

A novel cooling scheme for antiprotons

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Abstract. We propose a novel technique which uses laser-cooled negative osmium ions for sympathetic cooling of antiprotons. Temperatures down to the sub-millikelvin range might be achievable. These antiprotons could be used to form antihydrogen at ultra-cold temperatures, thus allowing efficient magnetic trapping of antihydrogen for high-resolution laser spectroscopy. Antihydrogen at sub-millikelvin temperatures might also enable first direct measurements of the gravitational acceleration of antimatter. Currently, no other technique exists which allows the cooling of large numbers of antiprotons to temperatures below that of the surrounding trap.

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1. Introduction

The antihydrogen ($\bar{\text{H}}$) atom offers unique opportunities for high-precision tests of CPT symmetry, the invariance of physical laws under the simultaneous inversion of charge, parity and time. CPT violation could manifest itself by the non-identity of any of the fundamental properties of a particle–antiparticle pair. Several such comparisons have already been carried out to high precision on pairs of subatomic particles [1]–[3]. It must be pointed out, however, that CPT violations, according to different theoretical models, would not necessarily be visible in all of a particle’s properties. Thus, the existing tests do not preclude deviations in other properties. Current efforts towards the production of cold antihydrogen were spawned by the immense progress, over the past decades, in precision Doppler-free two-photon laser spectroscopy on the hydrogen atom. Currently, the transition frequency between the 1s ground state and the 2s metastable excited state of hydrogen can be measured to about 2 parts in 10^{14} [4]. The same technique, or a variation thereof, could eventually allow a comparison of the hydrogen and antihydrogen atomic spectra to a similar or even higher relative precision.

In addition to these tests of the CPT symmetry, antimatter atoms can also be used to test general relativity. Quantum field theories of the gravitational force allow for the existence of scalar and vector gravitons, which would lead to a deviation from the inverse-square law of the gravitational force for antimatter, and thus a violation of the weak equivalence principle (WEP). While the universality of free fall is extremely well tested for ordinary matter [5], no corresponding measurements have ever been performed on the gravitational acceleration of antimatter. For the simpler quantum gravity models, however, a violation of the WEP can be constrained to the 10^{-6} level from existing high-precision experiments with ordinary matter [6]. Gross violations can be excluded on the grounds of energy conservation and other basic considerations [7]. Antimatter gravity studies have been hampered by the fact that only charged particles of antimatter were available, and that for those the electromagnetic forces—even from stray fields—by far outweighed the gravitational force. Neutral antihydrogen holds the promise of allowing the first-ever WEP test with anti-atoms. Such experiments could be carried out in a manner analogous to ordinary-matter atomic fountains [8] or using interferometric methods [9]. In order to achieve relative precisions at the 10^{-6} level, either method would require large numbers of cold $\bar{\text{H}}$ at temperatures well below 4 K and even below the Doppler cooling limit of $T_D = 2.4$ mK using the Lyman- α transition in antihydrogen.

The production of cold antihydrogen was first accomplished by two experiments installed at the CERN antiproton decelerator (AD) [10]. In 2002, both ATHENA [11] and ATRAP [12] observed large amounts of $\bar{\text{H}}$ produced from cold positrons and antiprotons (\bar{p}) confined simultaneously in Penning traps. Subsequent measurements by ATRAP of the $\bar{\text{H}}$ ’s velocity distribution [13] and by ATHENA of the spatial distribution of antihydrogen emission [14] both suggest that the temperature of the produced antihydrogen is several times larger than that of the positron plasma, which is in thermal equilibrium with the surrounding trap at liquid-helium temperature or slightly above. This probably means that the recombination rate is much higher than the antiproton cooling rate. While the anti-atoms produced by ATHENA and ATRAP are certainly ‘cold’ compared to those previously created in-beam [15, 16], they are most probably far from the temperatures required for precision tests. One of the most promising routes towards such ultra-cold antihydrogen is the cooling of the antiprotons prior to recombination.

ATHENA has attempted to achieve this by extending the sideband cooling technique [17] to non-neutral buffer gases [18], thus making it applicable to antiprotons, which would rapidly

annihilate with a neutral buffer gas. While the first results are encouraging, this technique allows at best the cooling of antiprotons to the temperature of the electron plasma and thus of the surrounding trap. The technique laid out in the present paper overcomes that limitation.

2. Indirect laser cooling

The simplest form of laser cooling is based on the directional absorption of a photon by excitation of an atomic or ionic system, followed by the spontaneous emission of a photon when the system de-excites at some later time. For this ‘Doppler cooling’ [19], the frequency of the light is detuned slightly to the red, or low-frequency side, of the transition frequency f_0 by Δf . Due to the Doppler effect, a photon can therefore only be absorbed by particles moving towards the light source with velocity $v = c(\Delta f/f_0)$, thereby reducing their momentum. The mean momentum transferred in the subsequent isotropic emission is zero; as a result of many absorptions and emissions the particle is cooled. The cooling action is countered by a smaller heating effect due to the spontaneously emitted photons. It limits the temperature that can be achieved with this technique to the Doppler temperature T_D , which is proportional to the natural linewidth of the transition.

Elementary particles cannot be directly laser-cooled. They can, however, transfer their kinetic and thermal energy to other charged particles which are in turn cooled—‘sympathetic cooling’. This technique has been instrumental in antihydrogen production, where antiprotons are cooled sympathetically with positrons which have thermalized by emitting synchrotron radiation [20]. In conjunction with laser cooling, it was first used to cool $^{198}\text{Hg}^+$ ions by the lighter laser-cooled $^9\text{Be}^+$ ions [21]. Recently, the sympathetic cooling of a species much lighter than the actively cooled ions has been demonstrated [22]. About 1000 positrons were cooled in a dense plasma of $^9\text{Be}^+$ ions confined in a room-temperature Penning trap. Due to a lack of reliable techniques for the measurement of the positron temperature, only an upper limit of the achieved temperature, 5 K, could be given.

If the two species contained in a two-component plasma have different mass, they centrifugally separate into shells [23]. At first sight, this can be viewed as detrimental to the sympathetic-cooling process. However, such a radial separation does not preclude further cooling of one population by the other, so long as the gap between the two species remains small with respect to the damping interaction, which is of course always the case with the Coulomb interaction. Furthermore, the centring of the lighter species is highly desirable, as it leads to a reduction of its rotational velocity. Without this effect, the velocity of particles near the edge of the plasma can be many orders of magnitude larger than the thermal velocity in the co-rotating frame.

The present proposal is based on a combination of the two techniques described above: a large collection of atomic ions is laser-cooled to temperatures near the Doppler limit. They in turn sympathetically cool a smaller number of antiprotons to the same temperature. Using this technique, a collection of antiprotons (or, for that matter, any other charged particles) can in principle be cooled to the Doppler temperature of the laser-cooled species. If antihydrogen is then produced from these cold antiprotons by use of a technique that avoids reheating, antihydrogen will be formed at essentially the \bar{p} ’s temperature due to the large mass ratio of the constituents antiproton and positron. Such a scheme for $\bar{\text{H}}$ production, based on multiple resonant charge-exchange processes [24], has recently been demonstrated by ATRAP [25].

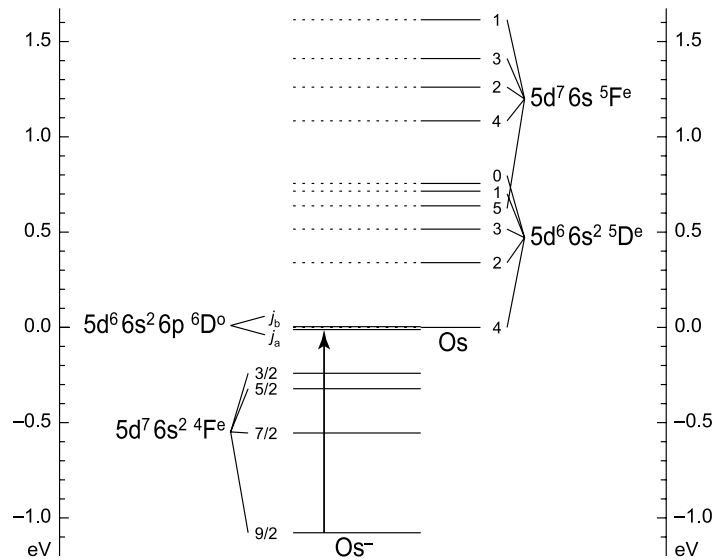


Figure 1. Energy level diagram for Os^- . The arrow indicates the transition applicable for laser cooling. Modified from figure 1 of [26] with permission © 2000 by the American Physical Society.

In order to minimize annihilation with the baryons contained in atomic ions, antiprotons can of course only be cooled by negative ions. Due to a lack of excited bound states in negative atomic ions with opposite parity compared to the ground state, no laser cooling of negative ions has ever been attempted or achieved. As we will discuss below, a system with such a configuration of states has recently been identified.

3. Negative osmium ions

In negative ions, the nuclear attraction is strongly suppressed. Most elements nevertheless form stable negative ions due to the correlation energy, the energy gained when all $Z + 1$ electrons adjust their motions such as to minimize the overlap of their wavefunctions in accordance with the Pauli exclusion principle (exchange correlation) and electrostatic repulsion (Coulomb correlation). When one of the electrons is excited, it essentially moves around a neutral atomic core. Negative ions therefore have no Rydberg series, but only a few, if any, bound excited states. Until very recently the only bound excited states of negative ions that had been identified had the same parity as the ground state. That means that electric-dipole transitions between them are forbidden and that their absorption cross-sections are correspondingly small.

Only a few years ago, the first bound state of a negative ion with opposite parity compared to the ground state was identified in the transition metal osmium [26]. The atomic level diagram of singly negative and of neutral osmium, with the tentative term assignments for the negative ion, is shown in figure 1, with the excited bound state identified as $5d^6 6s^2 6p \ ^6D^{\circ} j_a$. The excitation energy of the state was found to be 1.066 eV, which corresponds to a wavelength of 1163 nm, in the near infrared (IR) region. The absorption cross-section for this state is $\sigma_a \approx 6 \times 10^{-16} \text{ cm}^2$ and its Einstein A coefficient $A_a \approx 10^4 \text{ s}^{-1}$. It was deduced that the bound-bound transition is an electric-dipole transition, since all known magnetic-dipole transitions have Einstein A coefficients which are many orders of magnitude smaller. This electric-dipole transition should

be well-suited for the laser cooling of a collection of Os^- ions. With a natural linewidth of $\Gamma = A_a/(2\pi) \approx 10$ kHz, the Doppler cooling limit of the negative osmium ions, and thus in principle also of the sympathetically cooled antiprotons, is $T_D \approx 0.24 \mu\text{K}$.

4. Discussion

The proposed scheme, which we have outlined in the preceding sections, thus consists of the following discrete steps:

1. Produce antiprotons, load them into a trap, and pre-cool them sympathetically with electrons.
2. Produce negative osmium ions, load them into a (different) trap, and pre-cool them sympathetically with electrons.
3. Inject the antiprotons into the negative-ion plasma.
4. Laser-cool the negative ions and wait until the antiprotons are sympathetically cooled by the negative ions.
5. Remove the negative ions from the trap without reheating the antiprotons.

Of these, only the first has so far been experimentally demonstrated. In the following, we shall consider the practical implications of the remaining steps in detail and in particular discuss mechanisms which can lead to the premature loss of negative ions or antiprotons. In performing quantitative estimates, we shall assume that 10^6 negative osmium ions are used to cool about 10^4 antiprotons.

4.1. Production of an Os^- plasma

Current designs of high-intensity negative-ion sources are based upon the sputtering of atoms of a desired element from a surface by means of positive ions [27]. Commercially available negative-ion sources of the Middleton type use caesium as the sputtering ion. Measurements with a standard cathode loaded with osmium powder have shown that osmium readily forms a negative ion [28]. A current of $10 \mu\text{A}$ of $^{192}\text{Os}^-$ (41% abundance) was easily produced and sustained for many hours. It should therefore be straightforward to load about 10^6 Os^- ions into a high-field (3–6 T) Penning trap. These will form a spheroid plasma that performs a rigid rotation about the magnetic-field axis. With a characteristic trap dimension $d = 14$ mm, a magnetic-field magnitude $|\mathbf{B}| = 6$ T, a confining electric potential of $U_0 = 10$ V, and an azimuthal plasma area of $\pi r_0^2 = 10$ mm², the resulting half-length is $z_0 = 3.2$ mm, the plasma density $n = 2.4 \times 10^7$ cm⁻³ and the rotational frequency $\nu_{\text{rot}} = 5.8$ kHz.

Ions in a plasma can experience very strong electric fields as they interact in Coulomb collisions. While the Os^- ion is in its highest bound state, there is a risk of the excess electron being detached in these strong fields. The probability per unit time W for the breakup of atoms in static electric fields can be estimated according to the expression [29]

$$W = \frac{B^2(2L+1)}{2\gamma^{M_L}} \frac{M_L!(L+M_L)}{(L-M_L)!} \left(\frac{2\gamma^2}{F}\right)^{\frac{2Z}{\gamma}-M_L-1} e^{-\frac{2\gamma^3}{3F}}, \quad (1)$$

where B is a constant that describes the behaviour of the electron inside the atom, γ is related to the binding energy ϵ via $\gamma = (-2\epsilon)^{1/2}$, L is the orbital angular momentum, M_L is its projection, Z is the nuclear charge, and F is the electric-field magnitude. All quantities in equation (1) are in atomic units. For the negative osmium ion in the ${}^6D_a^o$ state ($Z = 0$, $L = 2$ and $\epsilon = 11.5$ meV) and using $B = 0.1$ and $M_L = 2$, the calculated breakup lifetime $1/W$ is practically infinite for fields up to 2 kV cm^{-1} , is of the order of 1 s for 3 kV cm^{-1} , then falls off to ms for $F = 4 \text{ kV cm}^{-1}$.

The ions' approach is closest in the case of a head-on collision, i.e., for an impact parameter $b = 0$, and that case can thus be used to obtain a lower bound on the separation. In such a collision, the distance of closest approach r_{\min} is reached when the initial kinetic energy in the centre-of-mass frame is fully converted to potential (Coulomb) energy. The electric field $F = e/(4\pi\epsilon_0 r_{\min}^2)$ experienced by the collision partners in Os^- – Os^- collisions reaches several 10 kV cm^{-1} at a temperature of 300 K, then falls off to about 6 kV cm^{-1} at 170 K and drops to about 2 kV cm^{-1} at 100 K. Since only a fraction of all ions is found in the excited state, the breakup rates are decreased by the same factor. These calculations show that pre-cooling of the Os^- ions to liquid-nitrogen temperature or slightly above is necessary in order to avoid breakup at the onset of laser cooling. This can be achieved by sympathetic cooling with an electron plasma preloaded into a cryogenic trap which has cooled to the trap temperature by emission of synchrotron radiation.

4.2. Injection of \bar{p} into the Os^- plasma

Once the Os^- ions have been pre-cooled to a temperature below which auto-neutralization can no longer occur, the smaller collection of \bar{p} can be injected into the plasma. During the injection process, the antiprotons will initially carry some kinetic energy. Due to the smaller number and mass of the antiprotons, heating of the osmium ions during this phase can be neglected. Until the antiprotons are fully thermalized, as well as during the subsequent indirect laser cooling phase, they have ample opportunity to be captured into negative osmium ions, and thus be lost irrecoverably. Such loss can occur either by collisional neutralization followed by capture into the neutral osmium atom:



or by direct capture according to the reaction



Since neutralized Os atoms rapidly leave the trap (see section 4.4 below), subsequent antiproton capture is highly unlikely. Depending on the initial antiproton energy, direct capture may, however, be a source for \bar{p} loss.

Calculations of the cross-section for direct capture according to equation (3) unfortunately only exist for the analogous formation of protonium in \bar{p} – H^- collisions [30, 31]. The latter of these were performed using the classical-trajectory Monte Carlo method with full four-body dynamics. Since classically the H^- ion is unstable, quantum-mechanical information was added by using an effective three-body potential to bind the second electron. The calculations yield a rather flat capture cross-section for centre-of-mass energies between 5 and 20 eV, with sharp cut-offs both to the low-energy and the high-energy sides [31].

While the capture cross-section for the negative osmium ion is certainly higher than that of the negative hydrogen ion for energies above threshold, the behaviour at low energy is dominated by the Coulomb repulsion. The threshold value itself should therefore only depend

on the ionic radius, which is almost the same for hydrogen and osmium (Os^- is actually smaller than H^-). The rate for the direct capture process is thus essentially zero for centre-of-mass energies below 2.5 eV. If some care is taken to transfer the \bar{p} without reheating them (i.e., by ballistic transfer), it should be possible to keep their temperature well below the threshold value of several 10^4 K.

4.3. Laser cooling and repumping

Cooling of Os^- ions from 80 K ($\approx 84 \text{ m s}^{-1}$) to 1 mK ($\approx 0.30 \text{ m s}^{-1}$) requires the absorption of about 3×10^5 photons, which at a transition rate per atom of $(dN_a/dt)_{\text{abs}} = 1000 \text{ s}^{-1}$ takes about 5 min. The wavelength for excitation from the ground to the excited bound state (1163 nm) is convenient for laser instrumentation, as tunable continuous-wave (cw) lasers based on high-power laser diodes and optical parametric oscillators (OPOs) are available. Using the cross-section $\sigma_a \approx 6 \times 10^{-16} \text{ cm}^2$ given above for the absorption into the excited bound state of Os^- and assuming a laser focal area of 10 mm^2 and full overlap of the laser beam with the confinement region, a desired transition rate per atom of $(dN_a/dt)_{\text{abs}} = 1000 \text{ s}^{-1}$ will require a laser power of about 30 mW.

The bound ${}^6\text{D}_a^0$ state in Os^- can not only decay to the ${}^4\text{F}_{9/2}^e$ ground state but also to some of the intermediate states of even parity. Due to the selection rules of the electric-dipole transition ($\Delta J = 0, \pm 1$), only the ${}^4\text{F}_{5/2}^e$ and the ${}^4\text{F}_{7/2}^e$ states can be populated. The branching ratios into these states and their lifetimes have not been measured. The lifetimes have, however, been calculated using the Dirac–Fock and relativistic configuration interaction methods [32] and found to be $\tau_{5/2} = 5.8 \text{ s}$ and $\tau_{7/2} = 0.46 \text{ s}$. These long lifetimes (with respect to the transition rate for laser cooling) mean that the states will probably have to be optically repumped to the j_a state. The laser wavelengths required for the repumping of the lower-lying bound states of negative osmium are $\lambda_{5/2} = 3.81 \mu\text{m}$ and $\lambda_{7/2} = 2.29 \mu\text{m}$. Assuming resonant cross-sections similar to that from the ground state and assuming furthermore that none of the states is populated with a rate of more than 1/10 the laser cooling rate, the required power for the repumping lasers is $P_{5/2} \approx 0.9 \text{ mW}$ and $P_{7/2} \approx 1.5 \text{ mW}$.

The absorption of a second photon by the ion in the ${}^6\text{D}_a^0$ state leads to a detachment of the excess electron and neutralization. This process has a cross-section about a factor 10 lower than the resonant excitation to the bound state, but at a laser power of 30 mW nevertheless takes place at a rate of about 90 s^{-1} . It therefore constitutes a competing ‘decay’ branch for the ${}^6\text{D}_a^0$ state, with a branching ratio of about 8%. In the course of the laser cooling, several 10^4 osmium ions can be lost in this way, but the recoil imparted on the neutralized Os atoms is small compared to the rotational velocity in the plasma. Therefore, again, the atoms will quickly leave the plasma radially and collisions with antiprotons or negative ions are unlikely. As the detachment rate scales linearly with the laser power, whereas the decay rate from the ${}^6\text{D}_a^0$ state to the ground state is independent from it, the laser power can be somewhat reduced if re-heating by the ejected osmium atoms turns out to be a problem.

4.4. Removal of Os^- ions

After a sufficient waiting time to allow for the indirect laser cooling of the antiprotons, the osmium ions will have to be removed in order to obtain only a collection of ultra-cold \bar{p} . This can be achieved very conveniently by resonant photo-neutralization of the negative ions. For this

purpose, the ions are resonantly excited to the unbound ${}^6D_b^o$ state with laser light at 1.081 eV (wavelength 1147 nm). Assuming that the laser employed for the laser cooling can be tuned to this slightly lower wavelength, it can be used for the neutralization despite the fact that the absorption cross-section into the j_b state is about an order of magnitude lower than that into the bound j_a state. At a transition rate of roughly 100 s^{-1} , all Os^- ions will be neutralized after a few tens of ms.

The neutralization process may, however, lead to antiproton loss via capture of \bar{p} into neutral osmium atoms. This capture reaction is, for instance, employed in the formation of antiprotonic helium atoms by ASACUSA [33]. The capture cross-section diverges as the centre-of-mass energy tends to zero. However, due to the rigid rotation of the plasma, the neutralized osmium ions are ejected radially outward at the velocity they had just prior to neutralization. The centrifugal velocity exceeds the thermal velocity and the recoil due to emission of the electron in the photo-neutralization (which are of the same order) for all radii greater than about $r_0/500$. If centrifugal separation is complete before the neutralization of the negative ions is carried out, the elongated \bar{p} cloud at the centre of the plasma is unperturbed by the removal of the Os^- . In order to consider the possibility of incomplete radial separation, we shall consider a uniform \bar{p} density throughout the Os^- plasma and an osmium ion which is neutralized close to the axis.

At the very low centre-of-mass energies discussed here, lower yet than those at which the adiabatic-ionization model applies [34], antiproton capture proceeds via Langevin orbiting (p wave) processes due to an induced dipole polarization, and the Langevin cross-section [35, 36] is

$$\sigma_{\text{orb}} = \pi \sqrt{\frac{2\alpha}{E_{\text{cm}}}}, \quad (4)$$

where α is the polarizability of the atom and E_{cm} is the centre-of-mass energy of the collision (all in atomic units). As this model requires that many angular-momentum partial waves occur, equation (4) is valid only as long as the centre-of-mass energy fulfils the relation [34]

$$E_{\text{cm}} \gg \frac{1}{\alpha\mu}, \quad (5)$$

where μ is the reduced mass. The cold collisions incurred at even lower energies, at radii below $r \approx r_0/10$, are dominated by s waves. Their cross-sections have no relationship to the Langevin cross-section, but they have the same energy dependence. Since we expect the p wave to have a \sin^2 distribution and the s wave to have an isotropic one, we may use twice the Langevin cross-section as an upper bound for the capture cross-section as E_{cm} tends to zero.

If we consider an ion neutralized at radius $r = r_0/100$ under the plasma conditions described above, and using $\alpha(\text{Os}) = 8.5 \text{ \AA}$, the capture cross-section is at most $\sigma_{\text{capt}} = 2 \times 10^{-10} \text{ cm}^2$ and the probability of a capture (per Os atom) is only $P = n_{\bar{p}}\sigma_{\text{capt}}r_0 \approx 10^{-5}$. Thus, under the worst possible assumptions, only 10 out of the original 10^4 antiprotons are lost at this stage.

5. Conclusion

Precision experiments to test antimatter gravity will most likely require anti-atoms colder than they can be produced with existing techniques, including laser cooling using Lyman- α radiation. We have presented a scheme based on indirect laser cooling of elementary particles which should allow the production of $\bar{\text{H}}$ at temperatures well below 1 mK. Quantitative estimates of

the negative-ion and antiproton losses incurred at the different steps of the scheme as well as calculations of the required laser power suggest that this novel method is feasible with currently existing technology. While not strictly required for spectroscopic tests of CPT symmetry using antihydrogen, the application of this technique should greatly increase the fraction of produced antihydrogen which can be trapped in a magnetic trap.

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