

$\Delta L \geq 4$ lepton number violating processesRenato M. Fonseca^{*} and Martin Hirsch[†]*AHEP Group, Instituto de Física Corpuscular, C.S.I.C./Universitat de València, Spain Edificio de Institutos de Paterna, Apartado 22085, E-46071 València, Spain*

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We discuss the experimental prospects for observing processes which violate lepton number (ΔL) in four units (or more). First, we reconsider neutrinoless quadruple beta decay, deriving a model independent and very conservative lower limit on its half-life of the order of 10^{41} ys for ^{150}Nd . This renders quadruple beta decay unobservable for any feasible experiment. We then turn to a more general discussion of different possible low-energy processes with values of $\Delta L \geq 4$. A simple operator analysis leads to rather pessimistic conclusions about the observability at low-energy experiments in all cases we study. However, the situation looks much brighter for accelerator experiments. For two example models with $\Delta L = 4$ and another one with $\Delta L = 5$, we show how the LHC or a hypothetical future pp collider, such as the FCC, could probe multilepton number violating operators at the TeV scale.

DOI: [10.1103/PhysRevD.98.015035](https://doi.org/10.1103/PhysRevD.98.015035)**I. INTRODUCTION**

So far, no lepton (L) or baryon (B) number violating process has been observed experimentally. However, there are good reasons to believe that neither of these quantities are actually conserved. In fact, even within the standard model (SM), nonperturbative effects such as the sphaleron [1] violate both B and L . More phenomenologically, also the observed baryon asymmetry of the Universe points to the existence of B violation at some point in its early history.

From the viewpoint of standard model effective theory, one can build nonrenormalizable operators which violate B and L [2,3]. The lowest dimensional operator of this kind, the Weinberg operator, appears at dimension 5 ($d = 5$) and violates L by two units. This operator generates Majorana neutrino mass terms which can be experimentally probed by the observation of neutrinoless double beta decay, $0\nu\beta\beta$: $(A, Z) \rightarrow (A, Z \pm 2) + 2e^\mp$ (for recent reviews see for example [4,5]). Next, at $d = 6$, one finds $\Delta B = \Delta L = 1$ operators. These operators cause proton decay in modes such as the famous $p \rightarrow e^+\pi^0$. This and other two-body nucleon decays are well known to arise in a variety of models, most notably in grand unified theories—see [6,7] and references contained therein.

The gauge structure of the SM and its field content is such that $\Delta L = 2n + \Delta B$ for all nonrenormalizable operators

(n being an integer). Thus, for example, even ΔL is associated to even ΔB , so no proton decay mode with $\Delta L = 2$ can exist. However, starting at $d = 9$ one finds $\Delta L = 3$ operators, associated to $\Delta B = 1$. Also, operators relevant for $\Delta B = 2$ processes, such as neutron-antineutron oscillations, appear first at $d = 9$. One would naively assume that the rates for $\Delta L \geq 3$ (or also $\Delta B \geq 2$) processes are necessarily much smaller than those corresponding to the lower dimensional nonrenormalizable operators. However, this may not be the case, and in fact it is possible that $\Delta B, \Delta L = 1$ processes are forbidden altogether. This is exactly what happens in the standard model, since sphalerons are $\Delta B = \Delta L = 3$ transitions (thus, sphalerons cannot destroy protons).¹

There is also the possibility that beyond the SM there exists some (so far unknown) symmetry such that lepton number and/or baryon number can only be created or destroyed in larger multiples. For $\Delta L = 3$ this has been recently discussed in [11]; see also [12]. In that case, for example, standard proton decay modes are absent and one is left with $p \rightarrow e^+\bar{\nu}\bar{\nu}$, $\pi^0 e^+\bar{\nu}\bar{\nu}$, $e^-\nu\nu\pi^+\pi^+$ and, more interestingly, $e^+e^+e^+\pi^-\pi^-$. As noted above, these processes are induced by $d = 9$ or higher operators, which implies that the proton decay rate is suppressed by many powers of the new physics scale Λ . Consequently, one can have $\Lambda \sim \text{TeV}$, making it possible for colliders to probe this hypothesis [11].

In this paper, we discuss $\Delta L = 4$ processes (addressing also the possibility of having $\Delta L \geq 5$), and analyze whether any of these processes can possibly be observed in the

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¹It is interesting to note that it has long been believed that sphaleron transitions are unobservable experimentally. However, some recent papers on sphaleron rates at accelerators have come to much more optimistic conclusions [8–10].

TABLE I. Some examples of $\Delta L = 4$ operators. Note that the lowest dimensional operator at $d = 10$ is unique. All odd-dimensional operators with $d < 23$ and $\Delta L = 4$ violate also B , and the smallest ones appear at $d = 15$.

ΔB	Operator dimension	Example operators
0	10	$LLLLHHHH$
0	12	$\partial\partial LLLLLHHHH, \bar{Q}\bar{u}^c LLLLLHHH, \bar{u}^c \bar{u}^c d^c LLLL$
0	14	$\partial\partial\bar{u}^c \bar{u}^c d^c LLLL, \bar{u}^c d^c \bar{e}^c \bar{e}^c LHHHHH$
2	15	$\bar{u}^c \bar{u}^c \bar{u}^c \bar{u}^c \bar{d}^c \bar{e}^c \bar{e}^c LL, \bar{u}^c \bar{u}^c \bar{u}^c \bar{u}^c \bar{u}^c \bar{e}^c \bar{e}^c \bar{e}^c$

foreseeable future. Specific example models are constructed where, due to the presence of some symmetry, operators involving fewer leptons are forbidden.

We start by noting that all $\Delta L = 4$ operators must have dimension greater or equal to 10 (see Table I for examples). Note that at $d = 10$ there is just one unique $\Delta L = 4$ operator.² At $d = 12$ there are already eight different operators (including derivatives), so we show only some examples for $d \geq 12$ [13]. Some operators require more than one generation of fermions, such as the operator at $d = 10$. Adding two derivatives to this operator, a very similar operator can be realized with just one generation of leptons at $d = 12$. We discuss these two operators in more detail in Secs. II and IV. The second example at $d = 12$ simply exchanges one Higgs boson from the operator at $d = 10$ for two quarks $\bar{Q}\bar{u}^c$, equivalent to a Yukawa interaction. In this way, higher dimensional operators with fewer Higgs fields can be constructed. This is important, as is seen in Secs. II and III, for experiments at low energies. Also at $d = 12$ we can find an operator involving only fermions (third example in Table I). However, for this particular operator the final state in any process always involves neutrinos, making it impossible to tag the lepton number experimentally.

We also show in the Table II examples of $\Delta L = 4$ operators with $d = 15$, both of which violate baryon number. The first one has two L fields; thus the $SU(2)_L$ contractions again lead to final states involving a neutrino. We are more interested in the other operator, with four charged leptons. In fact, its realization/decomposition necessarily involves new colored fields which can be searched at the LHC, as we point out in Sec. IV.

As far as we know, the only $\Delta L = 4$ process treated in some detail in the literature is neutrinoless quadruple beta

²We use the word *operator* to denote a combination of fields, independently of the number of contractions. In the case of $L_i L_j L_k L_l HHHH$ (subscripts denote the lepton flavors), there are two contractions: the square of the Weinberg operator, $\mathcal{O}_{ijkl} = \mathcal{O}_{ij}^W \mathcal{O}_{kl}^W$, and another one \mathcal{O}'_{ijkl} . Nevertheless, it is not necessary to include in the Lagrangian this last one because it is related to the \mathcal{O} contraction with the $ijkl$ indices permuted. Furthermore, note that there are several identities among the entries of the tensor \mathcal{O}_{ijkl} , so a total of six couplings/numbers encode all possible $L_i L_j L_k L_l HHHH$ interactions (for example, if three or more lepton indices are the same, the operator is identically 0).

TABLE II. List of lowest dimensional operators, after electro-weak symmetry breaking, involving charged lepton number violation in ΔL units. For $\Delta L > 4$, more than one generation of leptons and/or quarks is necessary (the alternative would be to add derivatives).

ΔL	Minimum operator dimension	Operators with smallest dimension
1	6	$e^1 \bar{d}^3, e^1 u^2 d^1$
2	9	$e^2 u^2 \bar{d}^2$
3	12	$e^3 u^4 \bar{d}$
4	15	$e^4 u^6$
5	21	$e^5 u^6 \bar{d}^3, e^5 u^8 d$
6	24	$e^6 u^8 \bar{d}$
...

decay ($0\nu 4\beta$), having been discussed for the first time in [15]. A simple power counting suggests that the decay rate associated to $0\nu 4\beta$ is extremely small if all particles which mediate this process are heavy [16]. However, the analysis in [15,16] leaves open the possibility of having an observable $0\nu 4\beta$ decay rate, if several of the mediator particles are very light [here light implies masses of the order of the nuclear Fermi scale, i.e., $\mathcal{O}(0.1)$ GeV]. Thus, in Sec. II we calculate a very conservative lower limit on the $0\nu 4\beta$ decay lifetime, based on collider searches for charged bosons. We find that this minimum lifetime is around 20 orders of magnitude larger than the current experimental limit [17], rendering $0\nu 4\beta$ virtually unobservable.

In Sec. III we extend the discussion to other $\Delta L = 4, 5, 6, \dots$ low-energy processes. In all cases our simple rate estimate is far below experimental sensitivities. The most optimistic case, dinucleon decay of the form $(A, Z) \rightarrow (A - 2, Z - 2) + 2\pi^- + 4e^+$, is expected to be at least some 8 orders of magnitude beyond the current Super-Kamiokande sensitivity. Thus it seems impossible for low-energy experiments to test lepton number violation in 4 or more units.

However, the prospects of observing $\Delta L \geq 4$ processes at colliders are good, as we discuss in Sec. IV.³ Indeed, because operator dimensionality becomes irrelevant at energies comparable to the new physics scale, the LHC is in a good

³See also [18], where the authors studied Higgs decays into four same-sign leptons in the minimal left-right symmetric model.

position to probe the creation or destruction of groups of four or more charged leptons. We construct two example models for $\Delta L = 4$ (and one for $\Delta L = 5$) and calculate production cross sections for the different particles in these models at pp colliders (the LHC and a hypothetical $\sqrt{s} = 100$ TeV collider). From these cross sections we derive lepton number violating event rates and estimate the scales up to which pp colliders can test such kinds of models. We then end the paper with a summary.

II. MINIMUM LIFETIME OF NEUTRINOLESS QUADRUPLE BETA DECAY

The possibility of observing $\Delta L = 4$ processes via neutrinoless quadruple beta decay ($0\nu 4\beta$) was put forward in [15]. Three candidate nuclei for the decay $(A, Z) \rightarrow (A, Z + 4) + 4e^-$ were identified: by far the most promising one is ^{150}Nd , which could decay into ^{150}Gd plus four electrons with a total kinetic energy of $Q = 2.08$ MeV.⁴ With 36.6g of the isotope, the NEMO-3 detector is well suited to measure this decay provided that it happens at a reasonable rate. Recently, based on an exposure of $0.19 \text{ kg} \cdot \text{y}$ of ^{150}Nd , the collaboration reported a lower limit of $(1.1\text{--}3.2) \times 10^{21}$ y on the half-life for this particular process, at a 90% confidence level [17]. The range in this number is explained mostly by the fact that, so far, no reliable theoretical calculation of the single electron spectra has been done for this process. Nevertheless, as can be seen, the result is fairly insensitive to such details.

Given the nonobservation of $0\nu 2\beta$ decay events so far, one has to wonder if $0\nu 4\beta$ decays will ever be observed. If the only contribution to the latter process is the conversion of a pair of neutrons into protons and two electrons, twice repeated (see Fig. 1), then one would expect the approximate relation

$$\tau_{0\nu 4\beta} \sim \left(\frac{\tau_{0\nu 2\beta}}{10^{26} \text{ y}} \right)^2 \left(\frac{q}{100 \text{ MeV}} \right)^2 \left(\frac{\text{MeV}}{Q} \right) 10^{88} \text{ y} \quad (1)$$

between the lifetimes of double and quadruple beta decay without neutrinos.⁵ In this expression $q \sim 100$ MeV stands for the typical momentum transfer in a nucleus. Using the experimental lower limit on $\tau_{0\nu 2\beta}$ for various nuclei of the order of 10^{26} years [20,21], we can extract the lower bound $\tau_{0\nu 4\beta} \gtrsim 10^{88}$ years.

⁴The isotope $^{126}_{50}\text{Sn}$ may decay into $^{126}_{54}\text{Xe} + 4e^-$, with $Q = 3.13$ MeV [19]. However, it can also undergo single beta decay, with a half-life of 2.3×10^5 years.

⁵This naive estimate is based on a simple dimensional analysis, assuming that both processes (involving potential different parent nuclei) have the same kinetic energy Q . This coarse assumption inevitably introduces a large error in the result which, nevertheless, is of no material significance in the face of such large lifetimes.

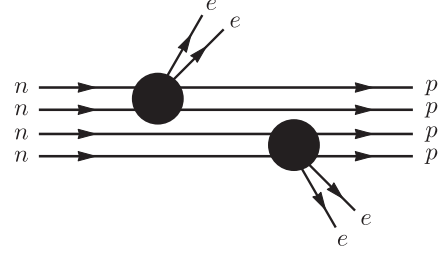


FIG. 1. Quadruple beta decay induced by two virtual double beta decays. This contribution to the decay rate is necessarily very small, given the current limits on the $0\nu 2\beta$ decay lifetime of various nuclei.

However, this estimate assumes that the main contribution to neutrinoless quadruple beta decay comes from two virtual double beta decays. This does not need to be the case; indeed, as previously mentioned, it is possible to forbid entirely $\Delta L = 2$ processes and still have those with $\Delta L = 4$.

In the following we argue that an important constraint on neutrinoless quadruple beta decay can be derived using data from collider searches of charged bosons. However, before proceeding, let us consider first what is the expected value of the $0\nu 4\beta$ decay rate from simple power counting (see also [15,16]). We start by noting that there are 12 fermions involved, so after electroweak symmetry breaking the relevant operator has dimension 18,

$$\mathcal{O}_{0\nu 4\beta} = \frac{\kappa}{\prod_{i=1}^{14} \Lambda_i} \bar{u} \bar{u} \bar{u} \bar{u} d d d d \bar{e} \bar{e} \bar{e} \bar{e}. \quad (2)$$

Here, κ is just some unspecified dimensionless coefficient. On the other hand, in the final state there are four electrons and a nucleus which stays essentially at rest, so the $0\nu 4\beta$ decay rate depends on the $(3 \times 4 - 1)$ th power of the available kinetic energy Q . With these considerations, inserting some numerical factors for the multibody kinematics, we obtain the formula

$$\tau_{0\nu 4\beta} \sim \kappa^{-2} \left(\frac{Q}{\text{MeV}} \right)^{-11} \left(\frac{q}{100 \text{ MeV}} \right)^{-18} \prod_{i=1}^{14} \left(\frac{\Lambda_i}{\text{TeV}} \right)^2 10^{110} \text{ y}. \quad (3)$$

Clearly, this is a very rough estimate for the lifetime of the process, with an uncertainty of a few orders of magnitude. Nevertheless, given the largeness of the numbers involved, it is good enough for the following discussion.

To obtain the lowest possible $\tau_{0\nu 4\beta}$, ideally one would need large, i.e., order $\mathcal{O}(1)$, couplings ($\kappa \sim 1$) and light mediator masses ($\Lambda_i \ll \text{TeV}$). Note that for very light particles, their mass becomes irrelevant when compared to the typical momentum transfer in the nucleus. Hence the best case scenario is $\Lambda_i^{\text{min}} \sim q$. One can see easily that, in the limit where all Λ_i are of the order of 1 GeV or lower, it becomes conceivable to have $\tau_{0\nu 4\beta}$ (^{150}Nd) smaller than 10^{26} years.

However, we now show that the $0\nu 4\beta$ decay diagram contains at least four propagators of charged bosons (scalar or vector). The lightest particle, that can play this role, is the W boson; hence the most optimistic scenario achievable is $\prod_{i=1}^{14} \Lambda_i = m_W^8 q^6$, which translates into

$$\tau_{0\nu 4\beta}^{\text{Min}}(^{150}\text{Nd}) \sim 10^{41} \text{ y.} \quad (4)$$

This corresponds to roughly one $0\nu 4\beta$ decay per year for a mass of $\sim 10^{17}$ kg of neodymium; hence the observation of this process would be extremely challenging even in the most optimistic scenario.

Independent on any concrete model, four or more charged boson propagators are needed for the following reason. Conservation of fermion number implies that the 12 external fermions which make up the $0\nu 4\beta$ operator must be arranged in six currents J_i . Out of the six possible pairings ($\bar{u}u$, $\bar{u}d$, $\bar{u}\bar{e}$, dd , $d\bar{e}$ and $\bar{e}\bar{e}$) none is electrically neutral. Thus all six J_i currents must exchange charge through scalar or vector bosons. (It is also conceivable that this charge exchange occurs through some intermediary vertex; yet this scenario does not lead to a minimal number of internal propagators.)

It is straightforward to see that the most economical setup is the one where pairs of currents J_i with opposite electric charged are connected by a single charged boson, in which case only three such propagators are needed. However, the only fermion bilinears with opposite charges are dd and $d\bar{e}$, and clearly one does not obtain the $0\nu 4\beta$ operator with three copies of these fermions. Hence, a minimum of four charged bosons are needed and the lower limit in Eq. (3) applies.

The six possible currents need to be coupled to scalars or vectors. These are charged or doubly charged particles, leptoquarks or diquarks. If one were to construct a loop model for $0\nu 4\beta$ decay and introduce also some new exotic fermions, more exotic scalars/vectors could, in principle, also appear. All of these states, however, necessarily couple to standard model fermions and thus can be searched at accelerators such as LEP and LHC. Again, considering the large numbers involved in Eqs. (3) and (4), a very rough argument suffices for our purpose. Thus, we only quote that LEP data rules out any electrically charged boson, decaying to SM fermions, below roughly 100 GeV [22].

The lower bound in Eq. (4) on the neutrinoless quadruple beta decay lifetime is therefore unavoidable. However, it is also very conservative. If one were to construct lower bounds individually for the six possible currents, (much) larger limits could be derived for the different individual cases. Instead, we now try to see whether it is actually possible to approach this bound, by considering some particularly promising models.

We have established already that $0\nu 4\beta$ decay is necessarily suppressed by the heavy mass of at least four charged bosons and, in order not to further reduce the decay rate,

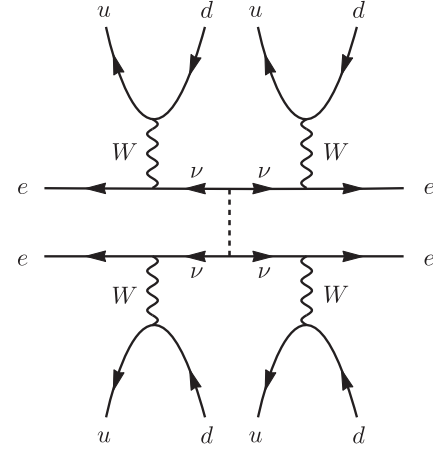


FIG. 2. Diagram for $0\nu 4\beta$ decay with only standard model fermions and a new neutral scalar (first considered in [15]). Note that all fermions drawn are left-handed.

one should avoid colored particles in internal lines.⁶ It is easy to check that, out of the 135 distinct ways of partitioning the $0\nu 4\beta$ operator in fermion bilinears/currents J_i , only $(\bar{u}d)(\bar{u}d)(\bar{u}d)(\bar{e}\bar{e})(\bar{e}\bar{e})$ makes it possible to have all internal bosons colorless. It is equally simple to arrive at the conclusion that, for this particular partition of the $0\nu 4\beta$ operator, the number of neutral and colorless internal bosons will be minimal (just 4) if and only if the corresponding diagram can be split into two halves, each with fermion bilinears $(\bar{u}d)(\bar{u}d)(\bar{e}\bar{e})$, connected by a neutral boson. A particularly interesting way of building such a diagram is the one shown in Fig. 2, where only SM fields plus a neutral scalar are used [15]. In the central part of the diagram, it is clear that four neutrinos are created from nothing; hence the neutrino-scalar interaction is critical for the violation of lepton number. Under the full standard model symmetry, and setting aside a complication which we mention later, this $\Delta L = 4$ operator must be of the form $LLLLHHHH$ (or perhaps higher dimensional). Crucially, this effective interaction does not need to be suppressed by the smallness of neutrino mass; indeed, the operator $LLHH$ might even be absent. One way of generating the $LLLLHHHH$ interaction is by adding to the standard model an extra $SU(2)_L$ scalar triplet Δ with one unit of hypercharge, as well as a real scalar singlet σ with no gauge interactions. The most general Lagrangian with these two new fields violates lepton number in two units, given the simultaneous existence of the couplings $LL\Delta$ and $HH\Delta^*$.⁷ It is possible to construct a model which violates lepton number in units of 4 only, and to do so we retain just the following interactions:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + (y_\Delta L L \Delta + \kappa H H \Delta^* \sigma + \text{H.c.}) - \mu_\sigma^2 \sigma^2 + (\text{other self-conjugate terms}). \quad (5)$$

⁶Bounds on colored particles from the LHC approach or exceed TeV masses, leading to further suppression of the rate.

⁷This setup recalls the seesaw type II [23], known for a long time.

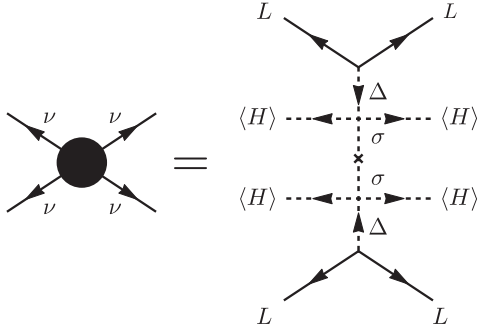


FIG. 3. One possible way of generating a sizable four-neutrino interaction which is not suppressed by $m_\nu/\langle H \rangle$, introducing a scalar triplet Δ and singlet σ . If more than two neutrinos have the same flavor, the amplitude of the process is suppressed by a factor (momentum/mediator mass)².

This Lagrangian is invariant under the $Z_4(L)$ lepton-number symmetry $\psi \rightarrow i^L \psi$, with $L(\Delta) = L(\sigma) = -2$. It is straightforward to arrange scalar potential parameters such that only the Higgs doublet acquires a nonzero vacuum expectation value, in which case there is no spontaneous breaking of the Z_4 symmetry. As such, the Weinberg operator is not generated, but $LLLLHHHH$ is [see Fig. 3]. Assuming without loss of generality that the coupling κ is real, σ mixes only with $\text{Re}(\Delta^0)$, resulting in two mass eigenstates with masses $m_{1,2}$ and a mixing angle θ which can be controlled at will. In particular, it is possible to (a) make one of these states very light, (b) have maximal mixing $\sin \theta$, and at the same time (c) keep all other new scalars [$\text{Im}(\Delta^0)$, Δ^+ and Δ^{++}] arbitrarily heavy. Neutrino masses can be accounted for by introducing right-handed neutrinos, or by the breaking of this Z_4 symmetry, for example by introducing a very small $HH\Delta^*$ coupling. Likewise, the $0\nu 2\beta$ decay rate can be 0 (absence of $HH\Delta^*$) or very small (small $\Delta L = 2$ coupling).

The complication mentioned earlier is that there is no $\frac{1}{\Lambda^2} \nu_{e,L} \nu_{e,L} \nu_{e,L} \nu_{e,L}$ local operator: given that the spinor $\nu_{e,L}$ has two components only, the Pauli exclusion principle forbids point interactions with more than two $\nu_{e,L}$. Instead, the four-neutrino interaction must be of the form

$$\begin{aligned} & \frac{1}{\Lambda^2} \nu_{e,L}(x) \nu_{e,L}(x) \nu_{e,L}(y) \nu_{e,L}(y) \\ & \approx \frac{(y-x)^2}{\Lambda^2} \nu_{e,L}(x) \nu_{e,L}(x) [\partial \nu_{e,L}(x)]^2, \\ & \approx \frac{1}{\Lambda^4} \nu_{e,L}(x) \nu_{e,L}(x) [\partial \nu_{e,L}(x)]^2, \end{aligned} \quad (6)$$

where Λ is a mass associated to the particle(s) mediating the effective interaction. We used the fact that the distance between the two interaction points x and y is inversely proportional to Λ . In practice, this means that there is an extra suppression factor $(\text{momentum}/\Lambda)^2$ due to Pauli

blocking on top of the expected $1/\Lambda^2$ suppression for this 4-fermion interaction,⁸ and that the relevant operator is $\partial\partial LLLLHHHH$ instead of $LLLLHHHH$.

As pointed out in [15], the uncertainty in the invisible decay width of the Z boson as measured indirectly at LEP [24–27] constrains the mass of the scalar boson in Fig. 2 to be heavier than roughly $\Lambda \sim 20$ GeV. In this latter process, the momentum p is of the order of the Z mass; hence $p \sim \Lambda$ and there is no Pauli blocking. However, the blocking is present in the $0\nu 4\beta$ decay diagram with 4 W 's, since $p \sim 100$ MeV is much smaller than the mass Λ of the neutral scalar. As a consequence, the $0\nu 4\beta$ decay lifetime is even larger than what has been assumed previously [15,16]; a simple order of magnitude calculation yields $\tau_{0\nu 4\beta}^{4W} \sim 10^{69}$ years, which is very far from the limit given in Eq. (4).

A possible way of avoiding this particular suppression factor is by introducing right-handed neutrinos ν_R and W_R gauge bosons such that lepton number is violated by a 4-fermion interaction $\frac{1}{\Lambda^2} \nu_{e,L} \nu_{e,L} \nu_{e,R} \nu_{e,R}$. This effective operator can be generated with the scalars Δ and σ mentioned earlier, keeping in mind that there is now a $\nu_{e,R} \nu_{e,R} \sigma$ interaction. Nevertheless, given the multi-TeV LHC mass limits on the new gauge bosons [28,29], the two diagrams imply a similar $0\nu 4\beta$ decay lifetime,

$$\tau_{0\nu 4\beta}^{2W+2W_R} \sim \left(\frac{g_R m_{W_R}}{g_L m_W} \right)^8 \left(\frac{q}{\Lambda} \right)^4 \tau_{0\nu 4\beta}^{4W} \lesssim \tau_{0\nu 4\beta}^{4W}. \quad (7)$$

Finally, we make some comments about the possible realizations of the $0\nu 4\beta$ decay operator in Eq. (2) with standard model fields (see also [30]). Due to Pauli's exclusion principle, fermion fields evaluated at the same space-time point anticommute, so for a generic 4-spinor

$$\Psi = \begin{pmatrix} \Psi_L^\uparrow \\ \Psi_L^\downarrow \\ \Psi_R^\downarrow \\ \Psi_R^\uparrow \end{pmatrix} \quad (8)$$

⁸This kind of amplitude suppression can equally appear in operators with no fermions. For example, consider a global $SU(2)$ symmetry under which the scalars $\phi = (\phi_1, \phi_2)^T$ and $\phi' = (\phi'_1, \phi'_2)^T$ transform as doublets, and S is a scalar singlet. The local operator $\phi(x)\phi(x)S(x)S(x)$ is identically 0, but $\phi(x)\phi(y)S(x)S(y)$ is not; hence it is possible to collide two S 's and create a ϕ_1 plus a ϕ_2 . This is done via two diagrams in which ϕ'_1 and ϕ'_2 are exchanged in the t and u channels. In the limit where the mass Λ of $\phi'_{1,2}$ is much larger than the momentum transferred p , the total amplitude varies with $1/\Lambda^2 \times (p/\Lambda)^2$, and not $1/\Lambda^2$.

we can only have operators of the form $\Psi^n[\dots]$ with $n \leq 4$ (if there are no derivatives).⁹ Furthermore, if we expand the Dirac indices of such an operator with $n = 4$, the only nonzero term must be proportional to $\Psi_L^\uparrow \Psi_L^\downarrow \Psi_R^\uparrow \Psi_R^\downarrow$. This means that the Pauli exclusion principle severely restricts local $0\nu 4\beta$ operators to the unique form

$$\mathcal{O}_{0\nu 4\beta} = \frac{\kappa}{\sum_{i=1}^{14} \Lambda_i} \bar{e}_L^\uparrow \bar{e}_L^\downarrow \bar{e}_R^\uparrow \bar{e}_R^\downarrow \times \bar{u} \bar{u} \bar{u} \bar{u} d d d d, \quad (9)$$

and higher dimensional $0\nu(2m)\beta$ decay operators, with $m > 2$, are forbidden entirely unless they have derivatives. Note that quarks have three colors; hence a similar issue arises for operators with more than six quarks of the same charge and chirality.

In this sense, quadruple beta decay (with or without neutrinos) is a borderline case between allowed and excluded local processes. An interesting consequence is that there are only three $0\nu 4\beta$ operators of minimal dimension (= 18),

$$\mathcal{O}_{0\nu 4\beta}^{(1)SM} \sim \bar{L} \bar{L} e^c e^c \bar{Q} \bar{Q} u^c u^c \bar{d}^c \bar{d}^c \bar{d}^c \bar{d}^c, \quad (10)$$

$$\mathcal{O}_{0\nu 4\beta}^{(2)SM} \sim \bar{L} \bar{L} e^c e^c \bar{Q} u^c u^c Q \bar{d}^c \bar{d}^c \bar{d}^c, \quad (11)$$

$$\mathcal{O}_{0\nu 4\beta}^{(3)SM} \sim \bar{L} \bar{L} e^c e^c u^c u^c u^c u^c Q Q \bar{d}^c \bar{d}^c, \quad (12)$$

This should be contrasted with the $0\nu 2\beta$ decay operators of dimension 9, of which there are six. On the other hand, the diagram shown above with four W 's [Fig. 2] and the equivalent one with $2W$'s + $2W_R$'s have dimensions 24 and 20 respectively,

$$\mathcal{O}_{0\nu 4\beta}^{(4W)} \sim \partial^2 \bar{Q} \bar{Q} \bar{Q} \bar{Q} Q Q Q Q \bar{L} \bar{L} \bar{L} \bar{L} \bar{H} \bar{H} \bar{H} \bar{H}, \quad (13)$$

$$\mathcal{O}_{0\nu 4\beta}^{(2W+2W_R)} \sim \bar{Q} \bar{Q} u^c u^c Q Q \bar{d}^c \bar{d}^c \bar{L} \bar{L} e^c e^c \bar{H} \bar{H}. \quad (14)$$

III. OTHER LOW-ENERGY PROCESSES WITH $\Delta L \geq 4$ INVOLVING CHARGED LEPTONS

We now move on to a brief discussion of other lepton number violating processes, involving low energies, with four or more charged leptons. Rough estimates for their rates are given in the following. We stress that for an experimental proof of L violation, final states should not contain neutrinos.

In Table II we give the lowest dimension at which a given $\Delta L \neq 0$ operator can appear, together with some examples.

⁹If only one chirality $X = R, L$ is involved, then $\Psi_X^n[\dots]$ with $n \leq 2$ are the only possibilities. For example, an electron-neutrino mass term $\nu_{e,L} \nu_{e,L}$ is allowed, but interactions of the form $\nu_{e,L} \nu_{e,L} \nu_{e,L}[\dots]$ or $\nu_{e,L} \nu_{e,L} \nu_{e,L} \nu_{e,L}$ are not.

(Note that, since we are interested here in low-energy processes, neither Higgs nor gauge bosons can appear as final states.) The table starts with the $\Delta L = 1$ operators which induce the standard proton decay modes (hence $\Delta B = 1$), and these are followed by the $\Delta L = 2$ operator associated to neutrinoless double beta decay. With larger ΔL , the dimension of the operators keeps rising and at some point one expects that the rates of associated low-energy processes becomes too small to be observed. We now discuss briefly this point, by focusing on the most promising processes.

It is important to distinguish those cases where baryon number is violated from those scenarios where $\Delta B = 0$. This is simply due to the fact that the available energy in $\Delta B \neq 0$ processes is fixed by the nucleon mass, of the order of $\sim \text{GeV}$, while kinetic energy of the charged leptons is much smaller ($\sim \text{MeV}$) in the $\Delta B = 0$ case.

Let us consider first the latter case, $\Delta B = 0$. This implies immediately that ΔL must be an even number. The relevant processes are then $0\nu(2n)\beta$ with $n > 2$. We discuss only β^- decays, since for quadruple beta decays it has been shown already in [15] that the positron emission or electron capture processes are even more hopeless, due to their smaller Q -values. For $0\nu(2n)\beta$ with $n > 2$ the same observation applies.

The process $0\nu 6\beta$ is induced by an operator with 18 fermions; hence the decay width Γ is suppressed at least by a factor $Q^{17} q^{29} / \Lambda^{46}$ relative to the nucleon Fermi momentum $q \sim 100 \text{ MeV}$. Moreover, we note that at most four electrons can be at a single point x ; therefore the operators for these $0\nu(2n)\beta$ decays require at least two derivatives, and consequently the decay width is suppressed by four more powers of q/Λ , compared to the simple-minded estimate quoted above. It is also straightforward to check that at least six electrically charged bosons are needed to mediate the process; hence, the same logic as discussed for neutrinoless quadruple beta decay applies.

Finally, kinematically $0\nu 6\beta$ and larger is only allowed for neutron-rich nuclides which are far from the valley of stability; hence these isotopes have a very short half-life. The longest-lived isotope seems to be $^{134}_{52}\text{Te}$, which can decay into $^{134}_{58}\text{Ce} + 6e^-$ with a Q value of 2.3 MeV, but also decays by single beta emission with a half-life of 41.8 minutes [19]. For $0\nu 8\beta$, considering only isotopes with an atomic mass below 200, we have $^{131}_{50}\text{Sn} \rightarrow ^{131}_{58}\text{Ce} + 8e^-$ ($Q = 2.4 \text{ MeV}$) and $^{132}_{50}\text{Sn} \rightarrow ^{132}_{58}\text{Ce} + 8e^-$ ($Q = 5.9 \text{ MeV}$) with half-lives $T_{1/2}(^{131}_{50}\text{Sn}) = 56 \text{ s}$ and $T_{1/2}(^{131}_{50}\text{Sn}) = 39.7 \text{ s}$. Thus, no realistic candidate for a $0\nu(2n)\beta$ experiment with $n > 2$ exists in nature.

Let us now turn to processes where baryons are destroyed and hence, the available energy is much larger. Here, we consider the cases $(\Delta L, \Delta B) = (4, \pm 2)$ and $(\Delta L, \Delta B) = (5, \pm 1)$; violation of lepton or baryon number in greater quantities is associated with even bigger minimum lifetimes.

The lowest dimensional operator with four charged leptons (after electroweak symmetry breaking) is $eeeeuuuuuu/\Lambda^{11}$ (see Table II). It leads, for example, to diproton decay, $(A, Z) \rightarrow (A - 2, Z - 2) + 2\pi^- + 4e^+$. In the most optimistic scenario, this operator can be built in such a way that only seven powers of Λ correspond to the mass of mediators with color (Λ_C), while the remaining four powers of Λ are related to the mass of fields with electroweak interactions only (Λ_{EW}). Hence,

$$\tau_{\min}^{-1} \sim 10^{-13} \frac{(2m_p)^{23}}{\Lambda_{EW}^8 \Lambda_C^{14}} \sim 10^{-40} \text{ y}^{-1}, \quad (15)$$

using the values $\Lambda_{EW} \approx 200 \text{ GeV}$ and $\Lambda_C \approx 2 \text{ TeV}$. The prefactor 10^{-13} takes care of the fact that this is a six-body decay. Super-Kamiokande has searched for other diproton decay modes, imposing limits of the order of 10^{32} years on the associated lifetimes [31]. It seems therefore very hard to probe $\Delta L = 4$ processes at low energies, even for those cases where baryon number is violated. Note, however, that due to the much larger energy release, the gap between the experimental sensitivity and the most optimistic expectation is only 8 orders of magnitude, compared to the (minimum of) 20 orders found for quadruple beta decay.

For $\Delta L = 5$ (or larger) the decay rates are necessarily even more suppressed. For $\Delta L = 5$, the lowest dimensional operators have $d = 21$, as shown in Table II; therefore the decay widths are suppressed by 34 powers of $\sim m_p/\Lambda$ when compared to m_p . In summary, we conclude that observation

of charged lepton number violation in four or more units in low-energy experiments is impossible in the foreseeable future.

IV. PROBING LEPTON NUMBER VIOLATION AT COLLIDERS

We now turn to a discussion of probing models with multiple lepton number violation at colliders. We do not aim at a full, systematic analysis of all possibilities. Instead, we discuss two simple models with $\Delta L = 4$ and then present one example for a model with $\Delta L = 5$. Models which lead to different $\Delta L = 4$ operators at low energy or models with larger ΔL violation can be easily constructed following the same principles that we use in our examples.

Our first model is inspired by the discussion on neutrinoless quadruple beta decay in Sec. II. In this model, called model I below, we add only two new fields to the standard model. Both are scalars: (i) $\Delta = S_{1,3,1,-2}$ and (ii) $T = S_{1,3,0,-2}$. Here, the subscripts stand for the transformation properties or charge under the SM gauge group and lepton number, $SU(3)_C \times SU(2)_L \times U(1)_Y, L$ (also we use an S for scalars and, later on, an F for 2-component Weyl spinors). The only change with respect to the model discussed in Sec. II is that we have replaced the singlet $\sigma = S_{1,1,0,-2}$ with the $Y = 0$ $SU(2)_L$ triplet field T .

As before, we enforce a $Z_4(L)$ symmetry which ensures that leptons can only be created or destroyed in groups of 4. The Lagrangian of the model is¹⁰

$$\begin{aligned} \mathcal{L} = & \mathcal{L}^{\text{SM}} + [Y_\Delta(L^T \epsilon \Delta \epsilon L) - \lambda_{HH\Delta T}(H^T \Delta^* T \epsilon H) + \text{H.c.}] + m_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) + m_T^2 \text{Tr}(T \epsilon T \epsilon) \\ & + \lambda_T [\text{Tr}(T \epsilon T \epsilon)]^2 - \lambda_{HT}(H^\dagger H) \text{Tr}(T \epsilon T \epsilon) - \lambda_{\Delta 1} [\text{Tr}(\Delta^\dagger \Delta)]^2 - \lambda_{\Delta 2} \text{Tr}(\Delta^\dagger \Delta \Delta^\dagger \Delta) \\ & - \lambda_{\Delta H 1}(H^\dagger H) \text{Tr}(\Delta^\dagger \Delta) - \lambda_{\Delta H 2}(H^\dagger \Delta \Delta^\dagger H) - \lambda_{\Delta T 1} \text{Tr}(\Delta^\dagger \Delta) \text{Tr}(T \epsilon T \epsilon) - \lambda_{\Delta T 2} \text{Tr}(\Delta^\dagger T \Delta T). \end{aligned} \quad (16)$$

Note that this Lagrangian is also $U(1)_B$ invariant. In other words, baryon number is preserved; hence processes such as dinucleon decay are completely absent in this model. Note that Δ is the same field that appears in the type-II seesaw mechanism. However, our symmetry forbids the term $H\Delta H$, which in seesaw type II is the source of $\Delta L = 2$. This implies that for $m_\Delta^2 \geq 0$, in our model there is no induced vacuum expectation value for Δ^0 and thus no Majorana neutrino mass term.

We have not written generation indices in Eq. (16). In general Y_Δ is a complex symmetric $(3,3)$ matrix. All terms in the Lagrangian, with the exception of those proportional to T^2 or T^4 , conserve lepton number. For the phenomenology

discussed below it is important that m_T^2 violates ΔL in four units.

The term proportional to $\lambda_{HH\Delta T}$ leads to mixing between the neutral and singly charged components in T and Δ after electroweak symmetry breaking (EWSB), as well as to a mass splitting between the CP -even and CP -odd components in Δ^0 . Thus, after EWSB the model has two new neutral CP -even scalars $S_{1,2}^0$ and two singly charged scalars $S_{1,2}^\pm$ plus one neutral CP -odd scalar, A^0 , and one doubly charged scalar, $\Delta^{\pm\pm}$. In our numerical calculations we always diagonalize all mass matrices and consider mass eigenstates correctly. However, in the following discussion we simply use T^0 and Δ^0 for S_2^0 and S_1^0 , respectively. (And similarly for the singly charged states T^\pm and Δ^\pm .) This is done only for the clarity of the discussion; it does not affect any of our conclusions. Note that for typical choices of masses m_T^2 and m_Δ^2 above $(500 \text{ GeV})^2$ mixing between the different states is small unless $m_T^2 \simeq m_\Delta^2$.

¹⁰We use the notation $\Delta \equiv \begin{pmatrix} \Delta^{++} & \frac{\Delta^+}{\sqrt{2}} \\ \frac{\Delta^+}{\sqrt{2}} & \Delta^0 \end{pmatrix}$, $T \equiv \begin{pmatrix} T^+ & \frac{T^0}{\sqrt{2}} \\ \frac{T^0}{\sqrt{2}} & T^- \end{pmatrix}$ and $\epsilon \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

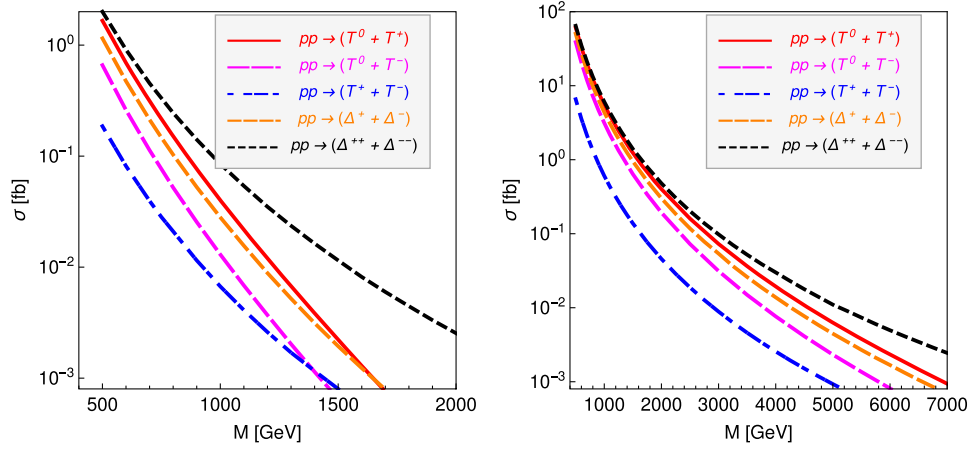


FIG. 4. Cross sections for various production modes for particles in our $\Delta L = 4$ model I. The plot to the left is calculated for the LHC, i.e., pp collisions at $\sqrt{s} = 13$ TeV. The plot on the right assumes a hypothetical future pp collider with $\sqrt{s} = 100$ TeV. Note the different scales in the two plots.

For the numerical calculations shown below, we have implemented the model into SARAH [32,33]. The implementation is then used to generate SPheno code [34,35] for the numerical generation of spectra. The UFO model files generated by SARAH are used for cross section and decay calculations with MadGraph [36].

Both Δ and T can be produced with sizeable rates at colliders. Figure 4 shows cross sections for the most important production modes in pp colliders for two values of \sqrt{s} : To the left $\sqrt{s} = 13$ TeV, to the right $\sqrt{s} = 100$ TeV. The numerically largest cross section is found for $\Delta^{\pm\pm}$ pair production. However, as we discuss below, for the observation of $\Delta L = 4$ processes the interesting production modes are $pp \rightarrow (T^0 + T^+)$ and $pp \rightarrow (T^0 + T^-)$. Both processes proceed through an off-shell W -diagram; see Fig. 5. The cross section for $pp \rightarrow (T^0 + T^+)$ is larger than for $pp \rightarrow (T^0 + T^-)$, reflecting the fact that the initial state is positively charged.

We first discuss the decays of Δ . The different components of Δ decay according to $\Delta^{\pm\pm} \rightarrow l_{\alpha}^{\pm} l_{\beta}^{\pm}$, $\Delta^{\pm} \rightarrow \nu_{\alpha} l_{\beta}^{\pm}$ and $\Delta^0 \rightarrow \nu_{\alpha} \nu_{\beta}$ with 100% branching ratio, when summed over α and β . Since cross sections are largest and background lowest for $\Delta^{\pm\pm}$, the most stringent constraints on m_{Δ} come from

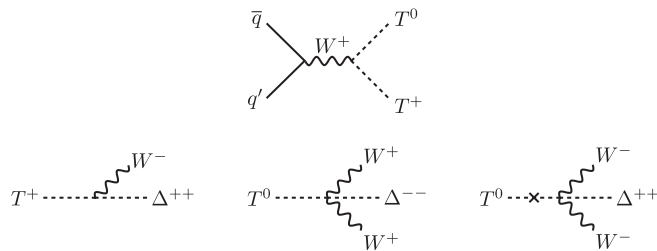


FIG. 5. Feynman diagrams for the production and decays of T^0 and T^{\pm} . This list of diagrams is not complete, showing instead only those diagrams relevant for the observation of a $\Delta L = 4$ process; see text.

searches for $\Delta^{\pm\pm}$. Both ATLAS [37] and CMS [38] have searched for doubly charged scalars decaying to charged leptons. Limits depend quite strongly on the flavor of the charged leptons. CMS [38] gives limits as low as $m_{\Delta^{\pm\pm}} \simeq 535$ GeV for a Δ^{++} decaying with 100% to pairs of taus, while limits are in the range of (800–820) GeV, if the Δ^{++} decays only to electrons or muons. ATLAS [37], on the other hand, has established lower limits on $m_{\Delta^{\pm\pm}}$ of roughly (600–800) GeV, for branching ratios to either electrons or muons in the range of (0.2–1). We therefore use two choices of m_{Δ} in our numerical examples below, namely, $m_{\Delta} = 0.6$ TeV and $m_{\Delta} = 1$ TeV. The former is allowed only for a Δ with coupling mostly to τ 's, while the latter is currently unconstrained. Note, however, that with the predicted $\mathcal{L} = 3/\text{ab}$ for the high luminosity LHC m_{Δ} in excess of $m_{\Delta} = 1$ TeV will be probed, while for a $\sqrt{s} = 100$ TeV collider we estimate from Fig. 4 that up to $m_{\Delta} \sim 5$ TeV could be tested with $\mathcal{L} = 3/\text{ab}$, in agreement with the numbers quoted in [39].

For the observation of $\Delta L = 4$ processes, we need to produce the hyperchargeless triplet T . We therefore now turn to a discussion of the decays of T^0 and T^{\pm} . First of all, note that all decay rates for these particles are proportional to the coupling $\lambda_{HH\Delta T}$. This is due to the fact that this term is the only one linear in T allowed by $Z(4)_L$. Mixing between T^0 and T^{\pm} with Δ^0 and Δ^{\pm} induces two-body decays for these states into leptonic final states. However, these always involve neutrinos and thus are not useful to establish experimentally lepton number violation. More important are then decays of T^0 and T^{\pm} to $\Delta^{\pm\pm}$ and gauge bosons. Figure 5 shows the most important Feynman diagrams. Apart from $T^+ \rightarrow W^- + \Delta^{++}$, T^+ can decay to $W^+ + A^0$, $W^+ + \Delta^0$,¹¹ as well as $\Delta^+ + h$ and $\Delta^+ + Z^0$. The branching ratio for $T^+ \rightarrow W^- + \Delta^{++}$ is always close

¹¹Recall that we use Δ^0 in this discussion synonymous for the CP -even scalar S_1^0 .

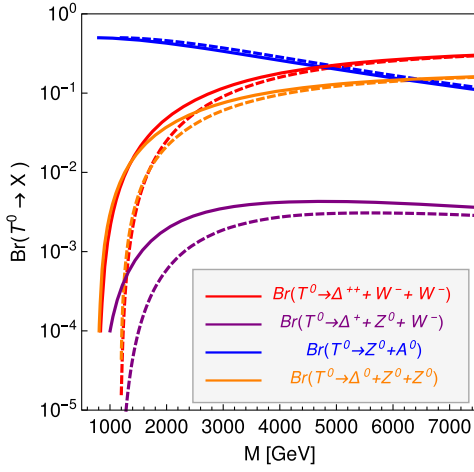


FIG. 6. Branching ratios for the decay of T^0 as a function of the mass of T^0 . Here, the full lines are for the choice $m_\Delta = 0.6$ TeV, the dashed lines for $m_\Delta = 1$ TeV. In addition to $T^0 \rightarrow A^0 + Z^0$ there is a second two-body decay mode for T^0 , $T^0 \rightarrow h + \Delta^0$. This one is not shown explicitly, since $\text{Br}(T^0 \rightarrow h + \Delta^0) \simeq \text{Br}(T^0 \rightarrow A^0 + Z^0)$ in all cases. Note that $\text{Br}(T^0 \rightarrow \Delta^{++} + 2W^-) = \text{Br}(T^0 \rightarrow \Delta^{--} + 2W^+)$. The coupling $\lambda_{HH\Delta T}$ was chosen $\lambda_{HH\Delta T} = 0.1$ in this example, while the entries in Y_Δ were arbitrarily put to be smaller than $\mathcal{O}(0.1)$. With these choices, decays to purely leptonic final states are negligible and therefore not shown.

to 25%, if the mixing between T^+ and Δ^+ is small. Similarly, T^0 can decay to a number of final states involving gauge (and Higgs) bosons. Figure 6 shows the most important branching ratios for T^0 decays as a function of m_T . As the plot shows, for small values of m_T the two-body decays $T^0 \rightarrow A^0 + Z^0$ (and $T^0 \rightarrow \Delta^0 + h$) have the largest branching ratios. However, for large values of m_T , the decay mode $T^0 \rightarrow \Delta^{\pm\pm} + 2W^\mp$ becomes dominant. T^0 decays to both Δ^{++} and Δ^{--} with equal rates. That both decays have the same rate can be understood as a mass insertion in the decay of T^0 ; see Fig. 5. Recall that m_T^2 violates L by 4 units.

At a pp collider one can therefore have lepton number violating (conserving) final states from the chain $pp \rightarrow T^+ + T^0 \rightarrow 2\Delta^{++} + 3W^- \rightarrow 4l^+ + 6j$ ($pp \rightarrow T^+ + T^0 \rightarrow \Delta^{++} + \Delta^{--} + 2W^+ + W^- \rightarrow 2l^+ 2l^- + 6j$). With the cross sections, see Fig. 4, and the decay branching ratios we can then calculate the number of $\Delta L = 4$ events as a function of m_T (and m_Δ). We have performed this exercise for the LHC and find that, taking into account the lower limit on m_Δ , there is less than 1 event even after 3/ab of data has been taken. However, the prospects look much brighter at a hypothetical $\sqrt{s} = 100$ TeV collider; see Fig. 7. In this figure we show the maximal number of $\Delta L = 4$ events attainable in model I for two choices of m_Δ and two different values for the expected luminosity. For $\mathcal{L} = 30/\text{ab}$ more than 10 $\Delta L = 4$ events could be found up to roughly $m_T \simeq 6$ TeV.

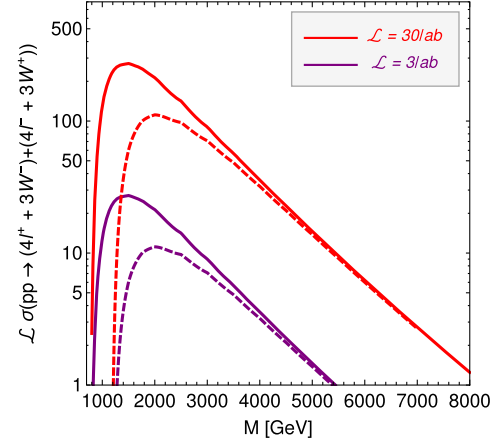


FIG. 7. Maximal number of $\Delta L = 4$ events attainable in model I at a future pp collider with $\sqrt{s} = 100$ TeV. Here, the event number sums over both, negatively and positively charged leptons.

Thus, for our first model we conclude that while the LHC can extend the search for the different particles in model I to above 1 TeV, observation of $\Delta L = 4$ seems not to be possible at the LHC. At a $\sqrt{s} = 100$ TeV collider $\Delta L = 4$ could be discovered in this model up to a scale of roughly 6 TeV.

The negative conclusions for the LHC can be simply understood from the fact that model I contains no new colored states. As shown in Table II, the smallest dimensional operator generating four charged leptons is 15 dimensional, $e^4 u^6$. We therefore choose to implement it in our second example, model II. Any model leading to this operator necessarily involves beyond-the-SM colored fields.

Model II introduces three new states, two fermions $O = F_{8,1,0,-2}$ and $D^c = F_{3,1,1/3,1}$ (together with its vector partner $\bar{D}^c = F_{3,1,-1/3,3}$) and one scalar, $S_d = S_{3,1,-1/3,1}$. By enforcing once again $Z_4(L)$ invariance, we obtain this time a Lagrangian with an enlarged accidental symmetry group $G_{\text{SM}} \times U(1)_{2B-L}$. It might not be immediately obvious that this latter group contains $Z_4(L)$, but this is nevertheless true. Indeed, the Lorentz and standard model group G_{SM} force all operators with SM fields to be $Z_2(B-L)$ invariant; hence we may write $L = 2n + B$ for some integer n . Together with $L = 2B$, it is then quite easy to see that L and B are forced to be multiples of 4 and 2, respectively. Crucially, unlike in model I, due to the $U(1)_{2B-L}$ symmetry it is not possible to break lepton number without breaking baryon number as well. As such, one can have small/unobservable dinucleon decays rates, but neutrinoless quadruple beta decay is strictly forbidden.

The Lagrangian contains the following terms,

$$\begin{aligned} \mathcal{L} \propto & Y_1 u^c e^c S_d^\dagger + Y_2 u^c D^c S_d + Y_3 Q L S_d + Y_4 \bar{D}^c O S_d \\ & + Y_5 D^c O S_d^\dagger + \text{H.c.} + m_O O O + m_D \bar{D}^c D^c. \end{aligned} \quad (17)$$

We have calculated the pair production cross sections for the new particles in our model II using again MadGraph [36].

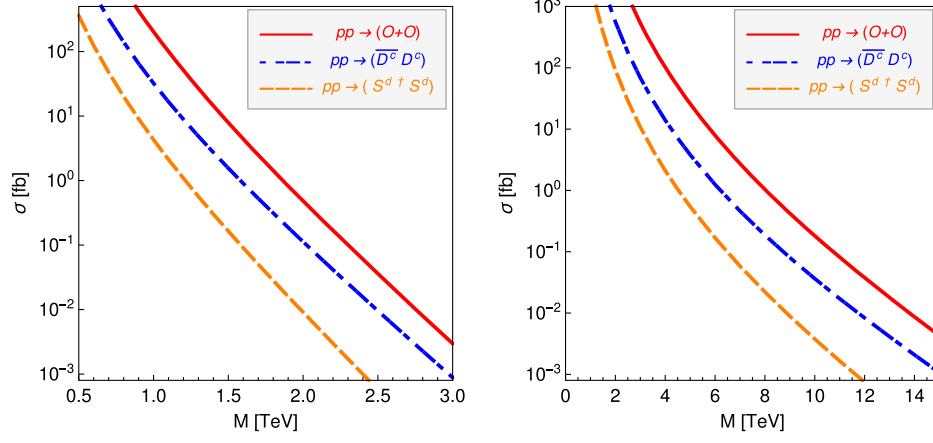


FIG. 8. Cross sections for various production modes for particles in our $\Delta L = 4$ model II. The plot to the left is calculated for $\sqrt{s} = 13$ TeV, the one on the right for $\sqrt{s} = 100$ TeV.

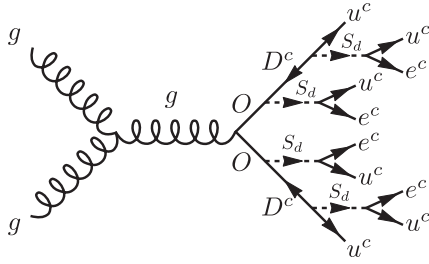


FIG. 9. Example diagram for production and decays of the color octet in model II.

The results are shown in Fig. 8. Again, the plot to the left is for the LHC; the one on the right is calculated for $\sqrt{s} = 100$ TeV. In all cases one expects that gluon-gluon fusion gives the largest contribution to the cross section; see Fig. 9. The largest cross section is found for the fermionic octet. More than ten events from O -pair production are expected in $\mathcal{L} = 3/\text{ab}$ for octet masses up to $m_O \simeq 3$ TeV. A $\sqrt{s} = 100$ TeV collider would be able to collect more than ten events for octet masses up to $m_O \simeq 15.5$ (18.5) TeV for $\mathcal{L} = 3/\text{ab}$ (30/ab).

In model II, since O is an electrically neutral state, it decays with equal branching ratios to $D_{1/3}^c + S_{d,-1/3}$ and $D_{-1/3}^c + S_{d,1/3}$. This implies that $\Delta L = 4$ final states $4l^+ + 6j$ have the same rate as the $\Delta L = 0$ final states $2l^+ + 2l^- + 6j$. Thus, the production cross section (event number) of O -pair production gives directly the limit on the scale up to which $\Delta L = 4$ can be tested in model II.

It is straightforward to use the ideas discussed above to construct also models, which lead necessarily to larger ΔL . We discuss only one example with $\Delta L = 5$. This model III introduces five new states. We need two copies of S_d , to which we assign different lepton numbers, $S_d^{L=1} = S_{3,1,-1/3,1}$ and $S_d^{L=0} = S_{3,1,-1/3,0}$. The vectorlike down quarks also now come in two copies. We have $D_{L=2}^c = F_{3,1,1/3,2}$; its vector partner is $\bar{D}_{L=3}^c = F_{3,1,-1/3,3}$, and also $D_{L=1}^c = F_{3,1,1/3,1}$

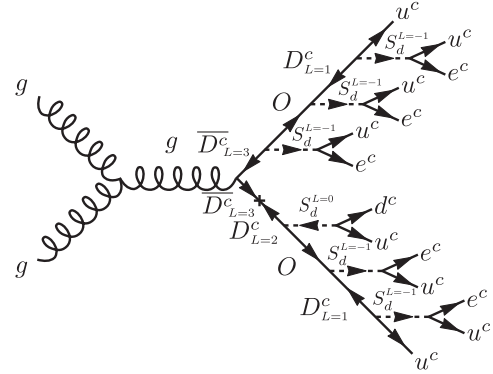


FIG. 10. Example diagram for a $\Delta L = 5$ process in model III.

with its vector partner $\bar{D}_{L=4}^c = F_{3,1,-1/3,4}$. Finally, the model also has the fermionic vector octet, $O = F_{8,1,0,2}$ and $\bar{O} = F_{8,1,0,3}$. With these lepton number charges, we then enforce a $Z_5(L)$ symmetry. Just as with model II, there is a bigger, accidental symmetry group in this model, $U(1)_{5B-3L}$. In other words, for each group of five leptons created, three new baryons should appear as well, and for this reason the proton is completely stable in this model III.

$\Delta L = 5$ processes at the LHC can then occur through diagrams such as the example shown in Fig. 10, where $\bar{D}_{L=3}^c$ is pair produced via gluon fusion. Note that the decay chains of both $\bar{D}_{L=3}^c$ and $(\bar{D}_{L=3}^c)^*$ end with the same number of SM fermions: 7. One can assign the source of lepton number violation in this diagram to the mass term $\bar{D}_{L=3}^c D_{L=2}^c$. If all other particles in the diagram are lighter than $D_{L=2}^c$ and all couplings the same order, $\Delta L = 5$ and $\Delta L = 0$ final states from these decay chains have similar rates.¹² Nevertheless, note that even if the $\bar{D}_{L=3}^c D_{L=2}^c$ mass term was switched off, lepton number would still be broken; in fact, B

¹²If all masses and all couplings are numerically the same, the branching ratio for $\Delta L = 5$ and $\Delta L = 0$ final states becomes equal.

and L conservation would be restored only if the vector masses of $D_{L=1}^c$ and O were absent as well.

We can estimate the mass reach of the LHC to test this kind of diagram from the cross sections shown in Fig. 8. We estimate that more than ten events (before cuts) would remain for masses of $D_{L=2}^c$ below 2.7 TeV in $\mathcal{L} = 3/\text{ab}$. At a $\sqrt{s} = 100$ TeV collider more than ten events would occur for m_{D^c} below 13.3 (15.5) TeV in $\mathcal{L} = 3/\text{ab}$ (30/ab). Thus, there is ample parameter space that could be tested in future colliders even for models with $\Delta L > 4$. Many different models of this kind can be readily constructed.

V. SUMMARY

Given the current experimental situation, the total number of leptons L might be a conserved quantity. Standard probes for $\Delta L \neq 0$ are proton decay ($\Delta L = 1, 3$) and neutrinoless double beta decay ($\Delta L = 2$) experiments. However, neither has found any signal so far. It is therefore possible that L is violated only in larger multiplicities, i.e., three, four or more units.

In this context, we have discussed that the decay of a nucleus into four electrons and no neutrinos will likely never be observed. This and other $\Delta L = 4$ low-energy processes where the leptons are electrically charged must necessarily be mediated by several heavy bosons; hence their amplitudes are severely suppressed, given the current accelerator constraints. We have calculated a very conservative lower limit on the half-life for neutrinoless quadruple beta decay, which is 20 orders of magnitude larger than current experimental sensitivities. The same conclusion is valid if the lepton number is broken in five or more units. The least pessimistic scenario that we have found is the one where baryon number is also violated. In this case two protons in a nucleus could decay into

four positrons plus pions, for example. The energy scale in this process is set by the proton mass, but even so, the high dimensionality of the operators involved implies that proton decay experiments would need to increase their exposure by at least 8 orders of magnitude before meaningful constraints could be derived experimentally.

Colliders, on the other hand, can explore the possibility that lepton number is violated in four or more units. The reason behind this observation is rather simple: Even though these processes involve many exotic particles, if the energy of the collider exceeds the mass of those exotics, the suppression associated with the high dimensionality of these $\Delta L \geq 4$ operators disappears. In this work, we presented two models for $\Delta L = 4$ and one for $\Delta L = 5$, which make use of this idea. We have calculated cross sections for the LHC and a possible future $\sqrt{s} = 100$ TeV collider, estimating the rates for $\Delta L = 4$ (and $\Delta L = 5$) processes. Naturally, for hadron colliders the expectations are highest for models which contain colored particles. In this case the LHC (the $\sqrt{s} = 100$ TeV collider) could probe $\Delta L = 4$ up to scales of roughly 3 TeV (18 TeV). We expect that the high multiplicity of these events associated with $\Delta L \geq 4$ will make them virtually background free, giving a rather spectacular signal.

ACKNOWLEDGMENTS

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