The Gravitational Dielectric Effect: D0-branes near a Black Hole Horizon

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Abstract

D-branes, having typical sizes of the order of the Planck length, are natural probes of quantum gravity. The D-brane dielectric effect is a phenomenon arising in string theory analogous to the polarization of uniform charges in a background E&M field; when an isolated collection of bound D0 branes is placed in a background electric flux, the electric force counteracts the collapse of the branes, supporting a configuration in the shape of a fuzzy sphere. We investigate whether the dielectric effect could arise from presence of non-trivial gravitational background curvature, such as in regions of space near the horizon of a black hole or in an expanding cosmological model. Barring a runaway potential in the non-Pauli matrix modes, we find a stable vacuum solution of the potential indicating a fuzzy sphere of D0-branes.

Contents

1 Introduction

General Relativity and Quantum Mechanics do not mix. Quantum mechanics has been successful in describing the physical world at characteristic scales of order $\hbar$, while General Relativity accurately describes large scale gravity as a geometrical curvature of space, having a characteristic scale of order G. The Standard Model has successfully unified the fundamental forces of E&M and the strong and weak force. However, attempts to incorporate gravity into the Standard Model have not succeeded, because the Standard Model is fundamentally a quantum theory, while general relativity is fundamentally classical. Thus, we are left with an unsatisfyingly splintered description of our physical universe. Enter string theory.

String theory is a promising attempt to combine all of the physical forces into one coherent framework. String theory sides with the Standard Model, assuming that gravity must be transformed into a quantum theory. One reason for this is that general relativity itself yields troubling results, such as physical gravitational singularities, indicating that the theory is somehow incomplete. The fundamental postulate of string theory is that an elementary particle is treated as a vibrating string extended in one or more spatial direction instead of a point particle. The different modes of vibration correspond to different elementary particles with different energies. This description holds promise for quantum gravity, since one of the vibrational states is the graviton. Strings can be closed, looping around on themselves, or open, in which case their ends are connected to dynamical"defects" in space or solitons known as Dp-branes, where p designates the dimensionality of the brane (D stands for Dirichlet, indicating that D-branes represent a physical boundary condition for the strings). For example, a D0 brane is a point-like, a D1 brane is string-like, and a D2 brane is like a two dimensional membrane. The world volume of a Dp-brane is the p+1 dimensional volume swept out by the brane across its time evolution.

String theory has been so promising in part because of one of its most bizarre implications: our universe contains more than the 3+1 spacetime dimensions that we observe. This would be possible because the extra dimensions are sufficiently compactified that we have not observed their signatures. However, such compactified dimensions may exhibit traces of their existence in our universe, for instance, if we are able to probe particles at higher energies and hence smaller length scales. Up to the mid-1990's, five distinct string theories we known to exist, each assuming that our universe contained 9+1 spacetime dimensions. Type I strings are open, containing electric charges on their ends. Type II strings are closed or end on D-branes, and are insulating; II_A strings allow for even dimensional D-branes while II_B allow for odd dimensional D-branes. Heterotic-O and Heterotic-E strings are closed, oriented, and superconducting. However, in 1995 these theories were united under one super-theory in 10+1 dimensions, M-theory. The five separate theories are simply facets of M-theory, and each are obtained from the details of the compactification from 11 to 10 dimensions. Although M-theory is an over-arching string theory, it is still not well-understood. Hence, much analysis in string theory is still performed using one of the simpler 9+1 dimensional theories.

One other unique signature of string theory is that, unlike quantum field theory, it contains no adjustable dimensionless parameters. Instead it contains scalar fields ϕ^i whose expectation values determine such parameters. Thus, one could presumably compute the dimensionless parameters based on the energies of the ϕ's. String theory has one dimensionful parameter, the string length l_s, which can be thought of as the size of fundamental strings. The string length is typically thought of as coinciding with the Planck length l_p

$$l_p = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-35} m \tag{1}$$

However, the only observable constraint on l_s is the signature of extra dimensions. l_s may be as large as 10^{-16} m and still produce dimensions sufficiently compact. It is possible that extra dimensions as large as one tenth of a millimeter may have up to now gone undetected!

2 The Gravitational Dielectric Effect

2.1 Background

The D-brane dielectric effect is a phenomenon arising in string theory analogous to the polarization of charges in a background E&M field; oppositely charged particles placed in a uniform electric field will separate to cancel the background field, forming a dielectric [1]. In string theory, a collection of D0 branes can be bound together with open strings ending on them. An isolated collection will collapse under its own tension, but when placed in a background magnetic field, the flux will support D0-branes from collapsing, causing them to puff up into the shape of a fuzzy spherical configuration. The fuzziness arises from the intrinsic quantum mechanical nature of spacetime on the Planck scale, incorporating non-commutative geometry analogous to the Heisenburg Uncertainty Principle.

While the dielectric effect has successfully been demonstrated to arise from a background E&M field [1], it remains an outstanding question as to whether it is possible to observe dielectric effect arising from a purely gravitational background. By unifying gravity with the electromagnetic force, string theory suggests that this should be possible, but until recently it was thought that the curvature of space alone was not sufficient to cause the polarization of D-branes. However, recent analyses [2], [3], [4], [5], have provided evidence that the gravitational dielectric effect may occur. One [3] suggests that a proper modification to the DBI action motivated from string theory might make static solutions possible. Recently, Sahakian [2] has used the original DBI action near the event horizon of a gravitational singularity, *i.e.* black hole; a point-like bound configuration of D0-branes might puff up into a fuzzy sphere as it falls inward toward the horizon. However, in looking for the desired static spherical configuration, the solution was at an extremum of the energy, and thus unstable. Thus, it remains an open question as to whether the curvature of space alone is sufficient to yield the dielectric effect in string theory.

2.2 The Dirac-Born-Infeld Action

The dynamics of Dp-branes are described by the Dirac-Born-Infeld (DBI) action as given in Myers [1]

$$S_{BI} = -T_p \int d^{p+1}\sigma Str \left\{ e^{-\phi} \sqrt{-det(P[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij} E_{jb}] + \lambda F_{ab}) det(Q^i{}_j)} \right\} \qquad (2)$$

where the integral is over $p+1$ spacetime dimensions, and σ represents the coordinates along the Dp-branes. Starting from the left, T_p represents the dimensionless tension of an individual Dp-brane, and is defined by

$$T_p \equiv \frac{2\pi}{g_s (2\pi l_s)^{p+1}} \qquad (3)$$

where g_s is a measure of the string coupling. The tension of a string scales its energy, which is relativistically equivalent to mass, thus T_p can also be thought of as a mass term.

Since the terms within the square-root are tensors, their indices must be traced over to produce a scalar over which to take the integral. However, because of the fundamental ambiguity in the ordering of tensors within the trace, the indices within the trace must be symmetrized over. For example, given matrices A,B, and C, the symmetrized trace of their product is given by

$$Str\{ABC\} \equiv Tr\{ABC + ACB + BCA + BAC + CAB + CBA\} \tag{4}$$

Next, ϕ represents the background dilation field, describing string coupling. $P[]$ is defined as the pullback of the background gravitational and E&M fields onto the world volume of the Dp-branes. Since II_A string theory exits in 9+1 dimensions, but branes may have any even number of dimensionality (up to 8), this terms represents the projection of a field in higher dimensions onto a lower dimensional brane. E_{ab} is defined as the sum of the gravitational metric tensor and the string gauge (magnetic) field

$$E_{ab} \equiv G_{ab} + B_{ab} \tag{5}$$

In addition, $Q^i{}_j$ is defined as

$$Q^i{}_j \equiv \delta^i{}_j + i\lambda[\Phi^i, \Phi^k]E_{kj} \tag{6}$$

where Φ^i are NxN coordinate matrices describing N Dp-branes. Diagonal entries in this matrix give the coordinates of spacetime that embed the branes, while off-diagonal terms describe interactions between a configuration of Dp-branes. The index $i = 1, ..., 9$ gives the dimension to which the coordinates refer. Next, λ is simply a constant defined by

$$\lambda \equiv 2\pi l_s^2 \tag{7}$$

and F_{ab} is gauge field on the world volume of the Dp-branes. Note that this is a two-dimensional object, and thus for D0-branes, having a one-dimensional world volume, this term is zero.

2.3 An Outline of Our Approach

Armed with the Myers version of the DBI action, we are ready to search for the existence of the gravitational dielectric effects in various non-trivial background curvatures. However, given that the relevant dynamical terms occur under a square root in the action, any non-trivial computations from this action will necessarily have to be expansions in λ. The recent effort of Sahakian [2] found static solutions to the DBI action to order λ^2. However, the analysis yielded an unstable (tachyonic) static extremum that quickly radiates away. In an effort to find more compelling evidence for the gravitational dielectric effect, we will expand upon this work and search for time-dependent solutions to order λ^4, noting that the DBI action is itself an approximation within string theory and is valid only to that order.

Because of their high level of intrinsic symmetric, two interesting and computationally tractable sources of background curvature to explore are the Schwarzschild metric near a black hole horizon and the Robertson-Walker metric of a de Sitter universe as an expanding cosmological model. Our computation will focus primarily on the dynamics of a point-like configuration of D0-branes infalling toward a black hole horizon. Using the Myers action (2), we will assume small curvature near the horizon, which will allow us to expand the action in Riemann Normal Coordinates into a useful form (Section 3). Because of spherical symmetry of the metric, we will postulate that SU(2) algebra will describe coordinate matrices of the D0-branes, and we will assume maximal symmetry of the metric, which will allow us to simplify the action to scalar form (Section 4). In Section 5, we extract the potential energy from the Lagrangian, analyzing its behavior to identify any stable configurations of the D0-branes. We find that, for positive curvature, the potential takes the shape of a modified Mexican hat potential, allowing for an extremum vacuum solution. This solution is stable barring a runaway potential in the non-Pauli matrix modes of the NxN D0-brane coordinate matrices Φ^i. In Section 6 we apply the results to the FRW cosmological model to explore the properties of D0-branes in a cosmological context, and in Section 7 we discuss the results.

Although our subsequent analysis is performed in II_A string theory, which exists in 9+1 spacetime dimensions, our computations will assume that 6 dimensions are sufficiently compactified so that we retrieve the familiar 3+1 dimensions. This is a natural assumption given our models in Schwarzschild and Robertson-Walker space.

3 Derivation of our Action

3.1 Simplification of the DBI Action for D0-branes

Starting with the DBI action (2)

$$S_{BI} = -T_p \int d^{p+1}\sigma Str\left\{e^{-\phi}\sqrt{-det(P[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij}E_{jb}] + \lambda F_{ab})det(Q^i{}_j)}\right\}$$

we will examine D0-branes in purely gravitational background fields. Thus, $T_p \rightarrow T_0$, which simplifies the string tension (3) to

$$T_0 = \frac{1}{g_s l_s} \tag{8}$$

and because we examine only gravitational fields, $E_{ab} \rightarrow G_{ab}$. Furthermore, we will let the background scalar field ϕ be constant, thus the exponent $e^-\phi$ can be absorbed into the string tension, and the action simplifies to

$$S = -T_0 \int d\sigma Str\left\{\sqrt{-det(P[G_{ab} + G_{ai}(Q^{-1} - \delta)^{ij}G_{jb}])det(Q^i{}_j)}\right\} \tag{9}$$

Noting that $Q^{-1}Q = 1$ by definition, and since $\lambda << 1$, one can expand Q^{-1} to find

$$\left(Q^{-1} - \delta\right)^{ij} = -i\lambda[\Phi^i, \Phi^k]G_k{}^j \tag{10}$$

The pullback of the gravitational field onto the worldline of the D0-brane is defined by

$$P[G_{ab}] \equiv G_{\mu\nu}\frac{\partial x^\mu}{\partial\sigma^a}\frac{\partial x^\nu}{\partial\sigma^b} \tag{11}$$

where the worldline of a D0-brane is one-dimensional, thus $a, b = 0$. We are employing the static gauge, and so $\sigma^0 = \tau$, the D0-brane proper time. Thus,

$$P[G_{ab}] = G_{00} + G_{ij}\dot{x}^i\dot{x}^j \tag{12}$$

where $\dot{x}^i \equiv \frac{\partial x}{\partial\tau}$.

We now allow the D0-brane coordinates to become matrices via the transformation

$$x^i \rightarrow \lambda\Phi^i \tag{13}$$

where the coordinate matrices Φ^i have units of inverse length, and λ has units of length squared. The pullback thus becomes

$$P[G_{ab}] = G_{00} + \lambda^2 G_{ij}\dot{\Phi}^i\dot{\Phi}^j \tag{14}$$

Letting $(Q^{-1} - \delta)^{ij} = A^{ij}$ for computational simplicity,

$$P[G_{ai}A^{ij}G_{jb}] = G_{\mu i}A^{ij}G_{j\nu}\dot{x}^\mu\dot{x}^\nu \tag{15}$$

Since the metric is symmetric, $G_{0i} = G_{i0} = 0$, the pullback scales as

$$P[G_{ai}A^{ij}G_{jb}] \sim A_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} \tag{16}$$

Note though that $A^{\mu\nu}$ is intrinsically antisymmetric because it scales as a commutator. Because (16) undergoes a symmetrized trace within the action, this term can be dropped. Furthermore, recalling that $a, b = 0$ for the D0-branes, the determinant of the pullback can also be dropped, and so the Lagrangian has simplified to

$$L = -T_0 Str\left\{(-(P[G_{ab}])^{1/2}\left(det(Q^i{}_j)\right)^{1/2}\right\} \tag{17}$$

3.2 Expansion in Riemann Normal Coordinates

In arbitrarily curved spacetime, one can always construct a locally inertial frame at some point P. In the near vicinity of P, spacetime is approximately flat, so the gravitational metric G_{ab} can be replaced by the flat space metric η_{ab}, and geodesics (free particle trajectories) are represented by straight lines. Further computations will employ a special realization of local flat space, Riemann Normal Coordinates (RNC). We choose P to be near the Schwarzschild horizon, where the RNC expansion is valid for sufficiently massive black holes, which have small curvature near the horizon.

The expansion of the metric in matrix coordinates (where $x^i \to \lambda\Phi^i$) is given compactly by

$$G_{ij}(x) = e^{\lambda\Phi^k\partial_k}G_{ij}|_{x=P} \tag{18}$$

Because spacetime is approximately flat near P, the first derivative of the metric vanishes, $\partial_i G_{\mu\nu} = 0$. Writing out the metric as $G_{\mu\nu} = G_{00} + G_{ij}$ and substituting in the above expansion to order λ^2

$$G_{00} = G_{00}|_P + \lambda G_{00,k}|_P\Phi^k + \frac{\lambda^2}{2}G_{00,kl}|_P\Phi^k\Phi^l \tag{19}$$

which simplifies to

$$G_{00} = \eta_{00} + \frac{\lambda^2}{2}G_{00,kl}\Phi^k\Phi^l \tag{20}$$

and similarly

$$G_{ij} = \eta_{ij} + \frac{\lambda^2}{2}G_{ij,kl}\Phi^k\Phi^l \tag{21}$$

Putting (20) and (21) into (11) yields

$$P[G_{ab}] = \eta_{00} + \frac{\lambda^2}{2}G_{00,ij}\Phi^i\Phi^j + \lambda^2\eta_{ij}\dot{\Phi}^i\dot{\Phi}^j + \frac{\lambda^4}{2}G_{ij,kl}\Phi^k\Phi^l\dot{\Phi}^i\dot{\Phi}^j \tag{22}$$

From [6], in RNC

$$
\begin{aligned}
G_{00} &\approx -1 - R_{0l0m}x^l x^m \\
G_{ij} &\approx \delta_{ij} - \frac{1}{3}R_{iljm}x^l x^m
\end{aligned}
\tag{23}
$$

to linear order in the curvature tensor R_{abcd}, which is a valid approximation since we are assuming small curvature. Taking the derivatives with respect to x^i,

$$
\begin{aligned}
G_{00,ij} &= -R_{0i0j} \\
G_{ij,kl} &= -\frac{1}{3}R_{ikjl}
\end{aligned}
\tag{24}
$$

and substituting the above into the pullback, we have

$$
P[G_{ab}] = \eta_{00} + \lambda^2\eta_{ij}\dot{\Phi}^i\dot{\Phi}^j - \frac{\lambda^2}{2}R_{0k0l}\Phi^k\Phi^l + \frac{\lambda^4}{2}R_{ikjl}\Phi^k\Phi^l\dot{\Phi}^i\dot{\Phi}^j \tag{25}
$$

Since the square roots in the Lagrangian (17) cannot be evaluated directly, they must be expanded in a Taylor series, where we will expand all terms to order λ^4. Using the expansion

$$
(1+\epsilon)^{1/2} \approx 1 + \frac{1}{2}\epsilon - \frac{1}{8}\epsilon^2
$$

we have

$$
\begin{aligned}
(-P[G_{ab}])^{1/2} &\approx \left(1 - \lambda^2(\dot{\Phi}^i)^2 + \frac{\lambda^2}{2}R_{0k0l}\Phi^k\Phi^l + \frac{\lambda^4}{6}R_{ikjl}\Phi^k\Phi^l\dot{\Phi}^i\dot{\Phi}^j\right)^{1/2} \\
&= 1 - \frac{\lambda^2}{2}(\dot{\Phi}^i)^2 + \frac{\lambda^2}{4}R_{0k0l}\Phi^k\Phi^l + \frac{\lambda^4}{12}R_{ikjl}\Phi^k\Phi^l\dot{\Phi}^i\dot{\Phi}^j) \\
&\quad -\frac{1}{8}\left(-\lambda^2(\dot{\Phi}^i)^2 + \frac{\lambda^2}{2}R_{0k0l}\Phi^k\Phi^l\right)\left(-\lambda^2(\dot{\Phi}^k)^2 + \frac{\lambda^2}{2}R_{0i0j}\Phi^i\Phi^j\right) \\
&= 1 - \frac{\lambda^2}{2}(\dot{\Phi}^i)^2 + \frac{\lambda^2}{4}R_{0k0l}\Phi^k\Phi^l + \frac{\lambda^4}{12}R_{ikjl}\Phi^k\Phi^l\dot{\Phi}^i\dot{\Phi}^j - \frac{\lambda^4}{8}(\dot{\Phi}^i)^2(\dot{\Phi}^k)^2 \\
&\quad +\frac{\lambda^4}{8}R_{0k0l}\Phi^k\Phi^l(\dot{\Phi}^i)^2 - \frac{\lambda^4}{32}R_{0k0l}\Phi^k\Phi^l R_{0i0j}\Phi^i\Phi^j + O(\lambda^6)
\end{aligned}
\tag{26}
$$

Next, to evaluate $(det(Q^i{}_j))^{1/2}$, we first expand out the metric in RNC, using (20) and (21)

$$
\begin{aligned}
Q^i_{\ j} &= \delta^i_{\ j} + i\lambda[\Phi^i, \Phi^k]G_{kj} \\
&= \delta^i_{\ j} + i\lambda[\Phi^i, \Phi^j] + i\frac{\lambda^3}{2}[\Phi^i, \Phi^k]G_{kj,lm}\Phi^l\Phi^m \\
&= \delta^i_{\ j} + i\lambda[\Phi^i, \Phi^j] - i\frac{\lambda^3}{6}[\Phi^i, \Phi^k]R_{kljm}\Phi^l\Phi^m \qquad (27)
\end{aligned}
$$

Making use of the identity

$$
(detA)^{1/2} = e^{\frac{1}{2}Tr(lnA)}
$$

as well as the expansions

$$
ln[(\delta + \lambda A)^{ij}] \approx \lambda A^{ij} - \frac{\lambda^2}{2}A^{ik}A^{kj} + \frac{\lambda^3}{3}A^{ik}A^{kn}A^{nj} - \frac{\lambda^4}{4}A^{ik}A^{kn}A^{np}A^{pj}
$$

$$
e^{\lambda A^{ij}} \approx \delta^{ij} + \lambda A^{ij} + \frac{\lambda^2}{2!}A^{ik}A^{kj}
$$

we see

$$
\begin{aligned}
ln(Q^{ij}) \approx\ & i\lambda[\Phi^i, \Phi^j] - i\frac{\lambda^3}{6}[\Phi^i, \Phi^k]R_{kljm}\Phi^l\Phi^m \\
& -\frac{1}{2}\left(i\lambda[\Phi^i, \Phi^n] - i\frac{\lambda^3}{6}[\Phi^i, \Phi^k]R_{klnm}\Phi^l\Phi^m\right)\left(i\lambda[\Phi^n, \Phi^j] - i\frac{\lambda^3}{6}[\Phi^n, \Phi^k]R_{kljm}\Phi^l\Phi^m\right) \\
& -i\frac{\lambda^3}{3}[\Phi^i, \Phi^n][\Phi^n, \Phi^p][\Phi^p, \Phi^j] - \frac{\lambda^4}{4}[\Phi^i, \Phi^n][\Phi^n, \Phi^p][\Phi^p, \Phi^q][\Phi^q, \Phi^j] \qquad (28)
\end{aligned}
$$

Plugging the above expansion into the exponent yields

$$
\begin{aligned}
(det(Q^{ij}))^{1/2} \approx\ & exp\left\{\frac{1}{2}Tr\left(i\lambda[\Phi^i, \Phi^j] - i\ \frac{\lambda^3}{6}[\Phi^i, \Phi^k]R_{kljm}\Phi^l\Phi^m + \frac{\lambda^2}{2}[\Phi^i, \Phi^n][\Phi^n, \Phi^j]\right.\right. \\
& -\frac{\lambda^4}{6}[\Phi^i, \Phi^n][\Phi^n, \Phi^k]\Phi^l\Phi^m R_{kljm} - i\frac{\lambda^3}{3}[\Phi^i, \Phi^n][\Phi^n, \Phi^p][\Phi^p, \Phi^j] \\
& \left.\left.-\frac{\lambda^4}{4}[\Phi^i, \Phi^n][\Phi^n, \Phi^p][\Phi^p, \Phi^q][\Phi^q, \Phi^j]\right)\right\} \qquad (29)
\end{aligned}
$$

Next, applying the trace will cause all tensor indices will be summed over, $A^{ij} \to A^{ii}$, which will cause the first two terms to drop, since they are antisymmetric, leaving

$$
\begin{aligned}
(det(Q^{ij}))^{1/2} \approx\ & exp\left\{\frac{\lambda^2}{4}[\Phi^i, \Phi^n][\Phi^n, \Phi^i] - \frac{\lambda^4}{12}[\Phi^i, \Phi^n][\Phi^n, \Phi^k]\Phi^l\Phi^m R_{klim}\right. \\
& \left.-i\frac{\lambda^3}{6}[\Phi^i, \Phi^n][\Phi^n, \Phi^p][\Phi^p, \Phi^i] - \frac{\lambda^4}{8}[\Phi^i, \Phi^n][\Phi^n, \Phi^p][\Phi^p, \Phi^q][\Phi^q, \Phi^i]\right\} \qquad (30)
\end{aligned}
$$

Finally, expanding out the exponential, we have

$$
\begin{aligned}
(det(Q^{ij}))^{1/2} \;\approx\; & 1+\frac{\lambda^2}{4}[\Phi^i,\Phi^n][\Phi^n,\Phi^i]-i\frac{\lambda^3}{6}[\Phi^i,\Phi^n][\Phi^n,\Phi^p][\Phi^p,\Phi^i] \\
& -\frac{\lambda^4}{12}[\Phi^i,\Phi^n][\Phi^n,\Phi^k]\Phi^l\Phi^m R_{klim}-\frac{\lambda^4}{8}[\Phi^i,\Phi^n][\Phi^n,\Phi^p][\Phi^p,\Phi^q][\Phi^q,\Phi^i] \\
& +\frac{\lambda^4}{32}\left([\Phi^i,\Phi^n][\Phi^n,\Phi^i]\right)^2
\end{aligned} \tag{31}
$$

Multiplying (26) and (31) and keeping terms to order λ^4 yields the fully expanded Lagrangian. Noting that the λ^3 term in (31) is antisymmetric and thus cancels out via the symmetrized trace in (17), the Lagrangian is thus given by

$$
\begin{aligned}
L \;=\; & -T_0 Str\Big\{1+\lambda^2\left(-\frac{1}{2}(\dot{\Phi}^i)^2+\frac{1}{4}R_{0k0l}\Phi^k\Phi^l-\frac{1}{4}[\Phi^i,\Phi^j]^2\right) \\
& +\lambda^4\Big(\frac{1}{8}(\dot{\Phi}^i)^2[\Phi^n,\Phi^p]^2-\frac{1}{16}R_{0k0l}\Phi^k\Phi^l[\Phi^n,\Phi^p]^2-\frac{1}{8}(\dot{\Phi}^i)^2(\dot{\Phi}^j)^2+\frac{1}{8}R_{0k0l}\Phi^k\Phi^l(\dot{\Phi}^i)^2 \\
& +\frac{1}{12}R_{ijkl}\Phi^k\Phi^l\dot{\Phi}^i\dot{\Phi}^j-\frac{1}{32}R_{0k0l}R_{0i0j}\Phi^k\Phi^l\Phi^i\Phi^j-\frac{1}{8}[\Phi^i,\Phi^n][\Phi^n,\Phi^p][\Phi^p,\Phi^q][\Phi^q,\Phi^i] \\
& -\frac{1}{12}R_{klim}[\Phi^i,\Phi^n][\Phi^n,\Phi^k]\Phi^l\Phi^m+\frac{1}{32}\left([\Phi^i,\Phi^n][\Phi^n,\Phi^i]\right)^2\Big)\Big\}
\end{aligned} \tag{32}
$$

We will assume that the configuration of D0-branes has small velocity, $\dot{\Phi}^i << \Phi^i$, and that the background gravitational field is weak, yielding small curvature, $R << 1$. Thus, the $-\frac{1}{8}(\dot{\Phi}^i)^2(\dot{\Phi}^j)^2$ and $-\frac{1}{32}R_{0k0l}R_{0i0j}\Phi^k\Phi^l\Phi^i\Phi^j$ terms can be dropped, leaving our final expression for the Lagrangian

$$
\begin{aligned}
L \;=\; & -T_0 Str\Big\{1+\lambda^2\left(-\frac{1}{2}(\dot{\Phi}^i)^2+\frac{1}{4}R_{0k0l}\Phi^k\Phi^l-\frac{1}{4}[\Phi^i,\Phi^j]^2\right) \\
& +\lambda^4\Big(\frac{1}{8}(\dot{\Phi}^i)^2[\Phi^n,\Phi^p]^2-\frac{1}{16}R_{0k0l}\Phi^k\Phi^l[\Phi^n,\Phi^p]^2+\frac{1}{8}R_{0k0l}\Phi^k\Phi^l(\dot{\Phi}^i)^2 \\
& +\frac{1}{12}R_{ijkl}\Phi^k\Phi^l\dot{\Phi}^i\dot{\Phi}^j-\frac{1}{8}[\Phi^i,\Phi^n][\Phi^n,\Phi^p][\Phi^p,\Phi^q][\Phi^q,\Phi^i] \\
& -\frac{1}{12}R_{klim}[\Phi^i,\Phi^n][\Phi^n,\Phi^k]\Phi^l\Phi^m+\frac{1}{32}\left([\Phi^i,\Phi^n][\Phi^n,\Phi^i]\right)^2\Big)\Big\}
\end{aligned} \tag{33}
$$

However, we are still left with the question of how to evaluate the symmetrized trace of the coordinate matrices, what to make of the commutation relations intrinsic to the action, and how to apply time-dependence to the solution. To proceed, we must make an ansatz as to the algebra that these coordinate matrices of D0-branes satisfy.

4 The SU(2) Algebra Ansatz

4.1 Applying the Algebra

We note that curvature arising from Schwarzschild (black hole) and FRW (cosmology) metrics will be maximally (spherically) symmetric, and thus in searching for a time-dependent solution to our action, we are led to a maximally symmetric algebra, namely SU(N). For computational simplicity, we will look to the SU(2) modes of the NxN Φ^i matrices, and thus we start with the ansatz

$$\Phi^i = a^i(t)\alpha^i \tag{34}$$

where $a^i(t)$ represent the time-dependent size of the configuration of D0-branes in the ith direction, and α^i are the generators of SU(2) (Pauli) algebra in an N-dimensional representation, satisfying the relation

$$[\alpha^i, \alpha^j] = 2i\epsilon^{ijk}\alpha^k \tag{35}$$

Despite the simplicity that this ansatz introduces, imposing dependence of direction on size still proves computationally intractable in solving for the dynamics. Thus we make the further ansatz of maximal symmetry in directionality, so the D0-branes puff out in a spherical configuration. We thus apply the simplified ansatz

$$\Phi^i = a(t)\alpha^i \tag{36}$$

Next, we assume that the gravitational source is a space of constant curvature, or maximally symmetric. Such symmetry is inherent to FRW cosmological models, and can be applied, for example, to a charged, rotating black hole. For such spaces, the curvature tensor obeys the relation [7]

$$R_{abcd} = \frac{R_0}{D(D-1)}(g_{ac}g_{bd} - g_{ad}g_{bc}) \tag{37}$$

where D is equal to the number of spacetime dimensions. Recall that we have moved into conventional spacetime territory with $D = 3+1$, under the assumption that the additional 6 dimensions are sufficiently compactified. Applying this relationship to the R_{0k0l} terms in the Lagrangian (33), we have

$$\begin{aligned} R_{0k0l} &= \frac{R_0}{12}(g_{00}g_{kl} - g_{0l}g_{k0}) \\ &= \frac{R_0}{12}(\eta_{00}\eta_{kl} - \eta_{0l}\eta_{k0}) \\ &= -\frac{R_0}{12}\eta_{kl} \end{aligned} \tag{38}$$

where $g_{ab} \approx \eta_{ab}$ because we have assumed small curvature, and because we are in 3-D space, $k, l = 1, 2, 3$, thus the η_{0l} terms vanish.

Rewriting the Lagrangian (33)

$$L = -T_0 Str\left\{1 + A\right\} = -T_0\left\{N + Str(A)\right\} \tag{39}$$

where for further computational simplicity, A is defined as

$$\begin{aligned} A \;\equiv\; & \lambda^2\left(-\frac{1}{2}\dot{a}^2(\alpha^i)^2 + \frac{1}{4}R_{0k0l}a^2\alpha^k\alpha^l - \frac{1}{4}a^4[\alpha^i, \alpha^j]^2\right) \\ & +\lambda^4\left(\frac{1}{8}\dot{a}^2a^4(\alpha^i)^2[\alpha^n, \alpha^p]^2 - \frac{1}{16}R_{0k0l}a^6\alpha^k\alpha^l[\alpha^n, \alpha^p]^2 + \frac{1}{8}R_{0k0l}\dot{a}^2a^2\alpha^k\alpha^l(\alpha^i)^2\right. \\ & +\frac{1}{12}R_{ijkl}\dot{a}^2a^2\alpha^k\alpha^l\alpha^i\alpha^j - \frac{1}{8}a^8[\alpha^i, \alpha^n][\alpha^n, \alpha^p][\alpha^p, \alpha^q][\alpha^q, \alpha^i] \\ & \left.-\frac{1}{12}R_{klim}a^6[\alpha^i, \alpha^n][\alpha^n, \alpha^k]\alpha^l\alpha^m + \frac{1}{32}a^8\left([\alpha^i, \alpha^n][\alpha^n, \alpha^i]\right)^2\right) \end{aligned} \tag{40}$$

Note that in (39) the trace over the identity simply yields the number of D0-branes, N. Plugging in the curvature tensor for spaces of constant curvature and the SU(2) algebra commutation relation yields

$$\begin{aligned} A \;=\; & \lambda^2\left(-\frac{1}{2}\dot{a}^2(\alpha^i)^2 - \frac{R_0}{48}a^2\eta_{kl}\alpha^k\alpha^l - \frac{1}{4}a^4\left(2i\epsilon^{ijk}\alpha^k\right)^2\right) \\ & +\lambda^4\left(\frac{1}{8}\dot{a}^2a^4(\alpha^i)^2\left(2i\epsilon^{ljk}\alpha^k\right)^2 + \frac{R_0}{192}a^6\eta_{kl}\alpha^k\alpha^l\left(2i\epsilon^{npr}\alpha^r\right)^2 - \frac{R_0}{96}\dot{a}^2a^2\eta_{kl}\alpha^k\alpha^l(\alpha^i)^2\right. \\ & +\frac{R_0}{144}\dot{a}^2a^2(\eta_{ik}\eta_{jl} - \eta_{il}\eta_{jk})\alpha^k\alpha^l\alpha^i\alpha^j - \frac{1}{8}a^8(2i)^4\epsilon^{ins}\epsilon^{npb}\epsilon^{pqr}\epsilon^{qit}\alpha^s\alpha^b\alpha^r\alpha^t \\ & \left.-\frac{R_0}{144}a^6(2i)^2(\eta_{ik}\eta_{lm} - \eta_{il}\eta_{mk})\epsilon^{ina}\epsilon^{nkb}\alpha^a\alpha^b\alpha^l\alpha^m + \frac{1}{32}a^8\left(2i\epsilon^{ijk}\alpha^k\right)^4\right) \end{aligned} \tag{41}$$

We next make use of the properties that the products of the totally antisymmetric tensor obey [7]

$$\begin{aligned} \epsilon_{abc}\epsilon^{pqn} \;=\; & \delta_a^p\delta_b^q\delta_c^n + \delta_a^q\delta_b^n\delta_c^p + \delta_a^n\delta_b^p\delta_c^q \\ & -\delta_a^p\delta_b^n\delta_c^q - \delta_a^n\delta_b^q\delta_c^p - \delta_a^q\delta_b^p\delta_c^n \end{aligned} \tag{42}$$

$$\epsilon_{abc}\epsilon^{aqn} = \delta_b^q\delta_c^n - \delta_b^n\delta_c^q \tag{43}$$

$$\epsilon_{abc}\epsilon^{abn} = 2\delta_c^n \tag{44}$$

$$\epsilon_{abc}\epsilon^{abc} = 6 \tag{45}$$

Thus, from (44) we have

$$(\epsilon^{ijk}\alpha^k)^2 = \epsilon^{ijk}\epsilon^{ijl}\alpha^k\alpha^l = 2\delta_{kl}\alpha^k\alpha^l = (\alpha^k)^2 \tag{46}$$

and from (43)

$$(\epsilon_{nsi}\epsilon^{npb})(\epsilon_{qrp}\epsilon^{qit}) = (\delta^p_s\delta^b_i - \delta^b_s\delta^p_i)(\delta^i_r\delta^t_p - \delta^t_r\delta^i_p) = \delta^t_s\delta^b_r + \delta^b_s\delta^t_r \tag{47}$$

Substituting these identities into the Lagrangian, noting that the $\frac{R_0}{144}\dot{a}^2a^2(\eta_{ik}\eta_{jl}-\eta_{il}\eta_{jk})\alpha^k\alpha^l\alpha^i\alpha^j$ term will cancel from the symmetrized trace in (39), we have

$$\begin{aligned} A &= \lambda^2\left(-\frac{1}{2}\dot{a}^2(\alpha^i)^2 - \frac{R_0}{48}a^2(\alpha^k)^2 + 2a^4(\alpha^k)^2\right) \\ &\quad +\lambda^4\left(-\dot{a}^2a^4(\alpha^i)^2(\alpha^k)^2 - \frac{R_0}{24}a^6(\alpha^k)^2(\alpha^r)^2 - \frac{R_0}{96}\dot{a}^2a^2(\alpha^k)^2(\alpha^i)^2\right. \\ &\quad \left. -4a^8(\alpha^r)^2(\alpha^t)^2 + \frac{R_0}{18}a^6(\alpha^b)^2(\alpha^m)^2 + 2a^8(\alpha^i)^2(\alpha^k)^2\right) \end{aligned} \tag{48}$$

4.2 Evaluating the Symmetrized Trace

Now we must now find a way to evaluate the symmetrized trace of $\alpha^i\alpha^i$. We use the result of Ramgoolam *et al.* [3], who have shown through knot theory that, for the ansatz $\Phi^i = a\alpha^i$ satisfying the SU(2) algebra, one can replace the symmetrized trace of $\alpha^i\alpha^i$ by the power series

$$Str(\alpha^i\alpha^i)^n = N\sum_{i=1}^{n-1} k^iC^{(n-i)} \tag{49}$$

where k^i is a constant and $C = (N^2-1)$. Having solved for the constants k^i, they find

$$Str(\alpha^i\alpha^i)^n = N\left(C^n - \frac{2}{3}n(n-1)C^{(n-1)} + \frac{2}{45}n(n-1)(n-2)(7n-1)C^{(n-2)} + ...\right) \tag{50}$$

Since our Lagrangian contains terms of highest order $n = 2$, the above simplifies to

$$Str(\alpha^i\alpha^i)^n = N\left((N^2-1)^n - \frac{2}{3}n(n-1)(N^2-1)^{(n-1)}\right) \tag{51}$$

and we thus make the following definitions

$$Str\left\{\alpha^i\alpha^i\right\} = N(N^2-1) \equiv F_1 \tag{52}$$

$$Str\left\{\alpha^i\alpha^i\alpha^k\alpha^k\right\} = N\left((N^2-1)^2 - \frac{4}{3}(N^2-1)\right) \equiv F_2 \tag{53}$$

Substituting this result into (48), and (48) into (39) yields the desired scalar expression for the Lagrangian

$$\begin{aligned} L &= T_0\Big(-N + \frac{\lambda^2}{2}\dot{a}^2F_2 + \frac{\lambda^2}{48}R_0a^2F_2 - 2\lambda^2a^4F_2 \\ &\quad + \lambda^4\dot{a}^2a^4F_4 + \frac{\lambda^4}{96}R_0\dot{a}^2a^2F_4 + 2\lambda^4a^8F_4 + \frac{7}{72}\lambda^4R_0a^6F_4\Big) \end{aligned} \tag{54}$$

For large N, $F_2 \approx N^3$ and $F_4 \approx N^5$ from (51), and we have the final expression for the Lagrangian,

$$\begin{aligned} L &= T_0\Big[\dot{a}^2\left(\frac{\lambda^2}{2}N^3 + \lambda^4a^4N^5 + \frac{\lambda^4}{96}R_0a^2N^5\right) \\ &\quad -N + \frac{\lambda^2}{48}R_0a^2N^3 - 2\lambda^2a^4N^3 + +2\lambda^4a^8N^5 + \frac{7}{72}\lambda^4R_0a^6N^5\Big] \end{aligned} \tag{55}$$

A most interesting feature of this Lagrangian is the radius-dependent mass term, indicating that the mass of the collection of D0-branes increases with the size of the fuzzy sphere. This result is expected, since given a fixed string tension between the branes, the energy of the collection, and hence its mass, should increase as the branes puff apart. We now must analyze whether a fuzzy sphere is in fact a stable configuration for the D0-branes.

5 The Vacuum Solution

5.1 Finding the Potential

Rewriting (55) in the Hamiltonian formalism

$$\begin{aligned} H &= T_0 \left[\dot{a}^2 \left(\frac{\lambda^2}{2} N^3 + \lambda^4 a^4 N^5 + \frac{\lambda^4}{96} R_0 a^2 N^5 \right) \right. \\ & \left. + N - \frac{\lambda^2}{48} R_0 a^2 N^3 + 2\lambda^2 a^4 N^3 - 2\lambda^4 a^8 N^5 - \frac{7}{72} \lambda^4 R_0 a^6 N^5 \right] \end{aligned} \tag{56}$$

where $H = E$, the total energy of the configuration. Adding E into the potential term and dividing by the mass term to scale the kinetic energy, we have a modified Hamiltonian

$$\dot{a}^2 + V(a) = 0 \tag{57}$$

$V(a)$ is the effective potential and is defined as

$$V(a) = \frac{-E_0 + N - \frac{\lambda^2}{48} R_0 a^2 N^3 + 2\lambda^2 a^4 N^3 - 2\lambda^4 a^8 N^5 - \frac{7}{72} \lambda^4 R_0 a^6 N^5}{\left(\frac{\lambda^2}{2} N^3 + \lambda^4 a^4 N^5 + \frac{\lambda^4}{96} R_0 a^2 N^5 \right)} \tag{58}$$

where E_0 is the "normalized" energy

$$E_0 = \frac{E}{T_0} \tag{59}$$

The behavior of the effective potential is dependent upon the sign of the Ricci (curvature) scalar R_0. If R_0 is negative, then the only stable configuration of the D0-branes is at $a = 0$, hence they remain bound together in a point-like configuration.

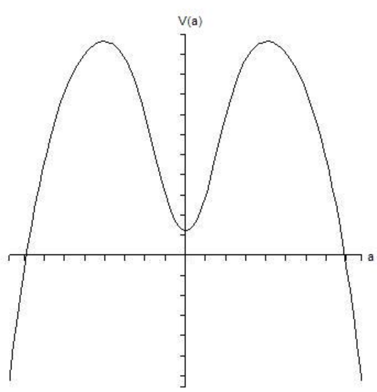

Figure 1: Normalized potential energy of spherical configuration vs. size, negative R_0

If however R_0 is positive, as expected from the Schwarzschild geometry of a black hole, the potential is similar to the Mexican hat potential, and it becomes energetically favorable for the D0-branes to puff up into a fuzzy sphere in the vacuum of the potential.

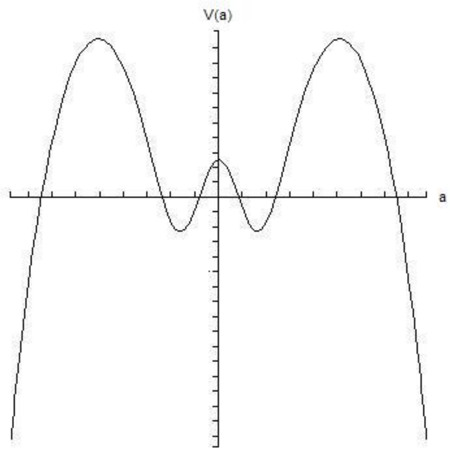

Figure 2: Normalized potential energy of spherical configuration vs. size, positive R_0

We use Maple to find the roots of the effective potential to linear order (small a), finding that the vacuum solution occurs at

$$a_v = \sqrt{\frac{R_0}{192}\left(\frac{2N}{E_0} - 1\right)} \tag{60}$$

Note however that a_v has dimensions of inverse length. The physical radius r of a spherical configuration of D0-branes is given generally in [1] by

$$r = \lambda\sqrt{\frac{Tr(\dot{\Phi}^i)^2}{N}} \tag{61}$$

Recalling that $\lambda = 2\pi l_s^2$, our configuration thus has a physical size

$$r_v = 2\pi l_s^2 \left(\frac{a_v^2 N}{N}\right)^{1/2} = 2\pi l_s^2 a_v \tag{62}$$

5.2 Limits to the Solution

The shape of the potential imposes limits on the normalized energy E_0 and the number of D0-branes N if we are to have a bound configuration at the vacuum solution. From (57), we have scaled the effective total energy to 0, and thus to ensure a stable minimum, the vacuum solution must occur for $V(a) < 0$, surrounded by regions of $V(a) > 0$. To satisfy the latter, we must have $N - E > 0$, and for the former, $N - E < \frac{N^3 R_0^2 \lambda^2}{18432}$. Combining these yields the restrictions

$$\left(1 - \frac{N^2 R_0^2 \lambda^2}{18432}\right) < \frac{E_0}{N} < 1 \tag{63}$$

In addition, the limits imposed by the initial expansion of the DBI action (26) are

$$N^2 \lambda^2 R_0 a^2 << 1 \tag{64}$$

$$N^2 \lambda^2 a^4 << 1 \tag{65}$$

and to satisfy the weak gravity/small curvature approximation (33)

$$R_0 \lambda << 1 \tag{66}$$

Combining these restrictions, we have

$$N \lambda R_0 << 1 \tag{67}$$

which, when applied to (63), means that $E_0 \approx N$. Recalling that $E_0 = \frac{E}{T_0}$, this means that the total energy of the spherical configuration of D0-branes is simply the sum of the individual brane energies

$$E = T_0 N \tag{68}$$

which appears to contradict the notion that the branes are bound since we observe no binding energy. However, this result is not new to string theory; Banks *et al.* [8] have shown that, assuming a duality between M-theory and II_A string theory, M-theory naturally allows for marginally bound states of any number of D0-branes. Also surprising is that, since the ratio $\frac{N}{E_0}$ is approximately fixed, the size of the fuzzy sphere (62) is independent of the number of D0-branes, N. This result is in seeming violation of the conjectured uncertainty principle inherent to the Planck scale [3].

6 Brief Application to de Sitter Cosmology

Having explored the behavior of D0-branes near a black hole horizon, we next turn to applying our analysis to an FRW cosmological model. The fundamental assumption we are using is that D0-branes are the building blocks of spacetime, and therefore even though their intrinsic size is on the order of the Planck length (10^{-35} m), they can be used to probe our universe as a whole. We assume a de Sitter model of the universe, in which space is homogeneous and isotropic, hence the Robertson-Walker metric is maximally symmetric, and can be expressed in the form

$$R_{abmn} = \frac{\Lambda}{3}(g_{am}g_{bn} - g_{an}g_{bm}) \tag{69}$$

which is in agreement with (37), where

$$\frac{R_0}{D(D-1)} = \frac{\Lambda}{3} \tag{70}$$

In addition, a de Sitter universe represents spacetime with a positive cosmological constant, Λ, and thus our vacuum solution (62) is still valid. Thus, for $N \approx E_0$, the size of the D0-brane fuzzy sphere r_v is approximately

$$r_v \approx \lambda\sqrt{\Lambda} \tag{71}$$

Note the curious result that the cosmological constant is scaling the size of the D0-brane configuration. We expect that the the D0-branes are naturally of size on order of the Planck length, yet they are dynamically affected by Λ, which fundamentally sets the scale and dynamics of the universe as a whole. This provocative concept of the mixing of energy scales has been proposed previously [10]. Since we are modeling the universe as a collection of D0-branes (or as one large Dp-brane), string theory alludes to the idea that the dynamics on the largest scale are fundamentally linked to those on the smallest scale. Future work in the field of string cosmology will hopefully shed more light on this intriguing link.

7 Discussion

We have found the vacuum solution to the potential of a spherical configuration of D0-branes near the horizon of a black hole. For positive curvature, the shape of the potential indicates that a possibly stable configuration of D0-branes of non-trivial size is possible, providing evidence that the D0-branes will puff up via the gravitational dielectric effect. However, while we found a stable vacuum solution, a_v, the stability is limited to the SU(2) modes (Pauli matrices) of the NxN coordinate matrices Φ^i. As a critical next step, we need to look at all modes not in the direction of the Pauli matrices. This is, however, a non-trivial computation; since Φ^i is NxN, we would need to analyze N^2 total perturbations. However, this analysis is vital, since if the non-Pauli modes are unstable, so is our vacuum solution. One can imagine this by extrapolating Fig. 2 into 3 dimensions, in which the potential minimum a_v becomes a saddle point. If our vacuum solution is in fact unstable, we have made little progress beyond [2].

One may ask what is the physical mechanism that causes the D0-branes to puff up. The puffing up specifically near the horizon suggests a fundamental limit to the number of branes that can exist in some patch of space, and that at the horizon represents a region of saturation of D0-branes, hence we would expect a dynamical bounce of the D0-branes off the horizon. This would indicate, contrary to General Relativity, that horizons represent more than just mere mathematical constructs beyond which escape from the gravitational potential of the singularity is not possible; they are an actual physical surface at which interesting dynamics can occur. More specifically, the horizon represents a physical boundary that the D0-branes cannot cross. This conjecture is supported by [3], which analyzed the dynamics of N D0-branes in flat space, finding that for finitely large N, a spherically collapsing configuration of D0-branes will bounce back. If this suggestion is true, one may ask what is the fundamental limit to the number of D0-branes in a given space. Unfortunately, though, our resulting lack of strong dependence of D0-brane spherical size on number of branes N (since always $N \approx E_0$, thus $a_v \sim \sqrt{R_0}$) appears to violate this suggestion.

To investigate this idea further, we would like to study the dependence of D0-brane configuration size vs. distance from horizon, which would shed light on the specific dynamics as the D-branes near the horizon. Ideally, this would be done through studying the equations of motion from the Lagrangian (55). Given its complexity, though, this may prove intractable, and we may have to resort to numerical analysis. Furthermore, to simplify the computations, we have assumed a spherical configuration of the D0-branes (36). However, an ellipsoidal geometry would likely be more realistic, in which the size in the direction normal to the horizon would differ from those in the other directions, as in [2]. We could then find a correlation between radial size of the ellipsoid and distance from horizon via the effects of relativistic tidal forces. While such an approach would be less direct then solving for the equations of motion, it may prove more computationally feasible.

Another direction for future work would be to generalize our result to any black hole solution. While the Robertson-Walker metric of cosmology is naturally a maximally symmetric space, the Schwarzschild geometry is not inherently so, and we had to assume a rotating, charged black hole with Ramond-Ramond fluxes on the D0 branes to simplify the geometry. Ideally, we would substitute into the Lagrangian (33) the individually curvature tensors R_{abcd} of the Schwarzschild metric, then solve for the dynamics. This approach may be made easier by the indication from symmetry that only the $R_{r\theta r\theta}$ component would be non-zero.

Finally, we would like to expand upon our analysis for an FRW cosmological model. Specifically, since the cosmology represents an expanding universe, we would like to use the time-dependence of our Lagrangian to study the dynamics of the D0-branes under cosmological expansion. Particularly interesting would be further study of the suggestions of mixing of energy between cosmological and Planck scales.

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