Some Theoretical Considerations Concerning the Neutrino Experiments

C. N. Yang
Institute for Advanced Study, Princeton, New Jersey

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We heard yesterday reports on the very successful experiment at CERN with the neutrino beam. Since the proposal in 1959-60, independently by Pontecorvo and by Schwartz, of the neutrino experiment, there has appeared in the literature much theoretical discussion of the subject. I shall therefore be brief today and I shall confine myself to the CERN experiment, mainly elaborating on a few points which were touched upon by Dr. Veltman yesterday.

The following topics can be illuminated by the neutrino experiment, as already emphasized in the literature.[1,2]

1) The question of two neutrinos.
2) The neutrino flip hypothesis.
3) The conservation of leptons.
4) The possible existence of a neutral lepton current.
5) The point structure of the lepton current.
6) Universatality of weak interactions involving $e^{\pm}$ and $\mu^{\pm}$.
7) The $|\Delta I| = 1$ rule.
8) Strange particle production in neutrino reactions.
9) Interaction with extremely large momentum transfers.
10) Possible production of $W^{\pm}$.
11) Matrix element of the weak interaction (form factors).

Most of these topics have been discussed yesterday. I shall concentrate today on topics 10) and 11).

<u>Possible $W^{\pm}$ Production.</u>

W production through the processes

$$\nu + Z \rightarrow Z' + \mu^- + W^+ \quad , \tag{1}$$

$$\bar{\nu} + Z \rightarrow Z' + \mu^+ + W^- \quad , \tag{1'}$$

has been considered in the literature.[3] (1) and (1') have the same total cross sections. The process utilizes one $\nu\mu W$ vertex. Excess momentum is taken through the electromagnetic field, by the nucleus and its constituent nucleons. The process can be divided into a coherent part in which the nucleus recoils unexcited as a whole, and an incoherent part in which the nucleus is broken up or excited. The minimum momentum transfer to a heavy nucleus for process (1) is

$$q_{min} = \frac{(m_W + m_\mu)^2}{2E_\nu} \tag{2}$$

where E_ν = lab. energy of the incoming ν. For a nucleus of radius R, if $q_{min} R >> 1$, to produce a W would require penetration of the nucleus. Thus the coherent process becomes unimportant. Therefore for low energies one expects the incoherent processes to dominate, while for high energies the coherent process dominates. Published numerical computation[3] shows that the cross section for (1) indeed exhibits these properties. The value of the total cross section is extremely sensitively dependent on the mass m_W and the energy E_ν, and quite sensitively dependent on the extra magnetic moment of W. At energies of a few hundred Mev above the threshold, processes (1) and (1') in general will dominate over all other neutrino processes (which are of the order of magnitude of $\sim 10^{-38}$ cm^2).

To identify process (1) in the CERN experiment one will have to look for $\mu^- e^+$ and $\mu^- \mu^+$ pairs in which the e^+ and μ^+ originate from the decay of W^+ (the lifetime of W is less than 10^{-17} seconds). Suppose experimentally neutrino induced events are observed in which $\mu^- e^+$ or $\mu^- \mu^+$ pairs are identified. Could there be neutrino reactions other than W production which also lead to these lepton pairs? The answer is yes, and theoretically it is reasonable to expect that the dominant neutrino induced lepton pair processes other than (1) are

$$\nu_\mu + Z \rightarrow Z' + \nu_\mu + \left\{ \begin{matrix} e^+ + e^- \\ \mu^+ + \mu^- \end{matrix} \right\} ,$$

and

$$\nu_\mu + Z \rightarrow Z' + \nu_e + e^+ + \mu^- \tag{3}$$

The cross section for these processes have been estimated by several authors.[4] A more exact calculation is being carried out by Walecka and Czyz. The order of magnitude of process (3) is, very roughly from these estimates,

$$d\sigma \sim 2 \times 10^{-42} \text{ per nucleon} \tag{4}$$

which is less than 1/20th of one percent of the dominant process

$$\nu + n \rightarrow p + \mu^- \tag{5}$$

to be discussed below. Thus if one percent of the neutrino induced reactions are identified as giving rise to lepton pairs, it would be hard to escape the conclusion that they are due to W production, i.e. process (1).

Form Factors for "Elastic" Processes.

Whatever the final conclusion will be concerning the W production process in the CERN experiment, it is already evident from yesterday's report that the present CERN experiment could not yield much more information on the W than its possible existence. It then seems that one of the most interesting quantitative questions that the experiment will throw light on concerns the form factors for the processes (5) and

$$\bar{\nu} + p \rightarrow n + \mu^+ \ . \tag{6}$$

There has been extensive treatment[1, 5, 6] of these processes in the literature. The interaction Lagrangian is taken to be of the form

$$\begin{aligned} -\mathcal{L}_{eff} &= J_\lambda j_\lambda + \text{herm. conj.} \\ j_\lambda &= -i\psi_\mu^\dagger \gamma_4 \gamma_\lambda (1+\gamma_5)\psi_\nu \end{aligned} \tag{7}$$

where J_λ represents the source, due to the strongly interacting particles, of the weak interaction.

Goldberger and Treiman[7] first applied the conditions of Lorentz invariance to the matrix element of J_λ relevant for processes (5):

$$< p|J_\lambda|n> = \frac{i}{\sqrt{2}} u_p^\dagger \gamma_4 [\gamma_\lambda (g_V + g_A\gamma_5) + i(n_\lambda + p_\lambda)(f_V + f_A\gamma_5) + i(n_\lambda - p_\lambda)(h_V + h_A\gamma_5)] u_n \tag{8}$$

where n_λ, p_λ are the four momenta of n and p, u_p and u_n are the spinor solutions of the free Dirac equation, and f, g and h's are complex functions of

$$q^2 = (n_\lambda - p_\lambda)^2 \geqq 0$$

The concluded that if J_λ consists only of $(1+\gamma_5)$ interactions for nucleon operators, then time reversal invariance of the strong interactions and I spin conservation together imply that

$$f_A = h_V = 0 \quad , \qquad g_V,\ f_V,\ g_A,\ h_A = \text{real} \tag{9}$$

For general forms of J_λ, conclusion (9) remains valid[8] provided time reversal invariance holds for all interactions, and provided J_λ and $J_\lambda^\dagger$ belong to an I = 1 multiplet.[9]

We shall in the following assume (9) to be true. The cross section for (5) and (6) then becomes (in the laboratory system)

$$d\sigma = \frac{d(q^2)}{4\pi} \frac{p}{p'(1+v)(v_\mu + v')} \Big\{ (g_A^2 - g_V^2)(1-v^2)^{\frac{1}{2}}(1-v'^2)^{\frac{1}{2}}(1-v_\mu \cos\theta) + (g_V \pm g_A)^2 (1+v_\mu v')(1+v) + (g_V \mp g_A)^2 (1+v_\mu v\cos\theta)(1+v'\cos\theta) + [(4+q^2)f_V^2 - 4f_V g_V]\xi + \frac{m_\mu^2}{4EE'}[q^2 h_A^2 - 4h_A g_A](1-v_\mu \cos\theta) \Big\} \tag{10}$$

where $$\xi = \tfrac{1}{2}\,\frac{vv_\mu}{v'}\left(2+\frac{v'}{v_\mu}+v'\cos\theta\right)^2 - \tfrac{1}{2}(1-v_\mu\cos\theta)\left(1+\frac{1}{EE'}-vv'\cos\theta\right) ,$$

v, v', v_μ = velocities of initial and final nucleon and μ in the centre of mass system,

E, E', E_μ = corresponding energies,

$p, p'(= p_\mu)$ = corresponding momenta,

θ = angle of production in the centre of mass system.

All energies are measured in units of m_p. The relationship between θ and q^2 is given by

$$q^2 = -2 + 2EE'(1-vv'\cos\theta)$$

The maximum and minimum values of q^2 are given by $\cos\theta = +1$ and -1. In (10) the upper and lower signs refer respectively to reactions (5) and (6). These differential cross sections have also been given in a different form in the literature.[6]

The conserved vector current hypothesis gives[10]

$$g_V = G_V\,[F_Q + (\mu_p - \mu_n)F_M] \quad ,$$
$$f_V = G_V\,\frac{1}{2m}\,(\mu_p - \mu_n)F_M \quad , \qquad (11)$$

where F_Q and F_M are the isovector part of the charge and magnetic form factors of the nucleon, normalized to unity at $q^2 = 0$, and $\mu_p = 1.79$, $\mu_n = -1.90$ are the extra magnetic moments of the nucleons.

If the weak interactions are transmitted through an intermediate Boson W, (11) should[6] be changed by dividing the right-hand side of both equations by $1 + m_W^{-2} q^2$.

Using the conserved vector current hypothesis and assuming time reversal invariance, one is left with only two undetermined form factors g_A and h_A. One of the tasks of the neutrino experiment is to determine these form factors.

To an accuracy of a few percent, the following two approximations can be made in the cross sections:

(A) The terms involving h_A can be dropped. These terms contribute less than 3 percent to the total cross section because of the following factors. (a) The coefficient m_μ^2/EE' in (10) is $\lesssim 10^{-2}$, (b) The magnitude of h_A at $q^2 = 0$ is, according to Goldberger and Treiman,[7] very large. But for values of $q^2 > m_\pi^2$, this large contribution deriving from the pion pole term rapidly disappears, while the form factors g_V and f_V remain substantial up to $q^2 \sim m_p^2$.

For example, in the limit of infinite neutrino energy, the cross section for (5) and (6) both approach[6]

$$\sigma \rightarrow \frac{1}{2\pi} \int_0^\infty d(q^2) \, [(g_V - 2m_p f_V)^2 + q^2 f_V^2 + g_A^2] \quad , \tag{12}$$

[which is independent of the form factor h_A] provided

$$\int_0^\infty d(q^2) \, [q^2 h_A^2] < \infty$$

(B) One can put $m_\mu = 0$. This is a good approximation for all $E_\nu >$ 200 Mev.

Using these two approximations, (10) reduces to[6]

$$d\sigma_{\nu(\bar{\nu})} = \frac{d(q^2)}{4\pi} \left\{ \frac{q^2}{2\nu^2} (g_A^2 - g_V^2) + (g_V \pm g_A)^2 + (1 - \frac{q^2}{2\nu})^2 (g_V \mp g_A)^2 \right.$$
$$\left. + \left[2 - \frac{q^2}{\nu} - \frac{q^2}{2\nu^2}\right] [(4+q^2) f_V^2 - 4 f_V g_V] \right\} \tag{13}$$

where all energies are measured in units of m_p, $\nu = E_\nu$ = lab. energy of ν and the upper and lower signs are for (5) and (6) respectively. The range of q^2 is

$$0 \leq q^2 \leq \frac{4\nu^2}{1+2\nu} \tag{14}$$

If the target nucleon were free, (13) would determine the form factor g_A. In the actual CERN experiment, and probably for some time to come, hydrogen targets will not be available, and the determination of g_A will be complicated by the following three factors:

(a) The motion of the target nucleon in the nucleus.

(b) The exclusion effect on the recoil nucleon.

(c) The recoil nucleon undergoes refractions at the nuclear surface, and occasionally may suffer a collision with or without meson production.

Some of these effects have been discussed[11] by Berman and by Lovseth. N. Byers and I are currently studying these problems. It seems to us that due to effect (c) it would be rather difficult to obtain accurate results for g_A from experiments not using hydrogen targets. It would be also difficult to obtain any information at all on h_A. Because of these difficulties, and because of the great intensity of the ν beam amply demonstrated at CERN, it is perhaps worthwhile to think of experiments with hydrogen targets. In this connection let us notice that equ. (10) or (13) correspond to the Rosenbluth formula for electron scattering. [Because of the small strength of the weak interactions (10) and (13) need not be corrected for higher order weak interactions though one should include corrections due to photon emission and a photon exchange between the leptons and the nucleons (and the W).] It is worth emphasizing that the hydrogen target experiment offers a way to probe the "weak interaction structure" of the nucleon at small distances in much the same way as the electron scattering experiment studies the electromagnetic structure of the nucleons.

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7. M. Goldberger and S. Treiman, Phys. Rev. 111, 354 (1958).
8. S. Weinberg, Phys. Rev. 112, 1375 (1958); T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960), Phys. Rev. 126, 2239 (1962).
9. To discuss these questions it is useful to construct examples of J_λ that may violate one or both of these assumptions. A simple example is

$$J_\lambda = (\Sigma_+)^\dagger O_\lambda \Lambda + \Lambda^\dagger O_\lambda' \Sigma_- \ .$$

10. R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).
11. S. Berman, 1961 CERN Conference report; J. Lovseth, Phys. Letters. July 1, 1963.