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NON-CHARM MECHANISM FOR NARROW STATES*

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ABSTRACT

A new three-body mechanism is proposed, which leads naturally to narrow states at energies easily calculable via a simple analytic formula. This (zero-parameter) formula predicts over a score of narrow resonances in remarkable coincidence with experiment.

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I. INTRODUCTION

Before recent events intensified theoretical interest in the quark model, a number of attempts were made to directly link two- and three-body resonances.[1, 2] In particular, several authors have performed relativistic three-body calculation based, in part, on the idea of "isobar dominance".[3-5] Despite some notable successes (e.g., a plausible $\Delta(1670)$ state in $N\pi\pi$ driven by the $\Delta(1236)$,[3] and a rather broad $\omega(783)$ driven by the $\rho(770)$ in the 3π system[4]), it would appear that this approach can "explain" only a limited fraction of the hadron spectroscopy, and is intrinsically incapable of generating narrow states (say, $\Gamma < 30$ MeV). Given the profusion of newly discovered, often very narrow, states, it seems evident that one must abandon such models and turn to an alternative description (e.g., quarks). Unfortunately, given our present ignorance of quark dynamics, this means that one must also abandon any hope of quantitatively understanding the mass spectrum, or of predicting new resonances.

However, it is not clear that all avenues of the "old-fashioned" approach have been fully explored. Thus, in this article I describe a new type of three-body mechanism which leads naturally to narrow states, at energies trivially calculable in terms of a simple analytic formula. Remarkably, this (zero-parameter) formula predicts over a score of resonances in surprising coincidence with the hadron spectroscopy. Moreover, it appears likely that some unusual properties associated with this mechanism could resolve some long-standing difficulties in this field.

II. THREE-BODY SUBENERGY SINGULARITY

The proposed mechanism arises from a singularity in the diagram shown in Fig. 1. This singularity corresponds to the process being realized as an on-shell sequential rescattering; i.e., to the case where the intermediate configuration characterized by k_1', p_2, k_3 is a physical state of invariant (three-body) energy $\sqrt{s}$. As a special case, the subenergies s_{12}, s_{23}' may coincide with bound state or resonant energies of the respective two-particle subsystems; this gives rise to the so-called Peierls singularity,[1] which received considerable attention some years ago. In fact, Peierls and I demonstrated that, taken completely on-shell, the diagram has two singularities $s_\pm$ on the physical sheet, corresponding to the extreme cases $\cos \theta_{k_3 k_1'} = \pm 1$.[2] However, if either the initial or final state is off-shell, only the s_- singularity is near the physical region. As we showed, the latter singularity (although only logarithmic in a given partial-wave) may generate a nearby pole, and hence a true three-body resonance. Unfortunately, if the input two-body resonances have typical widths on the order of 100 MeV, s_- turns out to be rather far from the real axis. For example, if one considers three identical particles in the nonrelativistic limit, the kinetic energies satisfy the relationship $W_3^- = 4W_2$ $(W_2 = W_r - i\Gamma/2)$, and hence the three-particle state would have a width comparable to 400 MeV, in accord with the general trend towards broad resonances noted above.

The situation of interest here is distinct in that, while we assume particles 1 and 2 to resonate at subenergy s_{12}^o, we take s_{23}' at the subenergy threshold $(\sqrt{s_{23}'} \simeq m_2' + m_3')$. This leads to a different formula for s_-, with quite different properties. Although we could proceed along the lines of Ref. 2 in deriving our result, it is simpler to argue as follows. Thus, we consider the diagram as a function of s_{23}', for fixed s and $\cos \theta_{k_3 k_1'} = -1$. It is apparent that the corresponding amplitude (T_s) develops a pole at that value of s_{23}' which satisfies the on-shell conditions

$$(k_1' + p_2)^2 = s_{12}^o ,$$

$$(p_2+k_3)^2 = s'_{23} \quad ,$$

$$(k'_1+p_2+k_3)^2 = s \quad , \tag{1}$$

for a mass-shell intermediate state ($k_1'^2 = m_1'^2$, $p_2^2 = m_2^2$, $k_3^2 = m_3^2$); i.e.,

$$T_s \simeq t_{12}(s_{12})\,(P^2-s)^{-1}\,t_{23}(s'_{23}) \quad , \tag{2}$$

where $P \equiv k'_1+p_2+k_3$ is the total 4-momentum. The singular value of s'_{23} (s^o_{23}) will be near threshold providing that s is near the critical value

$$s_c = (m_2+m_3)^2 + m_1'^2 + (m_2+m_3)\left(s^o_{12}-m_1'^2-m_2^2\right)/m_2 \quad , \tag{3}$$

which we obtain by solving Eq. (1) for s with particles 2 and 3 at rest.

In practice, s^o_{12} is complex (our input resonance is taken to have its physical width) and $s=s_c$ will not be achieved for physical values of s. However, in certain cases (depending on s^o_{12} and the mass ratios) the singularity in s'_{23} passes much closer to the subenergy threshold than one might suspect. Consider, for example, the $NN\pi$ system, in which particle 2 is the pion, and $\sqrt{s_{12}}$ is the complex mass ($\Gamma/2 = 50$ MeV) of the $\Delta(1236)$. This situation is illustrated in Fig. 2, where the dashed line shows the location of s^o_{23} in terms of the kinetic energy $T = \sqrt{s'_{23}} - (m'_2+m'_3)$. We note that its position varies rapidly with s, and is only 3.6 MeV below threshold for $\sqrt{s} = \sqrt{s_c} = 2640$ MeV. To illustrate the special nature of this result, we observe that for the alternative diagram in which particle 1 is the pion, the singularity lies 38 MeV below threshold.

Below we discuss the connection between this subenergy singularity and the generation of narrow resonances. However, before concluding this section, it is worth noting that our argument in no way depends on the on-shell character of the (initial) state described by k_1, k_2, k_3. Thus, we may have $(k_1+k_2+k_3)^2 \neq s$ and/or $(k_1+k_2)^2 \neq s^o_{12}$. In particular, the process depicted in Fig. 1 may be preceded by an arbitrary diagram, or series of diagrams, involving not only the three-body system, but coupled inelastic channels as well. Therefore, in the general case, one must integrate over k_1, k_2, k_3 in addition to p_2 in order to evaluate T_s. In this connection it is helpful to recall the following points: (1) the on-shell limit of Fig. 1 is correctly described by a Blankenbecler-Sugar type of propagator with mass-shell particles;[6] this means that 3-momentum is conserved at the right-hand vertex and we are left with an integration $\int dk_1 dk_2 d\vec{k}_3/\epsilon_3$, (2) from relativistic Faddeev theory we know that the blob at the left-hand vertex represents the off-shell amplitude $t_{12}(k_1k_2|k_1'p_2;s_{12})$, where $s_{12} = (k_1'+k_2'+k_3'-k_3)^2$, and

$$t_{12}(k_1k_2|k_1'p_2;s_{12}) \rightarrow g(k_1k_2)\, g(k_1'p_2)/(s_{12}-s_{12}^o) \tag{4}$$

in the vicinity of the resonance.[7] We then observe that two poles, arising from $(s_{12}-s_{12}^o)^{-1}$ and $(P^2-s)^{-1}$, respectively, pinch the $|\vec{k}_3|$ contour as $s_{23}' \rightarrow s_{23}^o$. In this way one verifies that our singularity is indeed on the correct sheet, and appears in every diagram which terminates in Fig. 1. The latter point is crucial if an associated resonance is to be generated.[2]

III. MECHANISM FOR NARROW RESONANCES

In order to see how this subenergy effect leads to a singularity in s, we consider the consequences of unitarity. For a three-body system it is advantageous to expand the full amplitude T_3 in the form $T_3 = \sum_{\alpha,\alpha'} \tau_{\alpha\alpha'}$, where α (α') labels the pair of particles interacting first (last). The unitarity relation for the $\tau_{\alpha\alpha'}$ (channel amplitudes) takes the form $\Delta\tau_{\alpha\alpha'} = -\sum_{\beta} \tau_{\alpha\beta}\, \Delta G_0(s)\, \tau^*_{\beta\alpha}$, where $G_0(s)$ is a suitable (mass-shell) propagator; i.e., $\Delta G_0(s) \propto \delta(\epsilon_1+\epsilon_2+\epsilon_3-\sqrt{s})$ in the c.m. In a partial-wave decomposition $\tau_{\alpha\alpha'}$ depends only on s and the subenergies $s_{\beta\gamma}$, $s_{\beta'\gamma'}$ ($\alpha \neq \beta \neq \gamma$, $\alpha' \neq \beta' \neq \gamma'$). Thus, for a subenergy pole in channel $\beta=1$, the corresponding contribution to $\Delta\tau_{\alpha\alpha'}$ takes the form

$$\Delta^{(1)}\tau_{\alpha\alpha'}(s_{\beta\gamma}, s_{\beta'\gamma'}) \simeq -2\pi i\, c_1^2\, \tilde{\tau}_{\alpha 1}(s_{\beta\gamma})\, \tilde{\tau}^*_{1\alpha'}(s_{\beta'\gamma'}) I_1 \;,$$

$$I_1 = \int_0^{\kappa_1^m} d\kappa_1 \kappa_1^2 / \left(\kappa_1^2 - \kappa_0^2\right)\left(\kappa_1^2 - \kappa_0^{*2}\right) \;. \tag{5}$$

Here we have taken the (pair) c.m. momentum (κ_1) as our variable and incorporated slowly varying kinematic factors into a constant, c_1^2; $\tilde{\tau}_{\alpha 1}(s_{\beta\gamma})$ is the residue of $\tau_{\alpha 1}(s_{\beta\gamma}, s'_{23})$ at the pole ($s'_{23}=s^o_{23}$). Ignoring the upper limit ($\kappa_1^m >> \kappa_0$), we have $I_1 = (\pi/4)\, |\mathrm{Im}\, \kappa_0|^{-1}$, depending solely on s.

We thus have shown that Im $\tau_{\alpha\alpha'}$ contains a term proportional to $|\mathrm{Im}\, \kappa_0|^{-1}$. However, for the typical situation illustrated in Fig. 2, Im κ_0 actually varies rather slowly with s. The circumstance which promotes our singularity into a significant effect is that we need <u>not</u> have $m'_2=m_2$, $m'_3=m_3$. For example, if the blob at the 23 vertex represents $n\pi^+ \to p\pi^o$ the threshold in T is shifted by 5.9 MeV to the left in Fig. 2; for $p\pi^- \to n\pi^o$ the shift is 3.3 MeV. Thus, the close proximity to threshold permits a final charge-exchange to shift the singularity into, or very near, the physical region; correspondingly, the "width" associated with the effect is essentially zero. In our $NN\pi$ example, this implies that Im $\tau_{\alpha\alpha'}$ is sharply peaked at $\sqrt{s} \simeq 2640$ MeV.

Actually, we have cheated slightly in taking cos $\theta_{k_3 k_1} \simeq -1$ in our partial-wave argument (one must integrate over $\theta_{k_3 k'_1}$). In a more careful treatment corresponding to a finite range of backward angles ($-1 \le \cos\theta_{k_3 k'_1} \le \bar{x} < 0$), one finds that I_1 is given by

$$I_1 = \int_0^{\kappa_1^m} d\kappa_1\, f(\kappa_1)\, f(\kappa_1)^* \quad ,$$

$$f(\kappa_1) = \ln\left[(\kappa_1-\kappa_0)/(\kappa_1+\kappa_0)\right] - \ln\left[(\kappa_1-\bar{\kappa})/(\kappa_1+\bar{\kappa})\right] \quad , \tag{6}$$

where $\bar{\kappa}$ ($\bar{x}$) is somewhat further from the physical region. An exact (numerical) evaluation shows that, to an excellent approximation

$$I_1(s) \simeq \left[(\sqrt{s} - \sqrt{s_c})^2 + \mu^2\right]^{-1/2} \quad , \tag{7}$$

where μ depends chiefly on Γ and the mass difference $(m_2+m_3) - (m_2'+m_3')$. Empirically, for our $NN\pi$ example (and $\bar{x}=-.5$), $\mu = 2$ MeV for $\pi^+n \to \pi^0 p$, and $\mu = 22$ MeV for $\pi^- p \to \pi^0 n$. One thus obtains substantially the same result, with some broadening in the s-dependence.

Given a production mechanism which emphasizes the geometry of Fig. 1, one would thus expect to see a sharp peak in the corresponding differential cross section. In addition, a true resonance may develop in one or more partial-waves. Consider, for example, a two-channel model in which channel 1 corresponds to the pair at threshold, and channel 2 to the pair at resonance. In the zero-width approximation, the β=2 contribution to $\Delta\tau_{\alpha\alpha'}$ takes on a form similar to that of Eq. (5), but with I_2 essentially equal to the c.m. momentum of the resonance-spectator system. In the spirit of the isobar model, one may factor out the subenergy singularities from $\tau_{\alpha\alpha'}$, and work with reduced (isobar) amplitudes $t_{\alpha\alpha'}$. The latter permit the simple unitary representation $t_{\alpha\alpha'} = N_{\alpha\alpha'}/D$, where

$$\begin{aligned} N_{\alpha\alpha'} &= \lambda_{\alpha\alpha'}(1 - \delta_{\alpha\alpha'}\,\gamma\rho_\beta) \; , \qquad \beta \neq \alpha \; ; \\ D &= 1 - \rho_1 - \rho_2 + \gamma\rho_1\rho_2 \quad , \end{aligned} \tag{8}$$

and $\gamma = 1-\lambda_{12}\lambda_{21}/\lambda_{11}\lambda_{22}$. Here ρ_α is given by a dispersion integral with $\mathrm{Im}\,\rho_\alpha = \pi\lambda_{\alpha\alpha}\, c_\alpha^2\, I_\alpha$; the functions $\lambda_{\alpha\alpha'}(s)$ have only left-hand cuts in s. Using the empirical form of Eq. (7) for I_1, one obtains (in the vicinity of $s=s_c$)

$$\rho_1 \simeq c_1^2 \lambda_{11}(s)\, I_1(s)\, (i\pi + \ln z) \quad ,$$

$$z = \frac{\left[\sqrt{s_c} - \sqrt{s} + I_1^{-1}(s)\right](\sqrt{s} - m_t)}{(\sqrt{s} - \sqrt{s_c})(\sqrt{s} - m_t) + I^{-1}(s)\, I^{-1}(m_t^2) - I^{-2}(s)} \quad , \tag{9}$$

where m_t is the three-body threshold. Given this expression, it is straightforward to verify that $\rho_1 \rightarrow 2\pi i c_1^2 \lambda_{11}(s)\, I_1(s)$ as $\sqrt{s} \rightarrow \sqrt{s_c} - i\mu$ (and hence becomes arbitrarily large), whereas ρ_1 is finite as $\sqrt{s} \rightarrow \sqrt{s_c} + i\mu$. Moreover, for real (physical) s, in the limit $\mu \rightarrow 0$, $\ln z \rightarrow \pm\infty$ for $s \lessgtr s_c$. Therefore, since μ is (empirically) quite small compared to m_t or $\sqrt{s_c}$, it is evident that both $\mathrm{Re}\,\rho_1$ and $\mathrm{Im}\,\rho_1$ are rapidly varying functions for real $s \simeq s_c$. Given the structure of Eq. (8), this behavior can cause a true three-body resonance to develop in one or more partial-waves.

Thus, we observe that a resonance pole corresponds to a complex zero of D, where

$$D = (1-\rho_2)\, D_0 \quad ,$$

$$D_0 = 1 - \sigma\rho_1 \quad , \tag{10}$$

$$\sigma = (1-\gamma\rho_2)/(1-\rho_2) \quad .$$

This leads us to consider the properties of $D_0 = 1-\sigma'\rho_1'$, where σ' is (approximately) a real constant (near s_c), and $\rho_1' \equiv \rho_1 / c_1^2 \lambda_{11}(s_c)$. From the above, it is clear that if σ'/μ is small, a pole <u>must</u> develop near $\sqrt{s} = \sqrt{s_c} - i\mu$; i.e., very close to the real axis and $\sqrt{s} = \sqrt{s_c}$. Physically, such a pole would indeed correspond to a quasi-stable state of the system (a near-eigenvalue of the Hamiltonian). However, the amplitudes $t_{\alpha\alpha'}$ (or T_3) would <u>not</u> exhibit simple Breit-Wigner behavior as s varied over physical values; e.g., in the extreme case $\sigma'/\mu \to 0$, D_0 would be indistinguishable from unity on the real axis. Of course, there is nothing mysterious about this result: the Breit-Wigner form assumes that the pole is the only nearby singularity, whereas we clearly have a branch point at $\sqrt{s_c} - i\mu$ in our example. An important consequence of this fact is that an experimental fit assuming a simple Breit-Wigner amplitude would obtain a "width" much larger than the imaginary part of $\sqrt{s}$ (and would provide a poor representation of the data). Although clearly exaggerated in the present example, it is worth recalling that far more modest complications in the analytic structure (associated with specific three-body effects) can be critical in interpreting experiments such as diffractive A_1 production.[8]

Similarly, further study of Eq. (8) leads one to the following conclusions: (1) as $|\sigma'|/\mu$ increases toward unity, both real and imaginary parts of the pole position will be shifted (e.g., for our $NN\pi$ system with $\sigma'=\mu= 2$ MeV, numerical studies give $M_{res} = 2660$ MeV, $\Gamma_{res} = 40$ MeV), (2) for $|\sigma'| >> \mu$ there will be no associated pole, (3) whether or not a pole is generated, it is quite possible to see a sharp peak in the elastic coupled channel reaction $(2 \to 2)$ arising from the presence of ρ_1 in N_{22}, without a comparable peak in $(1 \to 1)$. One would expect these qualitative aspects to survive in a more rigorous treatment at the three-body level, and suitably generalize to a situation with other types of inelastic channels.

In practice, the condition $|\sigma'| < \mu$ is likely to be satisfied, independent of the details of the dynamics, as a simple consequence of the fact that σ' incorporates the factor c_1^2, which in turn is proportional to $|t_{23}(s_{23}^o)|^2$; i.e., to a_{23}^2, where a_{23} is the (s-wave) scattering length for particles 2 and 3. Typically, for those systems in which a charge exchange can bring the subenergy singularity close to threshold (e.g., $p\pi^- \to n\pi^o$, $n\pi^+ \to p\pi^o$, $\pi^+\pi^- \to 2\pi^o$) the corresponding value of a_{23} turns out to be quite small. Other kinematic factors included in c_1^2 are also small because the intermediate state of Fig. 1 is near threshold. Thus, it is reasonable to anticipate a resonance near $s=s_c$ in those three-hadron systems for which s_{23}^o lies in or near the physical region (and in coupled inelastic channels). Furthermore, since the properties of the (12) resonances are presumed known, and the (23) system must be in an s-wave to take full advantage of the process illustrated in Fig. 1, it is possible to predict the quantum numbers of the effect with reasonable accuracy. In the next section we consider a variety of experimental evidence in support of these notions.

IV. APPLICATIONS TO MESON SPECTROSCOPY

In the preceding sections we have discussed a new type of three-body singularity, and have argued that it, in turn, could generate a nearby resonance. We have further shown that the intrinsic energy scale of the effect (μ) can be as small as a few MeV, in which case the width of the associated resonance can vary from a few to several tens of MeV (depending on the effective value of σ'). We now examine possible experimental evidence for such an effect, beginning with the $N\bar{N}\pi$ system produced in $\pi^- p \to p_F (p\bar{p}\pi^-)$.

From the above, we would expect a narrow resonance in $p\bar{p}\pi^-$ near 2640 MeV. In fact, this work was originally motivated by the apparent observation of such a state at 2660 MeV ($\Gamma < 20$ MeV) in a recent experiment at SLAC involving π^- on deuterium (preliminary results were reported by A. Rogers at the EMS-77 Conference[9]). A detailed analysis of this experiment is well underway, and it now appears certain that virtually all significant features of the data can be well understood on the basis of this mechanism.

Thus, the discussion above suggests a strong preference for a linear configuration in the final state (with $\cos\theta_{k_3 k_1'} \simeq -1$), which in turn suggests especially rapid t-dependence relative to background, a strong preference for backward Jackson angles, a marked tendency of $p\bar{p}$ to emerge back-to-back, etc. Moreover, the state with the largest transition amplitude corresponding to the diagram of Fig. 1 would have JPG=1+-, I=2, with intermediate $p\bar{p}$ interactions predominantly in the 1^- s-wave (I=1). The strong tendency of the latter state to decay via $p\bar{p} \rightarrow 4\pi$ is entirely consistent with the 5π decay mode (for the overall $p\bar{p}\pi$ system) reported by Rogers, whereas the prediction J=1 is at least compatible with the data in its present form. (Since the I=1, 1^- state is forbidden to the NN system, one would not expect a similar effect in, e.g., πD scattering.) It is interesting to note that the (exotic) possibility I=2 is supported by an apparent signal (of significantly poorer resolution) in $n\bar{p}\pi^-$ (Q=-2). This, however, would be somewhat puzzling from the standpoint of our theory, since the charge exchange mechanism cannot operate for Q=-2.

In view of its simplicity, one might expect that the general application of Eq. (3) would generate a multitude of spurious predictions, in which case the 2660 state could be dismissed as coincidence. However, if we restrict ourselves to stable particles (pseudoscalar nonet, baryon octet) and well-established resonances, the corresponding predictions are actually in remarkable coincidence with experiment. In fact, on the basis of the spectrum alone, there are no obvious contradictions. A simple calculation (including finite width corrections) yields the mass values displayed in Tables I and II, which we now consider in some detail.[10]

A. <u>The 1^+ nonet</u>: for purely mesonic three-body systems, only the transitions $\pi^+\pi^- \rightarrow \pi^0\pi^0$ and $\bar{K}_0K_0 \rightarrow K^+K^-$ can move the subenergy singularity to the physical threshold; this eliminates a sizable number of possibilities involving η's and $K\pi$ combinations. As a result, the <u>only</u> states which arise from such input correspond precisely to the 1^+ mesons, with masses in excellent agreement with either the established values (D, E) or the sharp (lower mass) edge of the associated peaks (A, Q).[11] In the absence of spin one can make virtually unique predictions, and verify both the 1^+ character and the correct isospin and hypercharge assignments. For example, in the A_1 (3π) system, the threshold interaction at the (23) vertex corresponds to $\pi^+\pi^- \rightarrow \pi^0\pi^0$, implying the I=0 s-wave state for the (23) subsystem; hence I=1 for the overall system. Moreover, in view of the available phase space, the most likely 3π state permitting both $\rho\pi$ and $\epsilon\pi$ configurations is the 1^+.

In addition to the very accurate predictions for the masses and quantum numbers of these states, this "explanation" of the 1^+ mesons may account for the great difficulties encountered in verifying their existence experimentally. Thus, as noted in previous work,[8] the existence of a complicated analytic structure related to strongly interactive "ρ" and "ϵ" channels can wipe out the simple Breit-Wigner signal (e.g., a large associated phase motion) anticipated in partial-wave analyses of the A_1 system. This is especially true for an amplitude possessing the unusual analytic properties discussed above. In fact, the absence of clearcut Breit-Wigner behavior is a common characteristic of all of the 1^+ states, and might well reflect such a mechanism. In this context we note that recent nondiffractive experiments indicate a narrower, lower mass (1050-1060 MeV) A_1 than has been obtained in diffractive analyses.[12]

B. The $N\bar{N}\pi$ system: if one grants the proposed connection between the 2660 state and our mechanism (operating in $\Delta\bar{N}$ or $\bar{\Delta}N$ configurations), it seems highly likely that other N and Δ resonances in place of $\Delta(1236)$ will produce similar effects at the corresponding values of s_c. In fact, since the ratio Γ/m_r tends to decrease as the N, Δ states increase in mass, the subenergy singularity moves closer to threshold (e.g., only 0.9 MeV below for N(1535)), and hence the effect should be enhanced. However, this trend is opposed by the increasing inelasticity of the N, Δ states of higher mass. One might thus expect a limited sequence of states significantly narrower than the 2660, corresponding to the more elastic of the N, Δ input resonances (in Table I we have approximated this sequence by taking $m_r < 1.9$ GeV). Surprisingly, this input generates a mass sequence in striking correspondence with the χ states observed in $\psi(3700)$ decays, and the spikes seen (e.g., at 4030, 4100, 4410 MeV) in the ratio $R(e^+e^- \to \text{hadrons}/e^+e^- \to \mu^+\mu^-)$,[13] especially if one slightly shifts the N(1500) masses within experimental uncertainties.

Of course, such an identification is highly speculative; these states could exist entirely independently, as yet unseen (and the mechanism need not always produce an observable effect). Nevertheless, the possibility of a connection is not quite as unlikely as it might first appear. In the first place, the states in question appear to have widths on the order of several MeV (as opposed to $\psi(3.1)$ or $\psi(3.7)$), and decay predominantly to ordinary mesons (4π, 6π, $K\bar{K}\pi\pi$, etc.). Secondly, the $N\bar{N}\pi$ configuration need not be a dominant decay mode. For example, consider the parametrization of Eq. (8) with channel 2 an arbitrary inelastic channel (e.g., $c\bar{c}$), and suppose that λ_{11} and λ_{22} are small compared to $\lambda_{12}=\lambda_{21}$. Then $t_{22}/t_{21} \propto \lambda_{12} I_1$; i.e., transitions from the "inelastic" channel to the $N\bar{N}\pi$ configuration are <u>suppressed</u> at the resonance.

(In addition, an $N\bar{N}\pi$ decay with Q=0 would involve at least one neutral, and would not be detected in the experiments to date.) In fact, from the standpoint of the quark model, a connection of this sort would actually be quite consistent with the picture of the 1^+ nonet suggested above (we shall return to this point in the Discussion). In any case, the model makes definite predictions concerning certain properties of these states, and may be rejected (or confirmed) on that basis (e.g., one indeed predicts that the $\chi(3410)$ and $\chi(3550)$ are 0^+, 2^+, respectively, but the other χ's are quite unlikely to have the 0^-, 1^+ quark model assignments).

C. <u>$S\bar{S}M$ systems, S=0</u>: using the relatively elastic (lower mass) strange baryon resonances, three additional strangeness S=0 states are predicted. In each case there exists a possible experimental candidate (seen in mesonic decays).

D. <u>$\bar{S}N\pi$, $S\bar{N}\pi$ systems, S=1</u>: if one considers the $\bar{N}\pi\Sigma$ system, the $\Lambda(1405)$ resonance in $\Sigma\pi$ generates a state with S=1 at 2600 MeV, which may already have been seen in $K(3\pi)$.[14] Similarly, the $\Sigma(1385)$ state of $\pi\Lambda$ implies an effect at 2730 MeV in $\bar{N}\pi\Lambda$; one might look for this directly in $\pi^- p \rightarrow \Lambda_F (p\bar{\Lambda}\pi^-)$, or via $K(3\pi)$ or $K(4\pi)$ states produced in $p\bar{p}$ annihilation. In addition, the $N\pi$ resonances generate a sequence of states in $\bar{\Sigma}\pi N$ comparable to those discussed in B; representative examples are given in Table II D. If these states (or a reasonable subset) were to be confirmed experimentally, one would have to seriously consider the unorthodox "interpretation" of the χ states, etc. suggested above.

E. <u>$N\bar{N}\pi$ system (N-exchange)</u>: taking particles 1, 2, 3 in Fig. 1 to be the $\bar{N}$, N, π, respectively, suggests a kind of $N\bar{N}$ bootstrap via coupling to the $N\bar{N}\pi$ channel; i.e., states at 1897, 2020, 2200 MeV generate resonances at 2020, 2190, 2360 MeV (all are seen experimentally[15]). Here, however, the input $N\bar{N}$ states must themselves be narrow for the effect to occur. Thus, unlike the π-exchange configuration discussed above, the mass ratios are such that s^o_{23} recedes rapidly below the subenergy threshold as Γ increases above 10-20 MeV (depending on m_r). In particular, if one takes $\Gamma = 90$ MeV as the width of the T(2190), $\sqrt{s^o_{23}}$ lies 25 MeV below threshold, and the sequence 1897, 2020, 2190 terminates (the input state at 2200 MeV is apparently a different effect, with $\Gamma \simeq 16$ MeV). Similarly, if one accepts the experimental width $\Gamma = 160$ MeV for the U(2350), it does not generate any higher mass effects (it would produce a state near 2520 MeV for $\Gamma \to 0$). There is, however, a subtlety here, in that (for us) $\Gamma/2$ is the imaginary part of the pole position, whereas our mechanism can result in an experimental "Γ" which is considerably larger. Thus, if the U(2350) is actually of this type, it might be appropriate to use $\Gamma \simeq 40$ MeV, leading to an effect at 2500 MeV (in this case finite Γ lowers s_c). This might correspond, in turn, to a resonance in $p\bar{p}$ reported at 2480 MeV.[15]

Finally, some of the N, Δ states employed as input may themselves be generated via the $N\pi\pi$ system; e.g., taking the $\Delta(1236)$ with particle 3 a pion in Fig. 1 yields a mass of 1488 MeV, which may correspond to the N(1470) and explain its odd properties (similar to the 1^+ mesons!). Taken in conjunction with more conventional three-body mechanisms, these results suggest that the hadron spectrum may be largely generated by threshold conditions involving observed physical hadrons. The corresponding implications for the quark model are discussed below.

V. DISCUSSION

The development given above falls naturally into three parts. In the first place, we have employed a rather general argument (which can be made quite rigorous) to deduce the existence of both a subenergy singularity (for fixed s), and a corresponding singularity in the total energy. Secondly, we have illustrated how this can lead to a resonance pole in one or more JPGI states, and given a qualitative, partly empirical, argument to the effect that this is actually quite likely to happen. Although it is clearly impossible to prove that this occurs in a model-independent way, it should nevertheless be possible to obtain reasonable estimates of quantities comparable to the $\lambda_{\alpha\beta}(s)$ for a particular system (e.g., by studying some variant of the relativistic Faddeev equations), and to thus determine the relative likelihood of an observable effect. The 2660 MeV $N\bar{N}\pi$ state is presently being studied as a representative example of this procedure.

The third aspect is entirely empirical, in that we have taken advantage of the simplicity of our mass formula to predict a spectrum of possible states, and have simply observed that a remarkable correspondence exists between this set of "resonances" and experiment. On the basis of the mass values alone, the significance of this "agreement" is certainly subject to debate, although the sheer number of possibilities cited would appear to stretch coincidence. One might also regard it as rather odd that a mechanism which generates a rapid energy-dependence at prescribed mass values should have no connection with observed resonances at those energies. However, the principal argument we would advance is based not so much on mass numerology, as on the correlation between the types of states predicted. Thus, it appears significant that the only mesons predicted below 2 GeV comprise the complete set of 1^+ mesons, and only that set, and that the unusual analytic properties associated with our effect could well reflect their odd experimental properties. Similarly, the only nucleon resonance which could be generated below 1.8 GeV in the $N\pi\pi$ system would correspond to the N(1470), which has similar odd properties. Finally, although a connection with the χ states is bound to be regarded as much more speculative, it should be noted that the χ's and 1^+ mesons are highly analogous from the standpoint of the quark model; i.e., both are p-wave triplet states of $q\bar{q}$. Thus, in addition to the highly accurate (but possibly coincidental) mass values, there are highly suggestive parallels between the kinds of states produced. Naturally, if the predictions for unit strangeness mesons above 2.7 GeV are confirmed, this empirical argument would be greatly strengthened.

Aside from the spectroscopy, there are a number of characteristic features of our mechanism which might signal its presence experimentally. Thus, if one is able to produce the three-body system itself (as in the $N\bar{N}\pi$ experiment reported by Rogers[9]), one might look for a high incidence of the driving (input) resonance between a pair in the final state, and for a marked tendency toward <u>linearity</u> (reflecting the geometry of Fig. 1). One might also look for an unusual degree of charge-dependence, corresponding to those combinations in which the singularity is particularly sharp. For example, in the Q=-1 $N\bar{N}\pi$ system, one should find a relatively large fraction of $\bar{p}n\pi^0$ charge states (in general, one would expect an unusual abundance of π^0's). This aspect might also be reflected in a tendency for associated resonances to be degenerate in isospin (for states and systems in which the isospin "overlap" of the two configurations bridged by Fig. 1 is independent of I). Finally, in cases where sufficient phase space is available, this mechanism might well produce closely degenerate resonances in two or more J^P states. One would not anticipate these general features on the basis of a more conventional dynamics, and hence they might well be employed to test our proposed effect experimentally. Of course, in the context of a specific model, one could make a number of additional predictions concerning branching ratios, production characteristics, etc.

In conclusion, we consider how such a mechanism might be reconciled with the quark model. If one were to take all of the predictions of the last section seriously, the most obvious point of conflict would be with regard to the χ states (and the higher mass SPEAR results). This is understandable, in view of the intensive efforts which have gone into constructing the charmonium model to explain this particular class of experimental facts. However, one should note that our "explanation" is in far better accord with the overall picture evident in hadron spectroscopy. Thus, there is ample empirical evidence correlating hadron resonances with inelastic thresholds. For example, Aaron and Amado have discussed the manner in which the opening of channels such as $N\rho$ or $\Delta\pi$ in $N\pi$ scattering can generate N and Δ resonances in the 1.6-1.7 GeV region.[3] One might also cite the effect of the $K\bar{K}$ threshold on the S*(993), and a calculation of the $\omega(783)$ as a 3π state driven by overlapping $\rho\pi$ combinations.[4] From such examples, one might well conclude that the masses actually attained by physical hadrons have very little to do with direct qq or $q\bar{q}$ forces.

From this point of view, our proposed mechanism for the 1^+ nonet falls well within the "traditional" framework. Moreover, on the grounds of consistency, one might then argue that the χ states should have a similar basis; i.e., if the triplet p-wave $q\bar{q}$ force is <u>not</u> instrumental in forming the A_1, then why should the $c\bar{c}$ analogue generate the 1^+ χ state? More specifically, our mechanism for the A_1 involves an overlap condition between a particular threshold combination of three pions (3q, $3\bar{q}$), and a $\rho\pi$ combination (2q, $2\bar{q}$). It is thus generated, in the quark language, by a strong, rapidly energy-dependent force connecting two multi-quark states (both of which are connected as well to the canonical $q\bar{q}$ configuration). While there is undoubtedly a direct interaction of some sort in the triplet $q\bar{q}$ p-wave, it would not by itself generate (in our picture) an A_1 resonance; our suggestion is that the $c\bar{c}$ (χ) situation should be similar. On the other hand, while this would eliminate the need for a rather exotic spin-orbit force in accounting for the χ masses, one would then have to explain the pattern of (electromagnetic) decays of $\psi(3.7)$ to these states, their branching ratios, etc.

This is clearly a nontrivial question, and we cannot resolve it here. However, it does not seen at all unlikely that one could construct a coupled channel model along the lines of Eq. (8), incorporating the $c\bar{c}$ p-wave in addition to multiquark components, in such a way that the singularity in the $(3q, 3\bar{q}) \rightarrow (4q, 4\bar{q})$ transition potential (i.e., $\bar{N}N' \rightarrow \bar{N}N\pi$) generates the resonance, whereas transitions from the (presumed s-wave $c\bar{c}$ state) $\psi(3.7)$ proceed via the p-wave $c\bar{c}$ component. One might argue, for example, that the coupling of $\chi(3414)$ to the 8 quark level $(4q, 4\bar{q})$ is in fact large, in view of the predominance of 4π and $K\bar{K}\pi\pi$ decay modes.[13] From the standpoint of the charmonium theory, what this amounts to is the suggestion that the effective p-wave force in $c\bar{c}$ has a sharp energy-dependence arising from coupled channels (as is well known, one can always reduce a multichannel problem to a single channel problem with a (complicated) s-dependent force). Again, our reason for taking such a hypothesis seriously is that ordinary mesons (not containing c or $\bar{c}$) do not seem to be generated as simple $q\bar{q}$ states, at least for angular momentum $\ell \geq 1$.

A possible scenario for understanding such a picture might go as follows. Suppose that there is a sharply rising, confining potential of some kind between $q\bar{q}$. For s-states, where the $q\bar{q}$ pair can get very close together, this potential may dominate the dynamics and, via a reasonable amount of spin-dependence, generate such states as the 0^- and 1^- nonets, $\psi(3.7)$, etc. However, as the angular-momentum barrier forces the q and $\bar{q}$ apart (for $\ell \geq 1$), it becomes far cheaper (in energy) to create one or more $q\bar{q}$ pair, and hence to separate by recombining to form ordinary mesons. In fact, it is hard to see why this should not happen. Under these circumstances the multiquark components may well dominate the interaction which generates the physical mass.

Of course, it is quite possible that $c\bar{c}$ pairs behave differently from other $q\bar{q}$ systems; the problem with quarks as constituents has always been that their interaction is a complete mystery. Nevertheless, the issues raised here deserve further consideration, and will be pursued in subsequent articles.

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10. In computing the predicted mass values one must correct Eq. (3) for $\Gamma \neq 0$ (particles 2 and 3 are not precisely at rest). The quoted values correspond to (real) s_c such that Im $s_{23}^{o} = 0$.
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Table I. Mass Predictions for Meson States

A. Mesonic Systems

System	Input Resonance	Predicted Mass	Experiment
3π	$\rho(770)$	1095	$A_1(1100)$
$K\pi\pi$	$K^*(892)$	1180	$Q_{1,2}(1200-1400)$
$K\bar{K}\pi$	$K^*(892)$	1430	E(1420)
$\eta\pi\pi$	$\delta(970)$	1270	D(1285)

C. $S\bar{S}M$ Systems (S=0)

System	Input Resonance	Predicted Mass	Experiment
$\bar{N}K\Sigma$	$\Lambda(1520)$	2790	2800, 2820
$\bar{\Sigma}\pi\Sigma$	$\Lambda(1405)$	2870	2850 [17]
$\bar{\Sigma}\pi\Sigma$	$\Sigma(1385)$	3055	3050 [17]

B. $N\bar{N}\pi$ System

$N\pi$ State	Predicted Mass	Experiment
$\Delta(1236)$	2640	2660 [9]
N(1470)	3440	$\chi(3415)$ [13]
N(1470)	3440	$\chi(3455)$ [13]
N(1535), (1500)*	3650, (3540)*	$\chi(3510)$ [13]
N(1520), (1510)*	3600, (3570)*	$\chi(3550)$ [13]
$\Delta(1650)$	4020	$\psi(4100)$ region
$\Delta(1670)$	4090	
N(1670)	4090	
N(1688)	4150	
N(1700)	4180	
N(1780)	4430	$\psi(4400)$ region
N(1810)	4520	

Table II. Mass Predictions for Meson States.

D. $\bar{S}N\pi$, $S\bar{N}\pi$ Systems (S=1)				E. $N\bar{N}\pi$ System (N-Exchange)		
System	Input Resonance	Predicted Mass	Experiment	Input $N\bar{N}$ Resonance	Predicted Mass	Experiment
$\bar{N}\pi\Sigma$	$\Lambda(1405)$	2600	2600 [14]	1897 [15]	2030	2020 [16]
$\bar{N}\pi\Lambda$	$\Sigma(1385)$	2730		1936 [15]	2080	$\rho(2100)$
$\bar{\Sigma}\pi N$	$\Delta(1236)$	2955		2020 [16]	2170	T(2190) [15]
$\bar{\Sigma}\pi N$	N(1470)	3860		2200 [16]	2360	U(2350) [15]
$\bar{\Sigma}\pi N$	N(1520)	4020				
$\bar{\Sigma}\pi N$	$\Delta(1650)$	4490				

FIGURE CAPTIONS

1. Rescattering diagram which generates the singularity discussed in the text. The vertex blobs correspond to off-shell scattering amplitudes.
2. Location of subenergy singularity in the kinetic energy (T) for the $N\bar{N}\pi$ system (dashed line) as a function of s (ticks indicate 20 MeV intervals in $\sqrt{s}$).

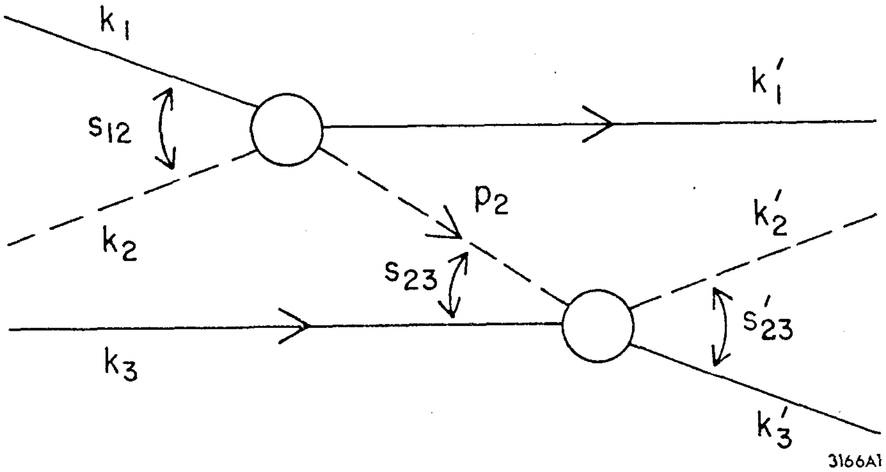

Fig. 1

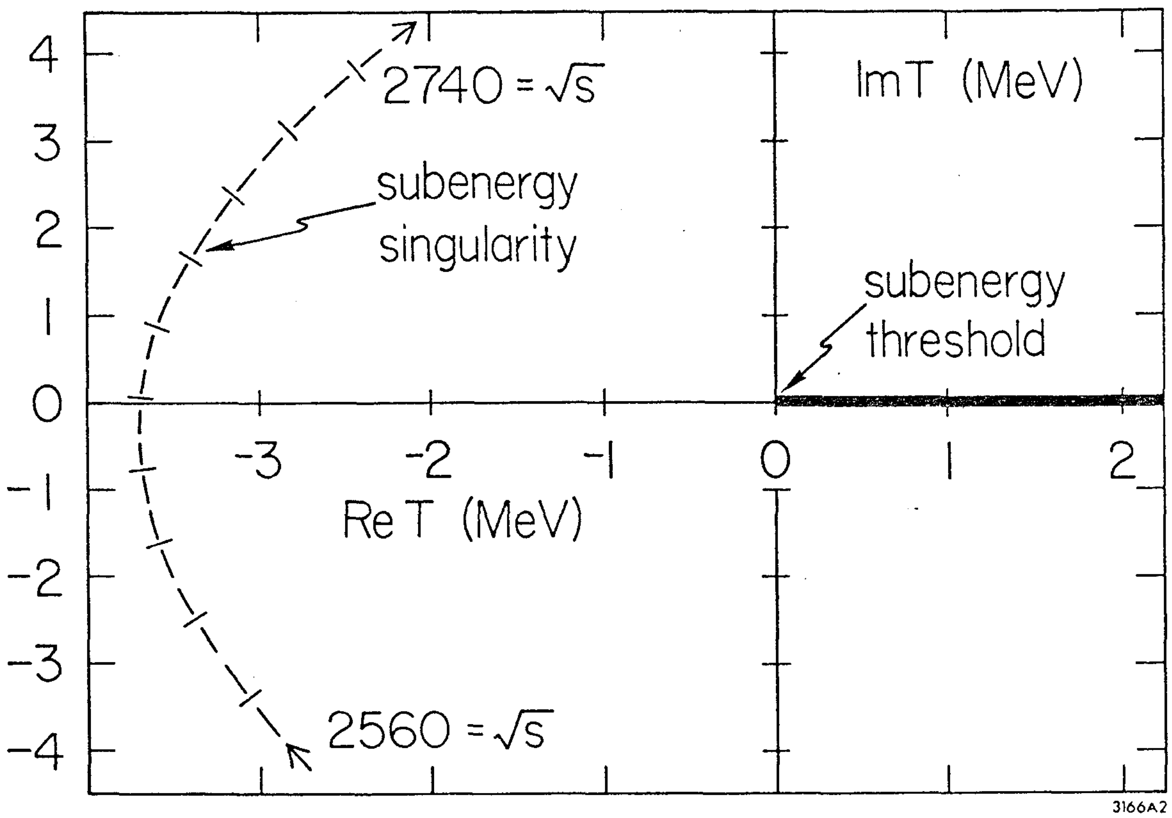
4
3
2
1
0
-1
-2
-3
-4
2740 = √s
ImT (MeV)
subenergy
singularity
subenergy
threshold
-3
-2
-1
0
1
2
Re T (MeV)
2560 = √s
3166A2

Fig. 2