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C symmetry of space time as origin of Majorana particles

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Abstract. We derive directly from the C symmetry of Lorentzian space-time a Majorana field with a non-standard pure-imaginary antisymmetric Majorana mass term and study some applications.

1. *C* invariant quantum states *ad hoc*

The description of Majorana fermions is one of the central topics of contemporary physics not only because of the spectacular phenomena of neutrino-oscillation or neutrinoless double beta decay, $(0\nu\beta\beta)$, but also because of the truly neutral fermion gauge fields (gauginos) predicted to appear as companions to ordinary gauge bosons in supersymmetric theories. From formal point of view, Majorana particles are treated in the textbooks differently but Dirac particles. While the Dirac field is systematically derived from the P symmetry of Lorentzian space time, Majorana particles are composed *ad hoc* at the quantum field level. The most prominent C invariant quantum states are those exploited in the description of neutrino oscillations, constructed as [1]

$$\nu_{DM} = \frac{1}{\sqrt{2}}(\nu \pm \nu^c)\,,$$
$$\nu = u_h(\vec{p})\mathbf{a}_h^+(\vec{p})\, e^{-ip\cdot x}\,|0\rangle\,, \qquad \nu^c = u_h(\vec{p})\,\mathbf{b}_h^+(\vec{p})\, e^{-ip\cdot x}\,|0\rangle\,,$$
$$\nu_{L/R} = \frac{1}{2}(1_4 \pm \gamma_5)\nu\,, \qquad (\nu^c)_{L/R} = \frac{1}{2}(1_4 \pm \gamma_5)\nu^c\,,$$

where $u_h(\vec{p})\mathbf{a}_h^+(\vec{p})|0\rangle$ and $u_h(\vec{p})\mathbf{b}_h^+(\vec{p})|0\rangle$ are in turn the states of neutrino of momentum $\vec{p}$ and spin-projection h, and its charge conjugate. The neutrino spinors ν_L and $(\nu^c)_R$ are the so called "electroweak active", while ν_R, and $(\nu^c)_L$ are the "electroweak sterile" ones. The C parity states in Eq. (1) underly neutrino oscillation data analysis and are frequently referred to as Majorana spinors. They satisfy the following system of two Dirac equations coupled by the Majorana mass term

$$\begin{pmatrix} \not p - M_D & -M_M \\ -M_M & \not p - M_D \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \end{pmatrix} = 0 \qquad (1)$$

Equation (1) is the Dirac equation for one neutrino generation. It has as two limiting cases the vanishing Dirac mass, $M_D = 0$, and the vanishing Majorana mass, $M_M = 0$, respectively. The quantum states (1) belong to the so called Dirac-Majorana fields, $\psi^{(\epsilon_j)}_{DM}(x)$, which are C invariant and constructed as the following combinations of the Dirac quantum field, $\psi_D(x)$, and its anti-partilce conjugate, $\psi^c_D(x)$,

$$\begin{aligned}
\psi^{(\epsilon_j)}_{DM}(x) &= \frac{1}{\sqrt{2}}\left(\psi_D(x) + \epsilon_j^*\psi^c_D(x)\right) , \\
C\psi^{(\epsilon_j)}_{DM}(x) = \epsilon_j\psi^{(\epsilon_j)}_{DM}(x) \,, &\qquad C = i\gamma_2 K \,,\ K = [\ ]^* \,, \\
\epsilon_1 = -\epsilon_2 = 1, \qquad &\text{or}, \qquad \epsilon_1 = -\epsilon_2 = i \,.
\end{aligned} \tag{2}$$

To obtain $\psi^c_D(x)$ one interchanges $\mathbf{a} \rightleftarrows \mathbf{b}$ in $\psi_D(x)$[3],

$$\psi^c_D(x) = \int \frac{d^3\vec{p}}{2p_0}\frac{1}{(2\pi)^{\frac{3}{2}}} \qquad \sum_h \left(u_h(\vec{p})\mathbf{b}_h(\vec{p})e^{-ip\cdot x} + u_h(\vec{p})^c\mathbf{a}^+_h(\vec{p})e^{ip\cdot x}\right) . \tag{3}$$

The one neutrino–anti-neutrino generation, *column* (ν, ν^c), in Eq. (1) is then described by means of the Dirac-Majorana Lagrangian which is cast into the following matrix form [2]

$$\begin{aligned}
\mathcal{L}_{DM}(x) &= \bar{\psi}^{(\epsilon_j)}_{(8)}(x)\begin{pmatrix} i\partial\!\!\!/ - M_D & -\epsilon_1 M_M \\ -\epsilon_1^* M_M & i\partial\!\!\!/ - M_D\end{pmatrix}\psi^{(\epsilon_j)}_{(8)}(x) \,, \\
\psi^{(\epsilon_j)}_{(8)} &= \begin{pmatrix}\psi_D(x) \\ \epsilon_j^*\psi^c_D(x)\end{pmatrix} \\
&= \int \frac{d^3\vec{p}}{2p_0}\frac{1}{(2\pi)^{\frac{3}{2}}}\sum_h \left[u_h(\vec{p})\begin{pmatrix}\mathbf{a}_h(\vec{p}) \\ \epsilon_j^*\mathbf{b}_h(\vec{p})\end{pmatrix}e^{-ip\cdot x} + u_h(\vec{p})^c\begin{pmatrix}\mathbf{b}^+_h(\vec{p}) \\ \epsilon_j^*\mathbf{a}^+_h(\vec{p})\end{pmatrix}e^{ip\cdot x}\right] ,
\end{aligned} \tag{4}$$

a field with one-storey "wave functions", and two-storey creation/annihilation operators. The diagonal part of the mass matrix is the standard Dirac mass term. We here call standard- and non-standard Majorana mass terms the ones corresponding to $\epsilon_1 = 1$, and $\epsilon_1 = i$, respectively. We further notice that the matrix defining the Majorana mass term can equally well be viewed as a metric in the $\psi^{(\epsilon_j)}_{(8)}(x)$ space according to

$$\widetilde{\psi^{(\epsilon_j)}_{(8)}}(x) = \left(\overline{\psi_D}(x), \epsilon_j\overline{\psi^c_D}(x)\right)\Gamma^{(\epsilon_j)}_8 \,, \qquad \Gamma^{(\epsilon_j)}_8 = \begin{pmatrix} 0_4 & \epsilon_1 1_4 \\ \epsilon_1^* 1_4 & 0_4\end{pmatrix} , \tag{5}$$

and write down the Lagrangian for $M_D = 0$ as

$$\mathcal{L}_{(8)}(x) = \widetilde{\psi_{(8)}}(x)\begin{pmatrix} -M_M\, 1_4 & \epsilon_1 i\partial\!\!\!/ \\ \epsilon_1^* i\partial\!\!\!/ & -M_M\, 1_4\end{pmatrix}\psi_{(8)}(x) \,. \tag{6}$$

Once having cast the Lagrangians into that form allows to search for field solutions beyond Eq. (4) and of the (generic) form

$$\psi_{(8)}(x) = \int dV\Big(\qquad \sum_k \lambda_k(\vec{p})\mathbf{d}_k(\vec{p})e^{-ip\cdot x} + \rho_k(\vec{p})\mathbf{d}^+_k(\vec{p})e^{ip\cdot x}\Big) . \tag{7}$$

Here dV is a properly chosen phase volume, the index k encodes in a proper way the required eight degrees of freedom, $\mathbf{d}^+_h(\vec{p})$, and $\mathbf{d}_h(\vec{p})$ are in turn creation and annihilation operators of one neutrino generation, while $\lambda_k(\vec{p})$, and $\rho_k(\vec{p})$ are the associated "wave functions". Our goal here is to derive systematically $\lambda_k(\vec{p})$, and $\rho_k(\vec{p})$ from the discrete C symmetry of space time. Compared to Eq. (4), we expect Feynman diagram rules for $\rho_k(\vec{p})$–, and $\lambda_k(\vec{p})$, following from $\psi_{(8)}(x)$ in Eq. (7) to be more comfortable in calculations of traces entering cross sections.

2. Local quantum fields for one neutrino–anti-neutrino generation

2.1. Restive C parities

The C invariant Lorentz representations for spin-1/2 (they will be denoted by $\Psi_M^{h;(\epsilon_j)}(\vec{p})$ in the following) diagonalize the charge-conjugation operator,

$$C\Psi_M^{h;(\epsilon_j)}(\vec{p}) \quad = \quad \epsilon_j \Psi_M^{h;(\epsilon_j)}(\vec{p})\,. \tag{8}$$

The existence of two types of C invariant Lorentz representations– with real, ($\epsilon_j = \pm 1$), and imaginary, ($\epsilon_j = \pm i$) C parities, contrasts the case of the P operator, γ_0, which allows only for real spatial parity spinors. In one of the possibilities, the four linearly independent rest-frame spinors that span the C invariant $(1/2,)\oplus(0,1/2)$ representation space

$$\frac{1}{2}\left(1_4 + i\epsilon_j^*\gamma_2 K\right)\Psi_M^{h;(\epsilon_j)}(\vec{0}\,) = \Psi_M^{h;(\epsilon_j)}(\vec{0}\,)\,, \tag{9}$$

can be chosen in the Cartesian frame as

$$\Psi_M^{\uparrow;(\epsilon_j)}(\vec{0}\,) \quad = \quad \sqrt{M_M}\begin{pmatrix} 0 \\ -\epsilon_j^* \\ 1 \\ 0 \end{pmatrix}, \qquad \Psi_M^{\downarrow;(\epsilon_j)}(\vec{0}\,) = \sqrt{M_M}\begin{pmatrix} -\epsilon_j^* \\ 0 \\ 0 \\ -1 \end{pmatrix}. \tag{10}$$

The basis spinors $\Psi_M^{h;(\epsilon_j)}(\vec{0}\,)$ result from the more general form given, among others, in Ref. [3], as well as in Ref. [4]

$$\begin{aligned} \Psi_M^{h;(\epsilon_j)} \quad &= \quad \begin{pmatrix} \epsilon_j^* i\sigma_2[\,\dot{\zeta}_h\,]^* \\ \dot{\zeta}_h \end{pmatrix} = \begin{pmatrix} \epsilon_j^*\zeta_{-h} \\ \dot{\zeta}_h \end{pmatrix}, \\ \epsilon_j = \pm 1, \qquad &\text{or} \quad , \quad \epsilon_j = \pm i\,, \quad h = \uparrow, \downarrow\,, \end{aligned} \tag{11}$$

which is no more but precisely the C symmetric left-chiral spinor $\chi_1 = \nu_L \oplus (\nu_L)^c$ introduced in Ref. [1]. Spinor (ζ), and co-spinor ($\dot{\zeta}$) are related via [5]

$$\begin{aligned} \zeta = \begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix}, \qquad & \dot{\zeta} = \begin{pmatrix} \xi_{\dot{1}} \\ \xi_{\dot{2}} \end{pmatrix}, \xi_\alpha = i(\sigma_2)_{\alpha\beta}\xi^\beta\,, \\ \xi_{\dot{\alpha}} : \quad &= \quad [\xi_\alpha]^*\,, \quad \zeta = \xi^1\,\zeta_\uparrow + \xi^2\,\zeta_\downarrow\,, \\ \zeta_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad & \zeta_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \end{aligned} \tag{12}$$

and $(i\sigma_2)_{ij} = \epsilon_{ij}$ is the Levi-Civita symbol which plays the role of metric in the $SL(2,C)$ spinor space. The equation of motion satisfied by the C parity spinors in any inertial frame is now obtained in subjecting Eq. (9) to the Lorentzian boost,

$$\begin{aligned} B(\vec{p})\frac{1}{2}\Big(1_4 \quad &+ \quad \epsilon_j^* i\gamma_2 K\Big)B(\vec{p})^{-1}\Psi_M^{h;(\epsilon_j)}(\vec{p}) = \Psi_M^{h;(\epsilon_j)}(\vec{p})\,, \\ B(\vec{p}) \quad &= \quad \frac{1}{\sqrt{2M_M(p_0 + M_M)}}(\not{p} + M_M\gamma_0)\gamma_0\,. \end{aligned} \tag{13}$$

To ones great surprise, due to $\gamma_\mu\gamma_2 = -\gamma_2\gamma_\mu^*$, the boosted C operator turns out to be *momentum independent*,

$$B(\vec{p})\,i\gamma_2 K B(\vec{p})^{-1} \quad = \quad i\gamma_2 K\,, \tag{14}$$

and *identical to the rest frame C parity operator*. This apparently contrasts the case of the rest-frame parity operator, $P=\gamma_0$, that upon boosting provides the kinetic term of the Dirac equation,

$$B(\vec{p})\,\gamma_0 B(\vec{p})^{-1}=\frac{p\cdot\gamma}{m}\,. \tag{15}$$

It seems that the construction procedure of a C based covariant local quantum field comes here to a stop. We emphasize on *covariant*, and *local*, because if one is willing to entertain non-local quantum fields based on the non-covariant $\Psi_M^{h;(\epsilon_j)}(\vec{p})$ propagation, one can have a plenitude of them in considering in place of C miscellaneous discrete symmetries. For example, the real C parity spinors are simultaneously $\gamma_5\gamma_1$ invariant and $B(\vec{p})\frac{1}{2}(1_4\mp\gamma_5\gamma_1)B^{-1}(\vec{p})\Psi_M^{h;(\pm1)}(\vec{p})=\Psi_M^{h;(\pm1)}(\vec{p})$, would be such an equation.[1]In general, C parities satisfy non-covariant equations of the type,

$$B(\vec{p})\frac{1}{2}(1_4+i\epsilon_j\gamma_2\mathcal{A})B^{-1}(\vec{p})\Psi_M^{h;(\epsilon_j)}(\vec{p})=\Psi_M^{h;(\epsilon_j)}(\vec{p})\,, \tag{16}$$

where $\mathcal{A}$ is the matrix that mimics complex conjugation in the basis of choice at rest. To be specific, let us consider imaginary C parity in the helicity frame, where

$$\zeta_\uparrow=\sqrt{m}\begin{pmatrix}\cos\frac{\theta}{2}e^{-i\frac{\varphi}{2}}\\ \sin\frac{\theta}{2}e^{+i\frac{\varphi}{2}}\end{pmatrix},\qquad \zeta_\downarrow=\sqrt{m}\begin{pmatrix}\sin\frac{\theta}{2}e^{-i\frac{\varphi}{2}}\\ -\cos\frac{\theta}{2}e^{+i\frac{\varphi}{2}}\end{pmatrix}.$$

Here, the operation of complex conjugation of, say, $\Psi_M^{h;(-i)}(\vec{0})$ takes the form $K\to\mathcal{A}:=$Diag $(-e^{+i\varphi},-e^{-i\varphi},e^{+i\varphi},e^{-i\varphi})$. Insertion of the latter expression into Eq. (16) allows to obtain the following frame dependent wave equation

$$\Big((\not p+m\gamma_0)\widetilde{\mathcal{A}}(\not p+m\gamma_0)-2m(p_0+m)\Big)\Psi_M^{h;(-i)}(\vec{p})\;=\;0,$$

where $\widetilde{\mathcal{A}}=\gamma_0\gamma_2\mathcal{A}\gamma_0$, and we used "$m$" as a generic mass notion in place of M_M. Reference [6] in fact reports same equation up to $\mathcal{A}$ being replaced by $-i\mathcal{A}$, a difference that comes about by the different definition of $C=\gamma_2K$ used there. In effect, as long as there is no universal matrix representation for the operation of complex conjugation, one encounters *a multitude of non-covariant equations* corresponding to multitude of frames related to each other by similarity transformations. We here take the position that such frames can not be interpreted as preferred frames in the Universe as occasionally done in the literature [6]. Non-local quantum states based upon non-covariant propagating Lorentz representations are in our opinion unphysical.

2.2. Taming C parity and two-storey spinors

In order to resolve the dilemma of static C propagators, we here for the sake of concreteness focus on imaginary C parity spinors and notice that spatial parity, the only covariant discrete symmetry, ladders between C parities according to $\gamma_0\Psi_M^{\uparrow;(i)}(\vec{0})=-i\Psi_M^{\downarrow;(i)}(\vec{0})$, $\gamma_0\Psi_M^{\downarrow;(i)}(\vec{0})=i\Psi_M^{\uparrow;(i)}(\vec{0})$, etc, and creates a new discrete symmetry. This allows one to obtain covariant wave

[1] Notice that also the Dirac $u_h(\vec{p})$ (or, $v_h(\vec{p})$) spinors can satisfy non-covariant equations. Suffices to recall that they are also eigenspinors to, for example, $\gamma_5\gamma_3$ and therefore solutions to $B(\vec{p})\frac{1}{2}(1_4\pm\gamma_5\gamma_3)B^{-1}(\vec{p})u_h(\vec{p})=u_h(\vec{p})$. Free spinors can satisfy a variety of occasionally non-covariant differential equations. It is the dynamics that selects the relevant one in according to data. As long as data require covariance and the only covariant discrete symmetry in spinor space is parity, the wave equations for spinors of other discrete parities need to be obtained from the action of γ_0 upon them.

equation in two-storey spaces of the type

$$\begin{pmatrix} 0_4 & -iB(\vec{p})\gamma_0 B(\vec{p})^{-1} \\ iB(\vec{p})\gamma_0 B(\vec{p})^{-1} & 0_4 \end{pmatrix} \Lambda(\vec{p}) = \Lambda(\vec{p})\,,$$

$$\Lambda(\vec{p}) = \begin{pmatrix} \Psi_M^{\uparrow;(i)}(\vec{p}) \\ \Psi_M^{\downarrow;(i)}(\vec{p}) \end{pmatrix}.$$

In substituting for $B(\vec{p})\gamma_0 B(\vec{p})^{-1} = \frac{p\cdot\gamma}{m} \equiv \frac{\not p}{m}$ results in

$$\begin{pmatrix} \not p & iM_M 1_4 \\ -iM_M 1_4 & \not p \end{pmatrix} \begin{pmatrix} \Psi_M^{\uparrow;(i)}(\vec{p}) \\ \Psi_M^{\downarrow;(i)}(\vec{p}) \end{pmatrix} = 0\,. \tag{17}$$

This is a Dirac like equation for a two-storey spinor. It differs from Eq. (1) through the *non-standard* Majorana mass term which is pure imaginary and anti-symmetric. In nullifying the determinant of the latter equation one obtains the standard time-like energy momentum dispersion relation, $p^2 - M_M^2 = 0$, and proves that such a mass term does not imply acausal spinor propagation. These spinors are self-orthogonal, $\overline{\Psi}_M^{h;(\pm i)}(\vec{p})\Psi_M^{h;(\pm i)}(\vec{p}) = 0$, and cross-normalized according to, $\overline{\Psi}_M^{h;(\pm i)}(\vec{p})\Psi_M^{-h;(\pm i)}(\vec{p}) = \pm 2iM_M(\delta_{h\uparrow} - \delta_{h\downarrow})$ (properties referred to as *bi-orthogonality* in Ref. [4]). We here notice that (i) self-orthogonality describes the limiting case of a vanishing Dirac mass term, (ii) cross-normalization corresponds to a non-vanishing Majorana-mass term, and conclude that fields based on top of $\Psi_M^{h;(\epsilon_j)}(\vec{p})$ will describe the limiting case of a pure Majorana field. Though the coupled equations (17) have been noticed (up to notational differences) already in Ref. [7], the identifications of the mass terms has not been established there.
We now introduce the following complete set of two-storey spinors corresponding to Eqs. (17)

$$\begin{aligned}
\Lambda_1^\tau(\vec{p}) &= \begin{pmatrix} u_\uparrow^R(\vec{p}) \mp iu_\uparrow^L(\vec{p})^c \\ u_\downarrow^R(\vec{p}) \mp iu_\downarrow^L(\vec{p})^c \end{pmatrix}, \\
\Lambda_2^\tau(\vec{p}) &= \begin{pmatrix} u_\downarrow^R(\vec{p}) \mp iu_\downarrow^L(\vec{p})^c \\ u_\uparrow^R(\vec{p}) \mp iu_\uparrow^L(\vec{p})^c \end{pmatrix}, \\
\Lambda_3^\tau(\vec{p}) &= \begin{pmatrix} u_\uparrow^R(\vec{p}) \mp iu_\uparrow^L(\vec{p})^c \\ -u_\downarrow^R(\vec{p}) \pm iu_\downarrow^L(\vec{p})^c \end{pmatrix}, \\
\Lambda_4^\tau(\vec{p}) &= \begin{pmatrix} u_\downarrow^R(\vec{p}) \mp iu_\downarrow^L(\vec{p})^c \\ -u_\uparrow^R(\vec{p}) \pm iu_\uparrow^L(\vec{p})^c \end{pmatrix},
\end{aligned} \tag{18}$$

with $\tau = \pm$. In defining now $\widetilde{\Lambda}_k^\tau(\vec{p})$ as

$$\widetilde{\Lambda}_k^\tau(\vec{p}) = \bar{\Lambda}_k^\tau(\vec{p})\Gamma_8^{(i)}\,, \tag{19}$$

allows for the construction of an orthogonal basis in the recently designed two-storey space as

$$\begin{aligned}
&\widetilde{\Lambda}_j^\tau(\vec{p})\Lambda_j^\tau(\vec{p}) = +4M_M\,, && \tau = +, \quad j = 1,4\,; \\
& && \tau = -, \quad j = 2,3\,; \\
&\widetilde{\Lambda}_k^\tau(\vec{p})\Lambda_k^\tau(\vec{p}) = -4M_M\,, && \tau = +, \quad k = 2,3\,; \\
& && \tau = -, \quad k = 1,4\,; \\
&\widetilde{\Lambda}_k^\tau(\vec{p})\Lambda_l^{\tau'}(\vec{p}) = 0\,, && \tau \neq \tau', \quad k \neq l\,.
\end{aligned} \tag{20}$$

Equation (20) shows that the space under consideration contains equal numbers of mutually orthogonal spinors of real positive–, and of real negative norms, much alike the Dirac space. This advantage allows for a canonical quantization *á la* Dirac when introducing the *local* $\psi_{(8)}(x)$ field operator as

$$\begin{aligned}\psi_{(8)}(x) &= \int dV\Big[\sum_{\tau=+,j=1,4;\tau=-,j=2,3}\Lambda^{\tau}_{j}(\vec{p})\mathbf{d}^{\tau}_{j}(\vec{p})\,e^{-ip\cdot x}\\ &+ \sum_{\tau=+,k=2,3;\tau=-,k=1,4}\Lambda^{\tau}_{k}(\vec{p})\mathbf{d}^{\tau\,+}_{k}(\vec{p})\,e^{ip\cdot x}\Big]\,. \end{aligned} \tag{21}$$

Here, dV is the appropriate phase volume. This local quantum field is built on top of Lorentzian C parity representations and describes one Majorana-neutrino generation. In this way we derived a truly neutral local spin-1/2 field directly from C invariant Lorentz representations. As long as above fields are eight-dimensional copies of the Dirac field, the Feynman diagram rules will be the eight-dimensional version of the rules valid for the standard Dirac spinors and can be exploited in calculations of cross-sections.

A decomposition of the textbook C parity spinors in Eq. (11) into Dirac spinors takes one to a further surprise. It turns out that those are not the most economic C invariant combinations of Dirac u and v spinors as they are superpositions of two smaller C invariant spinors of opposite spin-projections/helicities according to

$$\Psi^{\uparrow;(+i)}_{M}(\vec{p}) = \frac{1}{2}\Big((u_{\uparrow}-iu^{c}_{\uparrow})-i(u_{\downarrow}+iu^{c}_{\downarrow})\Big)\,, \tag{22}$$

etc. As a consequence, the quantum states,

$$\Lambda^{\tau}_{k}(\vec{p})\mathbf{d}^{\tau}_{k}(\vec{p})|\vec{p}\rangle\,, \tag{23}$$

are of unspecified spin-projections (helicities). However, above conduct only reflects independence of C parity of spin-projection, and is not Majorana neutrino specific as claimed in Ref. [6], but avoidable, a fact already manifest in Eqs. (1), (4) above. One could have started from the very beginning with the smaller single-helicity Majorana spinors and constructed a local quantum field on top of

$$\begin{pmatrix} u_h + i\eta\, u^{c}_{h}\\ \tau\,(u_h - i\eta\, u^{c}_{h})\end{pmatrix}\mathbf{d}_{h\eta\tau}(\vec{p})\,,\ \eta=\pm 1,\quad \tau=\pm 1. \tag{24}$$

However, this peculiarity has no impact onto physical observables such like widths and cross sections. Indeed, in Ref. [8] we calculated that traces including two-storey spinors of unspecified h label always reduce to standard Dirac traces including $u_{h'}(\vec{p})$ spinors of well defined h'. Notice, finally, that also P parity does not distinguish between h and $-h$, and one can write the Dirac equation as $(\not{p}-m)(u_h \pm u_{-h})=0$. That one favors the well known single helicity version is not a consequence of parity but a tribute to angular momentum conservation.

3. Summary and Discussion

The merit of our work as we see it is to have (i) revealed existence of either real, or, imaginary C parity fermions, (ii) shown how to derive pure Majorana *local* quantum fields from first principles on space time symmetries, exploiting covariant, parity based discrete symmetries in eight spinorial dimensions, (iii) argued non-physicality of non-local theories as artifact of a lacking universal matrix representation of complex conjugation, (iv) observed that the possible indefiniteness of the h label (due to helicity independence of C parity) does not show up in the physics observables such as widths and cross-sections. In Ref. [8] we further showed that unpolarized beta decays do not distinguish between Dirac and Majorana fields. In polarized single β decays, the non-standard Majorana mass term left a footprint in triggering the drop out of the neutrino mass from the trace.

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