DYNAMICAL SYMMETRY BREAKING DUE TO STRONG COUPLING YUKAWA INTERACTION *

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Abstract

Motivated by the top quark condensation scenario of the electroweak symmetry breaking ("top-mode standard model"), dynamical chiral symmetry breaking (χSB) due to strong coupling Yukawa interaction is studied in the framework of Schwinger-Dyson equations. In quenched approximation, we show existence of the dynamical χSB phase($\langle 0|\sigma|0\rangle = 0$, $\langle 0|\bar{\psi}\psi|0\rangle \neq 0$) in strong Yukawa coupling region. Introducing dynamical fermion (tadpole) in our framework, we still have a parameter region where χSB has its origin in the fermion condensate.

1. Introduction

The origin of electroweak symmetry breaking, which explains the masses of the weak bosons and fermions, is one of the most important problems in modern particle physics. In the standard model we introduce Higgs field $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\pi_2 + i\pi_1 \\ \sigma - i\pi_3\end{pmatrix}$ which is tuned to have a non-zero vacuum expectation value (VEV) $\langle 0|\sigma|0\rangle = v$. Here v is an order parameter of the electroweak symmetry breaking, through which the weak gauge bosons $W^{\pm}, Z^0$ become massive due to Higgs mechanism. Masses of fermions, though being in principle independent order parameters of the electroweak symmetry breaking, are also explained by v through Yukawa couplings with Higgs boson,

$$m_f = \frac{\eta_f}{\sqrt{2}}v.$$

This scenario is reasonable when all fermions have small masses, $m_f \ll v$. However if there exists a heavy fermion, $m_f \gtrsim v$, it seems rather awkward to assume that the fermion gets its large mass from a small VEV of Higgs field. In this case it would be more natural to consider a converse, i.e., the origin of $v \neq 0$ comes from a large m_f, the dynamical mass of the fermion.

In fact, in the low energy effective theory of the technicolor models, the mass of technifermion determines the value of the order parameter v.

More exciting possibility will be the top quark which may have a large mass $m_t \gtrsim v$ in the recent experimental situation. Actually, two of the authors (M.T. and K.Y.) and Miransky

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proposed some time ago the top quark condensation scenario[1] ("top-mode standard model"). In this model we no longer need elementary Higgs boson. We instead regard the large top quark mass as the result of certain short range dynamics of unspecified origin which breaks the chiral symmetry dynamically through the top quark condensate $\langle 0|\bar{t}t|0\rangle \neq 0$. Because of dynamical χSB we obtain composite Nambu-Goldstone (NG) bosons, which give rise to masses of the weak gauge bosons through dynamical Higgs mechanism. We predicted a very large mass of the top quark $m_t \sim 250\mathrm{GeV}$ and also a composite Higgs boson $H \sim \bar{t}t$ with a mass $m_H \simeq 2m_t$.* Similar ideas were also advanced by Terazawa[2] and Nambu[3] in somewhat different terminologies and with different results for the value of m_t. Further studies of the top-mode standard model have recently been done by various groups[4,5,6,7] and confirmed the very large top quark mass $m_t > 200\mathrm{GeV}$ in this model.

In our previous paper[1], we considered the case where the four-fermion interactions are responsible for triggering the top quark condensation at very high energy scale ($\gtrsim$ GUT scale), and in fact our arguments were based on the explicit solution of the gap equation for spontaneous χSB in the gauged Nambu–Jona-Lasinio model (four-fermion interaction plus gauge interaction).[7]

What is the origin of the four-fermion interactions, then? One might immediately think of exchange of heavy spin 1 bosons with mass m_V. In fact one finds[9,10,11] that the behavior of the χSB solution in this system is similar to that of the NJL model for $m_V \sim \Lambda$, based on the ladder SD equation[10] for $iS^{-1}(p) \equiv A(-p^2)\not{p} - B(-p^2)$ (with "gauge parameter" ξ $D_{\mu\nu}(p) = -i(p^2 - m_V^2)^{-1}[g_{\mu\nu} - (1-\xi)p_\mu p_\nu(p^2 - m_V^2)^{-1}]$);

$$B(x) = \frac{e^2}{(4\pi)^2}\int_0^{\Lambda^2} dy\, y\mathcal{K}_B(x,y)\frac{B(y)}{A^2(y)y + B^2(y)}, \tag{1a}$$

$$A(x) = 1 + \frac{e^2}{2(4\pi)^2}\int_0^{\Lambda^2} dy\, \frac{y}{x}\mathcal{K}_A(x,y)\frac{A(y)}{A^2(y)y + B^2(y)}, \tag{1b}$$

where

$$\mathcal{K}_B(x,y) = K_B(x,y;m_V^2)\left[(3+\xi) + \frac{(1-\xi)m_V^2}{\sqrt{(x+y+m_V^2)^2 - 4xy}}\right], \tag{2a}$$

$$\mathcal{K}_A(x,y) = 2K_A(x,y;m_V^2)\left[\xi + \frac{(\xi-1)m_V^2}{\sqrt{(x+y+m_V^2)^2 - 4xy}}\right], \tag{2b}$$

with K_A and K_B being defined in Eq.(6). Including gauge interaction in Eq.(1), we obtain a solution which is similar to that of the gauged NJL model in view of the top-mode standard model. (For detailed analysis, see Ref.[11].)

However, in the case of spin 1 boson exchange, we cannot obtain such an effective four-fermion interaction as

$$\epsilon_{ij}(\bar{\psi}_L^i t_R)(\bar{\psi}_L^j b_R),$$

through which ($g^{(2)}$ term in Ref.[1]) the bottom quark acquires its mass from a top quark condensation. We then must assume a bottom condensate independently of a top quark condensate

* Our recent analysis[8], including the effect of gauge interaction on the spectrum, implies $m_H \simeq \sqrt{2}m_t \simeq 350\mathrm{GeV}$.

in order to feed the mass to "down"-like fermions. It would be simple that the masses of fermions other than the top quark are also explained by the top quark condensate alone. So it does not seem to be the case that the interaction is mediated by spin 1 bosons.

Here, we wish to discuss another possibility that an attractive force due to a heavy spinless boson exchange through Yukawa interaction causes the top quark condensation. Even if we write the same $SU(2)_L \times U(1)_Y$ symmetric Yukawa interaction as the usual standard model, there will be in this picture an essential difference that the $SU(2)_L \times U(1)_Y$ breaking is mainly not due to the VEV of the spinless boson but to the top quark condensate caused by the attractive force of the strong Yukawa coupling.

Then, our task is to investigate the phase structure of the standard model with very large Yukawa coupling. In the following sections, we will investigate chiral phase transition of standard model in the framework of the Schwinger-Dyson (SD) equation and find the phase where χSB is dynamical.

2. The SD equations

In this section, we derive the SD equation for fermion propagator in the form of integral equation, which we can solve numerically and analytically. We discuss here $SU(2)_L \times SU(2)_R$ symmetric Yukawa interaction, for simplicity. Extension to other types of Yukawa interaction is straightforward. The lagrangian is given by

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi - \frac{\eta_0}{\sqrt{2}} \left[\bar{\psi}\psi\sigma + \bar{\psi} i\gamma_5 \vec{\tau}\psi \cdot \vec{\pi}\right] + \frac{1}{2}\left[(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2\right] - \frac{m_0^2}{2}\left[\sigma^2 + \vec{\pi}^2\right] - \frac{\lambda_0}{4}\left[\sigma^2 + \vec{\pi}^2\right]^2 .$$

From the equation of motion for ψ, $i\partial\!\!\!/\psi - \frac{\eta_0}{\sqrt{2}}[\psi\sigma + i\gamma_5\vec{\tau}\psi\cdot\vec{\pi}] = 0$, we obtain the SD equation for fermion propagator,

$$i\partial\!\!\!/ \langle 0|T\psi(x)\bar{\psi}(0)|0\rangle = i\delta^{(4)}(x) + \frac{\eta_0}{\sqrt{2}}\langle 0|T(\sigma(x) + i\vec{\tau}\cdot\vec{\pi}(x)\gamma_5)\psi(x)\bar{\psi}(0)|0\rangle .$$

Assuming $\langle 0|\vec{\pi}|0\rangle = 0$, we can rewrite the SD equation in momentum space

$$\begin{aligned} p\!\!\!/ - iS^{-1}(p) = \frac{\eta_0}{\sqrt{2}}\langle 0|\sigma|0\rangle + \frac{\eta_0}{\sqrt{2}}\int \frac{d^4k}{(2\pi)^4}\Big[&D_\sigma(k)S(p-k)\Gamma_\sigma(p-k,p) \\ &+ D_\pi(k) i\gamma_5\vec{\tau}S(p-k)\vec{\Gamma}_\pi(p-k,p)\Big], \end{aligned} \tag{3}$$

where D_σ, D_π and S are full propagators of σ, π and fermion ψ, respectively, and Γ_σ and $\vec{\Gamma}_\pi$ are full vertices of $\psi\bar{\psi}\sigma$ and $\psi\bar{\psi}\vec{\pi}$.

Under the approximation of boson propagators and vertices

$$D_\sigma = \frac{iZ_3}{k^2 - m_\sigma^2}, \qquad D_\pi = \frac{iZ_3}{k^2 - m_\pi^2},$$
$$\Gamma_\sigma = -i\frac{\eta_0}{\sqrt{2}}, \qquad \vec{\Gamma}_\pi = -i\frac{\eta_0}{\sqrt{2}}\vec{\tau}i\gamma_5,$$

we obtain the integral equation (SD equation), which reads after Wick rotation

$$B(x) = \frac{\eta}{\sqrt{2}}v + \frac{C}{2}\int_0^{\Lambda^2} dy\, y\mathcal{K}_B(x,y)\frac{B(y)}{A^2(y)y + B^2(y)}, \tag{4a}$$

$$A(x) = 1 + \frac{C}{4}\int_0^{\Lambda^2} dy\, \frac{y}{x}\mathcal{K}_A(x,y)\frac{A(y)}{A^2(y)y + B^2(y)}, \tag{4b}$$

where $C \equiv \eta^2/(4\pi)^2, x \equiv -p^2$, $y \equiv -k^2$ and $v \equiv Z_3^{-\frac{1}{2}}\langle 0|\sigma|0\rangle$, $\eta^2 \equiv Z_3\eta_0^2$. Here an ultraviolet (UV) cutoff Λ is introduced. The integral kernels $\mathcal{K}_B$ and $\mathcal{K}_A$ are defined as

$$\mathcal{K}_B(x,y) = 3K_B(x,y;m_\pi^2) - K_B(x,y;m_\sigma^2), \tag{5a}$$

$$\mathcal{K}_A(x,y) = 3K_A(x,y;m_\pi^2) + K_A(x,y;m_\sigma^2). \tag{5b}$$

K_B and K_A are given by

$$\begin{aligned} K_B(x,y;m^2) &\equiv \frac{2}{\pi}\int_0^\pi d\theta \frac{\sin^2\theta}{x+y-2\sqrt{xy}\cos\theta+m^2} \\ &= \frac{2}{x+y+m^2+\sqrt{(x+y+m^2)^2-4xy}}, \end{aligned} \tag{6a}$$

$$\begin{aligned} K_A(x,y;m^2) &\equiv \frac{4}{\pi}\int_0^\pi d\theta \frac{\sqrt{xy}\cos\theta\sin^2\theta}{x+y-2\sqrt{xy}\cos\theta+m^2} \\ &= \frac{4xy}{\left[x+y+m^2+\sqrt{(x+y+m^2)^2-4xy}\right]^2}. \end{aligned} \tag{6b}$$

Note that from Eq.(5a) σ gives repulsive force while π does attractive one.

For the case of Yukawa interaction with a discrete chiral symmetry, $\mathcal{L}_{Yuk} = -\frac{\eta_0}{\sqrt{2}}\bar\psi\psi\sigma$, the SD equations are given by Eq.(4) with the integral kernels

$$\mathcal{K}_B(x,y) = -K_B(x,y;m_\sigma^2), \tag{7a}$$

$$\mathcal{K}_A(x,y) = K_A(x,y;m_\sigma^2). \tag{7b}$$

For the case of $U(1)_L \times U(1)_R$ symmetric Yukawa interaction, $\mathcal{L}_{Yuk} = -\frac{\eta_0}{\sqrt{2}}[\bar\psi\psi\sigma + \bar\psi i\gamma_5\psi\pi]$, the SD equations are given by Eq.(4) with the integral kernels

$$\mathcal{K}_B(x,y) = K_B(x,y;m_\pi^2) - K_B(x,y;m_\sigma^2), \tag{8a}$$

$$\mathcal{K}_A(x,y) = K_A(x,y;m_\pi^2) + K_A(x,y;m_\sigma^2). \tag{8b}$$

In the case of the massless boson exchange, i.e., $m^2 = 0$, K_B and K_A become simple;

$$K_B(x,y;0) = \frac{1}{x}\theta(x-y) + \frac{1}{y}\theta(y-x),$$

$$K_A(x,y;0) = \frac{y}{x}\theta(x-y) + \frac{x}{y}\theta(y-x).$$

Then, in this case the integral kernels are the same as that of QED in ladder approximation.

3. Solution within Quenched Approximation

First we consider $v=0$ phase. In quenched approximation, $v=0$ does not mean $\langle 0|\bar\psi\psi|0\rangle = 0$. In fact, as we will see in the following, there exists a chiral phase transition even in the $v=0$ phase at strong Yukawa coupling region for $SU(2)_L \times SU(2)_R$ symmetric Yukawa interaction.

In the $v=0$ phase, σ and π have degenerate masses $m_\sigma = m_\pi \equiv m$. Then the integral kernels of the SD equation Eq.(4) are written simply as

$$\mathcal{K}_B(x,y) = 2K_B(x,y;m^2), \tag{9a}$$

$$\mathcal{K}_A(x,y) = 4K_A(x,y;m^2). \tag{9b}$$

Note that the integral kernels Eq.(9) are the same up to factor as those of the case of the massive vector boson exchange Eq.(2) in "Feynman gauge", $\xi = 1$. Then we obtain the χSB solution at strong coupling Yukawa region in the same way as the case of the massive vector boson exchange[10].

Note that we cannot obtain the χSB solution within this approximation (only ladder, without tadpole) in the cases of discrete chiral symmetric Yukawa interaction ($U(1)_L \times U(1)_R$ symmetric Yukawa interaction) because of absence (cancellation) of attractive force. (See Eq.(7a), Eq.(8a).)

Following Ref.[10], we approximate wave function renormalization $A \equiv 1$ for analytical calculation. This approximation is good if $\Lambda \sim m \gg B(0)$. Here we make a simple approximation for $(6a)$;

$$K_B(x, y; m^2) = \frac{1}{x + m^2}\theta(x - y) + \frac{1}{y + m^2}\theta(y - x).$$

To study a scaling relation near the critical point of chiral phase transition, it is sufficient to study the linearized integral equation (bifurcation technique)[12]. Then we obtain a simple integral equation;

$$B(x) = C\left[\int_{M^2}^{x} dy \frac{B(y)}{x + m^2} + \int_{x}^{\Lambda^2} dy \frac{B(y)}{y + m^2}\right], \tag{10}$$

where an infrared (IR) cutoff $M \simeq B(0)$ was introduced. Solving Eq.(10), we obtain a scaling relation[10];

$$\frac{M^2}{\Lambda^2 + m^2} = \exp\left[\frac{-4}{\sqrt{4C - 1}}\left(\frac{\pi}{2} - \tan^{-1}\sqrt{4C - 1}\right)\right] - \frac{m^2}{\Lambda^2 + m^2}. \tag{11}$$

Critical coupling constant C_c which separates χSB phase from the symmetric one corresponds to the solution of Eq.(11) in the limit of $M \to 0$.

Let us next consider $v \neq 0$ phase. In this phase m_π is zero because of the Goldstone theorem. Then our integral equation is given by the kernels

$$\mathcal{K}_B(x, y) = 3K_B(x, y; 0) - K_B(x, y; m_\sigma^2), \tag{12a}$$

$$\mathcal{K}_A(x, y) = 3K_A(x, y; 0) + K_A(x, y; m_\sigma^2). \tag{12b}$$

In this phase, we can define a renormalized ϕ^4 coupling λ as $\lambda \equiv m_\sigma^2/(2v^2)$. In the case of $\lambda = \infty$, $K_B(x, y; m_\sigma^2)$ can be neglected. Then the SD equations are

$$B(x) = \frac{\eta}{\sqrt{2}}v + \frac{3}{2}C\left[\frac{1}{x}\int_0^x dy \frac{yB(y)}{A^2(y)y + B^2(y)} + \int_x^{\Lambda^2} dy \frac{B(y)}{A^2(y)y + B^2(y)}\right], \tag{13a}$$

$$A(x) = 1 + \frac{3}{4}C\left[\frac{1}{x^2}\int_0^x dy \frac{y^2 A(y)}{A^2(y)y + B^2(y)} + \int_x^{\Lambda^2} dy \frac{A(y)}{A^2(y)y + B^2(y)}\right]. \tag{13b}$$

These integral equations are the same as those of QED in ladder approximation with gauge parameter $\xi = 3$. Here, we will discuss the behavior of solution only in the $\lambda = \infty$ case.

It is convenient to rewrite Eq.(13a) and Eq.(13b) into differential equations and boundary conditions;

$$\left[x\frac{d^2}{dx^2} + 2\frac{d}{dx} + \frac{3C}{2}\frac{1}{A^2(x) + B^2(x)}\right] B(x) = 0, \tag{14a}$$

$$\left[x\frac{d^2}{dx^2} + 3\frac{d}{dx} + \frac{3C}{2}\frac{1}{A^2(x) + B^2(x)}\right] A(x) = 0, \tag{14b}$$

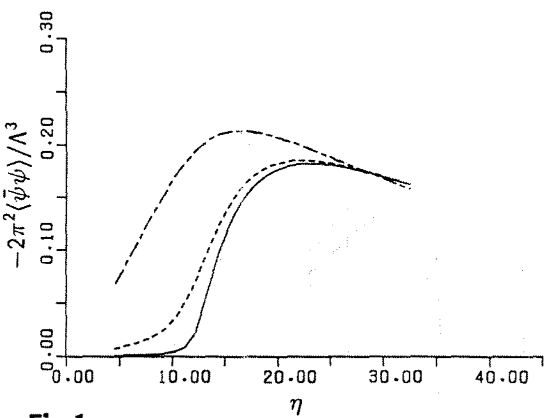

Fig.1a

Yukawa coupling dependence of the fermion pair condensation $\langle 0|\bar{\psi}\psi|0\rangle$ in the $v \neq 0$ phase. Dashed dotted line, dotted line and solid line correspond to $v/\Lambda = 2\times10^{-2}, 2\times10^{-3}, 2\times10^{-4}$, respectively.

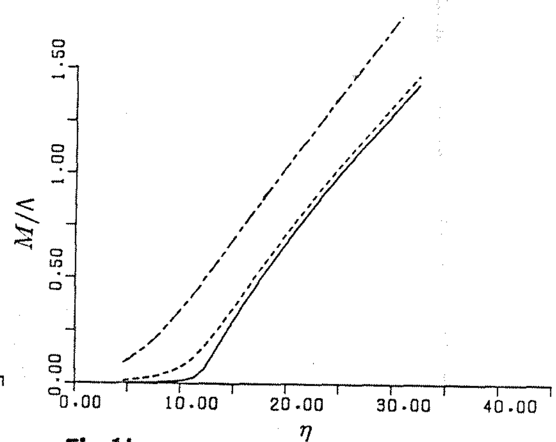

Fig.1b

Yukawa coupling dependence of the fermion mass $M \equiv B(0)/A(0)$ in the $v \neq 0$ phase. Dashed dotted line, dotted line and solid line correspond to $v/\Lambda = 2\times10^{-2}, 2\times10^{-3}, 2\times10^{-4}$, respectively.

$$
\begin{aligned}
&x^2\frac{d}{dx}B(x)\Big]_{x=0}, \quad \left(1+x\frac{d}{dx}\right)B(x)\Big]_{x=\Lambda^2} = \frac{\eta}{\sqrt{2}}v,\\
&x^3\frac{d}{dx}A(x)\Big]_{x=0}, \quad \left(1+\frac{x}{2}\frac{d}{dx}\right)A(x)\Big]_{x=\Lambda^2} = 1.
\end{aligned}
\tag{15}
$$

We define a local order parameter of the chiral phase transition:

$$
\langle 0|\bar{\psi}\psi|0\rangle \equiv -\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}S(p) = -\frac{1}{2\pi^2}\int_0^{\Lambda^2}dx\frac{xB(x)}{A^2(x)x+B^2(x)}. \tag{16}
$$

$-\langle 0|\bar{\psi}\psi|0\rangle$ is a positive definite function of η. In the case of $\eta = 0$, we obtain $\langle 0|\bar{\psi}\psi|0\rangle = 0$. In the strong coupling limit $\eta \to \infty$, $B(x)$ becomes large and dominates the denominator of Eq.(16) and we obtain $-\langle 0|\bar{\psi}\psi|0\rangle \sim 1/B(0)$ in that region. (This behavior is consistent with the strong coupling expansion which says $-\langle 0|\bar{\psi}\psi|0\rangle \sim 1/\eta$.) Then we have a turning-over point where the function $\langle 0|\bar{\psi}\psi|0\rangle$ takes the maximum value*. We in fact investigate the behavior of this function using a numerical solution of Eq.(14). The result is given at Fig.1a and Fig.1b. The turning-over point appears when the dynamical mass of fermion $M \equiv B(0)/A(0)$ has its value $M \sim \Lambda$. Note also $\langle 0|\bar{\psi}\psi|0\rangle$ is nonvanishing at the strong Yukawa coupling region even in the limit of $v/\Lambda \to 0$ (continuum limit).

Note that the behavior of $\langle 0|\bar{\psi}\psi|0\rangle$ is consistent with the result of lattice MC simulation[13].

We next investigate the "renormalized Yukawa coupling"[13] η_R, defined by $\eta_R \equiv \sqrt{2}M/v$. The result is shown in Fig.2. Because of nonvanishing M in the continuum limit ($v/\Lambda \to 0$), this value diverges at the strong Yukawa coupling region.

* Note here that this property of $-\langle 0|\bar{\psi}\psi|0\rangle$ (existence of a turning-over point and a maximum value) is universal[14] in our framework, i.e., it does not depend on details of the interaction which breaks the chiral symmetry. In fact, we can explicitly show the existence of a maximum value of $-\langle 0|\bar{\psi}\psi|0\rangle$ also in the cases of strong coupling QED and the NJL model in ladder approximation.

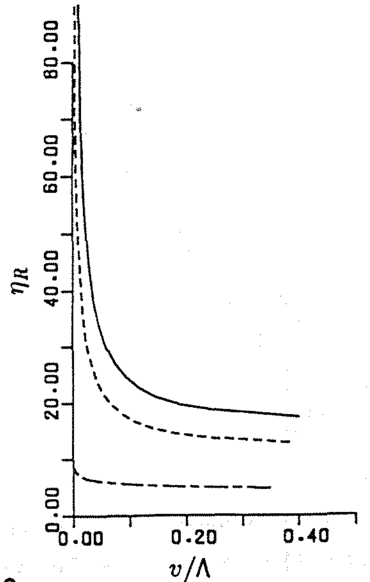

Fig.2

"renormalized Yukawa coupling" as the function of v. Dashed dotted line, dotted line and solid line correspond to $\eta = 4.59, 12.14, 16.54$, respectively.

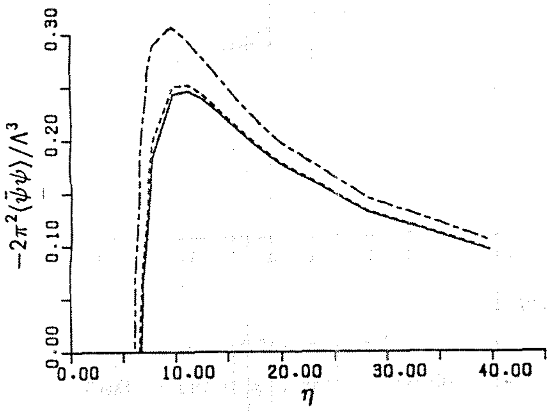

Fig.3

Scaling relation of the fermion pair condensation $\langle 0|\bar{\psi}\psi|0\rangle$ without quenched approximation when $Z_3 m_0^2 = \Lambda$. Dashed dotted line, dotted line and solid line correspond to $m = \Lambda, 10^{-1}\Lambda, 10^{-2}\Lambda$, respectively.

4. Effect of Dynamical Fermion

Finally, we discuss the effect of dynamical fermion (tadpole) on the above analysis using the SD equation.

Using the equation of motion of σ,

$$\Box\sigma + m_0^2\sigma + \lambda_0(\sigma^2 + \vec{\pi}^2)\sigma + \frac{\eta_0}{\sqrt{2}}\bar{\psi}\psi = 0,$$

we obtain the SD equation for VEV of σ,

$$m_0^2\langle 0|\sigma|0\rangle + \lambda_0\langle 0|\sigma(\sigma^2 + \vec{\pi}^2)|0\rangle + \frac{\eta_0}{\sqrt{2}}\langle 0|\bar{\psi}\psi|0\rangle = 0. \tag{17}$$

We wish to discuss χSB due to the effect of η_0, hence we disregard the effect of λ_0 here. Then, the value of v is determined by $\langle 0|\bar{\psi}\psi|0\rangle$;

$$v = Z_3^{-\frac{1}{2}}\langle 0|\sigma|0\rangle = -\frac{\eta}{\sqrt{2}}\frac{\langle 0|\bar{\psi}\psi|0\rangle}{Z_3 m_0^2}. \tag{18}$$

Unlike the case of quenched approximation, $v = 0$ means $\langle 0|\bar{\psi}\psi|0\rangle = 0$ in this unquenched case. From Eq.(4), Eq.(16) and Eq.(18), we obtain the SD equation

$$B(x) = \frac{4C}{Z_3 m_0^2}\int_0^{\Lambda^2} dy \frac{yB(y)}{A^2(y)y + B^2(y)} + \frac{C}{2}\int_0^{\Lambda^2} dy\, y\mathcal{K}_B(x,y)\frac{B(y)}{A^2(y)y + B^2(y)}, \tag{19}$$

where the kernel $\mathcal{K}_B$ is defined in Eq.(5a) and the SD equation for A is the same as Eq.(4b). Here we must note that m_σ and m_π are not independent quantities of η. For example, in the strong coupling phase of η where χSB occurs, we have $m_\pi = 0$ because of the Goldstone theorem. On the other hand, in the weak coupling phase of η where chiral symmetry is unbroken, m_π and m_σ should be degenerate. Such an η dependence of mass spectrum of bosons comes from the loop effect of fermion in the vacuum polarization in the σ and π propagators.

Especially in the strong coupling phase, the massless pole of π propagator comes from mixing with massless bound states of fermions,i.e., composite NG bosons. Then we must solve the SD equations for σ and π propagators and Nambu-Bethe-Salpeter equation for the bound state in a self-consistent manner. This is very difficult technically, however. We simply disregard the effect of dynamical fermion on the propagators of σ and π. We only consider the effect of dynamical fermion on the VEV of σ. Here we use the integral kernels $\mathcal{K}_B$, $\mathcal{K}_A$ given in Eq.(9).

In such an approximation we obtain the SD equation with the effect of the dynamical fermion,

$$B(x) = \frac{4C}{Z_3 m_0^2}\int_0^{\Lambda^2} dy \frac{yB(y)}{A^2(y)y + B^2(y)} + C\int_0^{\Lambda^2} dy\, y K_B(x,y;m^2)\frac{B(y)}{A^2(y)y+B^2(y)}, \quad (20a)$$

$$A(x) = 1 + C\int_0^{\Lambda^2} dy\, \frac{y}{x} K_A(x,y;m^2)\frac{A(y)}{A^2(y)y+B^2(y)}. \quad (20b)$$

We calculate this integral equation numerically. Fig.3 is the result of the chiral phase transition of this system. In this case it is difficult to say whether the χSB is dynamical or not, because we always have non-zero value of v whenever χSB occurs. Hence we next discuss a criterion of dynamical χSB.

The NG bosons couple to the axialvector current through its "decay constant" F_π,

$$\langle 0|J^a_{5\mu}(x)|\pi^b(q)\rangle = iq_\mu F_\pi \delta^{ab} e^{-iqx}.$$

The axialvector current is written as

$$J^a_{5\mu} = \bar\psi \frac{\tau^a}{2}\gamma_\mu\gamma_5\psi + \sigma\partial_\mu\pi^a - \pi^a\partial_\mu\sigma.$$

We divide the NG boson decay constant into two parts;

$$\langle 0|\bar\psi\frac{\tau^a}{2}\gamma_\mu\gamma_5\psi|\pi^b(q)\rangle = iq_\mu F^f_\pi \delta^{ab} e^{-iqx},$$
$$\langle 0|\sigma\partial_\mu\pi^a - \pi^a\partial_\mu\sigma|\pi^b(q)\rangle = iq_\mu F^b_\pi \delta^{ab} e^{-iqx}.$$

We call the χSB is dynamical, when *the fermionic part of the NG boson decay constant F^f_π is sufficiently larger than the bosonic part of the NG boson decay constant F^b_π.*

In this case the bosonic part F^b_π is written in terms of the VEV of σ, $F^b_\pi = Z_3 v$. On the other hand, the fermionic part F^f_π is written in terms of the mass function of the fermion and its value is order of M, $F^f_\pi \sim M$. Then our criterion of dynamical χSB is

$$M \gg Z_3 v = -\frac{\eta}{\sqrt{2}}\frac{\langle 0|\bar\psi\psi|0\rangle}{m_0^2}. \quad (21)$$

$\langle 0|\bar\psi\psi|0\rangle$ is given by

$$\langle 0|\bar\psi\psi|0\rangle \sim M^3\left(\frac{\Lambda}{M}\right)^\gamma,$$

where γ is determined from the high energy behavior of the fermion mass function (an analog of anomalous dimension[7]).

In our numerical calculation we obtain $\gamma \simeq 1.61$ for $m^2 = 10^{-3}\Lambda^2$, $Z_3 m_0^2 = \Lambda^2$, and $\gamma \simeq 1.97$ for $m^2 = \Lambda^2$, $Z_3 m_0^2 = \Lambda^2$. Noting $Z_3 < 1$, we find our criterion of dynamical χSB is fulfilled for small m and sufficiently large UV cutoff Λ.

5. Conclusions and Discussion

We have investigated the dynamical χSB due to strong coupling $SU(2)_L \times SU(2)_R$ symmetric Yukawa interaction in the framework of the SD equations. Within the quenched approximation we found the phase where χSB occurs while the VEV of elementary scalar field vanishes. In the approximation where the loop effect of the dynamical fermion affects the value of v, we discussed the criterion for dynamical χSB. We found the region where our criterion is fulfilled.

We discussed here only $SU(2)_L \times SU(2)_R$ symmetric Yukawa interaction. However, the Yukawa interaction with large isospin violation is important for the top quark condensation. More detailed analysis including the case of isospin violation will appear elsewhere.

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