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Superconducting Super Collider Laboratory

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Accelerator Physics in SSC Design

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ACCELERATOR PHYSICS IN SSC DESIGN*

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ABSTRACT

An elementary description of the accelerator physics considerations encountered in the design of the Superconducting Super Collider is presented. An attempt has been made to introduce the terminology and the basic physics issues from a user's point of view.

1. THE END PRODUCT

The end product of a storage ring collider can be summarized by three parameters: the type of colliding particles, the particle energy, and the luminosity. For the SSC, these parameters are chosen to be

$$\begin{aligned} \text{particle type} &= pp \\ \text{particle energy } E &= 20 \text{ TeV} \\ \text{luminosity } \mathcal{L} &= 10^{33} \text{ cm}^{-2}\text{sec}^{-1} \end{aligned} \quad (1)$$

We begin with a discussion on luminosity. Consider a certain type of high energy physics events of interest with cross section Σ . Obviously the counting rate $\mathcal{R}$ of these events in a collider is proportional to Σ . The proportionality constant is the luminosity, i.e.,

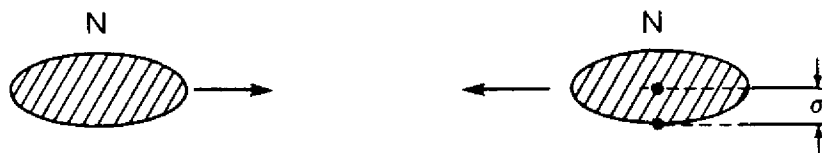
$$\mathcal{R} = \mathcal{L} \Sigma. \quad (2)$$

Consider two beam bunches with N particles each colliding head-on. Let the bunches have a round gaussian transverse distribution with rms size σ , as shown in Fig. 1. Let f be the frequency of collisions occurring at the collision point under consideration. Luminosity is given by^{1]}

$$\mathcal{L} = N^2 f / 4\pi \sigma^2. \quad (3)$$

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Fig. 1. Two colliding bunches.

The collision frequency is related to the revolution frequency f_{rev} by

$$f = f_{rev} B, \quad (4)$$

where B is the number of bunches in each beam.

These parameters for the present SSC design are^{2]}

$$\begin{aligned} N &= 1.4 \times 10^{10} \\ f_{rev} &= 3000 \text{ sec}^{-1} \\ \sigma &= 7 \text{ } \mu\text{m} \\ B &= 10000 \end{aligned} \quad (5)$$

With a circumference of 100 km, the spacing between bunches in each beam is 10 m.

We now ask what happens to a proton as it passes through the collision point. The probability that a proton actually collides with a proton in the on-coming bunch is $P = N \Sigma_{tot} / 4\pi \sigma^2$, where Σ_{tot} is the total cross section of collision. Assuming $\Sigma_{tot} = 137 \text{ mb}$, the probability of collision is found to be 3×10^{-10} per crossing, which is very small. The protons basically just pass through the on-coming beam without actual collisions.

However, there are two effects caused by beam crossings to be considered. The first is on the lifetime of the beam. With $N = 1.4 \times 10^{10}$, about 4 protons will find a partner to collide with per crossing. The beam lifetime due to pp collisions is 3.3×10^9 crossings, which corresponds to $1.8 \times 10^5 \text{ sec}$, or about 50 hours, since $f_{rev} = 3000 \text{ sec}^{-1}$ and there are 6 collision points per revolution.

The second, more important, effect caused by beam crossings is the perturbation on particle motion due to elastic scattering by the collective Coulomb field associated with the on-coming bunch. This perturbation is referred to as the beam-beam interaction,^{3]} which is described next.

2. BEAM-BEAM EFFECT

The beam-beam interaction constitutes one of the main limiting effects on the luminosity of a storage ring collider. To achieve a high luminosity, one needs a high beam intensity and a small beam area. These requirements, however, must be made so that the beam-beam effect is not made intolerably strong.

Consider a test particle in beam bunch 1 that passes through beam bunch 2 with a transverse displacement x from center, as shown in Fig. 2. In the relativistic limit, the electric field seen by the test particle points in the radial direction perpendicular to its direction of motion. Applying Gauss' law yields

$$E_r = (2Ne/\ell) [1 - \exp(-x^2/2\sigma^2)]/x, \quad (6)$$

where ℓ is the length of the beam bunch.

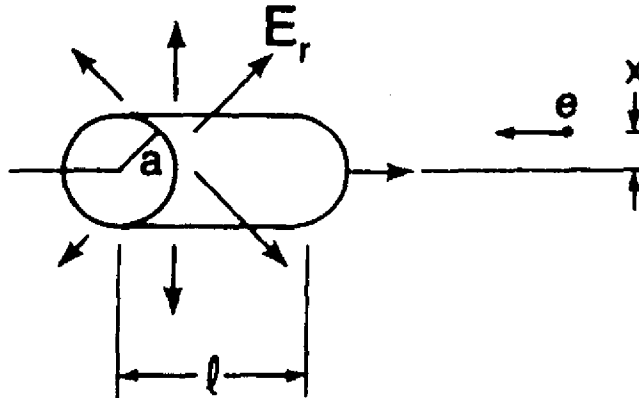


Fig. 2. The beam-beam encounter seen by a test charge.

In addition to the electric field, there is a magnetic field B_θ of the same strength (in cgs units) as the electric field. The Lorentz forces due to the electric and the magnetic fields add to give a force twice that due to the electric field alone. For the SSC with $a = 15\text{cm}$, the electric field is found to be 17 MV/m and the magnetic field is 0.5 kG evaluated at the edge of the beam, $x = 1.6\sigma$, where the fields are maximum.

The beam-beam interaction imposes limitation on luminosity not because it is extraordinarily strong but because it is extraordinarily nonlinear. The linear part of the force acts like a quadrupole magnet, whose effects can be compensated by adjusting the strengths and arrangements of the neighboring quadrupoles.

Figure 3 illustrates a comparison between a quadrupole magnet force and the beam-beam force. The linear quadrupole force provides transverse focussing which confines particles to execute simple harmonic oscillations, called the betatron oscillations.^{4]} The slope of the force directly relates to the betatron oscillation tune ν , which is defined to be the betatron oscillation frequency divided by

f_{rev} . Any additional linear force will cause a shift of the tune value. In particular, the linear part of the beam-beam force gives rise to a beam-beam tune shift given by^{3]}

$$\xi = N r_0 B^*/4\pi \sigma^2 \gamma, \quad (7)$$

where B^* is the β -function^{4]} at the collision point, γ is the relativistic factor, $r_0 = e^2/mc^2 = 1.53 \times 10^{-18} \text{m}$ is the classical radius of a proton. We will discuss more on the β -function later. As mentioned, the linear beam-beam tune shift is not difficult to compensate.

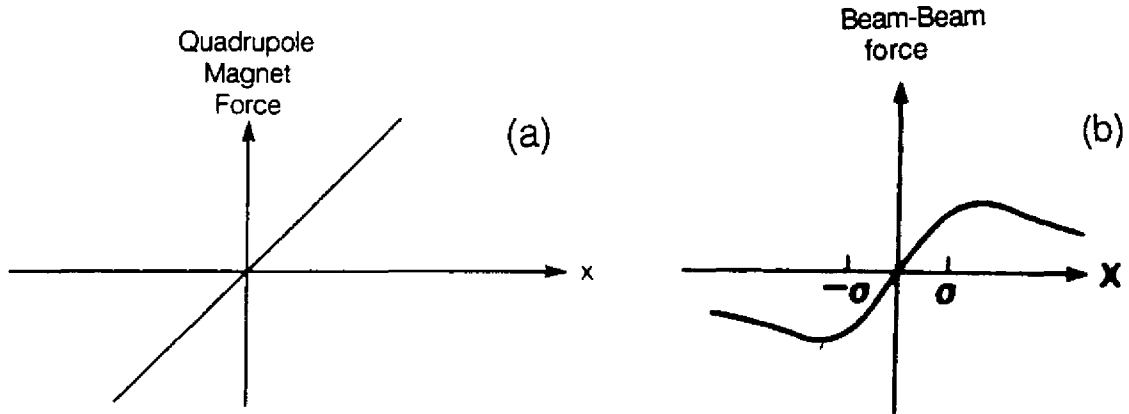


Fig. 3. (a) quadrupole magnet force as a function of the transverse displacement x , and (b) the same for the beam-beam force. The quadrupole linear force gives rise to the betatron tune. The dashed line is the linear part of the beam-beam force. It gives rise to a beam-beam tune shift.

The nonlinear part of the beam-beam force, on the other hand, is not so easy to deal with. As shown in Fig. 3(b), the beam-beam nonlinearity starts around $x > \sigma$. It turns out that the strength of the beam-beam nonlinearity is specified by the same quantity that specifies the linear part of the beam-beam force, namely the beam-beam tune shift ξ . (This is an important and nontrivial observation. It led to the discovery of low- β^* insertion,^{5,6]} which will be discussed later for the SSC.) To control the beam-beam nonlinear effects, it is therefore necessary to limit ξ . One of the fundamental constants in the design of a storage ring collider is in fact the maximum tolerable ξ in the presence of the nonlinear beam-beam perturbation. The conventional wisdom is that this maximum allowed for proton storage rings is somewhere between 0.003 and 0.005. For the SSC, we have

$$\xi = 0.0017 \text{ per head-on crossing.} \quad (8)$$

The fact that this is smaller than what is believed to be achievable will be explained later. Note that since there are 6 crossing points, the total beam-beam tune shift is $6 \times 0.0017 = 0.01$. Inserting the SSC parameters into Eq.(8), we find the needed β^* , β -function at the collision points, is 1.0 m.

3. LOW- β^* INSERTION

The β -function^{4]} has several physical meanings. The ones relevant to us here are

- β -function is a function of position around the storage ring.
- the equilibrium beam size at position s is proportional to $\sqrt{\beta(s)}$.
- the sensitivity of particle motion to perturbations at position s is specified by $\beta(s)$. The larger $\beta(s)$ is, the more sensitive is particle motion to perturbations.

At the collision point, we want to have a small beam size to enhance the luminosity. We also want to desensitize particle motion to the nonlinear beam-beam force. Both require small β^* .

The value of $\beta^* = 1$ m is to be compared with the average β -function in the storage ring which is determined by a convenient and economic spacing between quadrupole magnets in the lattice design. For the SSC, the average β -function is approximately 200 m. This means there needs to be a special lattice insertion consisting of a sequence of quadrupole magnets to focus the β -function from an average value of 200 m down to 1 m at the collision points. This special insertion is called the low- β^* insertion; it strongly enhances the luminosity of storage ring colliders. The price to pay is that it also strains the optics of the storage ring, as will be discussed in section.^{5]}

4. CROSSING ANGLE

Figure 4 is a sketch of the beam trajectories and lattice design around a collision point for the SSC. The two beams are designed to cross at a small angle α to avoid multiple head-on collisions in the region ± 65 m around the collision point. Without a crossing angle, there will be 26 head-on collisions per beam-beam crossing with bunch spacing of 10 m, yielding a clearly excessive beam-beam tune shift.

Even with a crossing angle, the beam-beam encounter includes 26 long range interactions, a few of them are illustrated in Fig. 5. The crossing angle must be chosen large enough to reduce the long range beam-beam effects. It must also be small enough so that the two beams can use common quadrupole magnets, i.e., the two triplets nearest to and on each side of the collision point.

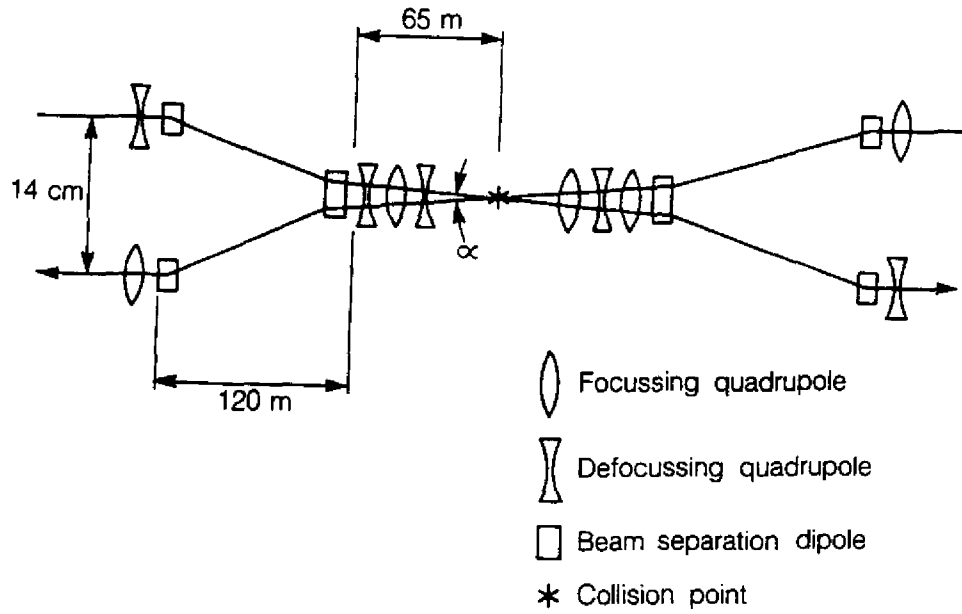


Fig. 4. Beam trajectories and a possible lattice design around a collision point for the SSC.

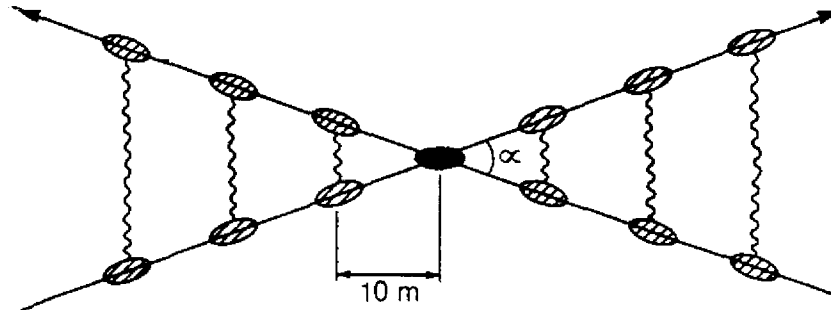


Fig. 5. Long range beam-beam encounters.

There are a few effects of a crossing angle. The first is that the effective overlapping of the colliding bunches is reduced by a factor $1/(1+f^2)$, where

$$f = \sigma_z \alpha / 2\sigma, \quad (9)$$

with σ_z the rms bunch length. Figure 6 shows the bunch overlapping as a function of f . To avoid loss of luminosity, f needs to be smaller than 1.

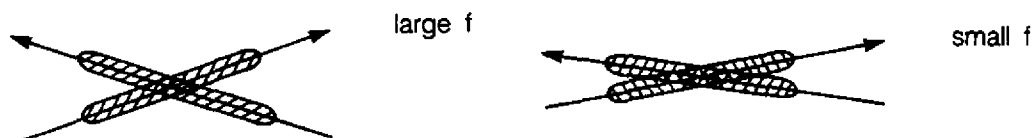


Fig. 6. Effect of crossing angle on the overlapping of the two beam distributions and thus its effect on luminosity.

The second effect is that the beam-beam interaction with a crossing angle will drive a whole set of potentially harmful resonances, called the synchro-betatron resonances.^{7]} The strength of these resonances is proportional to f . We like to have small f also for this reason.

The third effect is that the long range beam-beam force introduces a beam-beam tune shift term in addition to that due to the direct collision. The total tune shift is given by

$$(\text{total}) = (\text{direct}) + (\text{long range}), \quad (10)$$

where (direct) is given by Eq.(7). The long range contribution is related to the direct contribution by^{2]}

$$(\text{long range}) = g (\text{direct}), \quad (11)$$

where

$$g = 2n/(\beta^*\alpha/\sigma)^2$$

with n the number of long range encounters per crossing ($n = 26$ for SSC).

Assuming $\sigma_z = 7$ cm, $\sigma = 7$ μ m, $\alpha = 50$ μ rad for the SSC, we find $f = 0.25$ and $g = 1.0$. These values look quite acceptable. In particular, the total beam-beam tune shift is 0.0034, which is close to what is believed to be the beam-beam stability limit mentioned before.

5. CHROMATIC OPTICAL ABERRATIONS

There are several limitations in reducing β^* indefinitely in order to gain luminosity. Practical limitation on the strength of the low- β^* insertion quadrupole magnets is one example. Here we will discuss another limitation, i.e., the chromatic aberration of the storage ring optics which draws a substantial contribution from the low- β^* insertions.

At the start, a storage ring is composed of bending magnets and quadrupole magnets--bending magnets to guide the trajectory of particles and quadrupoles to provide the focussing. The β -function around the storage ring is sketched in Fig. 7. The insertion has produced a small β^* , but it also produces a large β -function, which for the SSC is $\hat{\beta} = 4000$ m, at the insertion quadrupole magnets.

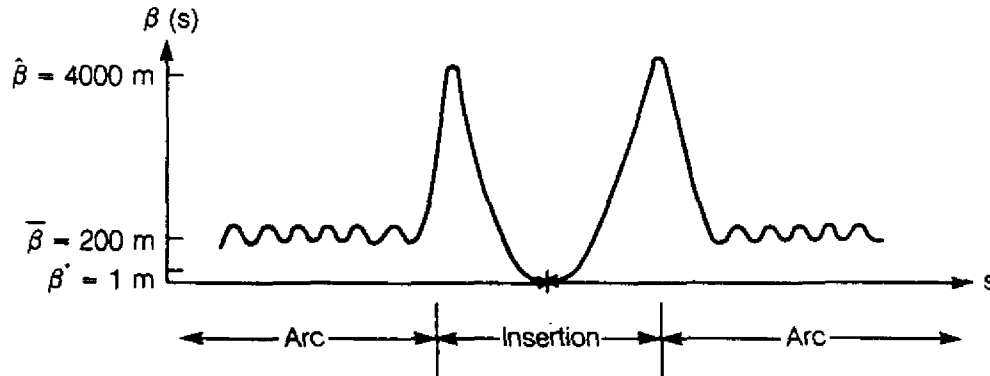


Fig. 7. Sketch of the β -function in SSC.

If the particles in the beam do not have any energy spread, a storage ring consisting of only bending and quadrupole magnets will satisfactorily produce the desired behavior shown in Fig. 7 and there will be no optical restriction on indefinitely reducing β^* . The motion of particle is linear and is perfectly stable.

The difficulty arises when the beam has a finite energy spread. To see that, consider the effect of a quadrupole magnet on the motion of an off-momentum particle with energy error $\delta = \Delta E/E$. The kick angle is given by

$$\begin{aligned}\Delta x' &= K x \\ &= K_0 x / (1 + \delta) \\ &= K_0 x (1 - \delta + \delta^2 - \dots),\end{aligned}\tag{12}$$

where K_0 is the quadrupole gradient seen by an on-momentum particle. The factor $1/(1+\delta)$ represents the rigidity in the kick to the off-momentum particle under consideration.

The rigidity factor is expanded in Eq.(12) into a power series to show its nonlinear behavior in δ . It is very nonlinear even it may not look like so. This is especially the case if β^* is small (and thus $\hat{\beta}$ is large) because significant chromatic aberrations comes from the insertion quadrupoles, which are strong to begin with and their effects are strongly enhanced due to the very large $\hat{\beta}$ there.

In a storage ring consisting of only bending and quadrupole magnets, therefore, particle motion is linear in x but nonlinear in δ . Figure 8(a) shows schematically the stability aperture diagram under this condition. In the aperture diagram, the maximum stable betatron

amplitude is called the dynamic aperture. The maximum stable energy width is called the momentum aperture. Figure 8(a) shows an infinite dynamic aperture and a very small momentum aperture in the case when the ring consists of only bending and quadrupole magnets. The momentum aperture is too small to accommodate the energy spread of the beam particles.

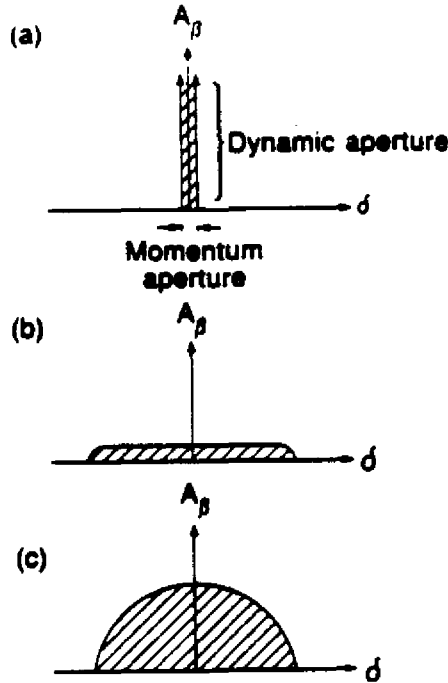


Fig. 8. Aperture diagram for a storage ring that consists of (a) dipole and quadrupole magnets only, (b) dipole, quadrupole and sextupole magnets, and (c) same as (b) but with special sextupole arrangements. A_β is the betatron amplitude, δ is the relative energy error. Shaded regions indicate region of stable motion.

6. SEXTUPOLES

To control the chromatic aberrations, sextupole magnets are installed in the storage ring in addition to bending and quadrupole magnets. Sextupoles have the property that they act like quadrupoles when the beam passes through them off-centered horizontally. We recall that a particle with $\delta \neq 0$ has its closed orbit displaced horizontally by an amount $\eta\delta$, where η is the horizontal dispersion function.^{4]} The kick given by a sextupole is therefore

$$\begin{aligned}\Delta x' &= S (x + \eta\delta)^2 \\ &= S x^2 + 2 S \eta x \delta + S \eta^2 \delta^2.\end{aligned}\tag{13}$$

The third term in Eq.(13) is not too significant. The middle term, which is the reason sextupoles are installed, is made to cancel to first order the chromatic nonlinearities due to the quadrupoles.

Unfortunately we still are left with the first term in Eq. (13). It produces a serious side effect due to its nonlinear nature in x . As a result, although we have removed to a large extent the chromatic

aberrations, we have introduced new nonlinearities in the betatron motion, which substantially suppress the dynamic aperture. The situation is sketched in Fig. 8(b). The achieved stable region is still not acceptable.

7. SEXTUPOLE SCHEMES

It is possible to improve the situation substantially by properly choosing the locations and strengths of the sextupoles. The idea is to make their x-nonlinearities cancel among themselves. There are a few schemes to do that; the simplest is the achromat scheme.^{8]}

In the achromat scheme, sextupoles are arranged in pairs in which two sextupoles of equal strength are spaced by a -1 transformation apart in the betatron motion, as shown in Fig. 9. It is easy to show that if an on-momentum particle enters the first sextupole with coordinate and slope of (x_0, x_0') , it will exit the second sextupole with $(-x_0, -x_0')$, independent of the existence of the sextupoles. The nonlinear effects of the sextupoles thus cancel each other as far as the betatron motion [the first term in Eq.(13) is concerned. On the other hand, the middle term is still active, yielding the needed control over chromatic aberrations.

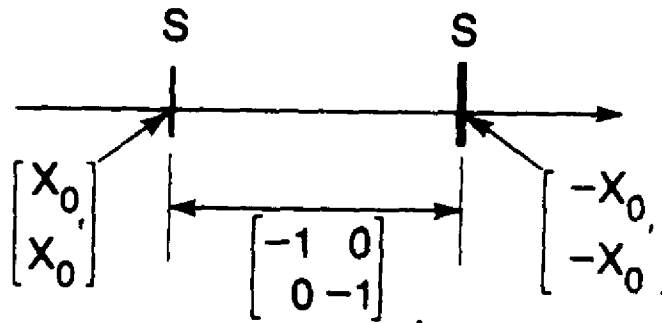


Fig. 9. A sextupole pair used in the achromat scheme.

Figure 8(c) shows the aperture diagram when sextupoles are arranged to minimize their betatron nonlinear effects. The arrangement does not affect the momentum aperture much but it increases the dynamic aperture substantially. The achromat arrangement have been adopted by the SSC design.

8. MAGNET FIELD ERRORS

We have so far discussed two sources of optical aberrations due to low- β^* and sextupoles, but there is another source, i.e., that due to the magnet field errors. Figure 10 is a sketch of two possible designs of the SSC superconducting bending magnets. One is a super-ferric design that reaches a field of 3 tesla. The other is a $\cos\theta$ design that gives about 6 to 6.5 tesla field.

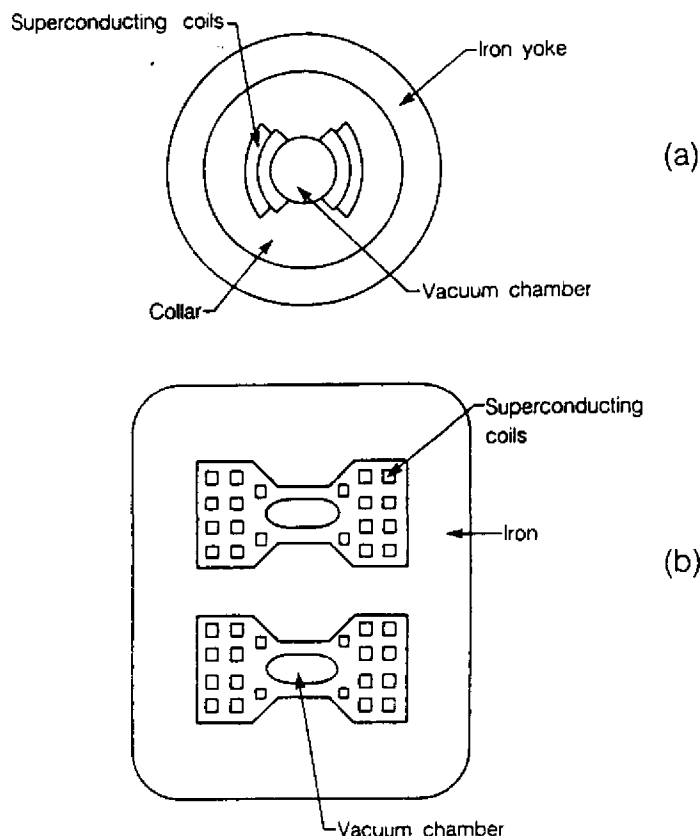


Fig. 10. Sketches of two SSC superconducting bending magnet designs. (a) the $\cos\theta$ design and (b) the superferric design.

The magnet field is determined by the placement of the superconducting coils. Field errors are produced due to design (systematic errors), or due to construction (random errors). The field error is specified by a set of error coefficients a_n and b_n defined by

$$\Delta(B_x + iB_y) = \sum_n (b_n + ia_n) (x + iy)^n. \quad (14)$$

To compare the nonlinearity due to magnet field errors to that of the beam-beam force, note that the beam-beam force deviates from linearity at a transverse distance of the order of the beam size at the interaction point (of the order of $7\mu\text{m}$) while a magnet field nonlinearity has the characteristic distance of the magnet coil or gap size (of the order of a few centimeters).

Extensive effort has been made on the magnet designs to make the error coefficients small. Table I gives a recent set of values of the random field error coefficients for the SSC $\cos\theta$ and the superferric magnet designs.^{9]} As a rough rule, a value larger than about 1 unit in the table means the multipole field error has the potential of causing instabilities in particle motion. However, closer studies must be performed to evaluate individual cases.

The magnet field errors are to be included in the optical aberrations just as the chromatic and sextupole aberrations discussed before. The stability aperture diagram for the $\cos\theta$ design, for example, is shown in Fig. 11 using results obtained by particle tracking.^{10]} The stable region is sufficient for the SSC beam to operate. In particular, the achieved dynamic aperture is about 5 mm and the momentum aperture is $\pm 0.15\%$.

Table I. A recent set of random multipole error coefficients of the $\cos\theta$ and the superferric magnet designs.^{9]} The unit of a_n and b_n is 10^{-4}cm^{-n} .

<u>coefficient</u>	<u>superferric</u>	<u>$\cos\theta$</u>
a_2	1.1	0.63
b_2	1.0	2.15
a_3	1.3	0.69
b_3	0.8	0.35
a_4	0.8	0.14
b_4	0.4	0.59
a_5	0.7	0.16
b_5	0.4	0.059
a_6	0.7	0.034
b_6	0.5	0.076
a_7		0.030
b_7		0.016
a_8		0.0064
b_8		0.021
a_9		0.0056
b_9		0.0030
a_{10}		0.0012
b_{10}		0.0071

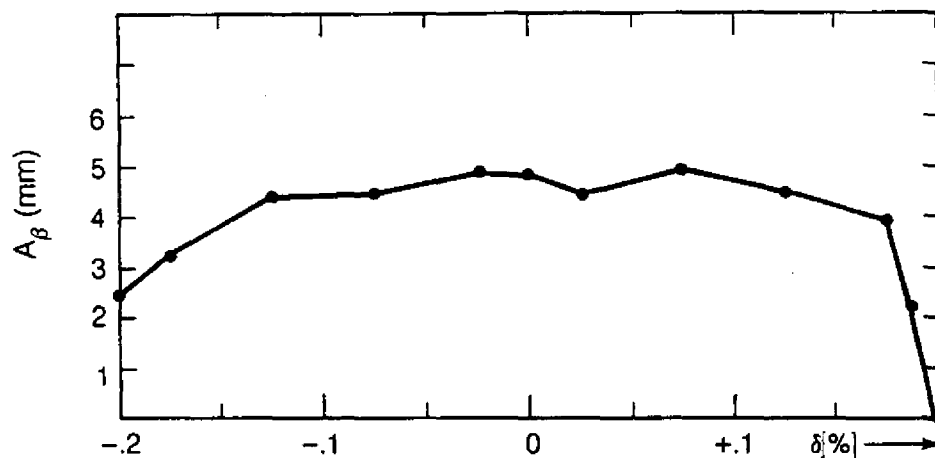


Fig. 11. The aperture diagram achieved by a $\cos\theta$ magnet design.

9. SYNCHROTRON RADIATION

Synchrotron radiation has been an important effect in electron circular accelerators. SSC will be the first proton storage ring in which synchrotron radiation plays a noticeable role. This is particularly because the radiation is deposited in a cryogenic environment. The following table gives the property of the radiation for the superferric and the $\cos\theta$ magnets.

<u>B (tesla)</u>	<u>typical photon energy</u>	<u>radiation power</u>
3	132 eV	3.9 KW
6.5	286 eV	8.5 KW

Synchrotron radiation power has to be removed by the cryogenic system. In addition to synchrotron radiation, there are other sources of heat generation:

- The collision points are heated by the colliding protons. With 4 protons in each bunch colliding per crossing, the associated energy release is about 700 watts. With 6 collision points, the total heat generation is 4.2 kW.
- The vacuum chamber inevitably has discontinuous joints. As beam bunches pass by these locations, parasitic energy losses occur (see next section). This contributes to a heating of about 4.5 kW for each ring.^{2]}
- Particles collide with residual gas molecules in the vacuum chamber, causing particle losses and generating heat. The heat load is estimated to be 0.2 kW per ring.

The above add up to a total heat load of 31 kW for the $\cos\theta$ magnet design. About 22 kW of this total load is deposited in the cryogenic part of the 2 storage rings. With a cooling efficiency of 1/500, this means the power needed to remove this heat load is $22\text{ kW} \times 500 = 11\text{ MW}$.

In addition to the radiation power, synchrotron radiation causes the beam emittance to shrink slowly.^{1,2]} The damping time of the beam size for the 6 tesla case is about 30 hours. This is a beneficial effect since it enhances the luminosity.

10. COLLECTIVE EFFECTS

In the SSC, there are several phenomena that depend on the beam intensity. Beam-beam effect is one of them. Another example occurs when particles within a single bunch Coulomb scatter with one another, yielding either particle losses from the bunch bucket^{11]} or a growth in the beam emittance.^{12]}

A discontinuous vacuum chamber is another important source of collective effects. Figure 12 illustrates what happens as a beam bunch passes by a cavity-like discontinuity in vacuum chamber. The bunch and the cavity interact in Fig. 12(b). A wake of electromagnetic field is generated. As the bunch leaves the region, Fig. 12(c), the wake field is trapped by the cavity. The bunch therefore has lost an energy that is equal to the energy stored in the wake field. The wake field can be either longitudinal or transverse. The longitudinal wake field retards beam motion and is directly related to the parasitic energy loss discussed in the previous section.

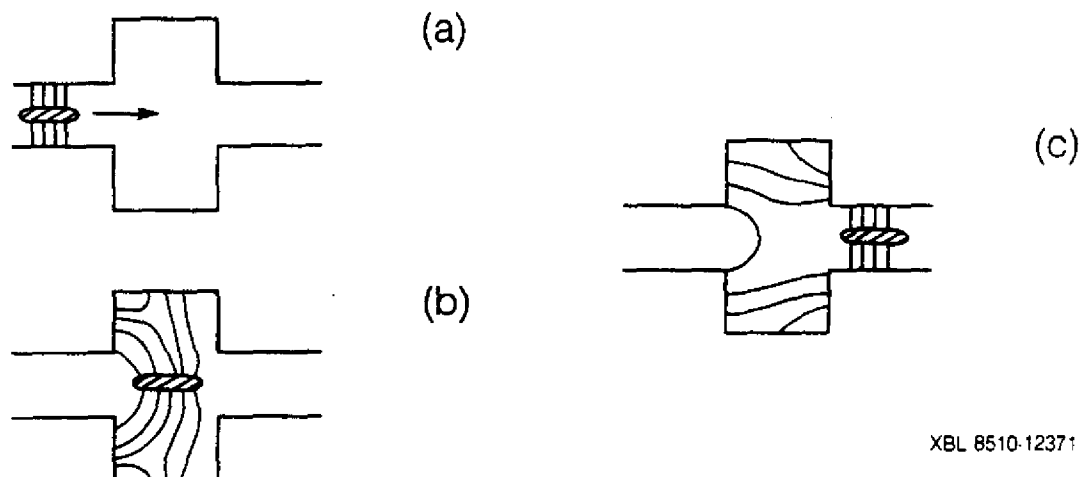
The wake field can also cause instabilities if it is strong enough. The strength of wake field is specified by basically two quantities. The longitudinal wake field is specified by the longitudinal impedance designated by Z_n/n . The transverse wake field is specified by the transverse impedance Z_t . Adding up contributions from all vacuum chamber discontinuities envisioned for the SSC, the estimated impedances for the superferric and the cos θ designs are given below:^{13]}

	<u>superferric</u>	<u>cosθ</u>
Z_n/n	0.35 Ω	0.35 Ω
Z_t	75 Ω/m	50 Ω/m

The parasitic loss of 4.5 kW per beam is obtained from the longitudinal impedance. As to the transverse impedance, its dominating effect is to cause an instability called single bunch transverse mode coupling instability, or sometimes called the fast head-tail instability.^{14,15]} To overcome this instability, the beam needs to have a minimum rms energy spread given by

$$\sigma_E/E = Ne^2c\beta Z_t/(45 ER\alpha), \quad (15)$$

where N is the number of particles per bunch, β is the β -function at the location of the transverse impedance, R is the storage ring radius and α is an optics parameter called momentum compaction factor.^{4]}



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Fig. 12. Wake field due to beam-cavity interaction. (a) before the beam passes the cavity. (b) during beam passage. (c) after passage.

In order to provide sufficient room for the beam to operate, it is necessary to have a momentum aperture of at least $\pm 4 \sigma_E/E$. Substituting parameters in Eq. (15), we find that the momentum aperture needed to stabilize the beam is

$$\begin{aligned} \pm 4 \sigma_E/E &= \pm 0.4 \times 10^{-3} \text{ for the cos}\theta \text{ design} \\ &\pm 1.3 \times 10^{-3} \text{ for the superferric design.} \end{aligned} \quad (16)$$

The needed momentum apertures for both designs are within the achieved values discussed in section 8 and Fig. 11.

11. RECAP

We started with the end product, in particular the luminosity, requirements. Beam dynamics is introduced by asking what happens to particle motion at the collision points. The beam-beam interaction causes a beam-beam tune shift which specifies the strength of the perturbation.

The beam-beam interaction in SSC has a long range contribution. Its strength is controlled by the crossing angle. The long range force also contributes to the beam-beam tune shift.

To reduce the beam-beam effect while maintaining a high luminosity, a low β^* is introduced by installing low- β^* insertions around the collision points. These low- β^* insertions introduce strong chromatic aberrations that complicate the optics. To compensate for these chromatic aberrations, sextupoles are introduced in the lattice. The sextupoles have their own nonlinear effects. This is taken care of by arranging them according to an achromat scheme so that their nonlinear effects cancel among themselves.

Synchrotron radiation begins to become a significant design consideration in the SSC. The power radiated must be removed by the cryogenic system. We also discussed the collective effects. Wake fields and impedances are introduced. The instability effects are found to be not very crucial for the SSC.

REFERENCES

1. M. Sands, SLAC-121/UC-28 (1970).
2. SSC Reference Design Studies for US Department of Energy, May 1984.
3. For a review, see for example, "Nonlinear Dynamics and the Beam-Beam Interaction," AIP Conf. Proc. No. 57, Brookhaven, 1979, edited by M. Month and J.C. Herrera.
4. E.D. Courant and H.S. Snyder, Ann. Phys. 3, 1 (1958).
5. K.W. Robinson and G.-A. Voss, Cambridge Electron Accel. report CEAL-TM-149 (1965) unpublished.
6. P.L. Morton and J.R. Rees, IEEE Trans. Nucl. Sci. NS-14, 630 (1967).
7. A. Piwinski, DESY Report 77/18 (1977).
8. K.L. Brown and R.V. Servranckx, Summer School on High Energy Accelerators, Stony Brook/Brookhaven, 1983, AIP Proc No. 127, page 62.
9. SSC Aperture Task Force, SSC-SR-1013 (1985).
10. Clustered Interaction Region Study Group, SSC-SR-1014 (1985).
11. C. Bernardini, G.F. Corazza, G.Di Giugno, G. Ghigo, J. Haissinski, P. Martin, R. Querzoli, and B. Touschek, Phys. Rev. Lett. 10, 407 (1963).
12. A. Piwinski, Proc. 9th Int. Conf. on High Energy Accelerators, 1974, page 405.
13. J. Bisognano, SSC-25 (1985) IEEE Trans. Nucl. Sci., NS-32, No. 5, 2392 (1985).
14. R. Talman, CERN/ISR-TH/811-17 (1981), unpublished.
15. R.D. Kohaupt, DESY Report M-80/19 (1980), unpublished.