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HEAVY FAMILIES AND $N = 1$ SUPERGRAVITY WITHIN THE STANDARD MODEL*

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ABSTRACT

We study the few features of $N = 1$ supergravity coupled to the gauge interactions $SU(2)_L \times U(1)_Y \times SU(3)^C$ of the standard model in the presence of heavy families. We assume the minimal set of Higgs fields, i.e., two $SU(2)_L$ doublets $H_{1,2}$, the desert between $M_W = 100\ GeV$ and $M_G = 2.10^{16} GeV$ and perturbative values of the dimensionless parameters throughout this region. Using the numerical as well as the approximate analytic solution of the renormalization group equations, we study the evolution of *all* the parameters of the theory in the case of large ($\gtrsim 0.5$) Yukawa couplings for the fourth family. Yukawa couplings and certain mass parameters of the theory exhibit an interesting infrared behavior. We also investigate the implications of heavy families on the low energy structure of the theory. The desired spontaneous symmetry breaking of the electroweak symmetry with $M_W = 100\ GeV$ takes place only for a rather unnatural choice of the initial values of certain mass parameters at M_G. The vacuum expectation pattern $\langle H_1 \rangle \approx \langle H_2 \rangle \approx 123\ GeV$ emerges necessarily in an interplay of the tree level Higgs potential and its quantum corrections. The quark masses of the fourth family are $\widetilde{m}_U \approx \widetilde{m}_D \approx 135\ GeV$, to an accuracy of 10%, while the mass of the fourth charged lepton has an upper bound $\widetilde{m}_E \leq 90\ GeV$. Further characteristic features of the model are one light neutral Higgs field of mass $m_{H^0} \lesssim 50\ GeV$ and gluino masses $m_{\lambda_3} \lesssim 75\ GeV$.

1. Introduction

Locally supersymmetric gauge theories[1] provide an attractive way of linking, though not truly unifying gravity with other forces of nature. It is very intriguing that $N = 1$ supergravity (SG) in ten dimensions ($d = 10$) arises[2] as an effective field theory of superstrings,[3] which naturally incorporate gravity. Dimensionally reduced $N = 1$ SG models in $d = 4$ are therefore promising candidates for the effective theory which crosses the desert between the physics at grand unification mass scale and the physics at the presently accessible energies. Those theories are very attractive because they provide the most satisfactory mechanism for spontaneous breaking of local supersymmetry (SS).[4] Realistic scenarios have been based on specific grand unified groups,[5] the left-right symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$[6,7] and the left-handed electroweak symmetry $SU(2)_L \times U(1)_Y$.[8−11] In these models the electroweak symmetry breaking is induced by the soft SS breaking terms which arise from the spontaneous breakdown of local SS. Mass parameters in these terms are of the order of the gravitino mass $m_{3/2}$ which therefore sets the weak scale, i.e., $M_W = \mathcal{O}(m_{3/2})$.[4,5,8,9]★

If the soft SS breaking parameters do not evolve substantially from their initial values at $\mu_R = \mathcal{O}(M_{Pl})$, one cannot break $SU(2)_L \times U(1)_Y$ at the tree level of the Higgs potential unless one is willing to introduce a highly unattractive Higgs singlet chiral superfield. Here μ_R is the renormalization scale and M_{Pl} is the Planck mass where local SS is broken. However, it is reasonable that the renormalization of these parameters is substantial and that it is this renormalization which at $\mu_R = \mathcal{O}(M_W)$ triggers the spontaneous symmetry breaking (SSB) of the electroweak symmetry. In one class of such models[9,10,6] a large Yukawa coupling of the top quark is responsible for this SSB. It also leads to a mass exceeding the mass of the top quark candidates seen by the UA1 collaboration. Another class[10,11] uses the idea that the parameters at M_W leave the vacuum expectation values (VEV's) of the Higgs fields undetermined at the tree level of the potential

★ In Ref. 7 $m_{3/2}$ sets the scale of the right-handed vector boson.

unless one also includes radiative corrections of the Coleman-Weinberg type[12] in the effective potential. In this case one ends up with a light Higgs particle in the mass spectrum.

The purpose of this paper is to study the effects of additional heavy families on the low energy $N = 1$ SG theory in $d = 4$ with the gauge group:

$$G \equiv SU(2)_L \times U(1)_Y \times SU(3)^C \ . \tag{1.1}$$

We assume the desert between M_W and the grand unification mass scale M_G as well as perturbative values of the dimensionless parameters throughout this region. In particular we investigate the evolution of all the parameters according to the renormalization group equations (RGE's) when in addition to the usual e-, μ- and τ-families one has a heavier fourth family with larger Yukawa couplings.† We assume a minimal Higgs sector with two $SU(2)_L$ doublet fields $H_{1,2}$. We also investigate the SSB pattern and the low energy mass spectrum of the theory.

A supersymmetric model with additional heavy families may arise from the family unification models or from the $E_8 \times E_8$ heterotic string theory.[13] Therefore analysis of the influence of such additional families may have implications for the low energy phenomenology of such theories.

In the non-supersymmetric theory based on the gauge group G a careful study[14] of the RGE's has shown that the Yukawa couplings of heavy families approach a stable infrared fixed point determined by the gauge couplings. As we shall see (see also Ref. 15) the supersymmetric case shows similar features. This in turn implies that the mass parameters of the theory evolve in a specific way, constraining the theory at the weak scale. Thus, the nature of the SSB pattern and the particle spectrum exhibit characteristic features which tightly constrain.

† In the case of more than four families we lose asymptotic freedom for the strong interactions and g_3 diverges below M_G.

The paper is organized as follows. In Sec. 2 we specify the model and the assumptions and fix the notation. We devote Section 3 to a study of the renormalization group evolution of the parameters, comparing the numerical results to the approximate analytic solution. In Sec. 4, the SSB pattern of the electroweak symmetry is investigated while the low energy mass spectrum is presented in Sec. 5. Conclusions are drawn in Sec. 6. For the sake of completeness we give the complete set of the RGE's for our model in Appendix A. The approximate analytic solution is presented in Appendix B.

2. The Model

In this section we shall present in detail the model and the assumptions used in the analysis.

Desert Hypothesis

We assume the group G of Eq. (1.1) to be the gauge symmetry of the theory between the weak scale $M_W = 100\ GeV$ and the grand unification scale $M_G = 2 \cdot 10^{16} GeV$. This enables us to study the undisturbed evolution of parameters over a wide range of energies from M_G down to M_W. This allows certain parameters to reach an infrared fixed point to a good accuracy as $\mu_R \to M_W$, independent of their initial values.

Local SS is broken at $M_{Pl} \sim 10^{18} GeV$, thus giving rise to the soft SS breaking mass parameters. We assume that the values of these mass parameters do not change substantially from M_{Pl} down to M_G. In that way the number of the initial values of the free parameters in the theory does not proliferate.

Perturbative Unification

We assume that all the dimensionless parameters have perturbative values between M_W and M_G. We are then allowed to analyze the RGE's using only one-loop beta functions.

Particle Content

We work with chiral superfields which transform under $SU(2)_L \times U(1)_Y \times SU(3)^C$

as follows:

$$(E_L)_f = (\underset{\sim}{2}, -\tfrac{1}{2}, \underset{\sim}{1}) \; ; (E_R)_f = (\underset{\sim}{1}, 1, \underset{\sim}{1}) \tag{2.1a}$$

$$(Q_L)_f = (\underset{\sim}{2}, \tfrac{1}{6}, \underset{\sim}{3}) \; ; (U_R)_f = (\underset{\sim}{1} \; -\tfrac{2}{3}, \underset{\sim}{\bar{3}}) \quad (D_R)_f = (\underset{\sim}{1}, \tfrac{1}{3}, \underset{\sim}{\bar{3}}) \tag{2.1b}$$

$$H_1 = (\underset{\sim}{2} \; -\tfrac{1}{2}, \underset{\sim}{1}) \quad H_2 = (\underset{\sim}{2}, \tfrac{1}{2} \; \underset{\sim}{1}) \; . \tag{2.1c}$$

Here $f = 1, 2, 3, 4$ denotes the family index. The fourth family therefore transforms in the *same* way as the first three families. The Higgs superfields (2.1c) are the minimal set for the supersymmetric extension of the standard model.

Superpotential

The most general renormalizable superpotential consistent with the particle content (2.1) has the following form:

$$g = E_R \Gamma_E E_L^T \epsilon H_1 + U_R \Gamma_U Q_L^T \epsilon H_2 + D_R \Gamma_D Q_L^T \epsilon H_1 + \mu H_1^T \epsilon H_2 \; . \tag{2.2}$$

Here $\epsilon = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $\Gamma_{E,U,D}$ are Yukawa matrices. Family indices are suppressed.

We neglect flavor-changing effects and therefore set the off-diagonal elements of $\Gamma_{E,U,D}$ to zero. The Yukawa couplings of the fourth family are assumed to be much larger than those of the other families.

$$(\Gamma_{E,U,D})_{44} \equiv h_{E,U,D} \gg (\Gamma_{E,U,D})_{ii} \; ; \quad i = 1, 2, 3 \tag{2.3}$$

However, $(\Gamma_{E,U,D})_{ii}$, $i = 1, 2, 3$, are not neglected in the RGE's.

Soft Supersymmetry Breaking Terms

In addition to the supersymmetric part of the Lagrangian we include the most general soft SS breaking terms as they arise from the spontaneous breakdown of

SG. These terms are of the following form:

$$\mathcal{L}_S = \mathcal{L}_g + \mathcal{L}_{S1} + \mathcal{L}_{S2} \tag{2.4}$$

where

$$\mathcal{L}_g = -\sum_{a=1}^{3} m_{\lambda_a} \lambda_a \lambda_a \tag{2.5a}$$

$$\begin{aligned} \mathcal{L}_{S1} = -\big[& E_R (m_E \Gamma_E) E_L^T \epsilon H_1 + U_R (m_U \Gamma_U) Q_L^T \epsilon H_2 + D_R (m_D \Gamma_D) Q_L^T \epsilon H_1 \\ & + m_{H_3} \mu H_1^T \epsilon H_2 \big] \end{aligned} \tag{2.5b}$$

$$\begin{aligned} \mathcal{L}_{S2} = -\big[& E_L^\dagger m_{E_L}^2 E_L + E_R^\dagger m_{E_R}^2 E_R + Q_L^\dagger m_{Q_L}^2 Q_L + D_R^\dagger m_{D_R}^2 D_R \\ & + U_R^\dagger m_{U_R}^2 U_R + H_1^\dagger m_{H_1}^2 H_1 + H_2^\dagger m_{H_2}^2 H_2 \big] \ . \end{aligned} \tag{2.5c}$$

The fields here denote the scalar components of the appropriate superfields. The subscript $a = 1,2,3$ refers to the gauge group $U(1)_Y$, $SU(2)_L$ and $SU(3)^C$, respectively and again we have suppressed the family indices. The mass matrices $(m_{E,U,D} \times \Gamma_{E,U,D})$ and $m^2_{E_L,E_R,Q_L,U_R,D_R}$ are chosen to be flavor diagonal. Here $m^2_{H_1}$, $m^2_{H_2}$ and $m^2_{H_3}$ denote the three mass parameters of the Higgs fields $H_{1,2}$.

In order to get as close as possible to the experimentally determined values for the gauge coupling constants as extracted from Ref. 16, we set

$$\sqrt{\frac{5}{3}}\, g_1^0 = g_2^0 = g_3^0 = g_0 = 0.96 \tag{2.6}$$

at $M_G = 2 \cdot 10^{16} GeV$. This value is determined to about 1% to 2% by integrating the RGE's for our particle content. We also assume that at M_G the soft SS breaking mass parameters have the following symmetry:

$$m_{\lambda_1} = m_{\lambda_2} = m_{\lambda_3} = m_\lambda^0 \tag{2.7a}$$

$$m_{H_3} = m^0_{H_3} \tag{2.7b}$$

$$m_E = m_U = m_D = m_0 \tag{2.7c}$$

$$m^2_{H_1} = m^2_{H_2} = m^2_{E_L} = m^2_{E_R} = m^2_{Q_L} = m^2_{U_R} = m^2_{D_R} = m^2_{3/2} \; . \tag{2.7d}$$

Here $m^0_{H_3}$ and m_0 are of the order of the gravitino mass $m_{3/2}$. The gaugino mass m^0_λ is a free parameter, which can be naturally smaller than $m_{3/2}$. This pattern of soft SS breaking mass parameters emerges from the hidden sector mechanism,[4] which spontaneously breaks the local SS at M_{Pl}; by assumption the pattern persists down to M_G.

3. Evolution of the Parameters

The coupling constants and the masses of our model evolve from the unification scale M_G to the weak scale M_W according to the renormalization group equations (RGE's) given in Appendix A. Their exact solution for the gauge couplings and gaugino masses are examined in Sec. 3.1. Numerical results and the approximate analytic expressions for the parameters of the superpotential and for the soft SS breaking mass parameters are presented in Sec. 3.2 and 3.3, respectively.

3.1 Gauge Couplings and Gaugino Masses

The solution of the RGE's for the gauge couplings and gaugino masses with initial conditions (2.6) and (2.7a) is of the following form:

$$g_1^2 = \frac{g_1^{02}}{1 - 2\left(\frac{10}{3} N_f + 1\right) g_1^{02}} \tag{3.1a}$$

$$g_2^2 = \frac{g_2^{02}}{1 - 2(2N_f - 5)\, g_2^{02}} \tag{3.1b}$$

$$g_3^2 = \frac{g_3^{02}}{1 - 2(2N_f - 9)\, g_3^{02}} \tag{3.1c}$$

and

$$m_{\lambda_i} = m_\lambda^0 \, \frac{g_i^2}{g_i^{02}} \qquad i = 1, 2, 3 \tag{3.2}$$

Here t is related to the renormalization mass scale μ_R in the following way:

$$t \equiv \frac{1}{16\pi^2} \, \ell n \, \frac{\mu_R}{M_G} \tag{3.3}$$

and N_f denotes the number of families. If $N_f > 4$ we lose asymptotic freedom for the strong interactions and g_3 diverges below M_G. This fact allows us to restrict our study to $N_f = 4$.

In principle one can use the running g_i's as parameters of other RGE's and numerically integrate those equations to find the evolution of other parameters. We have done that. But, we also find a way to approximate these equations. Figure 1 shows that g_2^2 and g_3^2 evolve slowly changing at most by a factor of two between M_G and M_W. One may then expect to obtain a reasonably good approximate analytic solution of the RGE's for other parameters of the theory if one set for all μ_R:

$$g_1 = \tfrac{1}{2}[g_1(\mu_R = M_W) + g_1^0] = 0.55 \tag{3.4a}$$

$$g_2 = \tfrac{1}{2}\left[g_2(\mu_R = M_W) + g_2^0\right] = 0.81 \tag{3.4b}$$

$$g_3 = \tfrac{1}{2}\left[g_3(\mu_R = M_W) + g_3^0\right] = 1.09 \tag{3.4c}$$

and

$$m_{\lambda_1} = \tfrac{1}{2}\left[\frac{g_1^2(\mu_R = M_W)}{g_1^{02}} + 1\right] m_\lambda^0 = 0.62\, m_\lambda^0 \tag{3.5a}$$

$$m_{\lambda_2} = \tfrac{1}{2}\left[\frac{g_2^2(\mu_R = M_W)}{g_2^{02}} + 1\right] m_\lambda^0 = 0.73\; m_\lambda^0 \tag{3.5b}$$

$$m_{\lambda_3} = \tfrac{1}{2}\left[\frac{g_3^2(\mu_R = M_W)}{g_3^{02}} + 1\right] m_\lambda^0 = 1.31 m_\lambda^0 \ . \tag{3.5c}$$

One also has $g_1^2 \ll g_{2,3}^2$ and g_1^2 usually appears in the RGE's with a smaller coefficient than $g_{2,3}^2$. In most cases we are allowed to neglect g_1^2 and $g_1^2 m_{\lambda_1}^2$ compared to $g_{2,3}^2$ and $g_{2,3}^2 m_{\lambda_2,\lambda_3}^2$, respectively. Together with (2.3) we also take the limits:

$$|Z| \equiv \left|\frac{h_U - h_D}{h_U + h_D}\right| \ll 1 \ , \qquad h_E \ll h_{U,D} \ . \tag{3.6}$$

It is natural to assume that also for the fourth family $h_E^0 < h_{U,D}^0$, where $h_{E,U,D}^0 \equiv h_{E,U,D}\,(\mu_R = M_G)$. Then both approximations in Eq. (3.6) are justified from the evolution of $h_{E,U,D}$ as given in Sec. 3.2. This will allow us to solve the RGE's for the other parameters analytically to an accuracy of 10% to 30%. In all cases, we have checked our analytic estimates against our exact numerical treatment.

3.2 Parameters of the Superpotential

For the Yukawa couplings of the fourth family $h_{E,U,D}$ one obtains the following analytic solution:

$$h_U^2 = \frac{\overline{g}^2}{1 - X}\left[1 + Z_0\left[\frac{X}{X_0}\left(\frac{1 - X_0}{1 - X}\right)\right]^{5/7} + \mathcal{O}(Z_0^2)\right] \tag{3.7a}$$

$$h_D^2 = \frac{\overline{g}^2}{1 - X}\left[1 - Z_0\left[\frac{X}{X_0}\left(\frac{1 - X_0}{1 - X}\right)\right]^{5/7} + \mathcal{O}(Z_0^2)\right] \tag{3.7b}$$

$$h_E^2 = h_E^{02}\left[\left(\frac{X}{X_0}\right)^{1-\eta}\left(\frac{1 - X_0}{1 - X}\right)\right]^{3/7} J(x)\left[1 + \mathcal{O}(Z_0^2)\right] \tag{3.7c}$$

where

$$X = X_0 \exp(14\overline{g}^2 t) \ , X_0 = 1 - \frac{\overline{g}^2}{h_0^2} \tag{3.8a}$$

$$h_0 = \frac{h_U^0 + h_D^0}{2} \; , \quad Z_0 = \frac{h_U^0 - h_D^0}{h_U^0 + h_D^0} \tag{3.8b}$$

$$\overline{g}^2 = \frac{1}{7}\left(\frac{16}{3} g_3^2 + 3g_2^2\right) \qquad \eta = \frac{g_2^2 + g_1^2}{\overline{g}^2} \tag{3.8c}$$

$$J(x) = \left\{1 + \frac{4}{7}\frac{h_E^{02}}{\overline{g}^2}\int_X^{X_0}\frac{dX}{X}\left[\left(\frac{X}{X_0}\right)^{1-\eta}\left(\frac{1-X_0}{1-X}\right)\right]^{3/7}\right\}^{-1} \tag{3.8d}$$

and t is defined in Eq. (3.3). Here 0 refers to the initial values of parameters at M_G. Evidently h_U and h_D approach the same infrared fixed point:

$$\lim_{\mu_R \to 0} h_{U,D} = \overline{g} = 1.09 \; . \tag{3.9a}$$

At $\mu_R = 100$ GeV one has $X/X_0 = 0.031 \ll 1$ and consequently as long as $h_0 \geq 0.5$ (i.e., $X_0 \gtrsim -4$), $h_{U,D}^0$ assume their asymptotic values (3.9a) at M_W to an accuracy of 10%. Therefore we define a heavy family by requiring $h_{U,D}^0 \geq 0.5$. For comparison $(\Gamma_U^0)_{33}$, for the top quark is of order 0.2.

For h_E from (3.7c) one obtains the following infrared fixed point:

$$\lim_{\mu_R \to 0} h_E = 0 \; . \tag{3.9b}$$

This value is not reached at M_W because $[(X/X_0)^{1-\eta}]^{3/7} \sim 0.75 \sim \mathcal{O}(1)$. Instead we obtain the following estimate:

$$h_E^2(\mu_R = M_W) = Min\,(h_E^{02}, \overline{g}^2) \times \mathcal{O}(\tfrac{1}{2}) \; . \tag{3.10}$$

In Figs. 2 and 3 we compare the approximate formulae (3.7) to the numerical solution of the full one-loop RGE's for $h_{U,D}$ and h_E, respectively. One sees that the two solutions are in good agreement.

The evolution properties of $h_{E,U,D}$ also justify approximation (3.6) which we use in order to get the analytic solution for the evolution of the soft SS breaking mass parameters.

The Yukawa couplings of the first three families are small compared to $\overline{g} = 1.09$, i.e., $(\Gamma_{E,U,D})_{ii} \ll \overline{g}$ where $i = 1, 2, 3$. Approximate expressions for these Γ's are given in Appendix B; these have the characteristic feature that when $\mu_R \to M_W$, $(\Gamma_E)_{ii}$ decrease while $(\Gamma_{U,D})_{ii}$ increase.

The approximate analytic solution for the evolution of the mass parameter μ of the superpotential (see Eq. (2.2)) has the following form:

$$\mu = \mu_0 \left[\left(\frac{X}{X_0} \right) \left(\frac{1 - X_0}{1 - X} \right) \right]^{3/7} \exp(-3 g_2^2 t) \qquad (3.11)$$

when X and X_0 are defined in Eq. (3.8a) and $\mu_0 = \mu\ (\mu_R = M_G)$. This result is in good agreement with the full numerical solution as seen in Fig. 4. From (3.11) it follows that μ approaches the infrared fixed point $\mu = 0$. However, at $\mu_R = M_W$, μ need not reach the fixed point, especially when μ_0 is large compared to the other mass parameters and $h^0_{U,D}$ are not much larger than $\mathcal{O}(\overline{g})$. In Sec. 4 we show that this feature allows us to break $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$ at $\mu_R \sim 100\, GeV$.

3.3 Soft Supersymmetry Breaking Mass Parameters

The RGE's for the soft SS breaking mass parameters are complicated (see Appendix A). In order to understand the structure of the numerical solutions it is necessary to study the approximate analytic results. The approximate equations are tractable if one makes use of (2.3), (3.4), (3.5) and (3.6).

We are especially interested in the infrared behavior, i.e., $\mu_R \to M_W$, of those parameters which are relevant for the proper breaking of $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$. These are the mass parameters m_{H_3}, $m^2_{H_1}$ and $m^2_{H_2}$ which appear in the terms with the doublet fields $H_{1,2}$, only (see Eqs. (2.5b,c)). However, we shall also comment on the evolution of other SS breaking parameters which are relevant for the particle mass spectrum of sleptons and squarks. The complete set of approximate analytic solutions is given in Appendix B.

The evolution of m_{H_3} is approximated by

$$m_{H_3} = m_{H_3}^0 - \frac{6}{7}\left[m_0 - \overline{m}_\lambda - \left(\overline{m}_\lambda - \frac{g_2^2}{2\overline{g}^2}\, m_{\lambda_2}\right) \ell n \left(\frac{X}{X_0}\right)\right]$$
$$+ \frac{6}{7}\,\frac{X/X_0}{1-X}\left[(1-X_0)\,(m_0 - \overline{m}_\lambda) + X_0 \overline{m}_\lambda\, \ell n \left(\frac{X}{X_0}\right)\right] \qquad (3.12)$$

where X and X_0 are defined in Eq. (3.8a),

$$\overline{m}_\lambda = \left(\frac{16}{3}\, g_3^2 m_{\lambda_3} + 3 g_2^2 m_{\lambda_2}\right) \Big/ \left(\frac{16}{3}\, g_3^2 + 3 g_2^2\right) = 1.18\, m_\lambda^0 \ , \qquad (3.13a)$$

and $m_{H_3}^0$, m_0 and m_λ^0 are the initial parameters at M_G given by Eqs. (2.7). The approximate analytic solution for $m^2_{H_1,H_2}$ to leading order in Z and h_E (see (3.6)) has the following form:

$$m_{H_1}^2 = m_{H_2}^2 = -\frac{2}{7}\, m_{3/2}^2 + \frac{6}{7}\, \tilde{m}^2 \ell n \left(\frac{X}{X_0}\right) + \frac{3}{7}\,\Sigma \qquad (3.14)$$

where

$$\Sigma \equiv \frac{1}{2}\, (m_{H_1}^2 + m_{H_2}^2) + m_{Q_L}^2 + \frac{1}{2}\, (m_{U_R}^2 + m_{D_R}^2) = (\overline{m}^2 - \overline{m}_\lambda^2)$$
$$+ \frac{X/X_0}{1-X}\, \Big\{ (1-X_0) \Big[\Sigma_0 - (m_0 - \overline{m}_\lambda)^2 \left(\frac{1 - X/X_0}{1-X}\right)$$
$$+ 2(m_0 - \overline{m}_\lambda)\overline{m}_\lambda\, \frac{1}{1-X}\, \ell n \left(\frac{X}{X_0}\right)$$
$$- (\overline{m}^2 - \overline{m}_\lambda^2)\Big] + X_0 \left[\overline{m}^2 + \overline{m}_\lambda^2\, \frac{1}{1-X}\, \ell n \left(\frac{X}{X_0}\right)\right] \ell n \left(\frac{X}{X_0}\right) \Big\} \qquad (3.15)$$

and

$$\overline{m}^2 = \left[\frac{32}{3}\, g_3^2\, m_{\lambda_3}^2 + 6g_2^2 m_{\lambda_2}^2\right] \Big/ \left(\frac{16}{3}\, g_3^2 + 3g_2^2\right) = 2.89\, (m_\lambda^0)^2 \tag{3.13b}$$

$$\widetilde{m}^2 = \left[\frac{16}{3}\, g_3^2 m_{\lambda_3}^2 - \frac{1}{2}\, g_2^2 m_{\lambda_2}^2\right] \Big/ \left(\frac{16}{3}\, g_3^2 + 3g_2^2\right) = 1.30\, (m_\lambda^0)^2\ . \tag{3.13c}$$

Here $m^2_{Q_L,U_R,D_R}$ refer to the mass parameters corresponding to the fourth family. A subscript 0 denotes again the values of the corresponding parameters at M_G and thus, $\Sigma_0 = 3m^2_{3/2}$. In Figs. 5 and 6 we plot the evolution of m_{H_3} and $m^2_{H_1,H_2}$, respectively. One sees that both the numerical and the analytic solution are in good agreement.

From Eq. (3.12) and (3.14) one observes that none of these parameters approach any infrared fixed point. They decrease as $\mu_R \to M_W$ and assume the following value at $\mu_R = 100\, GeV$:

$$m_{H_3} \approx m^0_{H_3} - 0.86\, m_0 - 1.89\, m^0_\lambda \tag{3.15a}$$

$$m^2_{H_1} \approx m^2_{H_2} \approx -\frac{2}{7}\, m^2_{3/2} - 3.86\ m^{02}_\lambda\ . \tag{3.15b}$$

The value of m_{H_3} depends linearly on its initial value $m^0_{H_3}$ at M_G. Since $m^0_{H_3}$ does not appear in the evolution equations for other mass parameters and thus its value is not restricted; thus m_{H_3} remains a free parameter of the model.

The mass parameters $m^2_{H_1}$ and $m^2_{H_2}$ approach the same value as $\mu_R \to M_W$ even if at M_G one has $h^0_U \neq h^0_D$ and $h^0_E \sim \mathcal{O}(\overline{g})$. This is a consequence of the fact that as $\mu_R \to M_W$, h_U and h_D assume the same fixed point value and h_E decreases. Also, in the RGE's h^2_E appears with a smaller coefficient than the one in front of $h^2_{U,D}$. The latter arises from the color degrees of freedom. Therefore if one assumed $h_U \approx h_D$ and $h_E \approx 0$ the RGE's for $m^2_{H_1}$ and $m^2_{H_2}$ become equivalent (see Appendix A) and then the evolution of these two parameters is the same.

From Eq. (3.16b) we also see that at $\mu_R \approx 100\,GeV$, $m^2_{H_1,H_2}$ are *negative.* This can be understood by examining the RGE's for $m^2_{H_1,H_2}$ and $m^2_{Q_L,U_R,D_R}$. Let us assume first that the gaugino masses are zero. In this case the relation between the beta functions for $m^2_{H_1,H_2}$ and $m^2_{Q_L,U_R,D_R}$ is the following:

$$\frac{d}{dt}\, m^2_{H_1,H_2} \approx \frac{3}{2}\, \frac{d}{dt}\, m^2_{Q_L,U_R,D_R} \geq 0 \ . \tag{3.17}$$

Since $\frac{d}{dt}\, m^2_{H_1,H_2} > \frac{d}{dt}\, m^2_{Q_L,U_R,U_D}$ it follows that $m^2_{H_1,H_2}$ decrease at a larger rate than $m^2_{Q_L,U_R,D_R}$ and therefore $m^2_{H_1,H_2} < m^2_{Q_L,U_R,D_R}$ for all $\mu_R < M_G$. On the other hand we see from Eq. (3.15) that

$$\Sigma \equiv \frac{1}{2}\,(m^2_{H_1} + m^2_{H_2}) + m^2_{Q_L} + \frac{1}{2}\,(m^2_{U_R} + m^2_{D_R}) \approx m^2_{H_1,H_2} + 2m^2_{Q_L,U_R,D_R} \to 0$$

as $\mu_R \to M_W$. This implies that at M_W, $m^2_{H_1,H_2} \approx -2m^2_{Q_L,U_R,D_R}$ and therefore the Higgs masses $m^2_{H_1,H_2}$ are necessarily *negative* while the squark masses are positive. This feature persists even in the case of nonzero gaugino masses because the beta function for $m^2_{Q_L,U_R,D_R}$ gets an additional negative contribution from gluino masses and it is therefore even smaller than the beta function for $m^2_{H_1,H_2}$. The above analysis is in agreement with the quantitative results of Fig. 6 and Fig. 7 where the evolution of $m^2_{H_1,H_2}$ and $m^2_{Q_L,D_R,U_R}$, respectively, are shown.

The result that at $\mu_R = 100\ GeV$, $m^2_{H_1,H_2}$ are approximately equal, negative and of order of the gravitino and/or gaugino masses has strong implications for the nature of SSB of $SU(2)_L \times U(1)_Y$. We discuss this in Sec. 4.

For the sake of completeness we also state the evolution of the soft SS breaking parameters $m_{U,D}$ for the fourth family:

$$m_U \approx m_D = \overline{m}_\lambda + \frac{X/X_0}{1-X}\ \left[(1-X_0)(m_0 - \overline{m}_\lambda) + X_0 \overline{m}_\lambda \ell n \left(\frac{X}{X_0}\right)\right] \ . \tag{3.18}$$

Here m_0 is defined in Eq. (2.7c), $\overline{m}_\lambda$ in Eq. (3.13a) while X and X_0 are defined in Eq. (3.8a). Thus, $m_{U,D}$ approach the infrared fixed point $\overline{m}_\lambda = 1.81\, m^0_\lambda$. Graphs of $m_{U,D}$ are given in Fig. 8.

The analytic expression for m_E and $m^2_{E_L,E_R}$ for the fourth family have a complicated form and are given in Appendix B. The value of m_E decreases as $\mu_R \to M_W$. If $h^0_E = \mathcal{O}(\bar{g})$ and $m^0{}_E \gg m^0{}_{E_L,E_R}$ it may be the case that at $\mu_R \sim 100\ GeV$ one ends up with $m^2_{E_L,E_R} < 0$. In this case the solution which preserves $U(1)_{em}$ is a saddle point, because the slepton masses are imaginary. One may avoid such a pathological behavior by choosing the initial conditions $h^0_E \lesssim \mathcal{O}(\bar{g})$ and/or $m^0_E \lesssim m^0_{E_L,E_R}$. Numerical solution for m_E and $m^2_{E_L,E_R}$ is presented in Figs. 9 and 10.

We briefly comment on the evolution of the soft SS breaking mass parameters for the first three families. The mass parameters $(m_{E,U,D} \times \Gamma_{E,U,D})_{ii}$, $i = 1,2,3$, are small compared to $m_{E,U,D} \times h_{E,U,D}$ because the Yukawa couplings $(\Gamma_{E,U,D})_{ii}, i = 1,2,3$, are smaller than $h_{E,U,D}$. The slepton and squark masses for the first three families $(m^2_{E_L,E_R,Q_L,U_R,D_R})_{ii}$, $i = 1,2,3$, evolve with a negative beta function which is in the leading order proportional to the product of the squares of gaugino masses and gauge couplings. Therefore at $\mu_R = 100$ GeV these masses are in general $m^2_{3/2} + \mathcal{O}(m^{02}_\lambda)$. Explicit expressions are given again in Appendix B.

4. Spontaneous Symmetry Breaking

The fixed point behavior of large Yukawa couplings determines to a large extent the magnitude and the symmetry pattern of the mass parameters in our model. In this section we study the implications for the spontaneous breakdown of $SU(2)_L \times U(1)_Y$ as they arise from the structure of the Higgs potential.

The SSB pattern should be compatible with the low-energy phenomenology, therefore it should ensure $Max[\langle H^0_1 \rangle, \langle H^0_2 \rangle] = \mathcal{O}(M_W)$ while the VEV's of other scalar fields must be zero. Here the superscript 0 denotes the neutral component of the field. For the sake of further discussion we shall give here the part of the tree level potential which depends on the Higgs fields $H_{1,2}$ fields, only:*

$$V_{TL} = (m^2_{H_1} + \mu^2)H_1^\dagger H_1 + (m^2_{H_2} + \mu^2)H_2^\dagger H_2 - \mu m_{H_3}(H_1^T \epsilon H_2 + h.c.)$$

$$\frac{g_2^2}{8} \sum_{a=1}^{3}(H_1^\dagger \tau_a H_1 - H_2^\dagger \tau_a H_2)^2 + \frac{g_1^2}{8} (H_1^\dagger H_1 - H_2^\dagger H_2)^2 \ . \qquad (4.1)$$

The mass parameters μ, m_{H_3} and $m^2_{H_1,H_2}$ are defined in Eqs. (2.2), (2.5b) and (2.5c), respectively, τ_a are Pauli matrices and $\epsilon = i\tau_2$. Since the RGE's lead to approximately equal values of $m^2_{H_1}$ and $m^2_{H_2}$ at $\mu_R = \mathcal{O}(M_W)$ (see Eq. (3.14)), the minimization of V_{TL} yields the VEV pattern:

$$|\langle H_1^0 \rangle| = |\langle H_2^0 \rangle| = H/2 \ . \qquad (4.2)$$

This pattern is correct up to order $(m^2_{H_1} - m^2_{H_2})\Big/(m^2_{H_1} + m^2_{H_2}) \lesssim 10\%$ (see also Fig. 6).

Potential V_{TL} as a function of the real VEV H is then given by:

$$V_{TL} = \frac{1}{2}\, m^2_{\mathcal{H}} H^2 \qquad (4.3a)$$

where

$$m^2_{\mathcal{H}} = \frac{1}{2}\,(m^2_{H_1} + m^2_{H_2}) + \mu^2 - |\mu m_{H_3}| \ . \qquad (4.3b)$$

Obviously, if $m^2_{\mathcal{H}} > 0$ the system has a minimum at $H = 0$, while for $m^2_{\mathcal{H}} < 0$, V_{TL} is unbounded from below. In such a situation one has to include quantum corrections to the tree level Higgs potential V_{TL}. This may be achieved by regarding $m^2_{\mathcal{H}}$ as a function of H, i.e., $m^2_{\mathcal{H}} = m^2_{\mathcal{H}}(\mu_R = H)$, or by improving the potential à la Coleman-Weinberg.[12] We thus observe that the model determines

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* All the squarks and sleptons should have zero VEV's. In the Higgs potential those fields appear in the bilinear combination, and therefore the extremum solutions are trivially satisfied. We shall prove later that such a VEV pattern for squarks and sleptons also satisfies constraints for the minimum.

the nature of the SSB of $SU(2)_L \times U(1)_Y$ down to $U(1)_{em}$; *the SSB is necessarily radiative*, i.e., quantum corrections to the tree level Higgs potential determine the magnitude of H.

The stable minimum of the potential occurs at the scale μ_R where $m_{\not{H}}^2 \sim 0$ and $H = \mathcal{O}(\mu_R)$. From Eq. (3.14) one observes that at $\mu_R = M_W$, $m^2_{H_1,H_2}$ are *negative* and large, i.e., of order of the gaugino and/or the gravitino mass. Also, μ approaches the fixed point value zero (see Eq. (3.11)). Therefore, $m_{\not{H}}^2$ is generally *negative* and *large*. This implies that the radiative SSB of $SU(2)_L \times U(1)_Y$ takes place typically at $\mu_R \gg M_W$, yielding $H = \mathcal{O}(\mu_R) \gg M_W$. This of course contradicts $H = 245$ MeV which is obtained from the experimentally observed $W^\pm$ and Z^0-boson masses. This implies that *without careful adjustments of the mass parameters at M_G, this model is not consistent with low energy phenomenology.*

In order to obtain a realistic scenario one has to choose $\mu_0 \equiv \mu(\mu_R = M_G)$ in such a way that $m_{\not{H}}^2(\mu_R = H)$ assumes a value close to zero, i.e., $m_{\not{H}} \ll \mathcal{O}(m_\lambda^0, m_{3/2})$ at M_W. From the expressions for $m_{\not{H}}^2$, μ, m_{H_3} and $m^2_{H_1,H_2}$ as given in Eqs. (4.3), (3.11), (3.12) and (3.14), respectively, one obtains the following constraint on μ_0 in terms of the initial values $h_0 = (h_U^0 + h_D^0)/2$, m_λ^0, m_0, $m_{H_3}^0$ and $m_{3/2}$ (see Eqs. (2.7)):

$$\mu_0 \approx 2.85\,(h_0)^{6/7}\left(\sqrt{0.28m_{3/2}^2 + 3.86\,m_\lambda^{02} + 0.25m_{H_3}^2} + 0.5m_{H_3}\right) \qquad (4.4a)$$

with m_{H_3} being defined in Eq. (3.12). In order to obtain the desired SSB pattern and to avoid the tuning of parameters one has to choose the following relations among the parameters at M_G:

$$h_{U,D}^0 \lesssim 5\ , \quad \mu_0 \gtrsim 3m_{3/2}\ , \quad m_\lambda^0 \lesssim \mathcal{O}(m_{3/2})\ . \qquad (4.4b)$$

Also, $m_{H_3}^0$ has to be adjusted to ensure $m_{H_3} \ll m_{H_3}^0$ at the weak scale. If we relax any of the above constraints we have to introduce additional mass hierarchies in the model. Different initial values of the parameters at M_G which yield $m_{\not{H}} \sim 0$ at M_W are given in Table I.

To our knowledge there does not exist any model that satisfies naturally constraint $\mu_0 \gtrsim 3\, m_{3/2}$. For example theories[17] starting from a superpotential containing only dimensionless couplings cannot accommodate heavy families.

In the following we shall present the form of the quantum corrections to V_{TL} and the minimization of the total potential. In the case when H is larger than the soft SS breaking masses one can use the mass independent renormalization and sum all the powers of the leading logarithms. In this case one obtains the so-called renormalization group improved potential which is of the following form:

$$V_T = \frac{1}{2}\, m_{\mathcal{H}}^2(\mu_R = H)H^2 \ . \tag{4.5}$$

In the leading logarithm approximation, V_T has the following form:

$$V_T^1 = \frac{1}{2}\, m_{\mathcal{H}}^2(\mu_R = \mu_{LL})H^2 + V_{LL} \tag{4.6}$$

where

$$V_{LL} = \frac{1}{2}\, \frac{dm_{\mathcal{H}}^2}{dt}\bigg|_{t_{LL}} \times H^2\ \frac{1}{16\pi^2}\ \ell n\ \frac{H}{\mu_{LL}}\ , \tag{4.7}$$

and $t = \frac{1}{16\pi^2}\, \ell n\, \frac{\mu_{LL}}{M_G}$.

At $\mu_R = \mathcal{O}(M_W)$ one may actually encounter a situation when H is of order of the largest soft SS breaking mass parameters. In this case the leading logarithm formula (4.6) is changed quantitatively and assumes the following Coleman-Weinberg form:

$$V_T^2 = \frac{1}{2}\, m_{\mathcal{H}}^2(\mu_R = \mu_{LL})H^2 + V_{CW} \tag{4.8}$$

where

$$V_{CW} = \frac{3}{16\pi^2}\ \left\{ \sum_{i=1}^{2} \eta_i^2 \ell n\ \frac{\eta_i}{\mu_{CW}^2} - \frac{1}{8}\, h^4 H^4 \ell n\, \frac{h^2 H^2}{4\mu_{CW}^2} \right\} \tag{4.9}$$

and

$$\eta_{1,2} = m_{S_+}^2 + \frac{1}{4}\, h^2 H^2 \pm \sqrt{m_{S_-}^4 + \frac{1}{4}\, h^2 H^2 (m_+ - \mu \mathrm{sign}\, m_{H_3})^2} \tag{4.10a}$$

$$h = \frac{h_U + h_D}{2} \tag{4.10b}$$

$$m_+ = \frac{m_U + m_D}{2} \tag{4.10c}$$

$$m^2_{S_\pm} = \frac{1}{2} \left[m^2_{Q_L} \pm \frac{1}{2} \left(m^2_{D_R} + m^2_{U_R} \right) \right] \tag{4.10d}$$

The free parameter μ_{CW} is related to μ_{LL} in such a way that V_{CW} is identical to V_{LL} when H is much larger than the soft SS breaking masses. In expression (4.9) we have included only the leading contribution to V_{CW}. All the parameters in V_{LL} and V_{CW} are taken at the renormalization scale μ_{LL}.

We choose μ_{LL} to be a few TeV. Then we may safely assume the mass independent RGE's between μ_{LL} and M_G while the leading logarithm potentials V_T^1 or V_T^2 still provide a good approximation at M_W. We compare V_T, V_T^1 and V_T^2 in Fig. 11. The location of the minimum is different in each case. However, this difference is not very significant; it can be countered by changing the initial value of the parameters at M_G by a few percent. We may neglect the difference between V_T, V_T^1 and V_T^2 safely, since the two-loop corrections are expected to be of order 10%. The minimization of V_T yields a local minimum for the values of the parameters at M_G given in Table I. We have checked *numerically* that this minimum is also the *global minimum*.

5. Mass Spectrum

We compute the mass spectrum by diagonalizing numerically the tree level mass matrices with renormalization group improved parameters. These are readily obtained form the Lagrangian described in Sec. 2 and have been given in the literature.[18]

Since we assume the Coleman-Weinberg mechanism to be operative, the neutral Higgs mass matrix calculated in that fashion has an imaginary eigenvalue which we replace by the square root of the curvature of the potential V_T (see Eqs.

(4.5), (4.6) and (4.8)) at its minimum. Examples of mass spectra are presented in Figs. 12.

Our model predicts the quark masses of the fourth family. In Sec. 3 we have seen that the Yukawa couplings h_U and h_D approach the same fixed point $\overline{g} = 1.09$ at the weak scale. Since the only possible spontaneous symmetry breaking pattern is $|\langle H_1\rangle| \approx |\langle H_2\rangle| = 123$ GeV, we obtain up- and down-quark masses of 135 GeV to an accuracy of 10%. From Eq. (3.7c) for the evolution of h_E we derive the inequality $h_E \leq 0.65\overline{g}$ and we get an upper bound on the mass of the fourth lepton of 90 GeV. In most examples this mass is below 50 GeV.

Since $SU(2)_L \times U(1)_Y$ is broken radiatively, one neutral Higgs fields ends up light, i.e., in the range of 20-40 GeV.

In our model we are able to accommodate photino masses $m_{\tilde{\gamma}} = 0$-40 GeV. As $m_{\tilde{\gamma}}$ rises the unpleasant mass hierarchy $\mu/m_{3/2} > 3$ at M_G becomes even larger, as we easily see from Eq. (4.4). The model therefore prefers $m_{\tilde{\gamma}} \leq 10\ GeV$. This in turn implies gaugino masses $m_{\lambda_3} \lesssim 75\ GeV$.

Barring any further fine-tuning of parameters all the other superparticles acquire masses of order $m_{3/2}$. The radiative symmetry breaking mechanism we employ decouples the value of $\langle H_1\rangle$ from that of $m_{3/2}$. This is illustrated by examples (a) and (e) of Table I and Figs. 12a and 12b where we have chosen $m_{3/2} = 100$ GeV and $m_{3/2} = 200$ GeV, respectively. In principle it is even possible to shift the masses of the superpartners of the ordinary particles into the TeV region. Apart from an increasingly difficult tuning of parameters we then have $\langle H_1\rangle / m_{3/2} \approx 0.1$ and two-loop effects must be taken into account. Also, since we may have $m_{H_1}^2 - m_{H_2}^2 = \mathcal{O}(m_W^2)$, we are faced with the formidable problem of the Coleman-Weinberg analysis of a potential that is stabilized by quantum corrections in more than one direction.

It is also interesting to observe that for the case with $m_{3/2} = 200$ GeV the mass of the lightest neutral Higgs field can be larger than the mass of Z^0 boson (see Fig. 12b). This is differs from the results of Ref. 19 where the lightest neutral Higgs field cannot be heavier than Z^0 even in the case of radiative SSB. However, in our example the relation $\langle H\rangle < m_{3/2}$ is different from the assumption of Ref. 19 where the soft super symmetry breaking parameters are all of order or smaller than M_W.

6. Conclusions

We have studied a standard model in the context of $N = 1$ supergravity when a heavy fourth family is present. The theory has a minimal set of Higgs fields with the two Higgs doublets. The fourth family is a replication of the first three families with the same gauge transformation properties, but its Yukawa couplings are chosen to be large, i.e., ≥ 0.5. We also assume that a desert extends between $M_W = 100\ GeV$ and $M_G = 2.10^{16}$ GeV and that dimensionless parameters are perturbative through this region.

First we studied systematically the evolution of all the parameters of the theory from M_G down to M_W. We obtained the numerical as well as approximate analytic solutions. The Yukawa couplings and certain other mass parameters of the theory have an interesting infrared behavior: (i) The Yukawa couplings h_U and h_D for the up and down quarks of the four family approach the same infrared fixed point $\overline{g} \sim 1.1$, (ii) the masses $m^2_{H_1}$ and $m^2_{H_2}$ of the Higgs fields H_1 and H_2 decrease and approach the *same negative* value which is of order of the gravitino or gaugino mass, and (iii) the mass parameter μ in the superpotential approaches the infrared fixed point 0. The fact that $m^2_{H_1} \approx m^2_{H_2} < 0$ at $\mu_R = M_W$ forces the spontaneous symmetry breakdown of the electroweak symmetry to occur in an interplay between the tree level Higgs potential and its quantum corrections. The spontaneous symmetry breaking pattern is then $\langle H_1\rangle \approx \langle H_2\rangle$. However, in order to obtain $\langle H_1\rangle \approx 123$ GeV one has to choose unnatural initial values of the parameters at M_G: $\mu_0 > 3m_{3/2}$, $h^0_{U,D} \lesssim 5$, $m^0_\lambda < \mathcal{O}(m_{3/2})$ and $m^0_{H_3}$ has to be chosen so that $|m_{H_3}| \ll m^0_{H_3}$ at M_W. It remains to be seen whether these constraints can be derived from a grand unified theory.

Our model also imposes interesting restrictions on the particle mass spectrum. The up and down quarks of the fourth family have the same mass 135 GeV to accuracy of 10%, while the mass of the lepton has an upper bound of 90 GeV. Because of the radiative nature of the spontaneous symmetry breaking one ends up with one relatively light neutral Higgs field with a mass below 50 GeV. The gluino masses tend to be light, i.e., below 75 GeV. Masses of other particles, except fermions of the first three families, are in general in the region of 100 GeV.

We conclude that the large Yukawa couplings of the fourth family *have* strong implications on the low energy structure of the standard model within $N = 1$ supergravity. They determine the spontaneous symmetry breaking pattern and restrict the particle mass spectrum.

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APPENDIX A. Renormalization Group Equations

In the following we present the renormalization group equations for the model described in Sec. 2. They have been partially given in the literature.[20,21] We have derived them by calculating the infinite parts of the one-loop diagrams in the superfield and component field formulation of the most general renormalizable softly broken super-Yang-Mills theory with chiral matter fields. The regularization method employed was dimensional reduction which is equivalent to dimensional regularization for our purposes. The difference between the two is proportional to $\epsilon = (4-d)$ and hence has no effect on the residues of the simple poles in ϵ. Higher poles do not appear in a one-loop calculation.

Recently an independent evaluation using the effective potential approach has been given in Ref. 21, with identical results.

In the following equations we regard $\Gamma_{E,U,D}$, $(m_{E,U,D} \times \Gamma_{E,U,D})$ and $m^2_{E_L,E_R,Q_L,D_R,U_R}$ as matrices with family indices. S is defined to be

$$S = -m^2_{H_1} + m^2_{H_2} - trm^2_{E_L} + trm^2_{E_R} + trm^2_{Q_L} + trm^2_{D_R} - 2trm^2_{U_R} , \qquad (A.1a)$$

and

$$t = \frac{1}{16\pi^2}\, \ell n\, \frac{\mu_R}{M_G} \qquad (A.1b)$$

while N_f denotes the number of families.

Gauge couplings

$$\frac{d}{dt}\, g_1 = \left(\frac{10}{3}N_f + 1\right)\, g_1^3 \qquad (A.2)$$

$$\frac{d}{dt}\, g_2 = (2N_f - 5)\, g_2^3 \qquad (A.3)$$

$$\frac{d}{dt}\, g_3 = (2N_f - 9)\, g_3^3 \; . \qquad (A.4)$$

Yukawa couplings

$$\frac{d}{dt}\,\Gamma_E = \Gamma_E(tr\,\Gamma_E\Gamma_E{}^\dagger + 3tr\,\Gamma_D\Gamma_D{}^\dagger - 3g_1^2 - 3g_2^2) + 3\Gamma_E\Gamma_E{}^\dagger\Gamma_E \qquad (A.5)$$

$$\frac{d}{dt}\,\Gamma_D = \Gamma_D\left(tr\,\Gamma_E\Gamma_E{}^\dagger + 3tr\,\Gamma_D\Gamma_D{}^\dagger - \frac{7}{9}\,g_1^2 - 3g_2^2 - \frac{16}{3}\,g_3^2\right)$$
$$+\,3\Gamma_D\Gamma_D{}^\dagger\Gamma_D + \Gamma_D\Gamma_U{}^\dagger\Gamma_U \qquad (A.6)$$

$$\frac{d}{dt}\,\Gamma_U = \Gamma_U\left(3tr\,\Gamma_U\Gamma_U{}^\dagger - \frac{13}{9}\,g_1^2 - 3g_2^2 - \frac{16}{3}\,g_3^2\right)$$
$$+\,3\Gamma_U\Gamma_U{}^\dagger\Gamma_U + \Gamma_U\Gamma_D{}^\dagger\Gamma_D \qquad (A.7)$$

The supersymmetric mass parameter μ

$$\frac{d}{dt}\,\mu = \mu(3tr\,\Gamma_U\Gamma_U{}^\dagger + 3tr\,\Gamma_D\Gamma_D{}^\dagger + tr\,\Gamma_E\Gamma_E{}^\dagger - g_1^2 - 3g_2^2) \qquad (A.8)$$

Gaugino masses

$$\frac{d}{dt}\,m_{\lambda_1} = 2\left(\frac{10}{3}\,N_f + 1\right)\,m_{\lambda_1}g_1^2 \qquad (A.9)$$

$$\frac{d}{dt}\,m_{\lambda_2} = 2\left(2N_f - 5\right)\,m_{\lambda_2}g_2^2 \qquad (A.10)$$

$$\frac{d}{dt}\,m_{\lambda_3} = 2\left(2N_f - 9\right)\,m_{\lambda_3}g_3^2 \qquad (A.11)$$

The Mass parameters of the bilinear soft term

$$\frac{d}{dt}\,m_{H_3} = 2tr\,\Gamma_E{}^\dagger(m_E\Gamma_E) + 6tr\,\Gamma_D{}^\dagger(m_D\Gamma_D) + 6tr\,\Gamma_U{}^\dagger(m_U\Gamma_U) - 2m_{\lambda_1}g_1^2 - 6m_{\lambda_2}g_2^2 \qquad (A.12)$$

Mass parameters of the trilinear soft terms

$$\frac{d}{dt}\,(m_E\Gamma_E) = 4\Gamma_E\Gamma_E{}^\dagger(m_E\Gamma_E) + 5(m_E\Gamma_E)\Gamma_E{}^\dagger\Gamma_E$$

$$+ (m_E\Gamma_E)\left(tr\,\Gamma_E{\Gamma_E}^\dagger + 3tr\,\Gamma_D{\Gamma_D}^\dagger - 3g_1^2 - 3g_2^2\right)$$

$$+ 2\Gamma_E\left[tr\,{\Gamma_E}^\dagger(m_E\Gamma_E) + 3tr\,{\Gamma_D}^\dagger(m_D\Gamma_D) - 3m_{\lambda_1}g_1^2 - 3m_{\lambda_2}g_2^2\right] \quad (A.13)$$

$$\frac{d}{dt}(m_U\Gamma_U) = 4\Gamma_U{\Gamma_U}^\dagger(m_U\Gamma_U) + 2\Gamma_U{\Gamma_D}^\dagger(m_D\Gamma_D)$$

$$+ 5(m_U\Gamma_U){\Gamma_U}^\dagger\Gamma_U + (m_U\Gamma_U){\Gamma_D}^\dagger\Gamma_D$$

$$+ (m_U\Gamma_U)\left(3tr\,\Gamma_U{\Gamma_U}^\dagger - \frac{13}{9}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right) + 2\Gamma_U\left[3tr\,{\Gamma_U}^\dagger(m_U\Gamma_U)\right.$$

$$\left. - \frac{13}{9}m_{\lambda_1}g_1^2 - 3m_{\lambda_2}g_2^2 - \frac{16}{3}m_{\lambda_3}g_3^2\right] \quad (A.14)$$

$$\frac{d}{dt}(m_D\Gamma_D) = 4\Gamma_D{\Gamma_D}^\dagger(m_D\Gamma_D) + 2\Gamma_D{\Gamma_U}^\dagger(m_U\Gamma_U)$$

$$+ 5(m_D\Gamma_D){\Gamma_D}^\dagger\Gamma_D + (m_D\Gamma_D){\Gamma_U}^\dagger\Gamma_U$$

$$+ (m_D\Gamma_D)(3tr\,\Gamma_D{\Gamma_D}^\dagger + tr\,\Gamma_E{\Gamma_E}^\dagger - \frac{7}{9}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2)$$

$$2\Gamma_D\left[tr\,{\Gamma_E}^\dagger(m_E\Gamma_E) + 3tr\,{\Gamma_D}^\dagger(m_D\Gamma_D) - \frac{7}{9}m_{\lambda_1}g_1^2\right.$$

$$\left. - 3m_{\lambda_2}g_2^2 - \frac{16}{3}m_{\lambda_3}g_3^2\right] \quad (A.15)$$

Soft mass squares

$$\frac{d}{dt}m_{E_L}^2 = {\Gamma_E}^\dagger\Gamma_E m_{E_L}^2 + m_{E_L}^2{\Gamma_E}^\dagger\Gamma_E + 2(m_E\Gamma_E)^\dagger(m_E\Gamma_E) + 2m_{H_1}^2{\Gamma_E}^\dagger\Gamma_E$$

$$2{\Gamma_E}^\dagger m_{E_R}^2\Gamma_E - 2g_1^2|m_{\lambda_1}|^2 - 6g_2^2|m_{\lambda_2}|^2 - g_1^2S \quad (A.16)$$

$$\frac{d}{dt}m_{E_R}^2 = 2{\Gamma_E}^*{\Gamma_E}^T m_{E_R}^2 + 2m_{E_R}^2{\Gamma_E}^*{\Gamma_E}^T + 4(m_E\Gamma_E)^*(m_E\Gamma_E)^T$$

$$+ 4m_{H_1}^2{\Gamma_E}^*{\Gamma_E}^T + 4{\Gamma_E}^*m_{E_L}^2{\Gamma_E}^T - 8g_1^2|m_{\lambda_1}|^2 + 2g_1^2S \quad (A.17)$$

$$\frac{d}{dt} m_{Q_L}^2 = \Gamma_D{}^\dagger \Gamma_D m_{Q_L}^2 + m_{Q_L}^2 \Gamma_D{}^\dagger \Gamma_D + \Gamma_U{}^\dagger \Gamma_U m_{Q_L}^2$$

$$+ m_{Q_L}^2 \Gamma_U{}^\dagger \Gamma_U + 2 m_{H_1}^2 \Gamma_D{}^\dagger \Gamma_D$$

$$+ 2 m_{H_2}^2 \Gamma_U{}^\dagger \Gamma_U + 2 \Gamma_D{}^\dagger m_{D_R}^2 \Gamma_D + 2 \Gamma_U{}^\dagger m_{U_R}^2 \Gamma_U \qquad (A.18)$$

$$+ 2 (m_D \Gamma_D)^\dagger (m_D \Gamma_D) + 2 (m_U \Gamma_U)^\dagger (m_U \Gamma_U)$$

$$- \frac{2}{9} g_1^2 |m_{\lambda_1}|^2 - 6 g_2^2 |m_{\lambda_2}|^2 - \frac{32}{3} g_3^2 |m_{\lambda_3}|^2 + \frac{1}{3} g_1^2 \ S$$

$$\frac{d}{dt} m_{U_R}^2 = 2 m_{U_R}^2 \Gamma_U{}^* \Gamma_U{}^T + 2 \Gamma_U{}^* \Gamma_U{}^T m_{U_R}^2 + 4 m_{H_2}^2 \Gamma_U{}^* \Gamma_U{}^T + 4 \Gamma_U{}^* m_{Q_L}^{2T} \Gamma_U{}^T$$

$$+ 4 (m_U \Gamma_U)^* (m_U \Gamma_U)^T - \frac{32}{g} g_1^2 |m_{\lambda_1}|^2 - \frac{32}{3} g_3^2 |m_{\lambda_3}|^2 - \frac{4}{3} g_1^2 \ S \qquad (A.19)$$

$$\frac{d}{dt} m_{D_R}^2 = 2 m_{D_R}^2 \Gamma_D{}^* \Gamma_D{}^T + 2 \Gamma_D{}^* \Gamma_D{}^T m_{D_R}^2 + 4 m_{H_1}^2 \Gamma_D{}^* \Gamma_D{}^T + 4 \Gamma_D{}^* m_{Q_L}^{2T} \Gamma_D{}^T$$

$$+ 4 (m_D \Gamma_D)^* (m_D \Gamma_D)^T - \frac{8}{9} g_1^2 |m_{\lambda_1}|^2 - \frac{32}{9} g_3^2 |m_{\lambda_3}|^2 + \frac{2}{3} g_1^2 \ S \qquad (A.20)$$

$$\frac{d}{dt} m_{H_1}^2 = 2 m_{H_1}^2 (tr\, \Gamma_E \Gamma_E{}^\dagger + 3 tr\, \Gamma_D \Gamma_D{}^\dagger)$$

$$+ 2 tr\, (m_E \Gamma_E)(m_E \Gamma_E)^\dagger + 6 tr\, (m_D \Gamma_D)(m_D \Gamma_D)^\dagger \qquad (A.21)$$

$$+ 2 tr\, \Gamma_E (m_{E_L}^2 + m_{E_R}^2) \Gamma_E{}^\dagger + 6 tr\, \Gamma_D (m_{Q_L}^2 + m_{D_R}^2) \Gamma_D{}^\dagger$$

$$- 2 g_1^2 |m_{\lambda_1}|^2 - 6 g_2^2 |m_{\lambda_2}|^2 - g_1^2 \ S$$

$$\frac{d}{dt} m_{H_2}^2 = 6 m_{H_2}^2 tr\, \Gamma_U \Gamma_U{}^\dagger + 6 tr\, (m_U \Gamma_U)(m_U \Gamma_U)^\dagger \qquad (A.22)$$

$$+ 6 tr\, \Gamma_U (m_{Q_L}^2 + m_{U_R}^2) \Gamma_U{}^\dagger - 2 g_1^2 |m_{\lambda_1}|^2 - 6 g_2^2 |m_{\lambda_2}|^2 + g_1^2 S$$

APPENDIX B. Approximate Analytic Solution

We give the approximate analytical solution for all the parameters of the model. The approximations are justified in Sec. 3 and are of the following form:

$$(\Gamma_{E,U,D})_{44} \equiv h_{E,U,D} \gg (\Gamma_{E,U,D})_{ii} \ ; \quad i = 1,2,3 \tag{B.1a}$$

$$g_i = \tfrac{1}{2}\left[g_i(\mu_R = M_W) + g_i^0\right] \ ; \quad i = 1,2,3 \tag{B.1b}$$

$$m_{\lambda_i} = \tfrac{1}{2}\left[\frac{gi(\mu_R = M_W)^2}{g_i^{02}} + 1\right] m_\lambda^0 \ ; \quad i = 1,2,3 \tag{B.1c}$$

$$|Z| \equiv \left|\frac{h_U - h_D}{h_U + h_D}\right| \ll 1 \ , \quad h_E \ll h_{U,D} \tag{B.1d}$$

Here $(\Gamma_{E,U,D})_{ii}$ denote the Yukawa couplings for the i$^{\text{th}}$ family while $g_{1,2,3}$ and $m_{\lambda_1,\lambda_2,\lambda_3}$ are the gauge couplings and the gaugino masses for $U(1)_Y$, $SU(2)_L$ and $SU(3)^C$ gauge groups, respectively. The renormalization mass μ_R spans the range form the unification scale M_G down to the weak scale M_W. We use the following notation

$$t = \frac{1}{16\pi^2}\,\ell n\,\frac{\mu_R}{M_G} \tag{B.2a}$$

$$X = X_0 \exp(14\overline{g}^2 t) \ , \quad X_0 = 1 - \frac{\overline{g}^2}{h_0^2} \tag{B.2b}$$

$$h_0 = \frac{h_U^0 + h_D^0}{2} \ , \quad Z_0 = \frac{h_U^0 - h_D^0}{h_U^0 + h_D^0} \tag{B.2c}$$

$$\overline{g}^2 = \frac{1}{7}\left(\frac{16}{3}g_3^2 + 3g_2^2\right) \qquad \eta = \frac{g_2^2 + g_1^2}{\overline{g}^2} \tag{B.2d}$$

$$J(x) = \left\{1 + \frac{4}{7}\,\frac{h_E^{02}}{\overline{g}^2}\int_X^{X_0}\frac{dX}{X}\left[\left(\frac{X}{X_0}\right)^{1-\eta}\left(\frac{1-X_0}{1-X}\right)\right]^{3/7}\right\}^{-1} \tag{B.2e}$$

Subscript 0 denotes the values of parameters at $\mu_R = M_G$.

Yukawa couplings of the fourth family $h_{E,U,D}$

$$h_U^2 = \frac{\overline{g}^2}{1-X}\left[1 + Z_0\left[\frac{X}{X_0}\left(\frac{1-X_0}{1-X}\right)\right]^{5/7}\right] \tag{B.3a}$$

$$h_D^2 = \frac{\overline{g}^2}{1-X}\left[1 - Z_0\left[\frac{X}{X_0}\left(\frac{1-X_0}{1-X}\right)\right]^{5/7}\right] \tag{B.3b}$$

$$h_E^2 = h_E^{02}\left[\left(\frac{X}{X_0}\right)^{1-\eta}\left(\frac{1-X_0}{1-X}\right)\right]^{3/7} J(x) \tag{B.3c}$$

Yukawa couplings $(\Gamma_{E,U,D})_{ii}$ of the first three families ($i = 1,2,3$)

$$(\Gamma_E)_{ii} = ({\Gamma_E}^0)_{ii}\left[\left(\frac{X}{X_0}\right)^{1-\eta}\left(\frac{1-X_0}{1-X}\right)\right]^{3/14} \tag{B.4a}$$

$$(\Gamma_{U,D})_{ii} = (\Gamma_{U,D}^0)_{ii}\exp(-4\overline{g}^2 t)\left(\frac{1-X_0}{1-X}\right)^{3/14} \tag{B.4b}$$

Mass parameter μ of the superpotential

$$\mu = \mu_0\left[\left(\frac{X}{X_0}\right)\left(\frac{1-X_0}{1-X}\right)\right]^{3/7}\exp(-3g_2^2 t) \tag{B.5}$$

Mass parameters m_{H_3}, $m_{H_1}^2$ and $m_{H_2}^2$

$$\begin{aligned} m_{H_3} = m_{H_3}^0 &- \frac{6}{7}\left[m - \overline{m}_\lambda - \left(\overline{m}_\lambda - \frac{g_2^2}{2\overline{g}^2}m_{\lambda_2}\right)\ell n\left(\frac{X}{X_0}\right)\right] \\ &+ \frac{6}{7}\frac{X/X_0}{1-X}\left[(1-X_0)(m-\overline{m}_\lambda) + X_0\overline{m}_\lambda\,\ell n\left(\frac{X}{X_0}\right)\right] \end{aligned} \tag{B.6a}$$

$$m_{H_1}^2 \approx m_{H_2}^2 = -\frac{2}{7}m_{3/2}^2 + \frac{6}{7}\widetilde{m}^2\ell n\left(\frac{X}{X_0}\right) + \frac{3}{7}\Sigma \tag{B.6b}$$

where

$$
\begin{aligned}
\Sigma \equiv & \frac{1}{2}\left(m_{H_1}^2+m_{H_2}^2\right)+m_{Q_L}^2+\frac{1}{2}\left(m_{U_R}^2+m_{D_R}^2\right)=\left(\overline{m}^2-\overline{m}_\lambda^2\right) \\
& +\frac{X/X_0}{1-X}\left\{(1-X_0)\left[\Sigma_0-(m-\overline{m}_\lambda)^2\left(\frac{1-X/X_0}{1-X}\right)\right.\right. \\
& +2(m-\overline{m}_\lambda)\overline{m}_\lambda\frac{1}{1-X}\,\ell n\left(\frac{X}{X_0}\right) \\
& \left.\left.-\left(\overline{m}^2-\overline{m}_\lambda^2\right)\right]+X_0\left[\overline{m}^2+\overline{m}_\lambda^2\frac{1}{1-X}\ell n\left(\frac{X}{X_0}\right)\right]\ell n\left(\frac{X}{X_0}\right)\right\}
\end{aligned} \tag{B.7}
$$

and

$$\overline{m}_\lambda=\left(\frac{16}{3}g_3^2 m_{\lambda_3}+3g_2^2 m_{\lambda_2}\right)\Big/\left(\frac{16}{3}g_3^2+3g_2^2\right) \tag{B.8a}$$

$$\overline{m}^2=\left[\frac{32}{3}g_3^2\, m_{\lambda_3}^2+6g_2^2 m_{\lambda_2}^2\right]\Big/\left(\frac{16}{3}g_3^2+3g_2^2\right) \tag{B.8b}$$

$$\widetilde{m}^2=\left[\frac{16}{3}g_3^2 m_{\lambda_3}^2-\frac{1}{2}g_2^2 m_{\lambda_2}^2\right]\Big/\left(\frac{16}{3}g_3^2+3g_2^2\right)\ . \tag{B.8c}$$

Here parameters m_{Q_L,U_R,D_R}^2 refer to the mass parameters corresponding to the fourth family. Subscript 0 denotes again the values of the corresponding parameters at M_G. Thus, $m_{H_3}^0$, m_0 and m_λ^0 are the parameters defined in Eq. (2.7) and $\Sigma_0=3m_{3/2}^2$.

Soft supersymmetry breaking mass parameters $m_{U,D}$ and m_{Q_L,U_R,D_R}^2 corresponding to the fourth family

$$m_U\approx m_D=\overline{m}_\lambda+\frac{X/X_0}{1-X}\left[(1-X_0)(m_0-\overline{m}_\lambda)+X_0\overline{m}_\lambda\ell n\left(\frac{X}{X_0}\right)\right] \tag{B.9a}$$

$$m_{Q_L}^2=\frac{1}{7}m_{3/2}^2+\frac{2}{7}\Sigma-\frac{3}{7}\left(\widetilde{m}^2+\frac{1}{2}\frac{g_2^2}{g^2}m_{\lambda_2}^2\right)\ell n\left(\frac{X}{X_0}\right) \tag{B.9b}$$

$$m_{U_R}^2\approx m_{D_R}^2=\frac{1}{7}m_{3/2}^2+\frac{2}{7}\Sigma-\frac{3}{7}\left(\widetilde{m}^2-\frac{1}{2}\frac{g_2^2}{g^2}m_{\lambda_2}^2\right)\ell n\left(\frac{X}{X_0}\right) \tag{B.9c}$$

Mass parameters $\overline{m}_\lambda$ and $\widetilde{m}^2$ are defined in Eqs. (B.8).

Mass parameters $(m_{U,D})_{ii}$ and $(m^2_{Q_L,U_R,D_R})_{ii}$, $i = 1,2,3,$

$$(m_U)_{ii} \approx (m_D)_{ii} = \frac{3}{7}\,\overline{m}_\lambda + \frac{4}{7}\,\left[m_0 - \overline{m}_\lambda\,\ell n\left(\frac{X}{X_0}\right)\right]$$

$$+\frac{3}{7}\,\frac{X/X_0}{1-X}\,\left[(1-X_0)(m_0-\overline{m}_\lambda) + X_0\overline{m}_\lambda\,\ell n\left(\frac{X}{X_0}\right)\right] \quad (B.10a)$$

$$(m^2_{Q_L})_{ii} = m^2_{3/2} - \frac{1}{2}\,\overline{m}^2\,\ell n\left(\frac{X}{X_0}\right) \quad (B.10b)$$

$$(m^2_{U_R})_{ii} \approx (m^2_{D_R})_{ii} = m^2_{3/2} - \frac{1}{14}\,(\overline{m}^2 + 12\widetilde{m}^2)\,\ell n\left(\frac{X}{X_0}\right)\ . \quad (B.10c)$$

Mass parameters m_E and $m^2_{E_L,E_R}$ corresponding to the lepton of the fourth family

$$m_E \approx \frac{3}{7}\,\overline{m}_\lambda + m_0\left[J(x) - \frac{3}{7}\right] + \frac{3}{7}\,\left(\overline{m}_\lambda - \frac{g_2^2}{g^2}\,m_{\lambda_2}\right)\,\ell n\left(\frac{X}{X_0}\right)$$

$$+\frac{3}{7}\,\frac{X/X_0}{1-X}\,\left[(1-X_0)(m_0-\overline{m}_\lambda) + X_0\overline{m}_\lambda\,\ell n\left(\frac{X}{X_0}\right)\right] \quad (B.11a)$$

$$m^2_{E_L} \approx m^2_{3/2}\left[\frac{1}{3} + \frac{2}{3}\,J(x)^{3/4}\right] + \frac{1}{3}\,K(X) - \frac{3}{7}\,\frac{g_2^2}{g^2}\,m_{\lambda_2}\,\ell n\,\frac{X}{X_0} \quad (B.12b)$$

$$m^2_{E_R} \approx m^2_{3/2}\left[-\frac{1}{3} + \frac{4}{3}\,J(x)^{3/4}\right] + \frac{2}{3}\,K(X) \quad (B.11c)$$

where

$$K(X) = \frac{3}{7}\int_{X_0}^{X} \frac{h_E^2}{g^2}\,\left[\frac{(m^2_{H_1} + m^2_{H_2})}{2} + m_E^2\right]\,\frac{dX}{X}\ . \quad (B.12d)$$

Mass parameters $(m_E)_{ii}$ and $(m^2_{E_L}, m^2_{E_R})_{ii}$, $i = 1,2,3$

$$(m_E)_{ii} = \frac{4}{7}\,m_0 + \frac{3}{7}\,\overline{m}_\lambda + \frac{3}{7}\,\left(\overline{m}_\lambda - \frac{g_2^2}{g^2}\,m_{\lambda_2}\right)\,\ell n\left(\frac{X}{X_0}\right)$$

$$+\frac{3}{7}\frac{X/X_0}{1-X}\left[(1-X_0)(m_0-\overline{m}_\lambda)+X_0\overline{m}_\lambda\,\ell n\left(\frac{X}{X_0}\right)\right] \qquad (B.13a)$$

$$(m^2_{E_L})_{ii}=m^2_{3/2}-\frac{3}{7}\frac{g_2^2}{\overline{g}^2}m^2_{\lambda_2}\,\ell n\left(\frac{X}{X_0}\right) \qquad (B.13b)$$

$$(m^2_{E_R})_{ii}=m^2_{3/2}\ . \qquad (B.13c)$$

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TABLE I

Examples for initial values of the parameters at $M_G = 2.10^{16}\ GeV$ which ensure the proper spontaneous symmetry breaking pattern $|\langle H_1^0\rangle| \approx |\langle H_2^0\rangle| \approx 123\ GeV$. Here $h^0_{E,U,D}$ denote the Yukawa couplings for the fourth family, μ_0 is the mass parameter of the superpotential (see Eq. (2.2)), m^0_λ and $m_{3/2}$ are the gaugino mass, and the gravitino mass, respectively while $m^0_{H_3}$ and m_0 denote the soft supersymmetry breaking mass parameters defined in Eq. (2.7b) and (2.7c). All the masses are in GeV.

	h_U^0	h_D^0	h_E^0	$m_{3/2}$	m_0	$m^0_{H_3}$	m^0_λ	μ_0
(a)	5	3	1	100	30	70	5	668
(b)	5	3	1	100	30	70	50	908
(c)	2	1	0.5	100	20	40	5	232.6
(d)	0.5	0.5	0.3	40	20	30	30	85
(e)	3	3	1	200	200	200	200	3050

FIGURE CAPTIONS

1. Evolution of the gauge couplings for the case of four families.

2. Evolution of h_U (solid line) and h_D (dotted line), the Yukawa couplings for the up- and down-quarks of the fourth family. The dashed line denotes the approximate analytic solution for $h = (h_U + h_D)/2$. The initial values for $h_{U,D}$ at M_G are taken from examples (a-d) of Table I.

3. Evolution of h_E, the Yukawa coupling for the lepton of the fourth family. The numerical and approximate analytic solution are plotted with the solid and dashed line, respectively. The values for the Yukawa couplings at $M_G = 2 \cdot 10^{16}$ GeV are chosen from examples (a-d) of Table I.

4. Numerical solution (solid line) and approximate analytic solution (dashed line) for μ, the mass parameter of the superpotential (see Eq. (2.2)). The initial values for $h_{E,U,D}$ and μ at M_G are chosen from the set (a), (c) and (d) of Table I.

5. Numerical solution (solid line) and approximate analytic solution (dashed line) for m_{H_3}, (see Eq. (2.5b)). The initial values for the parameters at M_G are from examples (a) and (b) of Table I.

6. Evolution of $m^2_{H_1}$ (solid line) and $m^2_{H_2}$ (dotted line) (see Eq. (2.5c)) for the values at M_G given in (a) and (b) of Table I. The dashed line denotes the approximate analytic solution for $m^2_{H+} = (m^2_{H_1} + m^2_{H_2})/2$.

7. Evolution of $m^2_{Q_L}$ (solid line), $m^2_{U_R}$ (dotted line) and $m^2_{D_R}$ (dot-dashed line), corresponding to the squark masses of the fourth family. The approximate analytic solution for $m^2_{S+} = \frac{1}{2}\left[m^2_{Q_L} + \frac{1}{2}(m^2_{U_R} + m^2_{D_R})\right]$ is plotted with the dashed line. The initial values of the parameters are taken from examples (a) and (b) of Table I.

8. The trilinear soft supersymmetry breaking mass parameter m_U (solid line) and m_D (dotted line) corresponding to the squarks of the fourth family (see Eq. (2.5b)). The dashed line presents the approximate analytic solution for

$m_+ = (m_U + m_D)/2$. The values of parameters at M_G are chosen from examples (a) and (b) of Table I.

9. Numerical solution for m_E, the trilinear soft supersymmetry breaking mass parameter corresponding to the slepton of the fourth family (see Eq. (2.5b)). The initial parameter at M_G are from examples (a) and (b) of Table I.

10. Numerical solution for $m^2_{E_L}$ (solid line) and $m^2_{E_R}$ (dotted line), the slepton masses of the fourth family (see Eq. (2.5c)). Parameters at M_G are taken from examples (a) and (b) of Table I.

11. Potentials V_T, V_T^1 and V_T^2 given by Eqs. (4.5), (4.6) and (4.8) as functions of $H = 2|\langle H_1^0 \rangle| \approx 2|\langle H_2^0 \rangle|$. The scale μ_{LL} is chosen to be 7 TeV. For aesthetical reasons we subtract a *constant* from V_T^1 and V_T^2 so that V_T, V_T^1 and V_T^2 have the same value at $H = 10$ GeV. The initial values of parameters are taken from example (a) of Table I.

12. Particle mass spectrum of the model. The initial values of the parameters are chosen from examples (a) and (e) of Table I, for Fig. 12a and Fig. 12b, respectively.

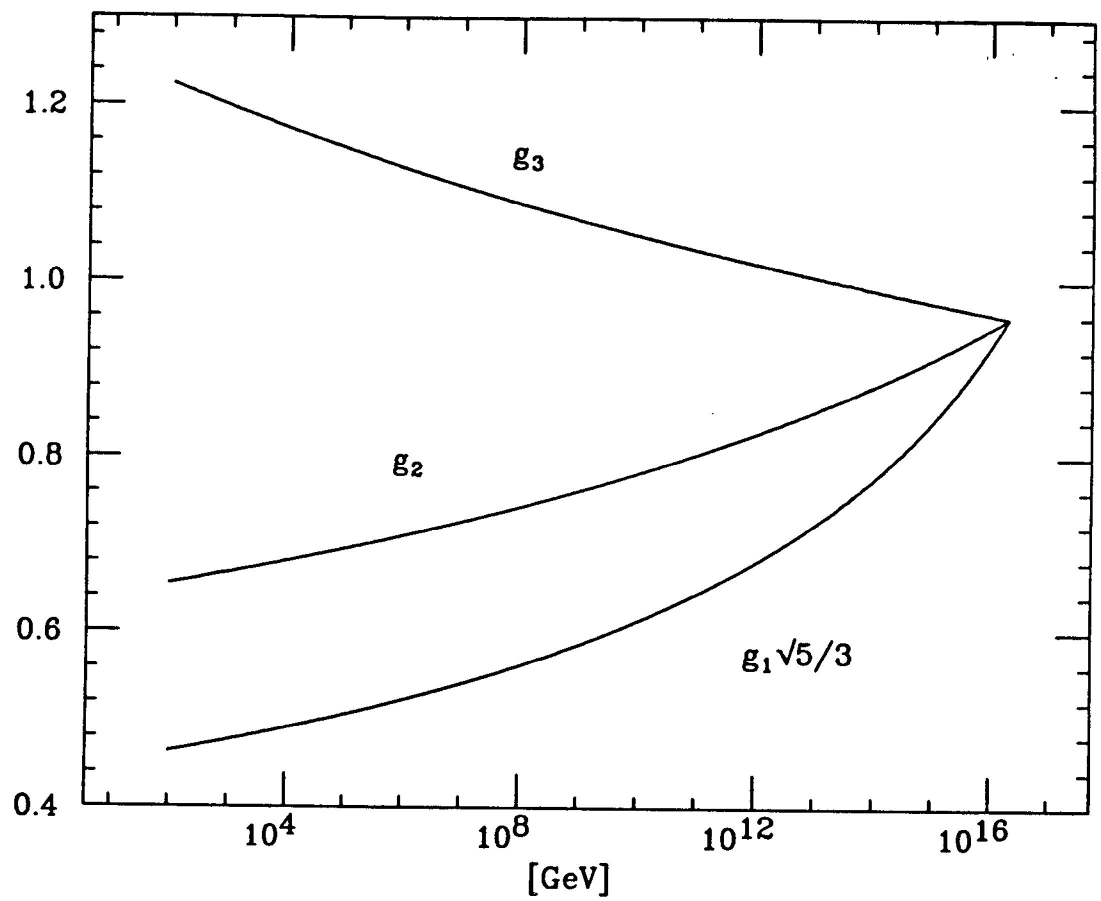


Fig. 1

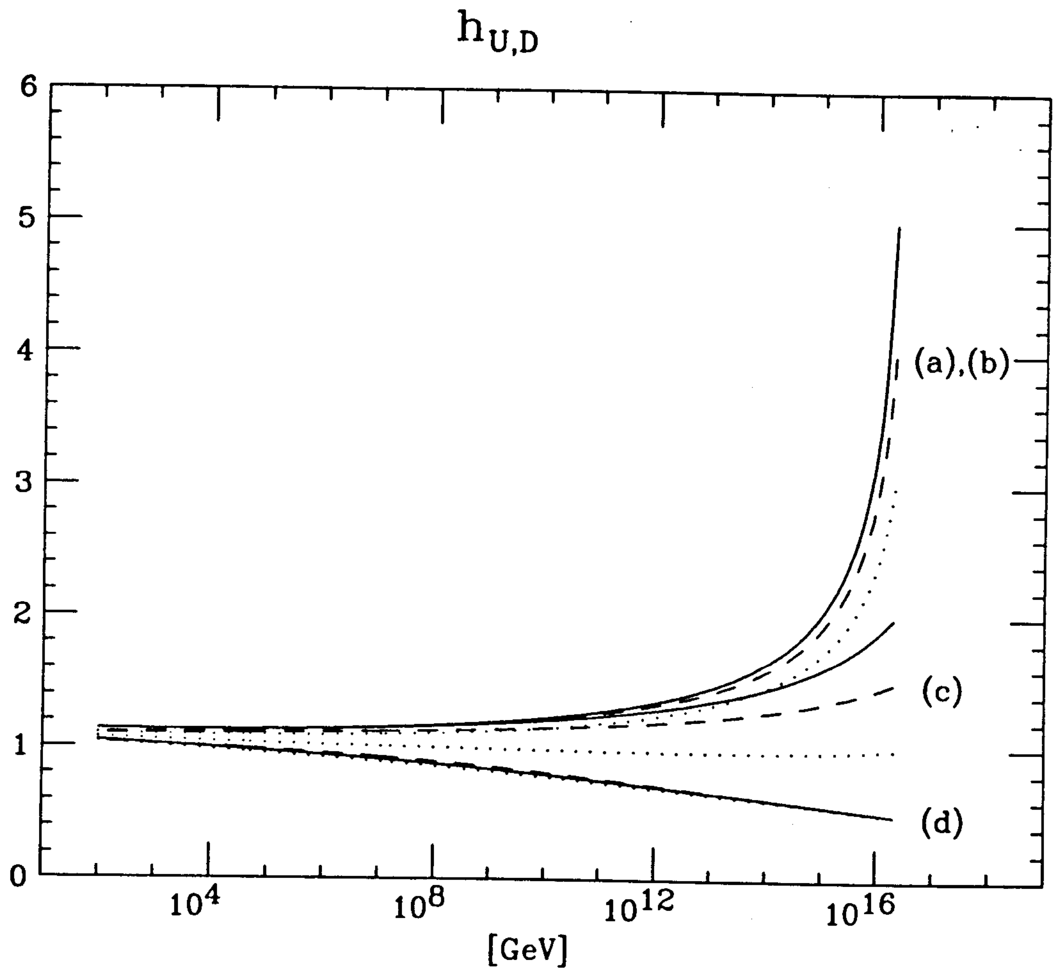


Fig.2

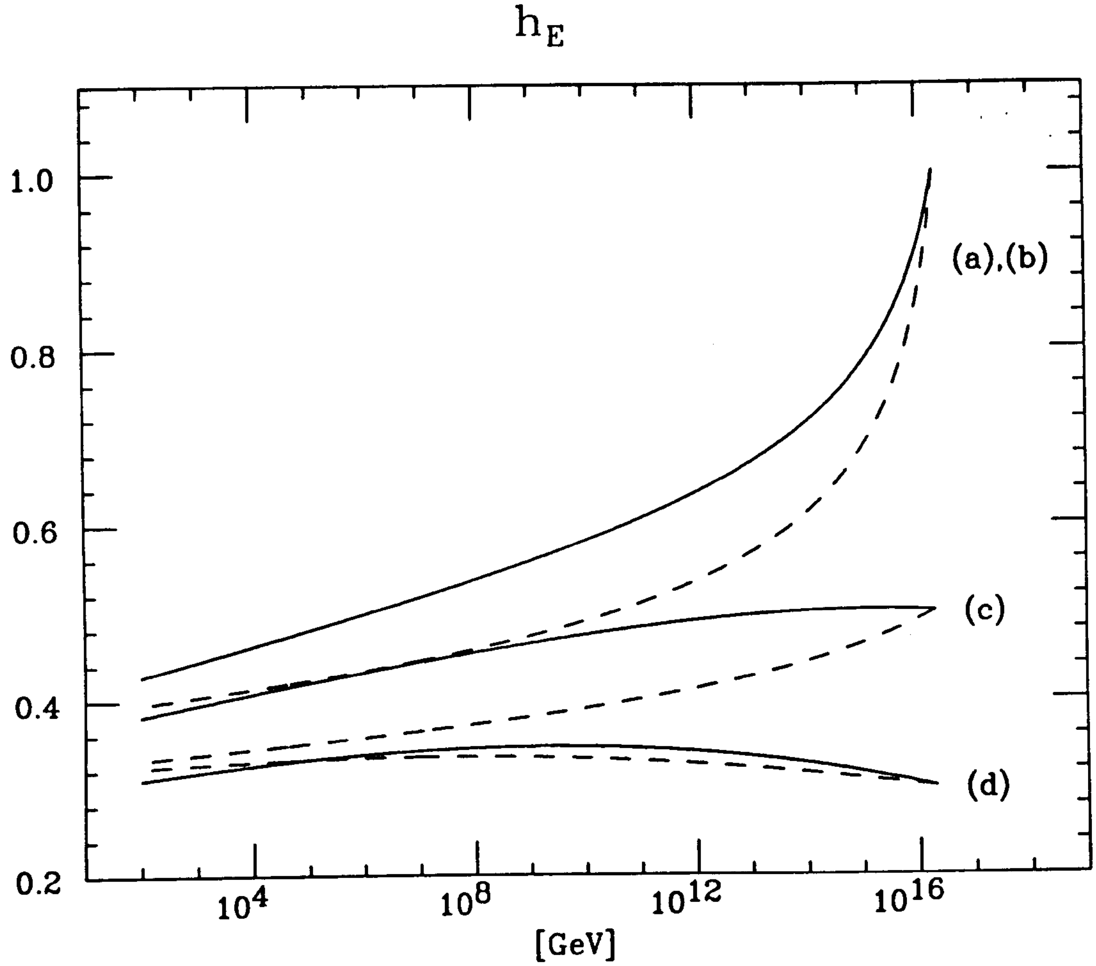


Fig.3

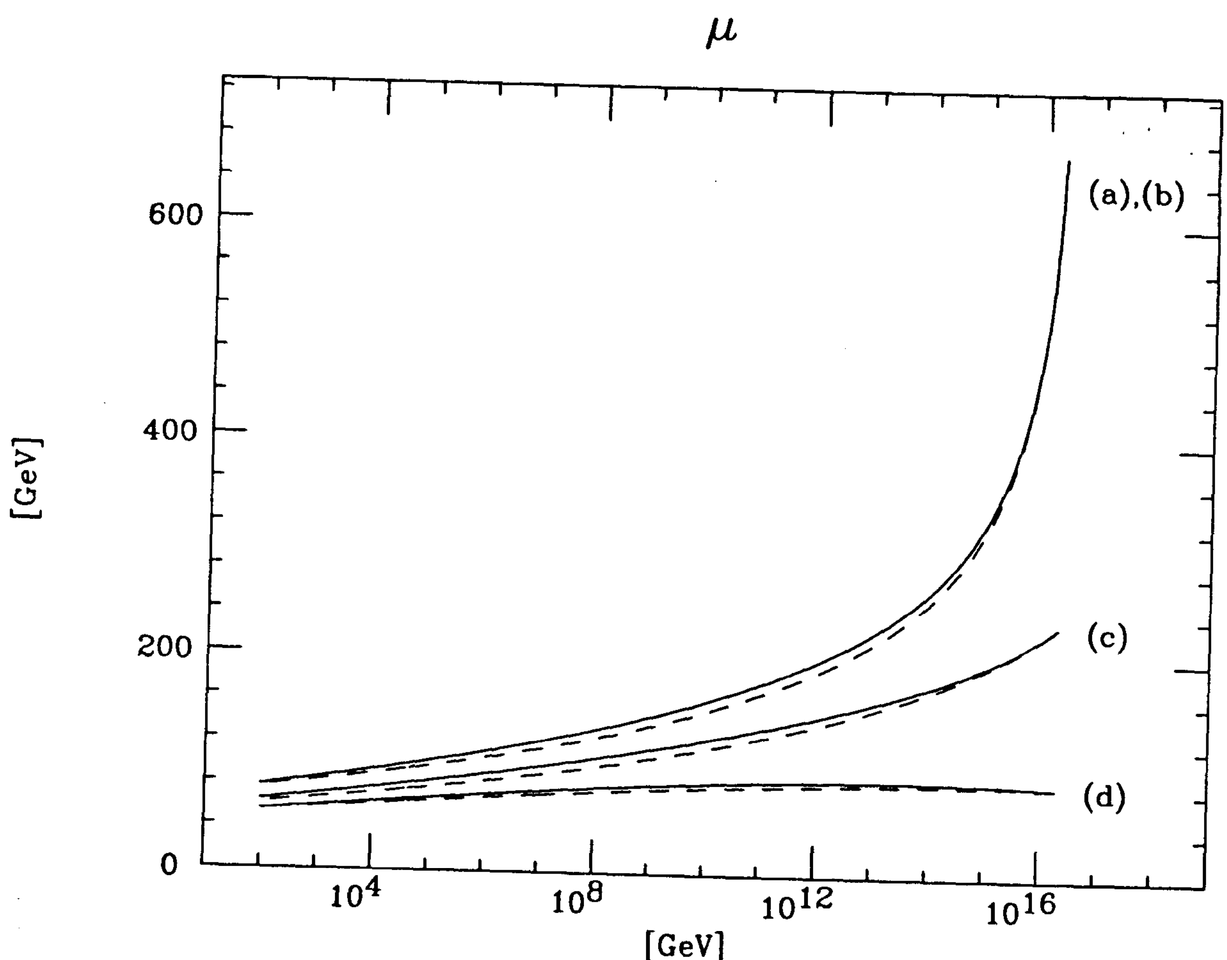


Fig.4

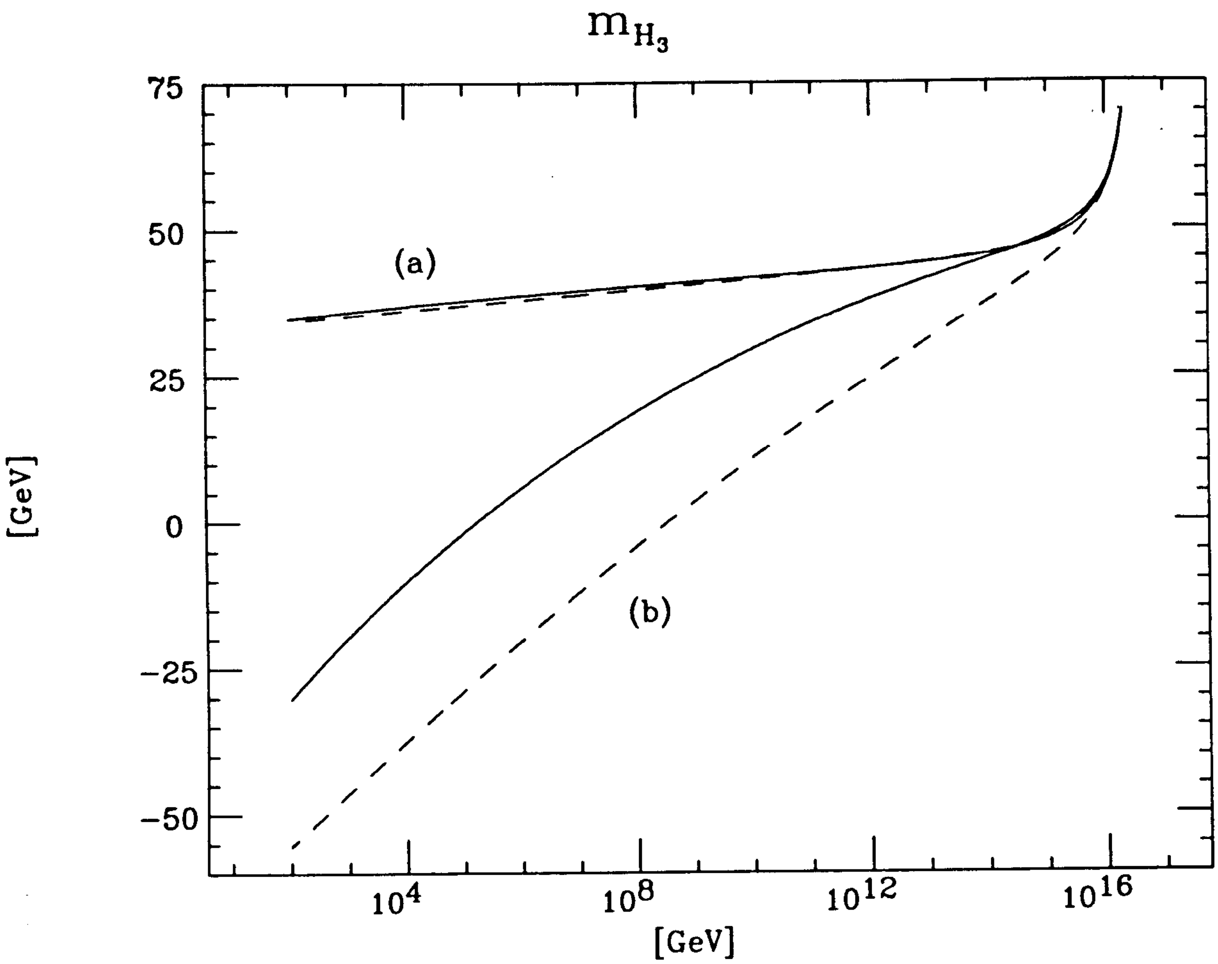


Fig.5

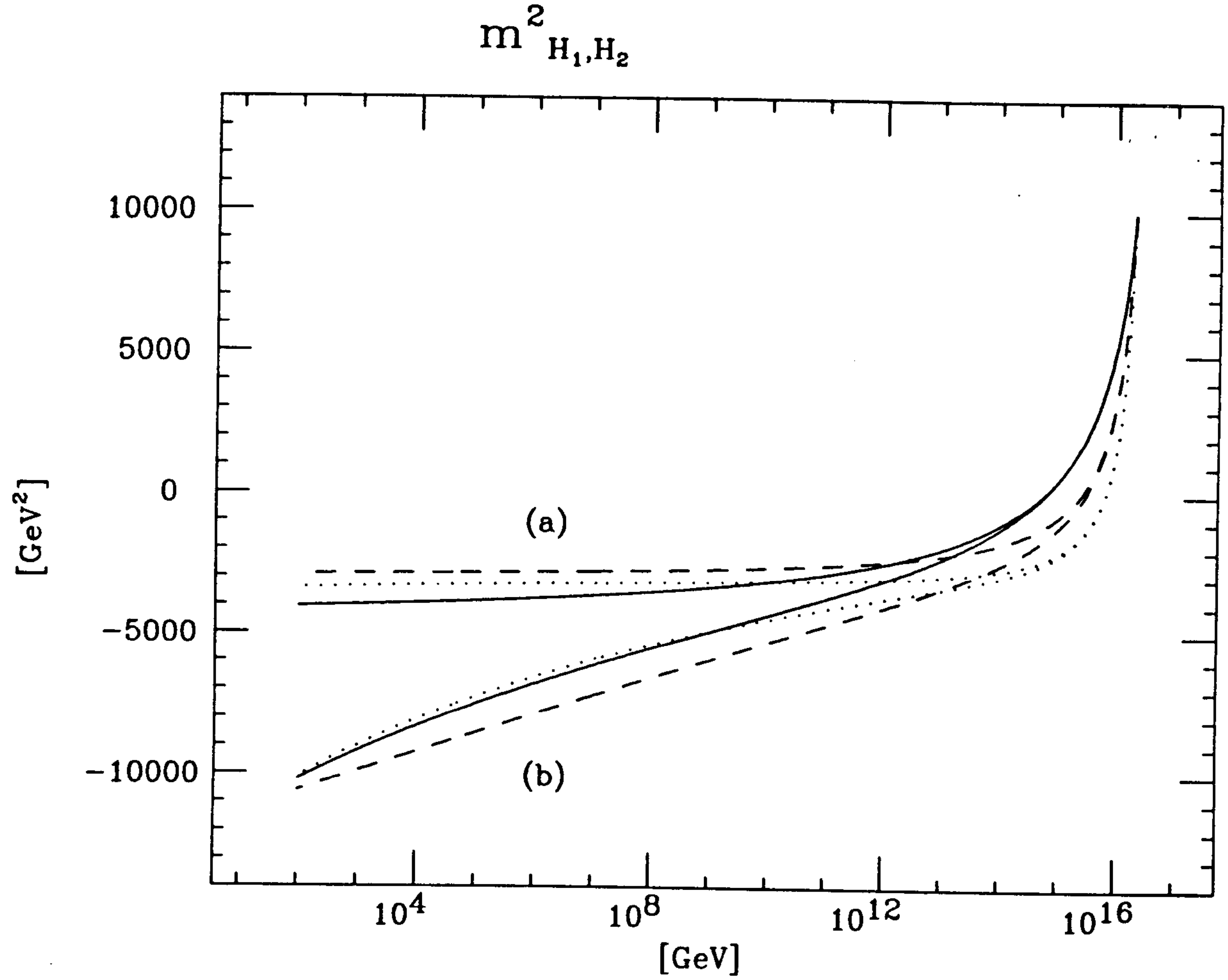
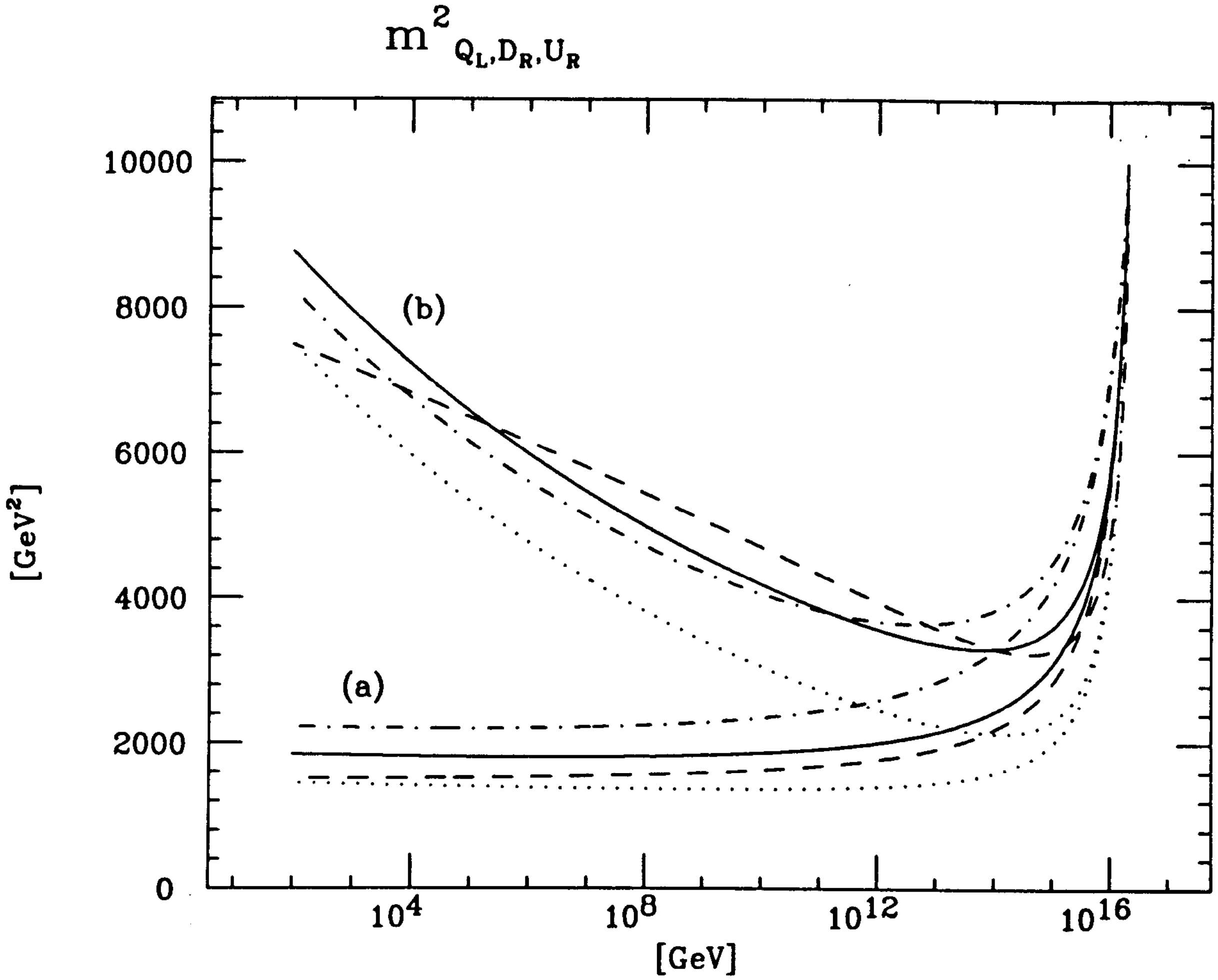


Fig.6



m²_{Q_L,D_R,U_R}
10000
8000
6000
4000
2000
0
[GeV²]
(b)
(a)
10^4
10^8
10^12
10^16
[GeV]

Fig.7

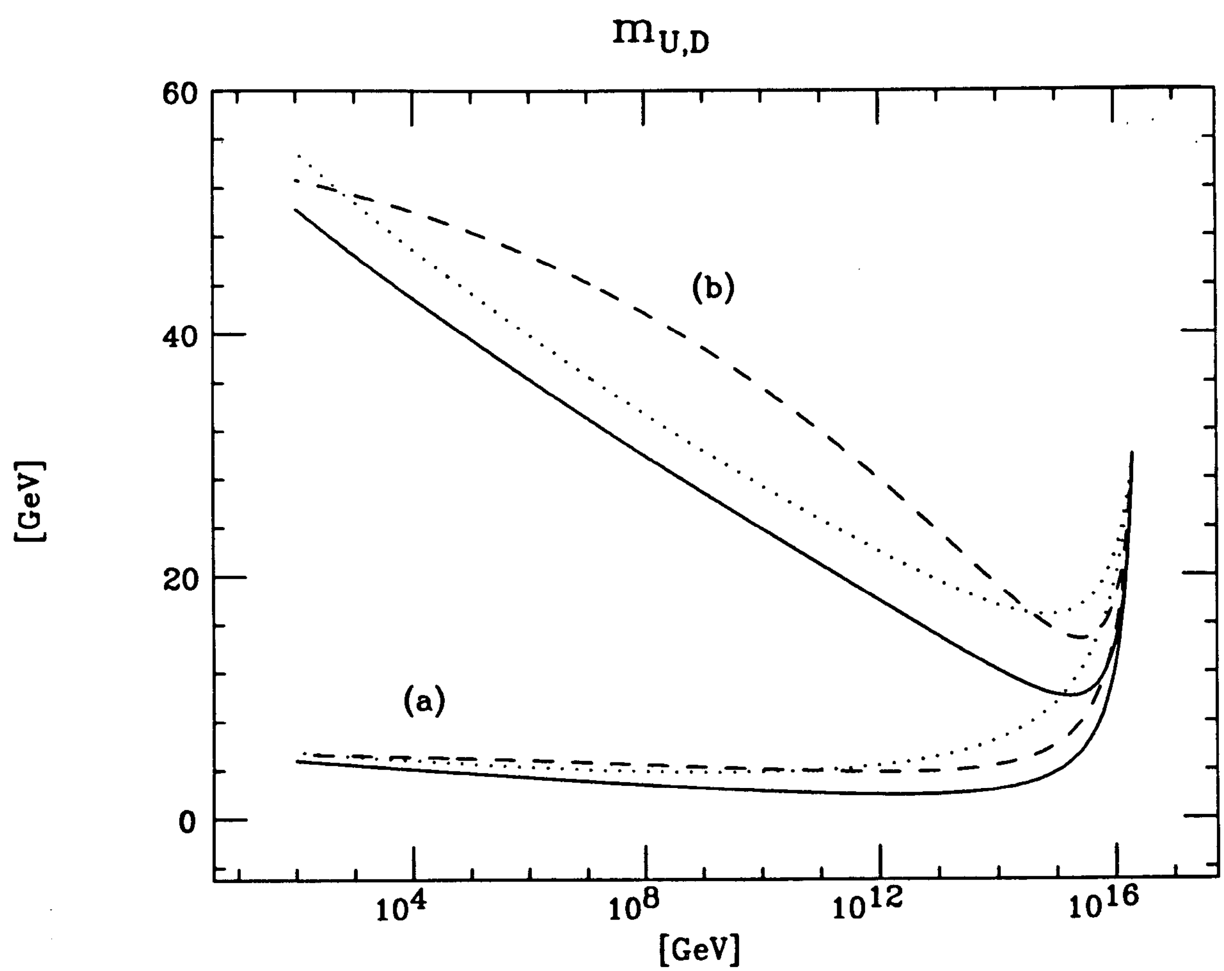


Fig.8

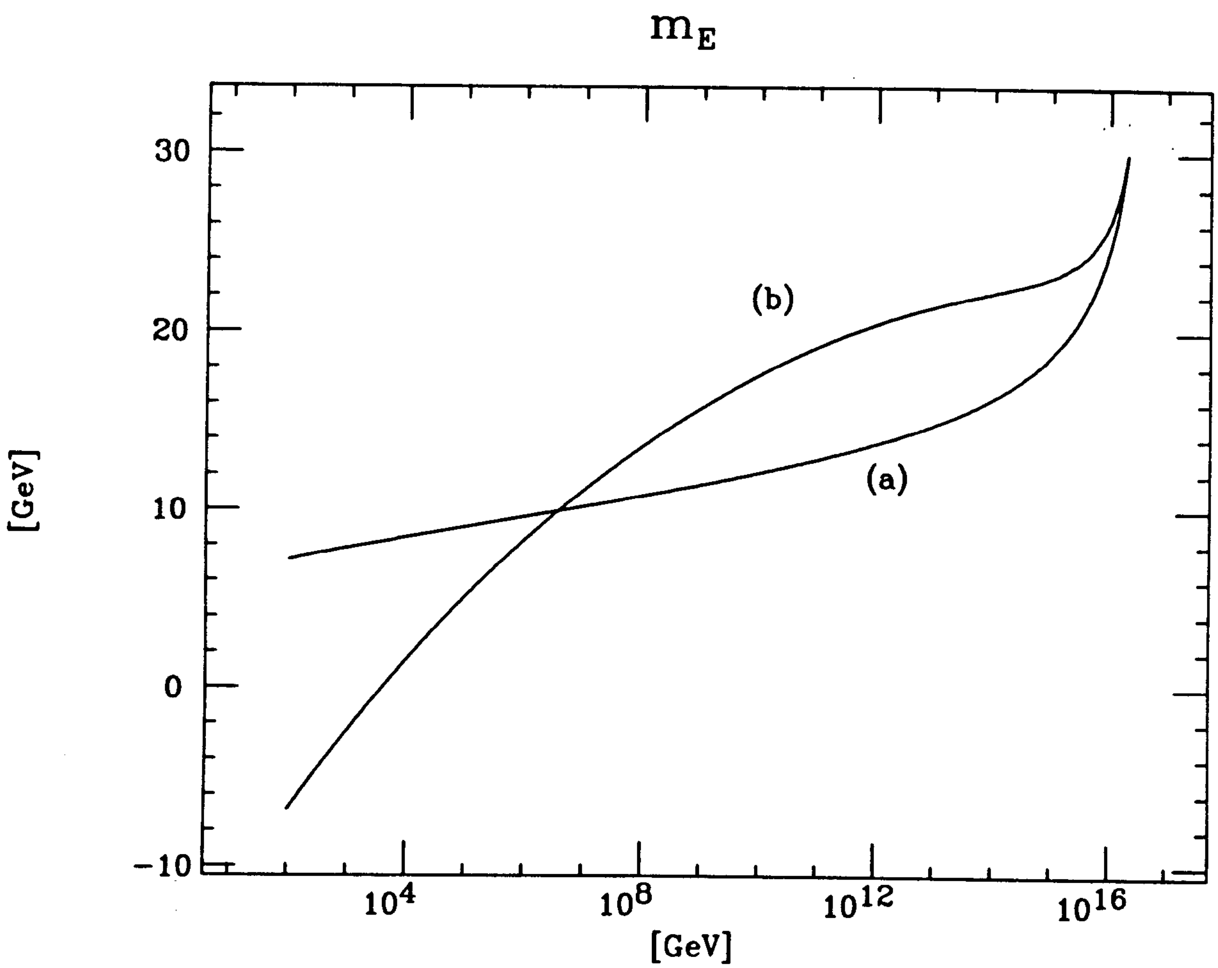
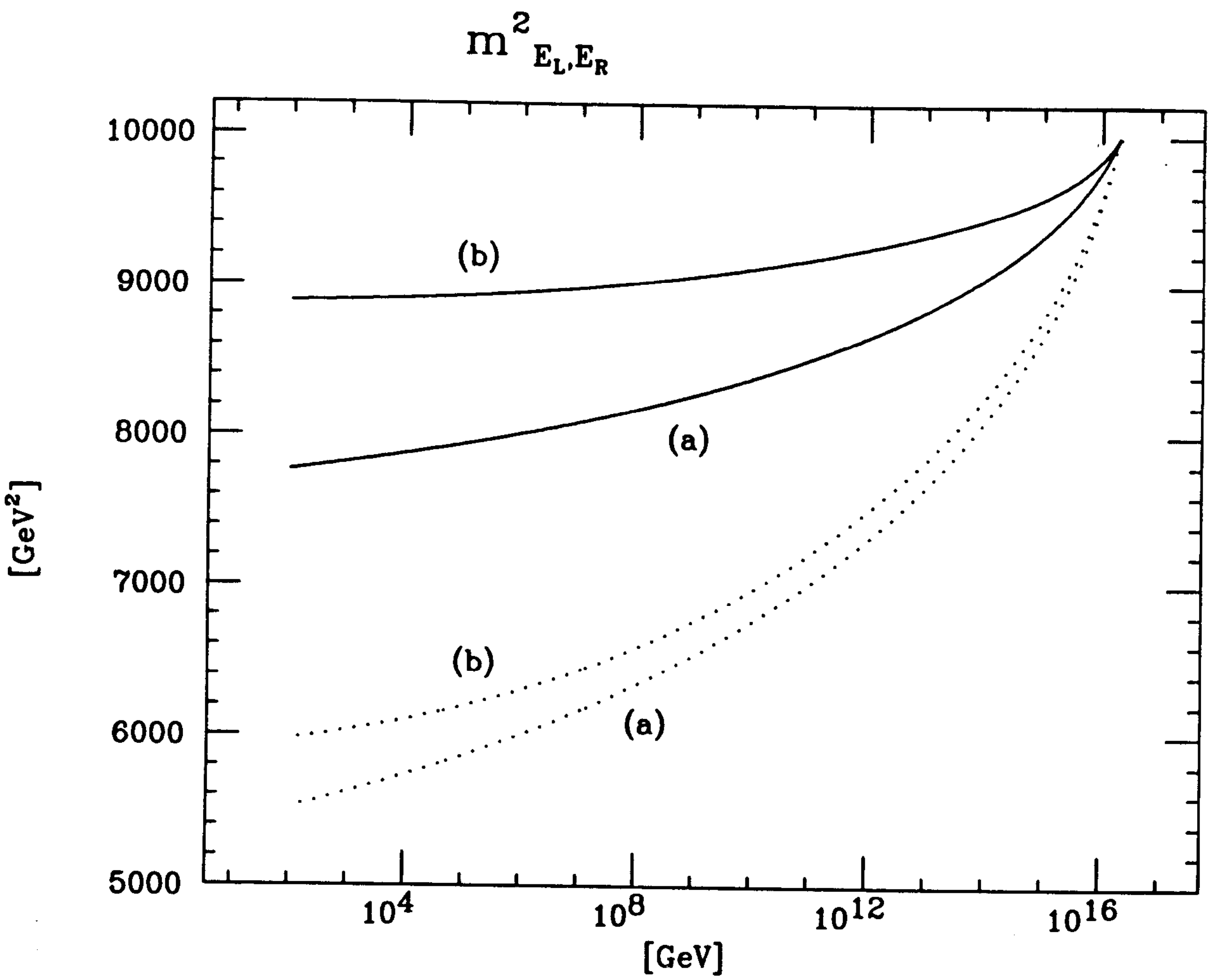


Fig.9



$m^2_{E_L,E_R}$
10000
9000
8000
7000
6000
5000
[GeV^2]
(b)
(a)
(b)
(a)
10^4
10^8
10^{12}
10^{16}
[GeV]

Fig.10

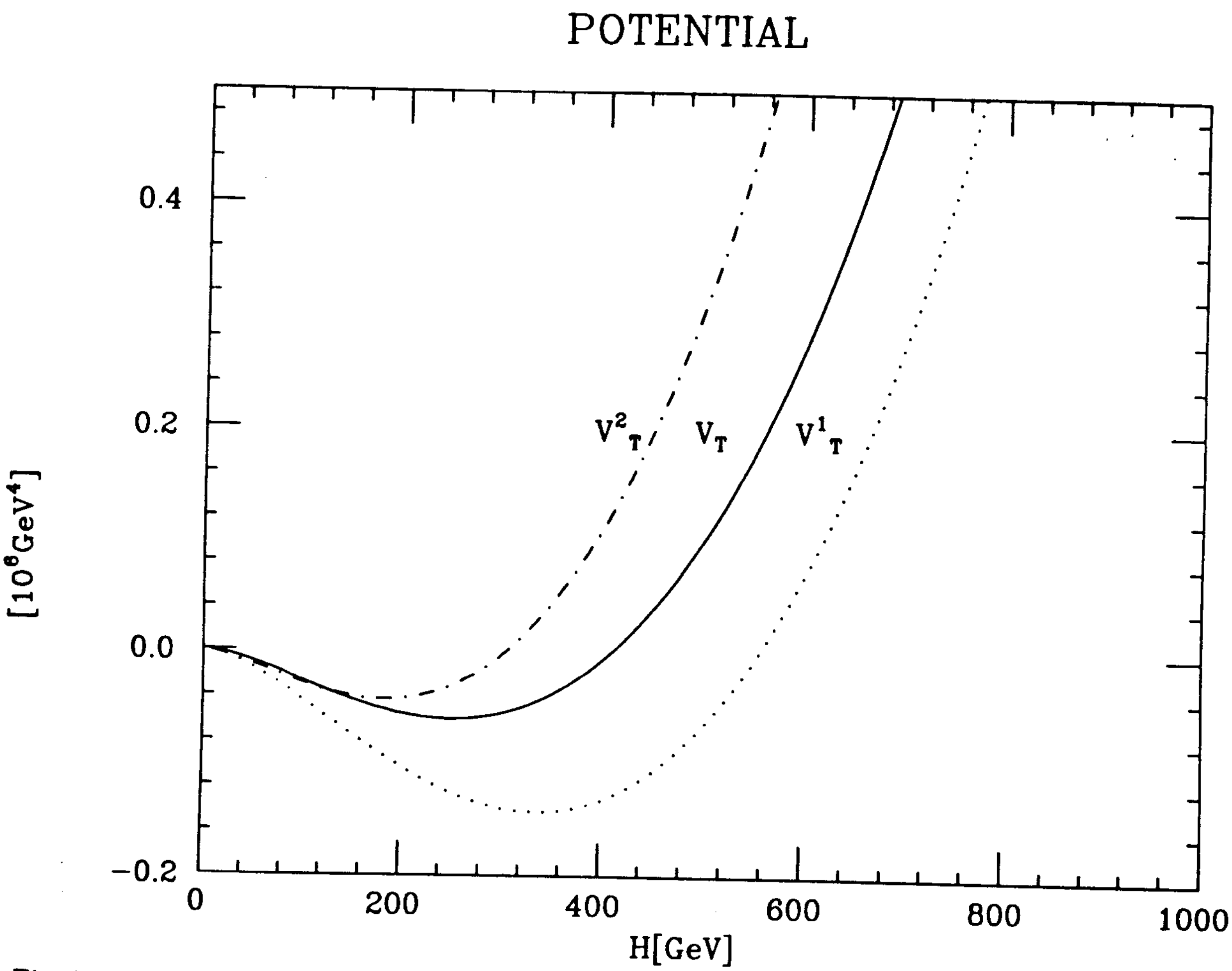
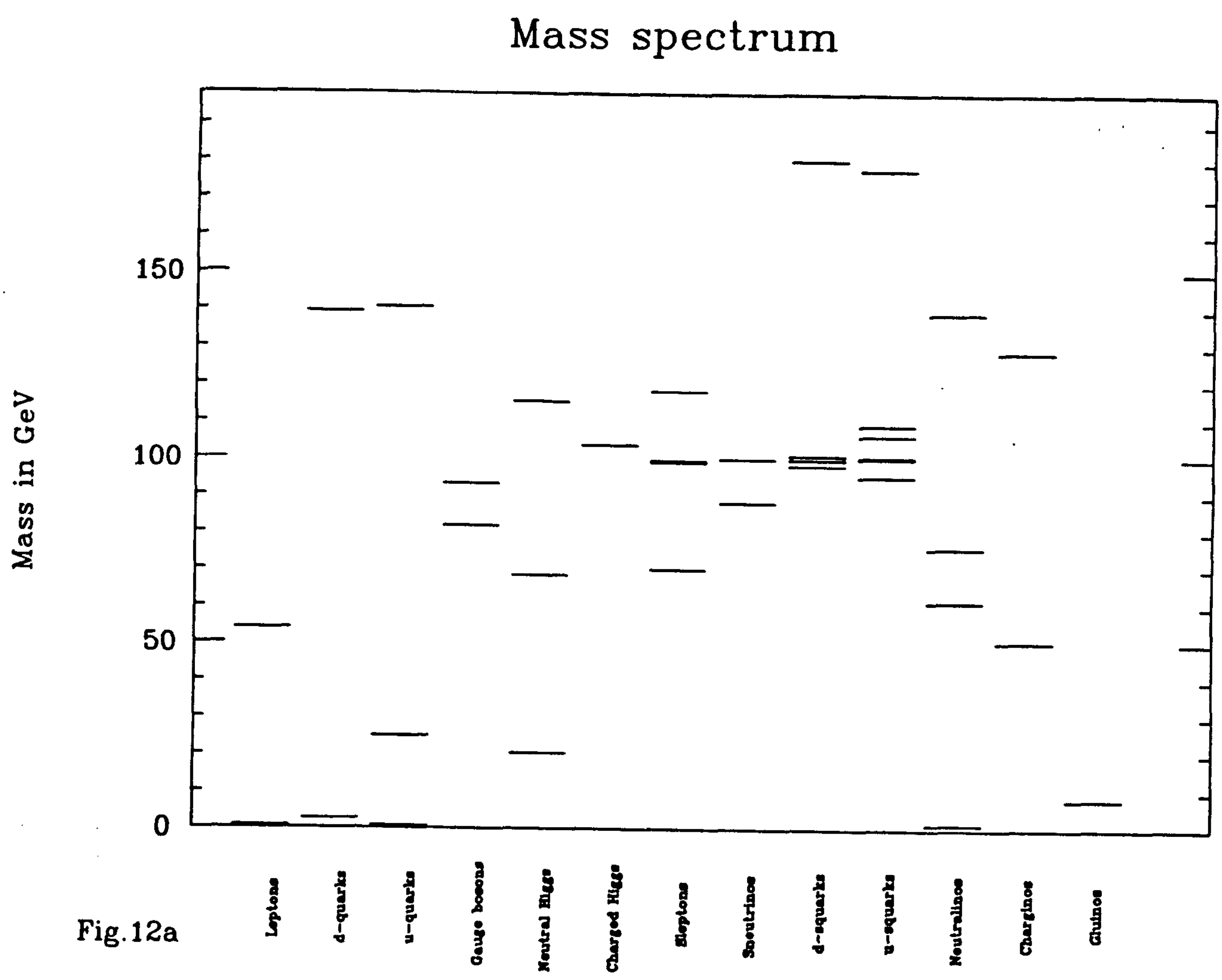

POTENTIAL
0.4
0.2
0.0
−0.2
[10^8 GeV^4]
0
200
400
600
800
1000
H[GeV]
V²_T
V_T
V¹_T

Fig.11



Mass spectrum
Mass in GeV
150
100
50
0
Leptons
d-quarks
u-quarks
Gauge bosons
Neutral Higgs
Charged Higgs
Sleptons
Sneutrinos
d-squarks
u-squarks
Neutralinos
Charginos
Gluinos

Fig. 12a

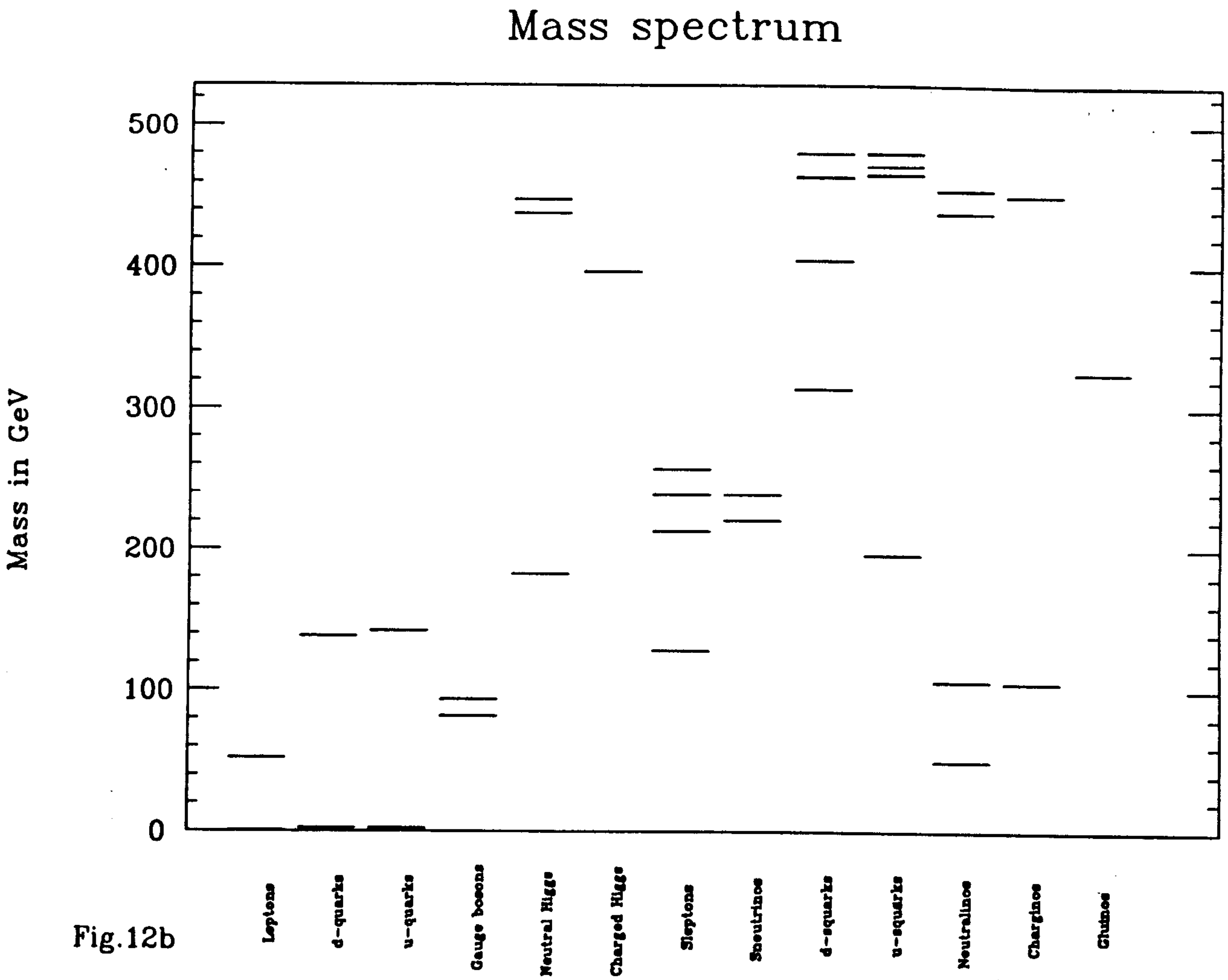

Mass spectrum
Mass in GeV
500
400
300
200
100
0
Leptons
d-quarks
u-quarks
Gauge bosons
Neutral Higgs
Charged Higgs
Sleptons
Sneutrinos
d-squarks
u-squarks
Neutralinos
Charginos
Gluinos

Fig.12b