сообщения
объединенного
института
ядерных
исследований
дубна

E4-85-878

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CONFRONTATION BETWEEN THE QUASIPARTICLE-PHONON NUCLEAR MODEL AND THE INTERACTING BOSON MODEL FOR DEFORMED NUCLEI

1985

1. Introduction

Doubly even deformed nuclei include low-lying two-quasiparticle and collective quadrupole and octupole vibrational states. Most of the two-quasiparticle states predicted in ref. [1] have later been measured experimentally. The collective $K^{\pi}_{n}=2^{+}_{1}$, γ-vibrational, $K^{\pi}_{n}=0^{+}_{2}$, β-vibrational, $K^{\pi}=0^{-}$, 1^{-}, 2^{-} and 3^{-} octupole states and $B(E\lambda)$-values of their excitation are well described as the one-phonon RPA states [2,3].

The generally accepted treatment (see ref. [4]) implies the existence of one-, two- and three-phonon states in doubly even spherical and deformed nuclei. The effect of the Pauli principle in the two-phonon components of the wave functions of excited states has been studied in refs. [5,6]. It was concluded in ref. [6] that the collective two-phonon states should not exist in deformed doubly even nuclei. Note that A.Bohr and B.Mottelson [7] support the idea of existence of the collective two-phonon states in deformed nuclei. Thus the contradiction arose between the results obtained in the quasiparticle-phonon nuclear model (QPNM) [8] and in the phenomenological models.

In recent years the interacting boson model (IBM) is widely used [9,10] for describing low-lying states of deformed nuclei [11,12]. It is interesting to compare the description of nonrotational states with $K^{\pi}=0^{+}$, 2^{+}, 3^{+} and 4^{+} in the energy interval (1.5-2.5) MeV within the QPNM and IBM. This problem is the aim of the present paper.

2. Description of nonrotational states of deformed nuclei in the QPNM and IBM

To compare how nonrotational states in deformed nuclei are treated in the QPNM and IBM, we first expound the basic assumptions of these models.

The starting Hamiltonian of the QPNM contains the Saxon-Woods potential describing an average field of the neutron and proton systems, pairing interactions as well as the isoscalar and isovector separable multipole and spin-multipole interactions.

The pairing constants are found from the difference of nuclear masses (see refs. [3,13]). The fitting of the parameters of the Saxon-Woods potential performed in 1968-74 implies the following zones of deformed nuclei in the mass number A: 155, 165, 173, 181, 229, 239, 247 and 255. This fitting consists of four stages: 1) the single-particle energies and wave functions are calculated with a certain set of parameters of the potential; 2) the equilibrium nuclear shape is found by the shell correction method thus fixing the parameters of the quadrupole β_2 and hexadecapole β_4 deformation; 3) phonons are calculated in the RPA; 4) the quasiparticle-phonon interactions are taken into consideration, the energies and wave functions of nonrotational states of odd nuclei are calculated and compared with the relevant experimental data. If necessary the parameters of the Saxon-Woods potential can further be changed and four stages of calculations are performed all over again. Such a procedure is repeated until rather a good description of the experimental data on low-lying nonrotational levels of odd nuclei is achieved. The parameters of the Saxon-Woods potential in the afore-mentioned zones are presented in refs. [13-16]. The parameters in zone A=181 have been modified in 1984 [17].

The isoscalar $\kappa_0^{(\lambda\mu)}$ and isovector $\kappa_1^{(\lambda\mu)}$ constants of multipole interactions are determined as follows: 1) the isovector constants $\kappa_1^{(10)}$ and $\kappa_1^{(11)}$ are found from the position of the giant E1-resonance; to exclude a spurious state the isoscalar dipole forces are introduced with the constant $\kappa_0^{(\lambda\mu)}$ which is obtained from the condition of vanishing of the first root of the secular equation; 2) the isoscalar constants $\kappa_0^{(2\mu)}$, $\kappa_0^{(3\mu)}$ and $\kappa_0^{(44)}$ are found from the energies of the first quadrupole, octupole and hexadecapole states; to reduce the number of parameters it is assumed that $\kappa_1^{(\lambda\mu)}/\kappa_0^{(\lambda\mu)} = -1.2$; 3) for $\lambda > 4$ phenomenological estimates given in ref. [4] are used. In the calculations of low-lying states phonons with $\lambda\mu$ =20, 22, 30, 31, 32, 33, 41, 43, 44 and in some cases up to $\lambda\mu$ =77 are taken into account. For each $\lambda\mu$ about 10 and in some cases up to 20 roots of the secular equation are taken into consideration.

In transforming the QPNM Hamiltonian by the canonical Bogolubov transformation one passes from the nucleon operators to the quasiparticle $\alpha_{q\sigma}$ and $\alpha^+_{q\sigma}$ operators. The pairs of operators $\alpha^+_{q\sigma}\alpha^+_{q'\sigma'}$ and

$\alpha_{q'\sigma'}\alpha_{q\sigma}$ are expressed through the phonon operators $Q_{g\sigma}$, $g=\lambda\mu i$ and the quasiparticle operators remain only in the form of $\alpha^+_{q\sigma}\alpha_{q'\sigma'}$. Then, the RPA equations are solved to determine the energies and wave functions of one-phonon states. All the model parameters are fixed at this stage. Using the solutions of the RPA secular equations the QPNM Hamiltonian is transformed to

$$H_{QPh} = \sum_{q\sigma} \varepsilon_q \alpha^+_{q\sigma}\alpha_{q\sigma} + H_v + H_{vq}, \qquad (1)$$

containing free quasiparticles and phonons and the quasiparticle-phonon interaction H_{vq}. Here ε_q is the quasiparticle energy, $q\sigma$ are the quantum numbers of single-particle states, $\sigma = \pm 1$. The explicit form of Hamiltonian (1) is presented in ref. [18].

In the low-lying states the isovector forces are insignificant, therefore they have not been considered in ref. [6]. In the present paper the calculations have been performed with the isoscalar and isovector forces so that not to miss isovector states in the energy interval 1.5-2.5 MeV. Moreover, it has recently been reported in ref. [19] that an isovector 1^+ magnetic collective state had been observed in ^{156}Gd. According to the IBM-2 calculations [20] the Sm isotopes contain isovector 2^+ states.

The nonrotational state wave function of a doubly even deformed nucleus is

$$\Psi_n(K_0^{\pi_0}\sigma_0) = \Big\{\sum_{i_0} R^n_{i_0} Q^+_{g_0\sigma_0} + \sum_{\substack{g_1\sigma_1\\ g_2\sigma_2}} \sqrt{\frac{1+\delta_{g_1g_2}}{2}}\, \delta_{\sigma_1\mu_1+\sigma_2\mu_2,\sigma_0K_0} P^n_{g_1g_2} Q^+_{g_1\sigma_1} Q^+_{g_2\sigma_2}\Big\}\Psi_0, \qquad (2)$$

where Ψ_0 is the ground state wave function, n=1, 2, 3,... is the number of a state with given $K_0^{\pi_0}$. Following ref. [21] we use phonons depending on the sign of projection K into the nuclear symmetry axis. In ref. [5] the function $\mathcal{K}^{K_0}(g_1g_2|g_2'g_1')$ has been introduced under the exact consideration of commutation relations between the phonon operators. Its explicit form is given in refs. [5,6,21]. It was shown in these papers that the function $\mathcal{K}^{K_0}(g_1g_2|g_2'g_1')$ is small if $g_1 \neq g_1'$, $g_2 \neq g_2'$. Therefore, in all the cases where possible only the diagonal terms $\mathcal{K}^{K_0}(g_1g_2|g_2g_1)$ are used. The normalization condition (2) in the $\mathcal{K}$-diagonal approximation has the form

$$1 = \sum_{i_0}(R^n_{i_0})^2 + \sum_{g_1g_2} \frac{1+\delta_{g_1g_2}}{2}\left(P^n_{g_1g_2}\right)^2\left\{1 + \frac{\mathcal{K}^{K_0}(g_1g_2|g_2g_1)}{1+\delta_{g_1g_2}}\right\}. \qquad (3)$$

If the Pauli principle is greatly violated, then at $g_1^\circ = g_2^\circ$ $\mathcal{K}^{K_0}(g_1^\circ g_2^\circ | g_2^\circ g_1^\circ) = -2$ and at $g_1^\circ \neq g_2^\circ$ $\mathcal{K}^{K_0}(g_1^\circ g_2^\circ | g_2^\circ g_1^\circ) = -1$, thus the component $g_1^\circ g_2^\circ$ turns out to be eliminated from the wave function (2).

Further, an average value of H_{QPM} over state (2) is calculated; the variational principle is used to derive equations for the energies $\mathcal{E}_n$ and coefficients R and P of the wave function (2). The explicit form of these equations is represented in ref. /18/.

The IBM is used to describe the low-lying states of deformed nuclei. Here we cite necessary formulae. With s- and d-bosons the IBM Hamiltonian has the form

$$H_{IBM\text{-}1} = -\kappa (Q \cdot Q) - \kappa' (L \cdot L) + \kappa'' (P^+ \cdot P),$$

where

$$Q = [d^+ \times s + s^+ \times \tilde{d}]^{(2)} + \chi [d^+ \times \tilde{d}]^{(2)},$$

$$L = \sqrt{10} [d^+ \times \tilde{d}]^{(1)}, \quad P = \frac{1}{2} \{ (\tilde{d} \cdot \tilde{d}) - (s \cdot s) \}, \tag{4}$$

$\tilde{d}_\mu = (-)^\mu d_{-\mu}$, the sign x means the tensor product.

In the harmonic approximation there are two types of excited states having the symmetry of β - and γ -vibrations and defined by the quantum numbers n_β and n_γ. The wave functions have the following form (see ref. /7/):

for the ground state

$$|g) = (N!)^{-1/2} (b^+)^N |0), \tag{5}$$

for the state $n_\beta = 1$ with $K_n^\pi = 0_2^+$

$$|n_\beta = 1) = [(N-1)!]^{-1/2} b_\beta^+ (b^+)^{N-1} |0), \tag{6}$$

for the state $n_\gamma = 1$ with $K_n^\pi = 2_1^+$

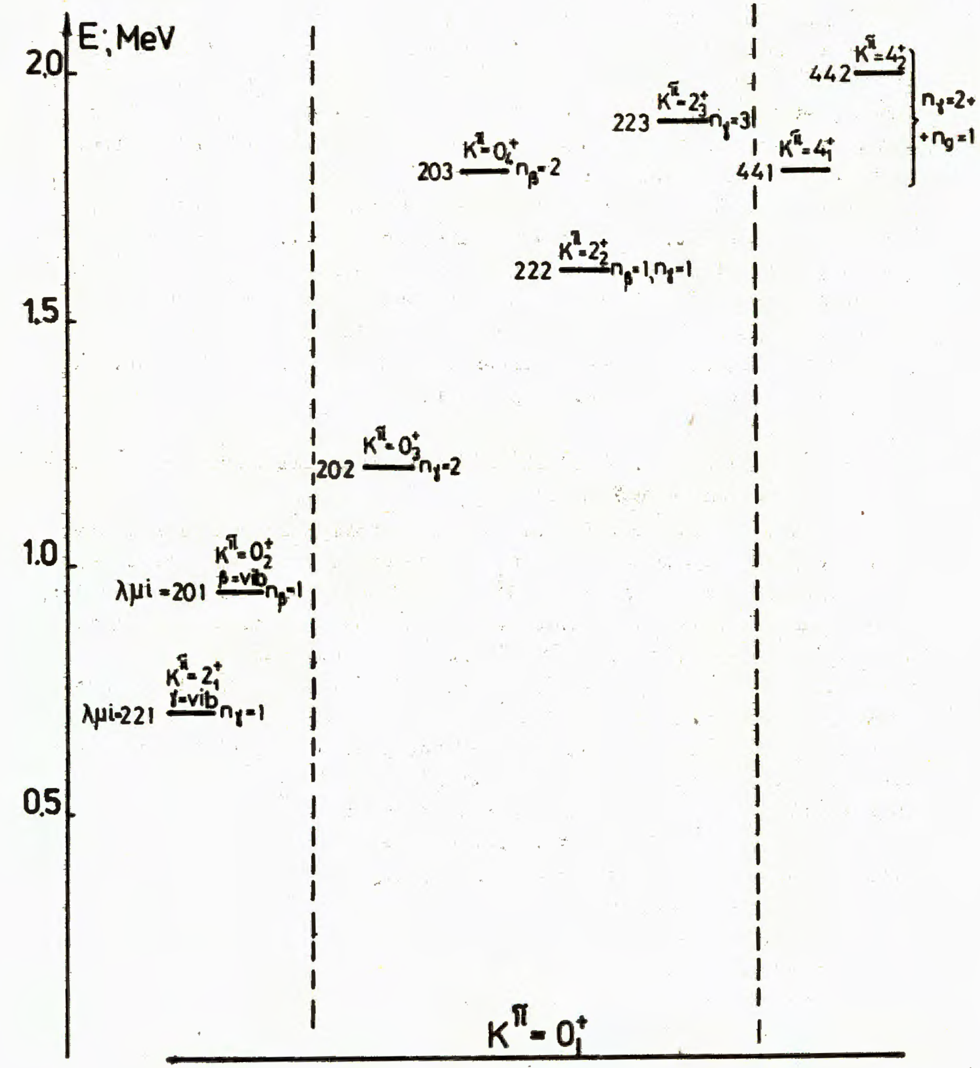
$$|n_\gamma = 1) = [(N-1)!]^{-1/2} d_2 (b^+)^{N-1} |0), \tag{7}$$

where N is the number of bosons equal to a half of the neutrons and protons,

$$b^+ = (1+\beta^2)^{-1/2} (s^+ + \beta d_0^+), \quad b_\beta^+ = (1+\beta^2)^{-1/2} (-\beta s^+ + d_0^+),$$

usually it is assumed that $\beta = 1.24$ (see ref. /7/). In the general case the wave function is

$$|Kq) = \sum_{N_d N_\beta} C^{Kq}_{N_d N_\beta} \{ (d_2^+)^{N_d}_K (b_\beta^+)^{N_\beta} \}_q (b^+)^{N - N_d - N_\beta} |0), \tag{8}$$



Scheme of nonrotational states of deformed nuclei with $K_n^\pi = 2_1^+, 0_2^+, 0_3^+, 0_4^+, 2_2^+, 2_3^+, 4_1^+$ and 4_2^+. For each level the dominating components are shown: on the left by the QPNM and on the right by the IBM.

where $N \geq N_\delta + N_\beta$. The anharmonic corrections in strongly deformed nuclei are not large. Therefore, the wave function (8) has one dominating component. These dominating components for the states with $K^\pi = 0^+$ and 2^+ are shown in the figure.

The study of the spectra of deformed nuclei necessitated an introduction of g boson. In this case the sd subspace is extended to the sdg subspace which has resulted in the introduction of some new parameters. The introduction of g-boson led to 3_1^+ band and in addition to the bands based on 0_3^+, 2_2^+ and 4_1^+ states to those based on 0_4^+, 2_3^+ and 4_2^+ states. In this case the sd dominance is conserved since the weight of g boson in the wave functions of the states $I < 6$ with an energy less than 2 MeV turned out to be small. Thus, in the figure the states 4_1^+ and 4_2^+ are defined by the mixture of two components $n_\gamma = 2$ and $n_g = 1$. The application of the IBM-2 in deformed nuclei encounters difficulties caused by the occurrence of some additional rotational bands which are not observed experimentally.

3. Comparison of the description of states with $K^\pi = 0^+$, 2^+ and 4^+ in the QPNM and IBM

For the decription of doubly even deformed nuclei within the QPNM the wave function (2) is usually taken as a sum of 5-10 one-phonon terms and a great number ($10^2 \div 10^3$) of two-phonon terms. According to our calculations the components $\lambda\mu i = 221$ and 201 contribute more than 80% to γ -vibrational $K_n^\pi = 2_1^+$ and β -vibrational $K_n^\pi = 0_2^+$ states. The states 2_2^+, 2_3^+, 2_4^+, 0_3^+, 0_4^+, 0_5^+, 4_1^+ and 4_2^+ have dominating components $\lambda\mu i$ = 222, 223, 224, 202, 203, 204, 441, 442. Up to the excitation energy of 2 MeV an admixture of two-phonon collective components does not exceed 10%. The states with $K_n^\pi = 3_1^+$ and 3_2^+ are as a rule two-quasiparticle. It can be stated that up to the energies (2.0-2.3) MeV the wave functions of nonrotational states have one dominating one-phonon component; they are shown in the figure.

Now we shall compare the description of β - and γ -vibrational states within the QPNM and IBM. Since the one-phonon components $\lambda\mu i$ = =201 and 221 are dominating[22], the QPNM does not provide a considerably better description of 0_2^+ and 2_1^+ states in comparison with the RPA calculations. Certainly, it is desirable to improve the description of 0^+ states. The consideration of only nonrotational states is unimportant. The rotational bands are described by taking account of the Coriolis interaction and by calculating single-particle states of the Saxon-Woods potential depending on rotation[23]. The wave functions of 0_2^+ and 2_1^+ states are the superposition of a large number of two-quasiparticle components. These states can be excited in one-nucleon transfer reactions. For the 2_1^+ states these are the particle-hole components. A correct description of the largest components of the 2_1^+ state wave functions in some nuclei has been confirmed experimentally. Note that a small part of the two-quasiparticle state space is taken into account while describing the 0_2^+ and 2_1^+ states.

In the IBM the one-boson components $n_\beta = 1$ and $n_\gamma = 1$ dominate in the 0_2^+ and 2_1^+ state wave functions. At the beginning of the region of deformed nuclei the main components of the wave functions of these states are particle-particle and at the end of the region are hole-hole[24]. A doubly even nucleus A+2 N is larger by unity than A. Due to the particle-particle structure of $n_\gamma = 1$ operator at the beginning of the region of deformed nuclei the 2_1^+ states can be well excited in reactions of the (dp) or (^{3}He,d)-type and should not be excited in the (dt) or (d,^{3}He)-type reactions. The whole space of two-quasiparticle states with $K^\pi = 2^+$ and 0^+ of the particle-particle - type is only that contained in the one-boson states $n_\gamma = 1$ and $n_\beta = 1$. Due to anharmonicity a fraction of strength of the $n_\gamma = 1$, $n_\beta = 1$ states is shifted into higher $K^\pi = 2^+$ and 0^+ states. Therefore, assertion about the excitation in the (dp) and (^{3}He,d)-type reactions and nonexcitation in the (dt) or (d,^{3}He)-type reactions can also be attributed to 2_2^+, 2_3^+, and 2_4^+ states. These reactions may verify the validity of the IBM in describing excited states as those containing only two-quasiparticle components of the particle-particle-type. The success of the IBM in describing the integral characteristics of 0_2^+ and 2_1^+ states is obvious especially as concerns E2 transitions to the ground state band and between the bands based on 0_2^+ and 2_1^+ states.

Compare the description of K_n^π = 0_3^+, 0_4^+, 0_3^+, 2_2^+, 2_3^+, 2_4^+ etc. states. In the QPNM the states 0_3^+, 0_4^+, 0_5^+ have large((80-95)%)one-phonon components $\lambda\mu i$ =202, 203, 204 and the states 2_2^+, 2_3^+, 2_4^+ large one-phonon components $\lambda\mu i$ =222, 223, 224. In some cases a mixture of one-phonon components is observed. It is to be noted that in describing these states the space of two-quasiparticle states became broader in comparison with that determining 201 and 221 phonons. According to ref.[6] these states should not have dominating collective two-phonon components and first of all those constructed of 201 and 221 phonons. It can be stated that the available experimental data[25-29] do not contradict this conclusion.

In the IBM the dominating components of the wave functions are: $\{n_\gamma = 1, n_\beta = 1\}$ for 2_2^+ states, $n_\gamma = 3$ for 2_3^+, $n_\gamma = 2$ for 0_3^+ and $n_\beta = 2$ for 0_4^+

states. The main part of the strength of one-boson states $n_\gamma=1$, $n_\beta=1$ is concentrated in 2_1^+ and 0_2^+ states and only a small fraction of their strength is attributed to 2_2^+, 2_3^+, 0_3^+ and 0_4^+ states. This means that the dominating components of the wave functions of 2_2^+, 2_3^+, 0_3^+ and 0_4^+ states are two- and three-boson ones and the contribution of one-boson and thereby two-quasiparticle components is very small. From the microscopic point of view, within the IBM only a small part of the space of two-quasiparticle states entering into the wave functions of β ($n_\beta=1$) - and γ ($n_\gamma=1$) -vibrational states is taken into consideration. With g-boson the space of two-quasiparticle states becomes broader. However, according to the calculations[30] for ^{168}Er the weight of g-boson in rotation bands based on 2_2^+, 2_3^+, 0_3^+ and 0_4^+ states does not exceed 30% and the two- and three-boson components are still dominating.

As a result we state that the wave functions of 2_2^+, 2_3^+, 2_4^+, 0_3^+, 0_4^+, 0_5^+ states in the IBM have large two- and three-boson components and in the QPNM they have large one-phonon components with $i=2, 3, 4$ and have no pronounced two-phonon collective components. In the QPNM the structure of these is mainly determined by the set of two-quasiparticle components that are absent in the IBM. As it has been mentioned in ref.[31] there is a fundamental difference in describing these states within the QPNM and the IBM.

We shall consider $K^\pi_n = 4_1^+$ and 4_2^+ states. According to the calculations within the QPNM the one-phonon components $\lambda\mu i$ =441 and 442 dominate in the wave functions of 4_1^+ and 4_2^+states; the contribution of two-phonon components {221,221} does not exceed (1-5)%. Obviously, the calculations in the approximation used in ref.[6] provide as large as possible shift of two-phonon poles. The inclusion of some corrections and three-phonon components of the wave functions will probably result in a small increase of the contribution of two-phonon components to 4_1^+ and 4_2^+ states. The situation cannot be changed cardinally. The statement about the absence of two-phonon collective 4^+ states is valid. According to the new experimental data[27-29] for instance, in ^{168}Er the state $I^\pi=4^+$ with an energy of 2.03 MeV which has been thought to have K=4 and treated in refs.[7,32] as the two-phonon state has $K^\pi=0^+$ and is not two-phonon.

In the IBM the 4_1^+ state is treated as the two-boson $n_\gamma=2$ state. The inclusion of g boson leads to the occurrence of $K=3_1^+$ and 4_2^+ states. According to the calculations of ref.[51] for ^{168}Er the weight of g-boson in the wave functions of 3_1^+, 4_1^+ and 4_2^+ states does not exceed 30%. This means that the two-boson $n_\gamma=2$ components are dominating. There is an essential difference in describing 4_1^+ as well as 3_1^+, 3_2^+ and 4_2^+ states within the QPNM and IBM. The wave functions of these states should contain either large two-quasiparticle components or large two-boson components.

4. Analysis of experimental data

What description of nonrotational states of deformed nuclei is more correct is to be checked experimentally. The experimental data [27-29,33-42] and the results of calculations within the QPNM[6,15,22,43] are given in the table. The experimental energies and available data on the contribution of one-phonon components and their structure are presented in the table, too. There phonons are denoted by $\lambda\mu i$ and their contribution to the wave function normalization is given in %. To denote neutron nn and proton pp components we have used the asymptotic quantum numbers $Nn_z\Lambda$ and ↑ for $K=\Lambda+1/2$ and ↓ for $K=\Lambda-1/2$. The B(Eλ)-values are given in the single-particle units. The energies and structure of β - and γ -vibrational states are described rather well within the QPNM and are not represented in the table.

In ^{156,158}Gd it has experimentally been observed[34-36] by three bands based on excited $K=0^+$ states. In both the nuclei the 2^+0_2 and 2^+0_3 states have large B(E2)-values for transitions into the ground states. According to the calculations[35,36] in the IBM a large value of B(E2) is attributed either to the 2^+0_2 or 2^+0_3 state but not to both the states simultaneously. By the experimental data the 0_3^+ and 0_4^+ states have large one-phonon components and the two-quasiparticle configuration pp411↑-411↓ in 0_4^+ in ^{158}Gd is clearly seen in (t,α) reaction. The states $K=4_1^+$ and 4_1^+ in both the nuclei have large two-quasiparticle components pp411↑+413↓ and nn521↓+523↓ respectively. The states ^{158}Gd presented in the table contain large one-phonon (two-quasiparticle) components and their structure is qualitatively well described in the QPNM. They cannot be described in the IBM with s- and d-bosons, according to which these states have no large one-boson components. In ref.[44] for the description of ^{158}Gd within the IBM there have been introduced additional s', d' and g bosons and the B(E2)-values have correctly been described for the 2^+0_2 and 2^+0_3 states. In this case the space of two-quasiparticle states is extended within the IBM. However, this modification of the IBM is hardly effective due to a large number of new parameters.

In ^{158}Dy[37] the state $K^\pi_n=2_2^+$ with an energy of 1.852 MeV is treated as the two-phonon {221,201} state on the basis of the data on inten-

Table. Structure of states 0_3^+, 0_4^+, 2_2^+, 2_3^+, 4_1^+ and 4_2^+

Nucleus	K_n^π	Experiment $\mathcal{E}$, MeV	Experiment structure	Calculations in the QPNM $\mathcal{E}$, MeV	Calculations in the QPNM structure
^{158}Gd	0_3^+	1.452	B(E2)=0.36	1.7	202 79%; B(E2)=0.4 202:nn521↑-521↑ 48%
	0_4^+	1.743	(t,α)pp411↑-411↑ is large	1.9	203 70%; B(E2)=0.006 203:pp411↑-411↑ 31%
	4_1^+	1.380	(t,α)pp411↑+413↓ is large	1.2	441 95%; {221,221} 3.3% 441:pp413↓+411↑ 85%
	4_2^+	1.9...	(dp)nn521↑+523↓ is large	1.7	442 98%; 442:nn521↑+523↓ 87%
^{158}Dy	2_2^+	1.852		2.4	222 86%; 223 2%; {221,202} 5% 222:nn642↑-660↑ 80% B(E2)=0.05
	4_1^+	1.895	logft=4.9 nn521↑+523↓ is large	2.2	441 92%; 442 6% 441:pp413↓+411↑ 36% nn521↑+523↓ 28%
^{160}Dy	4_1^+	1.694	logft=4.7 nn521↑+523↓ is large	1.7	441 98%, {221,221} 0.6% 441:nn521↑+523↓ 85%
^{162}Dy	4_1^+	1.535	(dt) nn521↑+523↓ is large	1.7	441 98% 441:nn521↑+523↓ 92%
^{168}Er	0_3^+	1.422	(tp) 10% of the ground state	1.5	202 75%; 203 6% {221,221} 5% 202:nn521↓-521↓ 54%
	0_4^+	1.834	(tp) 2.4% of the ground state (tα) pp411↓-411↓ is large	1.8	203 77%, 202 10%; {221,221} 2% 203:pp411↓-411↓ 28%
	2_2^+	1.848	(tp) 60% of the 2^+ ground band	1.7	222 98%; B(E2)=0.6 222:nn512↑-521↓ 90%
	2_3^+	1.930		1.9	223 96%
	4_1^+	2.055		2.2	441 94%; {221,221} 1%
^{170}Er	2_2^+	1.416		1.3	222 96%; B(E2)=0.45 222:nn512↑-521↓ 87%
^{172}Yb	0_3^+	1.405	B(E2)=0.01	1.6	202 77%; 201 3%; {221,221} 1% B(E2)=0.03
	0_4^+	1.795	B(E2)=0.14	1.8	203 81%; B(E2)=0.02
	0_5^+	1.893	B(E2)=0.33	1.9	204 90% B(E2)=0.1

continuation of table

Nucleus	K_n^π	$\mathcal{E}$, MeV	structure	$\mathcal{E}$, MeV	structure
	2_2^+	1.609	(dp),(dt) nn512↑-521↓ ≈ 25% B(E2)=0.4	1.7	222 90%; B(E2)=1.5 222:nn512↑-521↓ 40% pp411↑+411↓ 6%
	4_1^+	2.073	(p,)pp404↓+411↓ is noticable	1.9	441 99% 441:pp404↓+411↓ 98%
^{178}Hf	2_2^+	1.8		1.9	222: 86% B(E2)=0.002 222:nn624↑-642↑ 93%

sities of γ-transitions to the bands of the ground, β - and γ-vibrational states. According to the calculations within the QPNM the energy of this state turned out to be overestimated whereas the two-phonon configuration {221,201} small. This case needs a more thorough experimental and theoretical investigation. It is to be noted that ^{158}Dy is on the boundary of the region of deformed nuclei.

The states $K_n^\pi = 4_1^+$ in ^{158,160,162}Dy are described within the QPNM /43/ as the hexadecapole ones. They contain a large two-quasiparticle component nn521↑+523↓ which is clearly seen in β-decay and (dt) reaction.

The most complete experimental data are available for ^{168}Er /27-29,33/. The rotational band based on the states $K_n^\pi = 0_3^+$, 0_4^+ and 2_2^+ are strongly excited in (t,p) reaction; in (t,α) reaction the two-quasiparticle configuration pp411↓-411↓ is clearly seen in the 0_4^+ 1.834 MeV state. This indicates that the wave functions of these states have large one-phonon components. The state $K_n^\pi = 4_1^+$, 2.056 MeV decays into the band with $K^\pi = 4^-$, 1.094 MeV and according to refs. /27-29/ has no large two-phonon {221,221} components. By the calculations within the QPNM the wave functions of all the states presented in the table have large one-phonon (two-quasiparticle) components which is confirmed by the experimental data. The detailed calculations in the IBM for ^{168}Er have been performed in refs. /7,11,12,29/. By the calculations made in refs. /11,12,29/ within the IBM all the states presented in the table are formed of two and three bosons, i.e. have large components $n_\gamma = 2$, $n_\beta = 2$, $\{n_\gamma = 1, n_\beta = 1\}$ and so on and almost have no one-phonon (two-quasiparticle) components. With g-boson included in the IBM /30/ the space of two-quasiparticle states becomes breader though the situation is not improved due to the occurrence of the $K^\pi = 1^+$ state with an energy of 1.443 that is not observed experimentally. According to ref.[30], the calculations of excited states of ^{168}Er within the IBM-2 with neutron s_ν, d_ν and proton s_π, d_π bosons, have produced 8 extra bands (with band heads below 2 MeV) not observed experimentally, i.e. there is a contradiction with the experimental data. The theory providing extra states is worser than that not describing all the states.

Now we consider ^{172}Yb. According to the experimental data[39] the B(E2)-values for the excitation of 2^+_2 state are only by four times less than for the excitation of 2^+_1 state. The calculations within the QPNM provide for them almost equal values. The results of calculations of B(E2)-values are very sensitive to the behaviour of single-particle neutron levels near the Fermi levels. By the calculations[41] within the IBM B(E2)-values for the excitation of 2^+_2 state are by 12 times less than for the excitation of 2^+_1 state and the energy of 2^+_2 state equal to 2.268 MeV is about 0.6 MeV times as large as the experimental one. It follows from (dp) and (dt) reactions in ref.[40] that the wave function of 2^+_2 state has a large two-quasineutron component nn512↑−521↓ and a small two-quasiproton component from (p,α) reaction. By the calculations within the QPNM the wave function of 2^+_2 state in ^{172}Yb contains 90% of the one-phonon state $\lambda\mu i$=222 with a large two-quasineutron component nn512↓−521↓ and a small two-quasiproton component pp411↓+411↓ which may appear in (p,α) reaction. According to the calculations within the QPNM, ^{172}Yb contains four excited 0^+ states with energies less than 2 MeV, which is in agreement with the experimental data. There is no strong discrepancy between the calculated and experimental B(E2)-values for these 0^+ states.

The calculations within the QPNM show that the contribution of two-phonon components to the wave functions of 0^+_2, 0^+_3, 0^+_4, 2^+_2 and 4^+_1 states in ^{172}Yb does not exceed 5%. By the calculations of ref.[41] within the IBM the energies of two-boson $n_\beta=2$, $\{n_\gamma=1, n_\beta=1\}$ and $n_\gamma=2$ states were not fitted and they turned out to be higher than the experimental ones of the states 0^+_3, 0^+_4, 2^+_2 and 4^+_1. The states calculated in the IBM have another structure in comparison with the 0^+_3, 0^+_4, 2^+_2 and 4^+_1 states measured experimentally and they should not be compared with each other.

There are experimental indications to the existence of 2^+_2 state in ^{178}Hf. In the calculations[45] of the ^{178}Hf states within the IBM-2 the data on 2^+_2 state are not presented.

There are experimental data[42,46,47] on 0^+_3 and 2^+_2 states in ^{230,232}Th and ^{234}U at energies below 1.2 MeV. Following ref.[42] the excitation of 2^+_2 state in ^{230}Th results in rather a large B(E2)-value.

These states represent a scene for confrontation between the QPNM and IBM.

The experimental data on the $K^\pi_n = 0^+_3$, 0^+_4, 0^+_5, 2^+_2, 2^+_3, 4^+_1 and 4^+_2 states presented in the table indicate the presence in their wave functions of large one-phonon or two-quasiparticle components that are qualitatively well described within the QPNM and that are almost absent in the IBM. Hence, it follows that the states with large one-phonon components (except 0^+_2, 2^+_1 and 4^+_1 with g-boson) should be excluded from the states that are assumed to be correctly described within the IBM.

The hexadecapole excitations in deformed nuclei have been calculated within the QPNM in ref.[43]. It was shown that the Gd and Dy isotopes contain two-quasiparticle 4^+_1 states with an energy less than 2 MeV. The Er, Yb and Hf isotopes contain collective 3^+_1 states with B(E4)=0.8-1.6 for the excitation of $I K=4^+3_1$ rotational states. The Os isotopes include low-lying collective 4^+_1 states. The description of hexadecapole excitations is in agreement with the relevant experimental data.

5. Conclusion

The energies and wave functions of nonrotational two-quasiparticle and one-phonon states of doubly even deformed nuclei calculated in 1960-1975 provided rather a correct their description. This was confirmed by the latest experimental data. A rigorous consideration of the Pauli principle in the two-phonon terms of the wave function (2) within the QPNM allowed us to make a conclusion about the absence of two-phonon collective states in deformed nuclei. The conclusion was also made about small admixtures of two-phonon components in the wave functions of β- and γ-vibrational states. The calculations within the QPNM did not provide a considerably improved description of 0^+_2 and 2^+_1 states.

The results of calculations of one-phonon and two-quasiparticle states depend strongly on the behaviour of single-particle levels near the Fermi energy. The accuracy of calculations is limited by a rough description of an average field by the Saxon-Woods potential. The accuracy of the description of states with an excitation energy up to 2.5 MeV necessary for a detailed analysis of the relevant experimental data can be improved by fitting some model parameters for each nucleus and by introducing many corrections.

In the calculations within the QPNM with the wave function (2) one cannot pretend to the description of the fragmentation of two-phonon states. The fragmentation of two-phonon states can be correctly described by adding three-phonon terms into the wave function (2). It is possible to state that the introduction of three-phonon terms into (2) will not increase considerably the contribution of two-phonon components to the wave functions of $0^+_{2,3,4}$, $2^+_{1,2,3}$, $3^+_{1,2}$ and $4^+_{1,2}$ states. According to ref. /6/, an exact inclusion of the Pauli principle shifts the three-phonon poles towards higher energies. Note that the calculations in ref. /48/ with the phonons constructed of "ideal" bosons result in the same formulae for the shift of a two-phonon pole as the calculations in refs. /5,6/.

A crucial discrepancy in describing the structure of nonrotational states of deformed nuclei between the QPNM on the one hand and the IBM, the Bohr-Mottelson model and its microscopic analog as well as other phenomenological models on the other hand is the problem of existence of low-lying two-phonon collective states. The confrontation between the QPNM and IBM in describing 0^+_3, 0^+_4, 2^+_2, 2^+_3, 4^+_1 and 4^+_2 states is a consequence of the above-said discrepancy; here the states differ qualitatively in structure.

It may be stated, as it has been made in ref. /25/, that there are no reliable experimental data on two-phonon collective states in deformed nuclei. Attempts to explain their absence were made in two cases: for ^{168}Er and for the Th and U isotopes. According to refs. /7,32,49/, the absence of quadrupole two-phonon states with an energy less than 2 MeV in ^{168}Er is due to a large anharmonicity of γ-vibrational mode due to a triaxial shape of the ^{168}Er. However, the calculations in refs. /50,51/ indicate that though ^{168}Er is soft with respect to γ deformation, the energy minimum is achieved at γ_0=0. According to the experimental data /52/, there are no two-phonon octupole {301,301} 0^+ states in the Th and U isotopes. Their absence is usually explained by the existence of a stable octupole deformation experimentally observed in nuclei with A<228. According to the calculations /53/ a stable octupole deformation may occur in nuclei with A<228. Therefore, the absence of two-phonon {301,301} 0^+ states in ^{228,230}Th, ^{232,234}U and other nuclei is yet to be explained within the IBM, the Bohr-Mottelson model and other models.

The coupling of single-particle and collective motions or the quasiparticle-phonon interaction turned out to be essential for almost all nonrotational, except for the lowest ones, states of deformed nuclei. Extraction of a subspace of collective states breaks this coupling. Hence, one cannot expect a good description of the structure of states lying above β-, γ- and the first octupole states of doubly even nuclei and nonrotational states of odd and odd-odd nuclei. One should remember that the separation of the space of collective nonrotational states is ambiguous since in most of the cases there is no clear boundary between collective, less collective and weakly collective states.

We may state that in well deformed nuclei (in contrast with those of the transitional regions) in the states with an energy of (1.5-2.0) MeV the many-quasiparticle or many-boson components of the wave functions do not play a decisive role. There arises a question whether the IBM is valid for these nuclei. It should be clarified how correct is the SU(6) approximation. Thus, in ref. /54/ the SU(6) limit has been found for the QPNM Hamiltonian of spherical nuclei. The calculation of maximal numbers of bosons and the test of the conditions giving the SU(6) limit have been performed. The numerical calculations for the Zn isotopes indicate that the conditions following from the SU(6) approximation are fulfilled rather poorly.

The contradiction between the QPNM and the IBM in describing the structure of $K^\pi_n = 0^+_3$, 0^+_4, 2^+_2, 2^+_3, 4^+_1 and 4^+_2 states is mainly due to the drawback of the IBM - inclusion of a very small part of the space of two-quasiparticle states into consideration. It cannot be removed by an optimal fit of the IBM parameters. The inclusion of 2^+ states with large isovector components into the IBM /55/ does not enlarge the region of two-quasiparticle states, whereas the inclusion of s'-, d'- and g-bosons increases greatly the number of parameters. If the number of the parameters is comparable with that of the analysed experimental data, one can hardly judge about the efficiency of the model. It should be noted that in spherical nuclei the states of the type concidered in this paper lie above the two-phonon states and the aforesaid discrepancies are not yet obvious.

Further investigations of the structure of doubly even deformed nuclei need experiments on measurement of the contribution of two-quasiparticle components to the wave functions of rotational bands based on K^π_n = 0^+_3, 0^+_4, 0^+_5, 2^+_2, 2^+_3, 3^+_1, 3^+_2, 4^+_1, 4^+_2 and other states lying at energies 1.5-2.5 MeV and search for two-phonon collective states.

Acknowledgements

The author is grateful to V.O.Nesterenko, A.V.Sushkov and N.Yu. Shirikova for help in numerical calculations.

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Received by Publishing Department on December 6, 1985.

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