar{\nu}p$ measurement as well as the difficulty inherent in measuring such a process.

The cross section for νp elastic scattering has been determined by both the HPW and CIR experiments. The number quoted is the ratio of the neutral current elastic to the quasi-elastic process $\nu_\mu n \to \mu^- p$. They quote (see also the reports of H. Williams and of W. Y. Lee in these Proceedings)

$$\frac{\sigma_{\nu p \to \nu p}}{\sigma_{\nu n \to \mu^- p}} = \begin{cases} .23 \pm .09 & \text{CIR} \\ .17 \pm .05 & \text{HPW} \end{cases} \tag{5.50}$$

(These are cut over ranges of Q^2 and/or θ_p; see the reports of H. Williams and W. Lee for the details.)

The theoretical expectation is somewhat lower, but not seriously so, as shown in Fig. 19. The model-independent analysis we gave suggests it may not be completely simple to find (within the assumption of a local V-A effective Lagrangian), any choice compatible with deep-inelastic data, a small value of $\sigma^{el}_{\bar{\nu}p}/\sigma^{el}_{\nu p}$, and a value as large as above. This underlines my belief that as the elastic scattering data becomes more accurate, it will become a powerful tool is disentangling the possible neutral current couplings in a largely model-independent way.

D. Resonance Production. Careful study of the production of single-pion and double pion production by the neutral current should reveal additional information on its isospin structure. Regrettably there was not the time (nor did I have the energy) to review the subject in these lectures in detail. In addition to

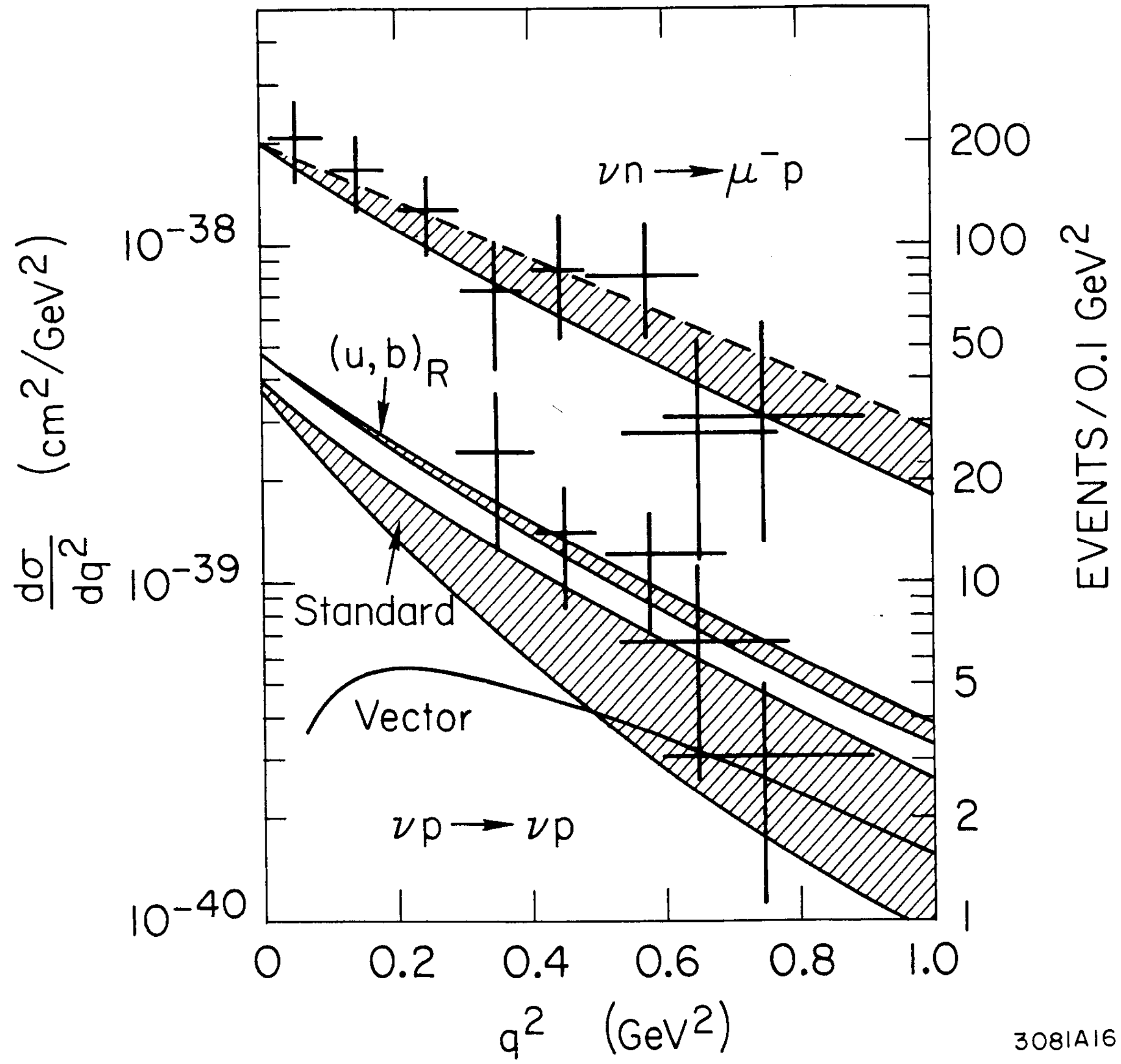

Fig. 16. Data of HPW for νp elastic scattering, along with the calculations of Albright et al. for various models and choices of m_A.

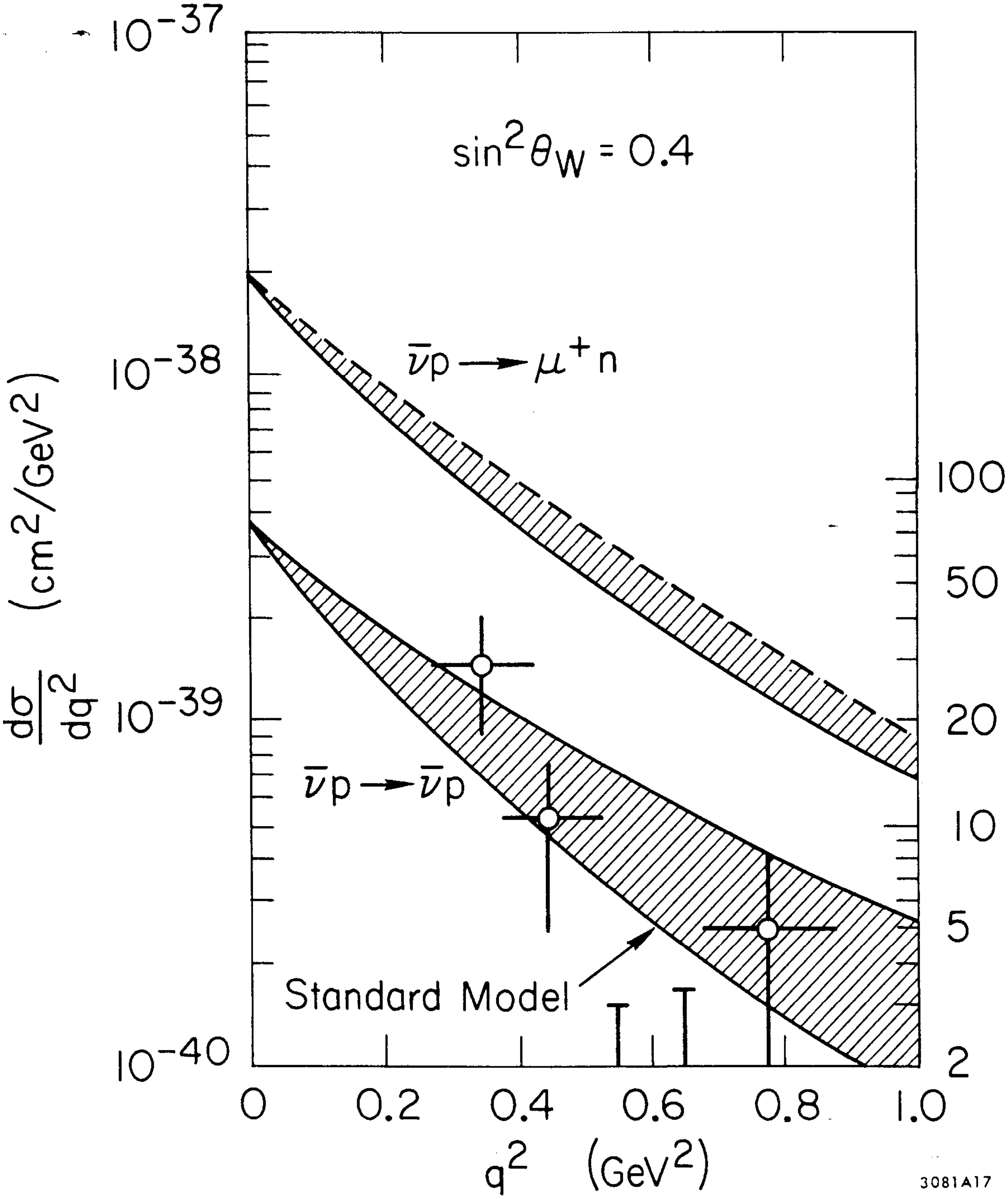

Fig. 17. Data of HPW for $\bar{\nu}$p elastic scattering, along with the calculations of Albright et al. for the standard model and various choices of m_A.

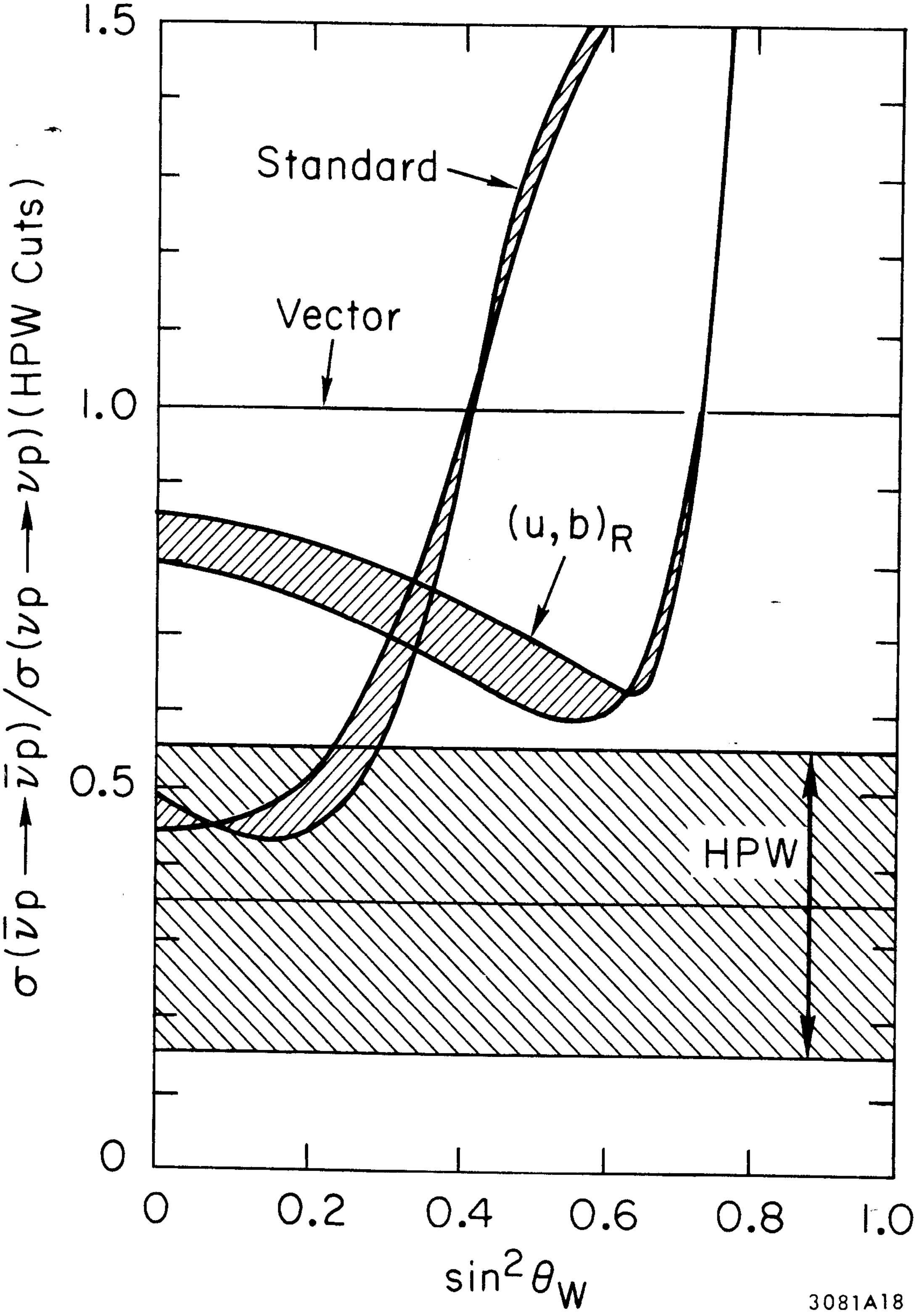

Fig. 18. Ratio of $\sigma_{\bar{\nu}p}$ to $\sigma_{\nu p}$ as calculated by Albright et al. compared with the HPW measurement.

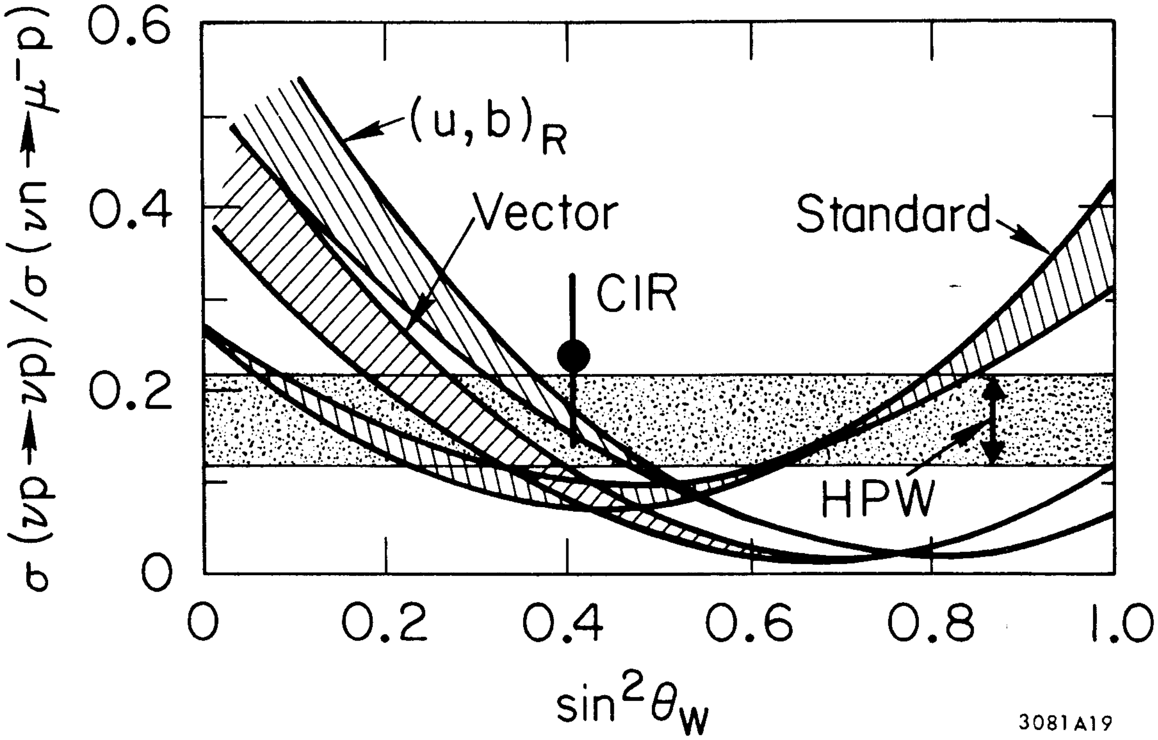

Fig. 19. Expected ratio of $\sigma(\nu p \rightarrow \nu p)/\sigma(\nu n \rightarrow \mu^- p)$ as calculated by Albright et al. compared with the HPW and CIR measurements.

a considerable body of data, a large amount of very careful and thorough work has been done by Stephen Adler and his collaborators on the theory.[88] We reiterate that Adler's lectures at the Hawaii Summer School last year serve as an excellent introduction to the subject. We note here only a few salient features from the experiments:

1. Δ production: The CIR counter experiment at BNL, while observing a clear Δ peak in charged-current processes, do not have any clear evidence for a Δ peak in the neutral-current process[89] $\nu p \rightarrow \nu p \pi^0$. Absence of this peak would be quite disquieting, given the indication of a large isovector left-handed neutral current from the elastic neutrino scattering. It is not easy to see how this would lead to a small Δ-production cross section by the neutral current.

2. Evidence for isovector neutral current: The Gargamelle experiment[90] has measured the π^0/π^- ratio for single pion production by the neutral current (in heavy liquid). Were the neutral current pure isoscalar this ratio should be unity, whereas it is found to be 1.8 ± 0.4 for ν incident and 2.5 ± 0.6 for $\bar{\nu}$ incident (after background corrections). (For pure isovector they expect a value of 2.6.)

3. Evidence for isoscalar-vector interference: In an exposure of the BNL seven-foot bubble chamber filled with D_2, double-pion production by incident neutrinos has been observed.[91] The measurement of the n/p ratio is

$$\frac{\sigma(\nu n \rightarrow \nu n \pi^+ \pi^-)}{\sigma(\nu p \rightarrow \nu p \pi^+ \pi^-)} = .49 \pm .19 \tag{5.51}$$

It should be unity if the neutral current is either pure isovector or isoscalar. This follows from the fact that the two amplitudes are equal by charge symmetry (up to an overall sign) if the neutral current has definite properties under isospin rotations (either I=1 or I=0 but not both). In addition, at Argonne National Laboratory a deuterium exposure of the 12-foot chamber to neutrinos[92] yielded events of the single-pion production processes

$$\begin{aligned} \nu n &\rightarrow \nu p \pi^- \\ \nu p &\rightarrow \nu n \pi^+ \end{aligned} \tag{5.52}$$

The rates for these processes are in reasonable agreement with the calculations of Adler et al. using the SU(2) ⊗ U(1) model, but the π^- and π^+ momentum spectra, which should be similar if isovector amplitudes dominate (as expected in this region) look rather different, again indicating some isoscalar interference.

E. Parity Violation in Atomic Physics. The presence of a parity-violating neutral current coupling the electron to hadrons, as exists in the standard SU(2) ⊗ U(1) model, can give rise to observable effects in atomic radiative transitions. Three experiments which are in progress were reported[93] recently at the International Conference on Atomic Physics held at the University of California at Berkeley. Two experiments, both involving the same radiative transition in Bi, have reached a sensitivity which has impact on the neutral current models which we have been discussing.

The relevant effective Lagrangian is constructed from Eq. (1.4). Before writing it all out, we note that since the experiments attempt to detect a parity-violating effect, only the axial electron current × vector hadron current (or vice versa) need to be considered. Furthermore, only the time component of the vector hadron current gives a contribution which is coherent over the A nucleons in the bismuth (Z=83, N=126). (This is one reason a heavy nucleus is used; another is the large value of $|\psi(0)|^2$.) Thus we need only keep the time-component of the electron-axial current multiplied with the time component of the hadron vector current. Noticing that

$$\bar{e}(x)\gamma_0\gamma_5\, e(x) \approx -i\, e^+(x) \left[\frac{\sigma\cdot\vec{\nabla}}{2m_e} - \frac{\sigma\cdot\overleftarrow{\nabla}}{2m_e}\right] e(x) \tag{5.53}$$

we obtain (up to a difficult overall minus-sign, which we do not here attempt to trace) the effective Hamiltonian density

$$H'_{PV} = (-)^{?} \frac{iG}{2\sqrt{2}\,m_e} e^{+}(x) \left[\sigma\cdot\overrightarrow{\nabla} - \sigma\cdot\overleftarrow{\nabla}\right] e(x) \begin{bmatrix} (2Z+N)\,\epsilon(e,u)_{A,V} \\ +\,(Z+2N)\,\epsilon(e,d)_{A,V} \end{bmatrix} \tag{5.54}$$

with

$$\begin{aligned} \epsilon_{A,V}(e,u) &= \epsilon_{RL}(e,u) + \epsilon_{RR}(e,u) - \epsilon_{LL}(e,u) - \epsilon_{LR}(e,u) \\ \epsilon_{A,V}(e,d) &= \epsilon_{RL}(e,d) + \epsilon_{RR}(e,d) - \epsilon_{LL}(e,d) - \epsilon_{LR}(e,d) \end{aligned} \tag{5.55}$$

$$? = 0 \text{ or } 1$$

There is little reason at this stage to try to hold to the general case; there isn't enough information to warrant such generality. Within an SU(2) ⊗ U(1) picture we may reduce the couplings further. For example, using the $\left(T_3 - Q\sin^2\theta_W\right)$ rule of Eq. (3.37)

$$\begin{aligned} \epsilon_{RL}(e,u) &= 2\left(T_3 + \sin^2\theta_W\right)_e \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) \\ \epsilon_{LL}(e,u) &= 2\left(-\frac{1}{2} + \sin^2\theta_W\right)_e \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) \end{aligned} \tag{5.56}$$

Only the difference of terms as above occur in Eq. (5.55) for H'_{PV}, because only the axial electron current contributes. Thus if, as in the standard model, e^-_R is a weak isosinglet we get

$$\begin{aligned} \epsilon_{RL}(e,u) - \epsilon_{LL}(e,u) = \epsilon_L(u) &= \left(T_3 - Q\sin^2\theta_W\right)_u \\ &= \frac{1}{2} - \frac{2}{3}\sin^2\theta_W \end{aligned} \tag{5.57}$$

On the other hand, if e^-_R is a member of a weak isodoublet $(E^0, e^-)_R$, the electron current is vector-like, the terms in Eq. (5.56) for H' cancel pairwise, and H'_{PV} vanishes. A sufficient condition for the absence of a parity-violating effect in this atomic transition is a vector-like electron theory. If this is the case, there is of course little more to say theoretically. We hereafter restrict our attention to the standard SU(2) ⊗ U(1) model for leptons, which gives, according to Eqs. (5.55) and (5.57)

$$\begin{aligned} \epsilon(e,u)_{A,V} &= \epsilon_L(u) + \epsilon_R(u) \\ \epsilon(e,d)_{A,V} &= \epsilon_L(d) + \epsilon_R(d) \end{aligned} \tag{5.58}$$

with $\epsilon_L(u)$ and $\epsilon_L(d)$ defined in the previous discussion of neutrino processes. This gives,[94] finally

$$H'_{PV} = -\frac{iG}{4\sqrt{2}\,m_e}\, e^{+}(x)(\vec{\sigma}\cdot\overrightarrow{\nabla} - \vec{\sigma}\cdot\overleftarrow{\nabla})\; e(x)\Big|_{x=0}\, Q_W(Z,N) \tag{5.59}$$

with the coupling-strength Q defined by

$$Q_W(Z,N) = 2(2Z+N)\left[\epsilon_L(u)+\epsilon_R(u)\right] + 2(Z+2N)\left[\epsilon_L(d)+\epsilon_R(d)\right] \tag{5.60}$$

For the four SU(2) ⊗ U(1) variants we discussed before, we may evaluate the coupling strength

$$\begin{array}{ll} \text{Standard model} & Q_W = Z\left(1-4\sin^2\theta_W\right) - N \\ (u\ \ b)_R & Q_W = Z\left(3-4\sin^2\theta_W\right) \\ (t\ \ d)_R & Q_W = -3N - 4Z\sin^2\theta_W \\ \text{Vector-like} & Q_W = 2Z\left(1-2\sin^2\theta_W\right) - 2N \end{array} \tag{5.61}$$

These are plotted in Fig. 20. In general an effect at least as large as the standard model is anticipated.[95]

The relevant transition is an M1 transition between $^2D_{3/2}$ and $^4S_{3/2}$ levels in the $(6p)^3$ shell. One looks for an E1 admixture caused by mixing of the atomic $(6p)^3$ wave functions with $(6p)^2$ ns states. The atomic physics calculations[96] are considered reliable at least to a factor two, although it is also said that not nearly enough theoretical work has yet been done. Any E1 mixing with M1 leads to a preference of one circular polarization over the other in the radiative decay.

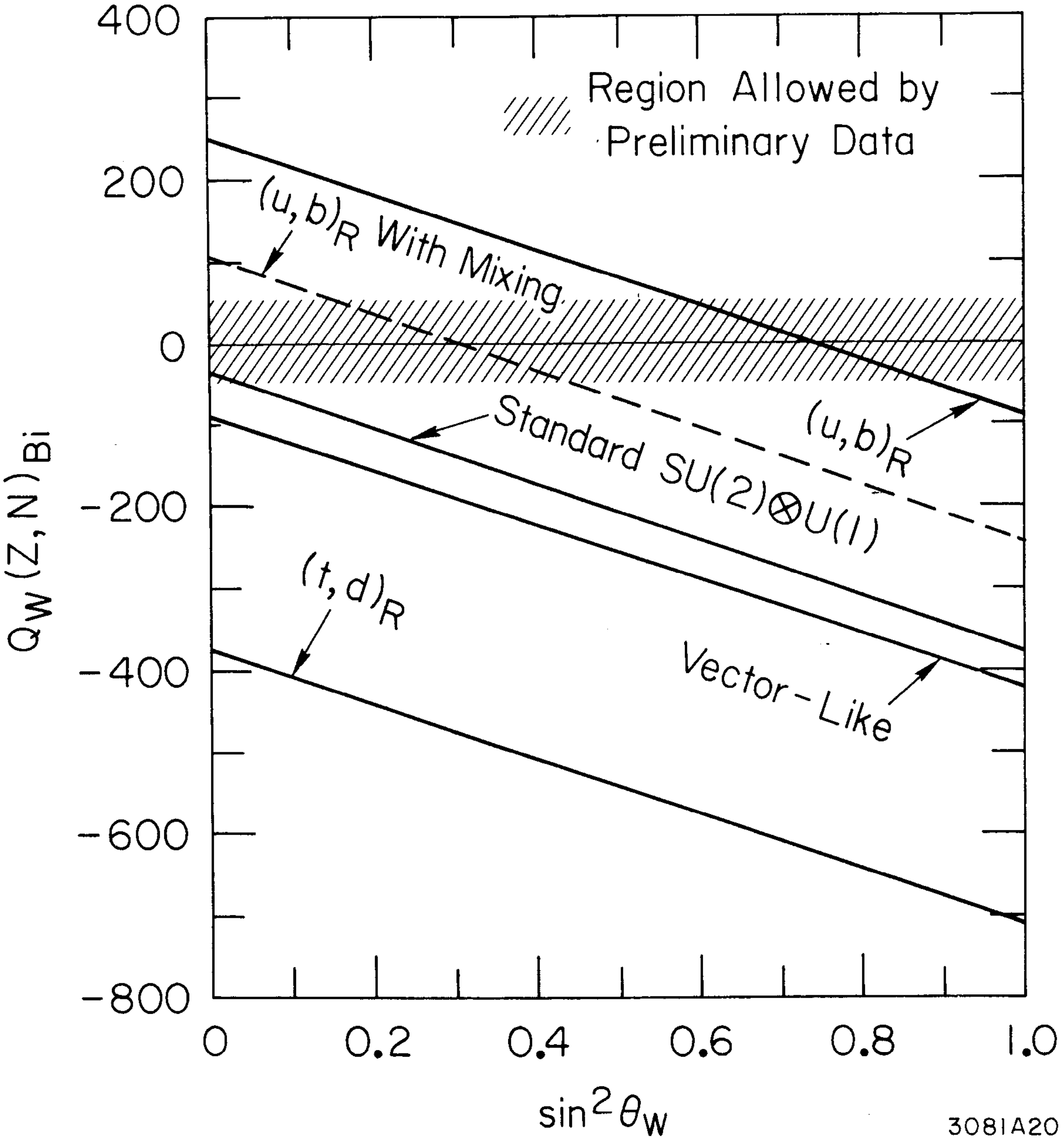

Fig. 20. Values of Q(Z,N), the effective nuclear charge, in the hamiltonian for parity-violating electron-nucleus couplings for various models. The shaded area is the region allowed by experiment. The dashed line is an SU(2) ⊗ U(1) model with mixing between c_R and u_R. See Eq. (5.65) and the adjacent words.

This in turn means that the resonant absorption of left and right circularly polarized light will differ slightly. Rather than measure the absorption, the experimentalists[97] prefer to observe the difference in the anomalous dispersion by utilizing the Faraday effect. Linearly polarized laser light (modulated by a standard Faraday cell) is passed through bismuth vapor, thereafter through a crossed polariod, and then detected. One then searches for a rotation of the polarization vector by the Bi vapor in addition to that provided by the Faraday-cell modulator. Using the standard SU(2) $\otimes$ U(1) model as a prototype, the calculations predict a rotation of the polarization vector of about 3.5×10^{-7} radians/absorption length. Both experimental groups (University of Washington, Seattle and Oxford University) set an upper limit of 1×10^{-7} for any parity violating effect,[93] with residual effects observed at a level 2 or 3-times lower. However these residual effects may be attributable to systematic errors which are not yet understood. At present these experiments pose a serious challenge to the correctness of the SU(2) $\otimes$ U(1) scheme.

It is probably a little too soon to draw far-reaching conclusions from these experiments, which are still not complete. Nevertheless, if we accept uncritically the result, what does it mean? As we saw in Fig. 20, none of the four basic SU(2) $\otimes$ U(1) options survive. However, it is possible in the $(u\ b)_R$ option to, for example, mix the u quark with some other weak isosinglet with charge 2/3, for example, the charmed c-quark. If we do this, the only change in the neutral current coupling which involves nonstrange quarks is as follows:

$$\mathscr{L}_{\text{eff}} \rightarrow \frac{G}{\sqrt{2}}\, \bar{\nu}_\mu \gamma_\lambda (1-\gamma_5)\, \nu_\mu \begin{cases} \bar{u}' \gamma^\lambda (1+\gamma_5)\, u' \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W\right) \\ + \bar{c}' \gamma^\lambda (1+\gamma_5)\, c' \left(-\frac{2}{3} \sin^2 \theta_W\right) \end{cases} \tag{5.62}$$

with

$$\begin{aligned} u' &= u \cos \alpha + c \sin \alpha \\ c' &= -u \sin \alpha + c \cos \alpha \end{aligned} \tag{5.63}$$

This implies

$$\epsilon_R(u) \rightarrow \frac{1}{2}\cos^2\alpha - \frac{2}{3}\sin^2\theta_W \qquad (5.64)$$

and consequently

$$Q_W(Z,N) = Z\left(1+2\cos^2\alpha - 4\sin^2\theta_W\right) + N(\cos^2\alpha - 1) \qquad (5.65)$$

This clearly is an interpolation (cf., Eq. (5.61)) between the standard model ($\cos\alpha = 0$) and the $(u\ b)_R$ option ($\cos\alpha = 1$), each of which do reasonably well in accounting for other neutral-current and charged-current phenomena. Inasmuch as these models predict atomic parity-violating effects of opposite sign, it is possible to choose the mixing angle such that $Q_W(Z,N) = 0$. A choice of $\alpha \sim 45^\circ$ does quite well; this is plotted in Fig. 20 as the dashed line.

Such a mixed neutral-current coupling has other implications, the most direct being the existence of charm-changing neutral current processes, e.g., $D^+ \rightarrow \pi^+ e^+ e^-$; $D^0 \rightarrow e^+ e^-$, etc. The nonleptonic effective Lagrangian could also contain terms with $\Delta C=1$, $\Delta S=0$ leading to hadronic final states not containing strange particles as well as $\Delta C=2$, $\Delta S=0$ which induces D-$\bar{D}$ mixing.

We have already discussed another alternative compatible with the atomic physics results which is to assign e_R^- to a weak isodoublet $(E^0 e^-)_R$. In this case the parity-violating atomic effect vanishes. This choice changes the predicted cross section for $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$ scattering, but the revised value is compatible with the data, as we have already discussed. The predicted cross section for $\bar{\nu}_\mu e^-$ scattering is lowered, but the data are sufficiently sparse that this does not yet pose an insurmountable problem (unless the Aachen-Padua numbers are accepted).

A third alternative is to abandon $SU(2) \times U(1)$ and posit a pure isovector neutral-current coupling.[99] If one does this, assumes single-Z^0 exchange, and assumes the axial coupling of Z^0 to the electron has the same strength as the axial coupling of Z^0 to ν_μ, one finds the result (for bismuth)

$$Q_W(Z,N) = (Z-N)\left[\epsilon_L(u)+\epsilon_R(u)\right] = -43\left[\epsilon_L(u)+\epsilon_R(u)\right] \tag{5.66}$$

compared with the experimental limit of $|Q_W| \lesssim 50$. Given the limits on $\epsilon_L(u)$ and $\epsilon_R(u)$ from the deep-inelastic data (Eq. (5.29)), there is evidently no problem with experiment.

Finally, one might ask what might be said in a model-independent way. It is not too much. Assuming that the electron neutral current coupling is related to the ν_μ neutral current coupling by the previous assumptions of single-Z^0 and equal axial couplings of Z^0 to e and ν_μ, it is easy to see that given any choice of $\epsilon_R(u)$ and $\epsilon_R(d)$ compatible with deep inelastic data, it is possible to find a very small range of values of $\epsilon_L(u)$ and $\epsilon_L(d)$ in the vicinity of the pure isovector point $\epsilon_L(u) = -\epsilon_L(d) \sim 0.35$ such that $|Q_W| < 50$. Varying over all possibilities for $\vec{\epsilon}_R$, the estimated bound on a left-handed isoscalar coupling is not very strong.

$$|\epsilon_L(u) + \epsilon_L(d)| \lesssim 0.4 \tag{5.67}$$

This is to be compared with

$$|\epsilon_L(u) - \epsilon_L(d)| \sim 0.7 \tag{5.68}$$

VI. CONCLUSIONS

Given the rapid experimental progress taking place in this field at the present time, it is probably the wrong time to try to draw any firm conclusions. But certainly the $SU(2) \otimes U(1)$ model has survived the first round of reasonably quantitative neutral-current experiments remarkably well. At present, only the atomic-physics experiment provides a serious challenge to its validity.

If the $SU(2) \otimes U(1)$ model can be trusted for a basis of weak interaction phenomenology (and that is a big if), then we can use the neutral current information to help limit the nature of new flavor degrees of freedom, i.e., hadronic constituents beyond the charmed quark. Here this field appears to be as yet tied to the phenomenology: new quarks are added as required by the data, without any underlying Grand Scheme. Even the question of the necessity of more than four quarks remains open.

This is not to say that there are not candidates for Grand Schemes—only that as yet the evidence does not clearly point to any particular one. The Grand Principle[100] of the vector-like models, based on the idea that all constituents be weak isodoublets, appears to be in trouble with experiment. Another Grand Scheme[72] is that of Gursey and collaborators. It is a superunified theory based on the exceptional group E7. It does not have trouble with experiment as yet (it essentially belongs to the $(u\ b)_R$ option), and has nice esthetic features: a natural place for the color degrees of freedom, in particular. But the E7 representations are big; predicted are two new charged heavy leptons, five (4-component) neutral leptons, 35 weak-electromagnetic gauge bosons, and 133 leptoquarks, presumably of very high mass ($\gtrsim 10^{15}$ GeV). Leptoquark masses are necessarily large in order to protect the stability of the proton. Even if such a scheme is true, we have a long way to go to find out.

Among these conclusions, there is one that is unassailable: since the emergence of the existence of neutral currents from the Gargamelle data a few years ago, the field has made truly remarkable progress. The impact of neutral currents on physics has been similar to the impact of the θ-τ parity violation puzzle on β-decay: revolutionary progress has been made in many fields on many different energy scales. In the case of the neutral currents, we have seen that important information is coming from experiments using photon beams of energy 10^{-10} GeV, reactor neutrino beams of energy $\sim 10^{-3}$ GeV, BNL and ANL neutrino beams of energy ~ 1 GeV, as well as the higher energy CERN and FNAL beams of energy 5-100 GeV. It is a good reminder of the unity of physics: progress comes from experimentation across a broad front, not only from the cutting edge of the very highest energies.

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