PION-NUCLEON SCATTERING AND PION-PION INTERACTIONS

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We examine what information can be obtained about the $T = 0$ and $T = 1$ $\pi - \pi$ interactions by using the dispersion relations for the s- and p-wave $\pi - N$ partial waves. These partial waves have the singularities shown in Fig. 1 where

$$s = [(M^2 + q^2)^{\frac{1}{2}} + (\mu^2 + q^2)^{\frac{1}{2}}]^2$$

is the square of the energy in the c.m. system $\pi + N$. The real part of the amplitudes and the contributions from the cuts $(M+\mu)^2 \leqq s \leqq \infty$ and $0 \leqq s \leqq (M-\mu)^2$ are evaluated from the known $\pi - N$ phase shifts [1]. The method of calculation has been discussed in detail elsewhere [2].

The dispersion relations are evaluated for $(M+\mu)^2 = 59.6 \leqq s \leqq 80$ (i.e. 0 to 210 MeV pion lab. energy) on the physical region (we use the units $\hbar = c = \mu = 1$). They are also evaluated for $20 \leqq s \leqq (M-\mu)^2 = 32.7$ on the "crossed cut", by using the crossing theorem. The evaluation on the crossed cut, as well as the physical cut, is a great advantage of our method, and it is the main fact which makes it possible to separate out the effect of the $\pi - \pi$ interactions.

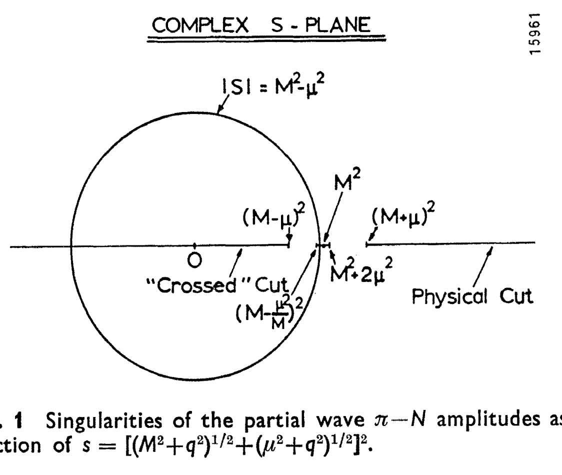

Fig. 1 Singularities of the partial wave $\pi - N$ amplitudes as a function of $s = [(M^2+q^2)^{1/2} + (\mu^2+q^2)^{1/2}]^2$.

This analysis yields experimental values of the "discrepancies" $\Delta(s)$. These give the sum of the contributions to the partial wave amplitudes coming from the circle $|s| = M^2 - \mu^2$ and the cut $-\infty \leqq s \leqq 0$. These have the following physical interpretation:

(i) the discontinuity across the circle $|s| = M^2 - \mu^2$ is given by the absorptive part of the amplitude for $\pi + \pi \to N + \bar{N}$, and by unitarity this is related to $\pi + \pi \to \pi + \pi$. Low energy $\pi - \pi$ interactions contribute to the front of the circle (i.e., the portion near $s = M^2 - \mu^2$) and give a term in $\Delta(s)$ which may vary rapidly with s over the range $20 \leqq s \leqq 80$. Thus the long range effect due to low energy $\pi - \pi$ interactions should show up strongly in low energy $\pi - N$ scattering.

(ii) the cut $-\infty \leqq s \leqq 0$ gives contributions to the $\pi - N$ partial wave amplitudes which vary slowly with s over $20 \leqq s \leqq 80$. We can regard this part of $\Delta(s)$ as being due to the short range ($\lesssim 0.3 \; 10^{-14}$ cm) interactions in $\pi - N$ scattering.

The marked difference in the behaviour of these two parts of the functions $\Delta(s)$ enables us to separate out the effect of the $\pi - \pi$ interactions.

RESULTS

(a) The low energy $\pi - \pi$ effects should give some characteristic features for $\Delta(s)$. For example, $\Delta_0^{(+)}(s)$ and $\Delta_0^{(-)}(s)$ relate to the s-wave $\pi - N$ amplitudes, and $(+)$ and $(-)$ are charge combinations corresponding to the $T = 0$ and $T = 1$ $\pi - \pi$ interactions respectively. If the low energy $T = 0$ $\pi - \pi$ interaction is important we would expect $\Delta_0^{(+)}(s)$ to be a hump shaped curve with the centre of the hump near $s = M^2 = 46$. The experimental values shown in Fig. 2 show this behaviour. Also if the T = 1 $\pi-\pi$ interaction is important $\Delta_0^{(-)}(s)$ should be fairly constant in the low energy physical region $s \geqq (M+\mu)^2$, and it should change abruptly on going over to the crossed physical region $s \leqq (M-\mu)^2 = 32.7$. The experimental values of $\Delta_0^{(-)}(s)$ do show this behaviour.

(b) Using Menotti's results [3] for the $T = 0$ $J = 0$ $\pi+\pi \to N+\bar{N}$ amplitude, we can compute the effects to be expected for various forms of the $T = 0$ $J = 0$ $\pi-\pi$ phase shift δ_0^0. Figs. 3 and 4 show that reasonably good agreement with the s, $p_{1/2}$ and $p_{3/2}$ $\pi-N$ data is obtained by using a N/D form for the $T = 0$ $J = 0$ $\pi-\pi$ amplitude on using a single pole for N. Fig. 5 shows the effective range plot for δ_0^0 corresponding to these solutions. The phase shift δ_0^0 is attractive. It rises to a maximum around 25° or 30° for $t = (\pi-\pi \text{ energy})^2$ in the region 5-7, and falls off for higher $\pi-\pi$ energies. A resonance in the $T = 0$ $J = 0$ $\pi-\pi$ amplitude at low energies is ruled out by the p-wave π-N data.

(c) Our solutions for δ_0^0 can be compared with recent [4] solutions of the Chew-Mandelstam equations for $\pi-\pi$ scattering. Some of the latter are shown in Fig. 5. From this comparison we deduce the values $a_0 = 1.3 \pm 0.4$, $\lambda = -0.18 \pm 0.05$ for the scattering length a_0 and $\pi-\pi$ coupling parameter λ.

(d) *Other information on* δ_0^0. Recent work by Jacob *et al.*[5] shows that the results of the $p+d$ experiment [6] are in agreement with a $T = 0$ $J = 0$

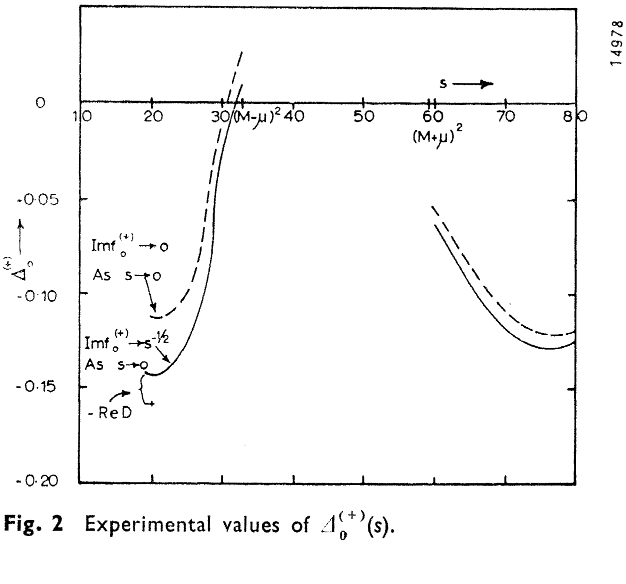

Fig. 2 Experimental values of $\Delta_0^{(+)}(s)$.

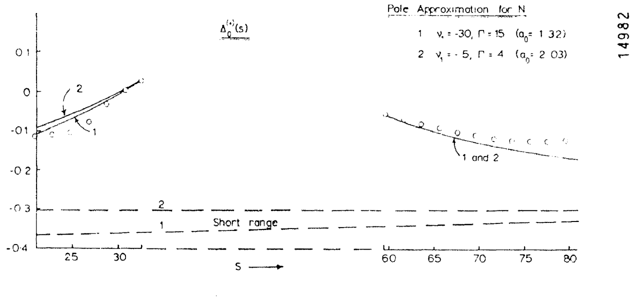

Fig. 3 Calculated values of $\Delta_0^{(+)}(s)$ for two N/D pole solutions for the $T = 0$ $J = 0$ $\pi-\pi$ amplitude. Experimental values are shown by open circles.

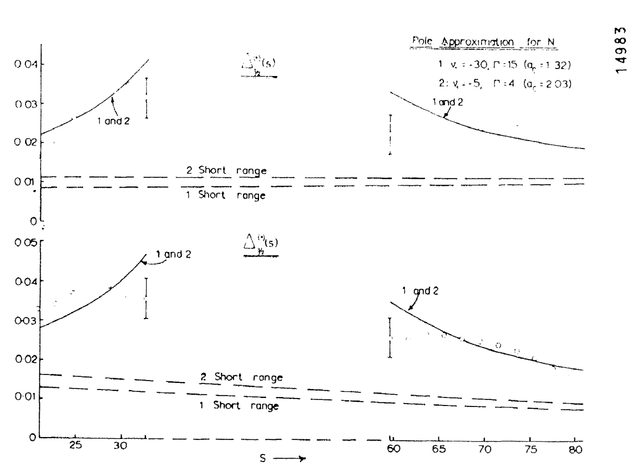

Fig. 4 Comparison of calculated and experimental values (open circles) for the functions $\Delta_{3/2}^{(+)}(s)$ and $\Delta_{1/2}^{(+)}(s)$ relating to the $p_{3/2}$ and $p_{1/2}$ $\pi-N$ states.

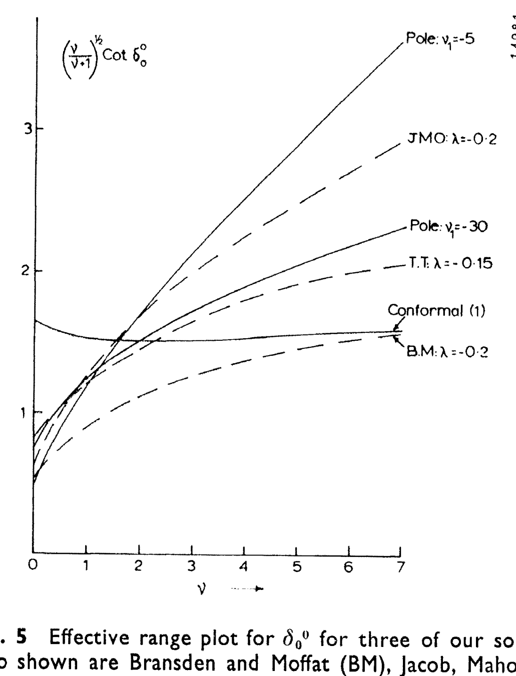

Fig. 5 Effective range plot for δ_0^0 for three of our solutions. Also shown are Bransden and Moffat (BM), Jacob, Mahoux and Omnès (JMO), and Taylor and Truong (TT) solutions of the Chew-Mandelstam equations.

$\pi-\pi$ low energy attraction as given by the solution (JMO)[5], $\lambda = -0.20$ (Fig. 5). This is in good agreement with the results of our $\pi-N$ analysis. Recent work[7] on the analysis of τ-decay events show that a positive value of a_0 is now allowed, and the value $a_0 = 1.3 \pm 0.4$ appears to be in agreement with the τ-decay data. The experiments on $\pi+N \rightarrow \pi+\pi+N$ at low energy are difficult to analyse theoretically in an accurate way. (Work to date has suggested $a_0 < 1.0$.)

(e) We conclude that the $T=0$ $J=0$ $\pi-\pi$ interaction is fairly strong and attractive at low energies. It plays a very important role in s-wave π-N scattering and also gives noticeable effects in p-wave π-N scattering. The behaviour which we deduce for the phase shift δ_0^0 is in agreement with other sources of information about this $\pi-\pi$ interaction.

(f) The three functions $\Delta^{(-)}(s)$ for the $s, p_{1/2}$ and $p_{3/2}$ $\pi-N$ states are reproduced well by assuming there is a narrow $T=1$ $J=1$ $\pi-\pi$ resonance (ρ) at $t_R \sim 30$ (Figs. 6 and 7). We represent the helicity amplitudes for $\pi+\pi \rightarrow N+\bar{N}$ by the δ-function approximation $C_i\delta(t-t_R)$ $(i=1, 2)$, and determine the parameter C_1 from our $\pi-N$ data. This value of C_1 is related, via the nucleon form factors, to the

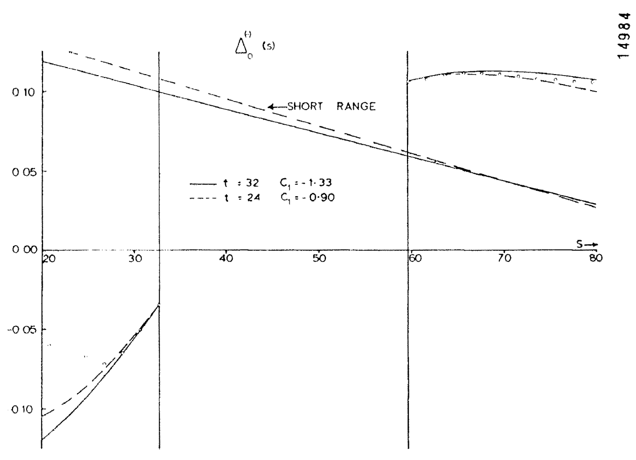

Fig. 6 Calculated and experimental values (open circles) for $\Delta_0^{(-)}(s)$ (for s-wave $\pi-N$ scattering).

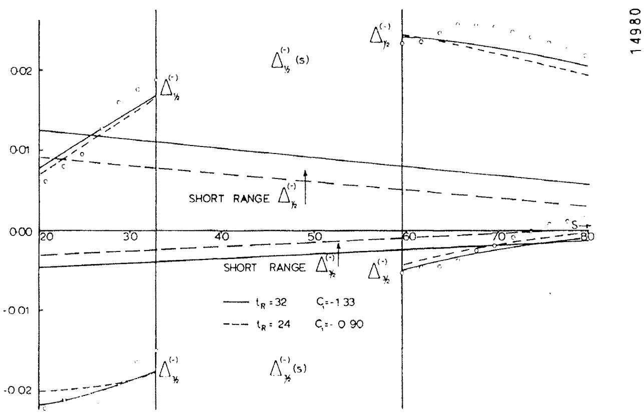

Fig. 7 Calculated and experimental values (open circles) for $\Delta_{3/2}^{(-)}(s)$ and $\Delta_{1/2}^{(-)}(s)$ referring to the $p_{3/2}$ and $p_{1/2}$ $\pi-N$ states.

width of the $\pi-\pi$ resonance ρ. Our value of C_1 is consistent with a half width of around 45 MeV. Direct calculation of C_1 from the dispersion relations for $\pi+\pi \rightarrow N+\bar{N}$ is not yet satisfactory. This is probably due to the large values of t which are important in the Omnès solution.

(g) Our analysis makes it possible to distinguish the various contributions which generate low energy s- and p-wave π-N scattering—Born term (long range part) crossed (3/2, 3/2) resonance, $\pi-\pi$ interactions, short range (or core) effect. For the s-wave case the results have been published[2]. For p-waves the $\pi-\pi$ effects are smaller, but are not negligible. We give one example. The (3/2, 3/2) amplitude is, of course, dominated by the strong long range Born attraction. The crossed (3/2, 3/2) resonance term and the $T=1$ $\pi-\pi$ effect are very small in this case. The short range (core) term is attractive and varies very little with energy. The $T=0$ $\pi-\pi$ effect is also attractive. It is twice the size of the core term at threshold and is still larger than the core term at pion lab. energy 250 MeV. We deduce that any attempt to predict the position of the (3/2, 3/2) resonance must take account of the $T=0$ $\pi-\pi$ interaction as well as the short range term.

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DISCUSSION

ZÖLLNER: I should like to point out that it is very difficult to explain the low energy pion production with a scattering length a_0 of the order 1. To which consequences for the $\pi-N$-scattering should lead an a_0-scattering length of the order 0.3 as one can get from low energy pion production?

HAMILTON: A scattering length of the order of 1 is in fact necessary to get enough attractive interaction to explain the slopes of the S-wave pion-nucleon phase-shifts as a function of momentum in the physical and crossed physical regions.

MANDELSTAM: Could not these slopes be explained by a short range force?

HAMILTON: No. The shape and height of the hump could not easily be got from short-range effects.

OMNÈS: Moreover, it is certainly impossible to get more than a very rough order of magnitude information on the pion-pion scattering length from pion production at threshold, once there is competition between the opposite terms of about the same magnitude: the peripheral interaction and direct pion production in $P_{1/2}$ state.

CINI: I wish to stress that I also consider it unreliable to attempt to deduce the pion-pion S-wave scattering length from pion production experiments at threshold. The peripheral contribution alone, evaluated with double spectral techniques by Cassandro in Rome, turns out to come almost completely from regions outside the strip, namely from regions with $t>16\mu^2$.

SAKURAI: Am I correct in saying that essentially all of the isospin dependence is due to the ϱ-meson?

HAMILTON: Almost all.

SAKURAI: So there is no isospin dependence from the short range term.

HAMILTON: This appears to be approximately true.

CINI: I would like to ask how sensitive is the value of $f^{(-)}$ at threshold to the value of the 33 pion-nucleon phase-shift at energies above resonance. This is because if one wants to obtain the parameters of the P-wave pion-pion resonance by using a more refined relativistic version of the Bowcock-Cottingham-Lurié (" B.C.L. ") work, Carrassi and Passatore have found that the high-energy tail of the dispersion integral contributes considerably to the scattering lengths at threshold. One has therefore to make a subtraction and determine the pion-pion parameters by means of the energy variation of $f^{(-)}$ rather than by its value at threshold. The value of C_1 obtained in this way turns out to be about one-third of what you have found, but subject to big variations due to the uncertainty on experimental S-wave phase-shifts.

HAMILTON: In the case of $f^{(+)}$ the contribution of the $D_{3/2}$ resonance to the left-hand cut appears in fact as more important than the tail of the $P_{3/2}$ resonance. The case of $f^{(-)}$ is less clear cut, but the situation in the partial wave method and here is more favourable than in the B.C.L. method.

ROSS: I want to ask Hamilton why Haber-Schaim obtained good results in the $f^{(-)}$ case without taking into account pion-pion interaction?

HAMILTON: If I understand the question correctly Haber-Schaim used a relation like a sum-rule. It related (a_1-a_3), f^2 and $(\sigma_- - \sigma_+)$. This is in no way in contradiction with our results. The ϱ meson contribution is hidden in the values of (a_1-a_3) and $(\sigma_- - \sigma_+)$.