# SEARCH FOR A COMPOSITION DEPENDENT FIFTH FORCE: RESULTS OF THE VALLOMBROSA EXPERIMENT

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ABSTRACT

A search for fifth–force effects has been performed on the side of a mountain, by means of a differential accelerometer consisting of a floating sphere in a stratified solution. The sensitivity of the instrument has been carefully tested with *ad hoc* measurements. From the upper limit to the terminal velocity of the sphere ($v < 10\mu m/h$), limits to the strength of the force are derived in the frame of different theoretical models, for values of the range $\lambda$ from 50 m to 10 km.

## 1. Introduction

We report here the results of a search for the "fifth force" performed on a mountain slope (at Vallombrosa, near Florence) with a floating-body accelerometer. Preliminary results of a first series of measurements have already been shown in the last Moriond workshop [1]. In the meantime, a new series of measurements has been performed with a somewhat different (and hopefully improved) set-up and – what we consider most important – all auxiliary measurements and tests, which were announced one year ago, have been completed. We are therefore in the condition to derive, from the experimental results, limits for the strength of composition dependent forces. Due to the nature of the source – a big mountain, whose composition is only approximately known – our results are subject to the common uncertainties of geophysical measurements, unless the "fifth-force charge" is roughly proportional to the mass (as in the original Fischbach's proposal [2] or in the "Cosmon model" of Peccei, Solà and Wetterich [3]); but, in this frame, they give information also in the region of ranges $\lambda > 1$ km and (with somewhat increasing uncertainty) up to 10–15 km, *i.e.* in a region not completely covered by previous experimental works.

## 2. Experimental set-up

The principle of the floating-body method for measuring the horizontal component of composition dependent forces was proposed long ago by Rózsa and Selényl[4] and independently rediscovered by several groups[5,6] in the context of the "fifth force" researches. In brief, a horizontal acceleration field, such as that of the fifth force from a mountain cliff, acting differently on the material of the floating body and on that of the surrounding liquid, would apply to the body a net unbalanced force. The body should therefore start to move and eventually reach the terminal velocity at which the viscous forces balance the external one. In the present version of the method, a solid sphere of homogeneous plastic (commercial "Nylon 12"), of radius $R$=5 cm, is floating freely *inside* a fluid (a solution of electrolyte salts in water) having approximately the same density as the solid. To stabilize the vertical position of the floating body (no parts of which, in our case, do emerge from the liquid surface), a weak gradient of density is established in the fluid due to the variable concentration of the solute, uniformly decreasing from bottom to top. This vertical gradient of density is also useful as it prevents, to some extent, the formation of thermal–convection cells[7].

With respect to other set-ups[4,5], we avoid, in this way, problems related to surface tension and all sort of surface phenomena. Moreover, it is possible to cover the liquid almost completely with a horizontal plate, in order to decouple the liquid surface from the convective movements of the atmosphere (we have actually exploited this possibility in the second series of measurements performed in the last few months). To obtain these advantages, however, one has to pay with some more uncertainties in the relation between the applied force and the terminal velocity of the sphere. We had actually some doubts [1] about the *exact* validity of Stokes' law for a sphere moving in a stratified fluid, but the loss of sensitivity due to this

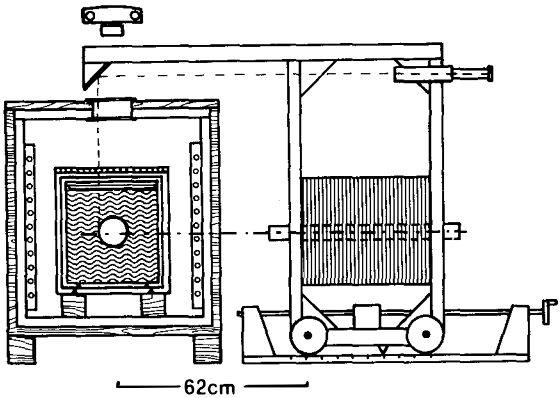


*Fig. 1: – Experimental set-up.*

cause resulted (as will be shown in §3) even worse than expected, although it was not such as to spoil the significance of the results. Similarly, the choice of having a *solid* homogeneous sphere and, therefore, a safer situation for what concerns the position of its centre of mass and the effect of gravitational gradients [6], makes it impossible to use test materials with density far from 1 $g/cm^3$.

The present experimental set up is shown schematically in fig. 1. The inner vessel has lateral walls of plexiglass and a 6 mm thick Al bottom, internally coated with a thin teflon layer. The liquid contained in the vessel is covered by a float composed of a flat plexiglass plate (6 mm thick) with emerging lateral borders, and the vessel itself is covered by a glass plate. Two shielding boxes, made of 6 mm thick Al walls and glass covers, separate the inner vessel from the outer container, whose temperature is stabilized to better than $0.1^0$C by water circulation in four heat exchangers, disposed symmetrically around the lateral walls. A copper grid, made of 19 vertical strips (1 mm thick and 10 mm high), inserted in a thick copper frame, covers the upper glass plate of the external shielding box in order to improve the temperature uniformity on the surface (this grid, as well as the plexiglass float, has been added for the second series of measurements). Pictures of the sphere and of the reference marks are taken at fixed time intervals by means of a camera placed outside the thermalized container and two flashes inserted in the outer cover. To determine the horizontal coordinates of the centre of the sphere, pictures are projected on a digitizing table connected to a computer (the apparent diameter of the sphere gives also a rough estimate of the depth).

In the first series of measurements, reported last year, a solution of KBr in water (average concentration $\approx$ 5.5% in weight) was used. This solution turns out to have a magnetic susceptibility almost exactly equal to that of Nylon 12. As a consequence, the system was

remarkably insensitive to magnetic fields. This was in some aspect an advantage, because it removed one of the possible sources of instrumental effects which could have simulated the action of a fifth force. But, having not observed any effect at all, there was no way to check the sensitivity of the system by applying to the sphere a known external force.
For the new series of measurements, a small amount of the paramagnetic salt $MnSO_4$ was added to the stratified solution. More exactly, we started from two equal quantities of a homogeneous solution* of $MnSO_4$ and added to them different quantities of KBr to obtain the densities desired for the top and the bottom layer of the stratified fluid. Then, they were mixed together in different proportions to obtain different solutions of graded density. We have slowly poured the liquid along four pipes descending along the corners to the bottom of the vessel, using four phleboclysis sets. First, the sphere was placed upon the bottom of the vessel; then, equal layers of fluid of *increasing* density have been inserted one under the others, to reach a total thickness of the fluid close to 33 cm. Finally, the plexiglass float was deposed on the liquid surface, and centered by means of teflon screws against the lateral walls. About 15 days from the preparation of the system were necessary in order to achieve a good stabilization of the temperature, and for damping at least the faster part of transient currents in the fluid.

## 3. Test of Stokes' law

Thanks to the different susceptibilities of Nylon 12 ($\kappa < 0$) and solution ($\kappa > 0$), in the presence of a non homogeneous magnetic field $B$ the floating sphere is subject to an external force corresponding to the differential acceleration field

$$\Delta \vec{a}' = \Delta\kappa\,(\vec{B}\cdot\vec{\nabla})\vec{B} \qquad [1]$$

where $\Delta\kappa$ is the difference of the mass susceptibilities of Nylon 12 and solution (which, for this purpose, are assumed to have equal density). To produce the magnetic field we used an iron-free coil, mounted as shown in fig. 1. As the differential acceleration field $\Delta a'$ at the location of the sphere is strongly dependent on the distance from the source of the field**, it was necessary, in order to maintain the force on the sphere constant, to follow its movement with an equal displacement of the coil. For this purpose, the coil was mounted on a carriage moving on rails. The position of the sphere, relative to the carriage, was monitored by means of a horizontal telescope looking at the far border of the sphere through a mirror at $45^0$ to the telescope axis. In this way, the distance of the sphere from the front of the coil ($\approx$ 67 cm) was maintained constant, within less than 2 mm, during all measurements involving the magnetic field.

* The concentration of $MnSO_4$ salt in the base solution was 1%, in weight, for the test measurements carried out in Florence and 0.9% for the new series of measurements in Vallombrosa.

** For a point body on the axis of a magnetic dipole, at a distance $r$ from it, the force would decrease as $r^{-7}$.

In principle, it would be possible to calculate the force exactly, for every given current in the coil, if $\Delta\kappa$ had been exactly known. Actually, polarization of water can alter the molar susceptibilities of $Mn^{++}$ and $SO_4^{--}$ ions with respect to the crystal form of the salt, and small ferromagnetic (or even paramagnetic) contaminations in the sphere could substantially affect the value of the force. We preferred, therefore, to measure statically the force applied to the sphere by the magnetic field of the coil itself – although, necessarily, at a smaller distance (either 19.2 cm or 15 cm from the front of the coil). The method used is shown schematically in fig. 2. The sphere was hold in the solution, somewhat higher than its equilibrium position, by means of a thin (10 $\mu$m diameter) nylon wire, 2 m long, hanging from the arm of a torsion balance, which measured the external vertical force $F_v$ acting on the sphere.

The coil was placed close to the vessel, with its axis directed horizontally and passing close to the center of the sphere. When the current in the coil was switched on, the sphere was displaced in the direction opposite to the coil, and the horizontal displacement $\Delta x$ for a given point of the sustaining wire was measured by means of a theodolite equipped with a parallel-plate micrometer, with a precision of a tenth of a mm.

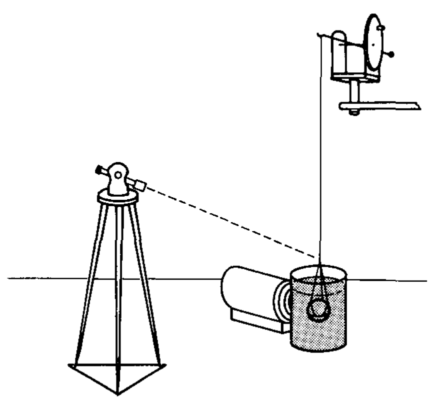

*Fig. 2: – Set-up for the static measurement of the force produced by a magnetic field.*

The horizontal force $F_h$ acting on the sphere can be deduced as

$$F_h = F_v \; \Delta x/\ell \qquad [2]$$

where $\ell$ is the length of the wire above the point whose displacement is measured. The magnetic field $\vec{B}$, its gradient $\vec{\nabla}\vec{B}$ and the product

$$(\vec{B}\cdot\vec{\nabla})\vec{B} \equiv I^2\vec{\mathcal{G}}(\mathrm{P}) \qquad [3]$$

have been calculated numerically on the basis of the geometrical shape of the coil.

As a check, the magnetic field distribution around the coil was also mapped by means of a Hall probe, and found to be in agreement with the calculated one. By using the calculated values of $\vec{\mathcal{G}}$, it is now possible to scale the value of the force due to the magnetic field, from the distance at which the static measurements have been done to that chosen for the dynamic measurements.

For this purpose, the force acting on the sphere, with its centre on the axis of the coil at a distance $x$ from its front, has been approximated as

$$\vec{F}_h(x) \equiv \rho\ \Delta\kappa\ I^2 \int_V \vec{\mathcal{G}}\ dV \approx M\ \Delta\kappa \left[\vec{\mathcal{G}}(x) + \frac{R^2}{10}\frac{\partial^2\vec{\mathcal{G}}}{\partial x^2}\right] I^2 \qquad [4]$$

(the force does not change appreciably if the sphere is not exactly on the axis of the coil).

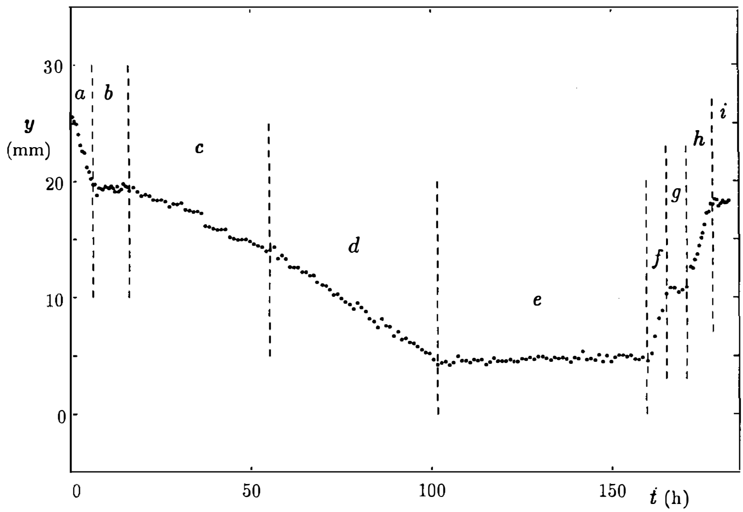


*Fig. 3: – Results of part of measurements with the magnetic field. The coordinate of the centre of the sphere, in the direction of the axis of the coil, is shown as a function of time, with different values of the current: I=+5A (region a), I=+2A (c), I=−2A (d), I=−5A (f), I=+5A (h). The current was zero in regions b, e, g, i. During the time intervals f and h, the coil was placed at the opposite side of the vessel, in order to reverse the direction of the applied force.*

The measured static force turned out to be consistent with the value one should obtain if the susceptibility of the solution would correspond to the weighted mean of those of the components. In order to test Stokes' law in the velocity range relevant for fifth–force measurements, very small values of the applied force must be considered. In this case, the terrestrial magnetic field $\vec{B}_\oplus$ cannot be neglected compared to the field $\vec{B}$ of the coil (while its gradient $\vec{\nabla}\vec{B}_\oplus$ is certainly negligible) and the force $\vec{F}_h$ contains a term proportional to

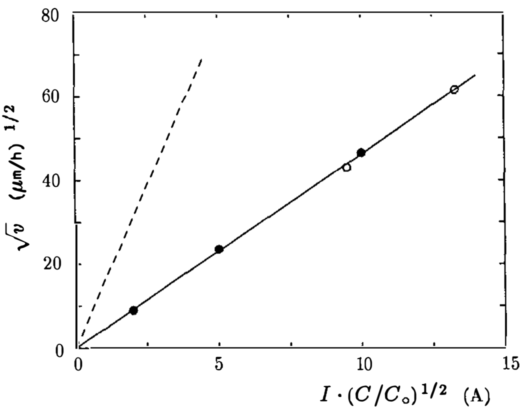


*Fig. 4: – Test of Stokes' law. Full points show the square routh of the terminal velocity of the sphere, as a function of the current I in the coil, for measurements performed in Florence (MnSO*$_4$ *concentration* $C_0$*=1.0* $10^{-2}$*). The full line corresponds to eq. 5 with* $\zeta$*=0.3, the dashed one to Stokes' law predictions. Results obtained during the latest run in Vallombrosa (MnSO*$_4$ *concentration C=0.9* $10^{-2}$*), are shown as open circles as a function of* $I \cdot (C/C_0)^{1/2}$.

the first power of $I$. Therefore, critical measurements have been performed two times with opposite signs of the current, so as to average out the influence of $\vec{B}_\oplus$.

Results of measurements performed with different values of the current $I$ are partially shown in fig. 3. From these data, the terminal velocity $v$ corresponding to each value of $I$ has been deduced by least–square fitting. In fig. 4, the square root of $v$ is displayed as a function of $I$. Experimental points fit well on a straight line passing through the origin, as it would be expected. The slope is however different from that predicted by Stokes' law.

In conclusion, our measurements show that, in the relevant region of velocities and forces, a "generalized Stokes' formula"

$$v = \zeta \frac{2}{9\nu} R^2 \frac{F}{M} \qquad [5]$$

(where $\nu = \eta/\rho$ is the kinematic viscosity of the liquid* and $M$ the mass of the sphere) can be used, with a correction factor $\zeta \geq 0.3$. Actually, the value $\zeta = 0.3$ is what we deduce from the results shown in fig. 4, which have been obtained a short time after preparation of the stratified solution. If, with the time going, the density gradient decreases, the value of the correction factor cannot get worse. One could expect, however, that the transition from the region of validity of eq. [5] with $\zeta = 0.3$ to that of Stokes' law ($\zeta = 1$) does not take place as long as viscous forces are much smaller than buoyancy forces**.

* The kinematic viscosity $\nu$ of the solution has been measured with a capillary type viscosimeter, in the temperature interval $4^0$C$\div 9^0$C. It deviates not more than 10% from that of pure water.

** As a rough estimate, we can fix the scale of forces per unit area at $g\rho R^2(1/\rho)|d\rho/dz|$ for buoyancy and at $\eta v/R$ for viscous forces, respectively.Their ratio comes out to be $\approx 10^7$ for $-(1/\rho)(d\rho/dz) \approx 10^{-3}$ cm$^{-1}$ and $v \approx 1$ mm/h. Although the above estimate might be in error by several orders of magnitude, our assumptions seem therefore to be largely verified for the relevant values of $v$ and of the density gradient.

4. New measurements in Vallombrosa

Measurements in the Paradisino building in Vallombrosa had to be suspended in spring, to leave room for the summer lectures of the Faculty of Forestal Sciences, and could start again in october. In this second run, we chose to employ the same kind of solution which had been used to test Stokes' law in Florence, *i.e.* a stratified KBr solution including in addition a constant concentration (0.9% in weight) of $MnSO_4$, and to change from time to time the position of the sphere by means of an external magnetic field.
The results of five weeks of measurements are shown in fig. 5. In the absence of magnetic field, the sphere remained substantially stable, with residual drifts which, although not so small as in the previous series with a pure KBr solution, were still of the order of $10\mu$m/h or less, and did not show any preferred direction. This fact, and the results of measurements with the magnetic field gradient, exclude the possibility that the sphere had been kept stable by convective currents of a single convection cell, converging to the center in the upper part of the fluid, as one could have hypothesized to explain the stability of its position even in the presence of an external force. Measurements with the magnetic field were performed with the same procedure described in §3. The measured values of the terminal velocity (shown as open circles in fig. 4) agree with those obtained in Florence, if one takes into account the different concentration of paramagnetic salt.
Average velocities of the sphere in the absence of magnetic field, obtained by least-square fitting in time intervals of about 10 days (and after damping of the initial currents) are shown in Table I, together with corresponding values obtained in the first series of measurements[1]. If, on this basis, we take $v < 10\mu$m/h as the upper limit for the terminal velocity which could originate from an external force, by means of eq. (5) we deduce the corresponding limit for the differential acceleration field $\Delta a' < 2.4 \cdot 10^{-9}$ cm/s$^2$.

**TABLE I**

Residual drifts in the S→N and in the E→W direction, observed in the 1987 measurements (ref. 1]) and in present work. Average values of the four series and their r.m.s. deviations (as deduced from the dispersion of the sample) are shown in the last line.

| year | time interval (hours) | $\bar{v}_{S\to N}$ ($\mu$m/h) | $\bar{v}_{E\to W}$ ($\mu$m/h) |
|---|---|---|---|
| 1987 | 195-410 | −0.4±0.3 | 3.2±0.2 |
| " | 410-615 | −2.2±0.2 | 2.1±0.2 |
| 1988 | 300-540 | 1.1±0.4 | −6.1±0.7 |
| " | 550-780 | 8.8±0.3 | −4.6±0.3 |
| average | | 1.8±2.4 | −1.4±2.3 |

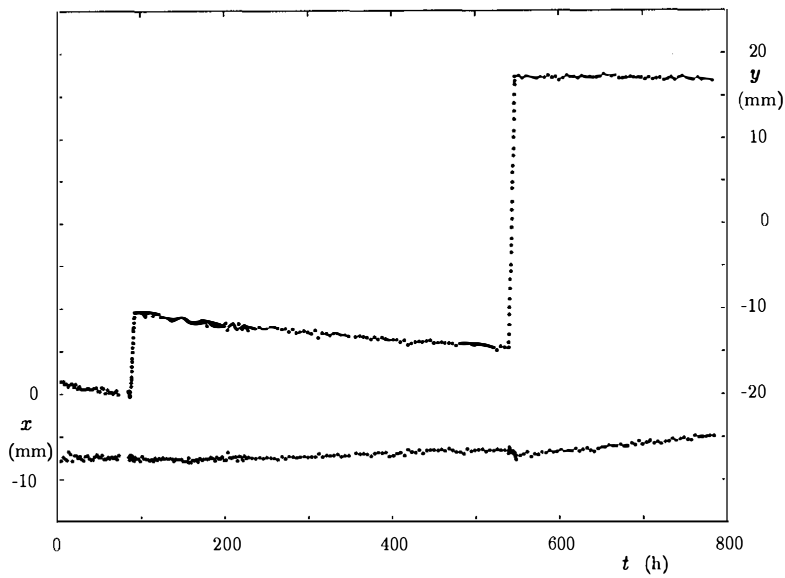


*Fig. 5: – Horizontal coordinates of the centre of the sphere, as a function of time, for the latest measurements in Vallombrosa. The y direction corresponds to the axis of the coil, and makes an angle of 18 $^0$ with the E→W direction.*

5.Limits on the strength of the force

According to current approaches to a self-consistent quantum theory of gravitation, corrections [8] to the Newtonian law are related to *massive* scalar and/or vector partners of the graviton, whose exchange would produce Yukawian forces of range $\lambda$ given by the inverse of the mass.

It has been customary to discuss the fifth–force experiments in terms of simple models involving one single Yukawian, having range $\lambda$ inside the "geophysical window" ($\sim 10^2$ m to $\sim 10^3$ m). However, in order to reconcile the otherwise conflicting results of several recent experiments, one has also considered [9,10] the possible existence of two competing parts of the new interaction: a vector force (repulsive, between pieces of ordinary matter) and a scalar one (the latter, of attractive nature, seems to be required to account for the gravimetric measurements by Eckhardt *et al* [11]). If this was the case, ranges of the force significantly longer than originally proposed should be explored, to complement the no longer exhaustive[10] information provided by the comparison of the Earth gravitation at the surface with that at satellite orbits.

A sufficiently general form of the gravitation–like potential between two point bodies $i, j$ of mass $M_i, M_j$ at distance $r$ seems therefore to be

$$V(r) = -G\,\frac{M_1 M_2}{r}\left[1 + \alpha_0\,\frac{q_i^{(0)}}{\mu_i}\,\frac{q_j^{(0)}}{\mu_j}\,\exp(-\frac{r}{\lambda_0}) + \alpha_1\,\frac{q_i^{(1)}}{\mu_i}\,\frac{q_j^{(1)}}{\mu_j}\,\exp(-\frac{r}{\lambda_1})\right] \qquad [6]$$

where $G$ is the gravitational constant, $\mu_i$ the atomic masses and $\alpha_0 > 0$, $\alpha_1 < 0$ the normalized coupling constants for the scalar and the vector part of the interaction. The various models differ in the assumptions concerning the "atomic charges" $q^{(0)}$ (scalar) and $q^{(1)}$ (vector).

For vector forces, the charge $q^{(1)}$ should correspond to a conserved quantum number. In addition to the initially proposed charge $q^{(1)} = B$, the alternative choice[12] $q^{(1)} = N - Z$, or a linear composition of the two, $q^{(1)} = B\cos\beta + (N - Z)\sin\beta$, have been considered. For scalar forces, which do not depend on conserved currents, a similar property does not hold and the atomic scalar charge is not necessarily[13] a linear combination of $B$ and $N - Z$. However, a linearized expression of the form $q^{(0)}/\mu = 1 + c_B[B + \gamma(N - Z)]$ has been proposed, *e.g.*, in ref. 3]. The differential acceleration between solid and liquid in our experimental apparatus is, therefore, presumably related to the corresponding difference in $B/\mu$ and/or in $(N - Z)/\mu$, as it results from their chemical composition.

The composition of the commercial Nylon 12 plastics has been determined by elemental analysis (for elements lighter then oxygen) and by PIXE (for elements heavier than sodium). The latter showed the presence of sizeable quantities of sulfur, which is known to be present in a plasticizer (Santicizer 8) often used in the preparation of nylon bars. The results of the analysis are in agreement with a content of 93.8% (in weight) of pure Nylon 12 – *i.e.* $(C_{12}H_{23}NO)_n$ – and 6.2% of Santicizer 8 – *i.e.* $C_{10}H_{15}NO_2S$. The water solution used in the first measurement[1] contained, in the average, 5.5% (in weight) of KBr, while in the latest measurements a solution containing 4.6% of KBr plus 0.9% of $MnSO_4$ was used. With values of $B/\mu$ and $(N - Z)/\mu$ given in ref. 14] one obtains, in both cases, $\Delta\frac{B}{\mu} \approx -0.40 \cdot 10^{-3}$ and $\Delta\frac{N-Z}{\mu} \approx -1.3 \cdot 10^{-2}$.

We have now to evaluate, from eq. [6], the fifth–force field produced by our *extended* source. Let us consider first the simplest case, *i.e.* a single Yukawian force of range $\lambda$ (either $\alpha_0$ or $\alpha_1$ equal to 0). If the density $\rho_g$ of the ground and its average $< q/\mu >_g$ ratio are assumed to be constant in the relevant region, the integral over the volume of the source can be transformed into a surface integral[1], and the acceleration field of the fifth force for a body (of given $q/\mu$) located at the point P turns out to be

$$\vec{a}' = -G\,\alpha\lambda \cdot (q/\mu) \cdot \rho_g < q/\mu >_g \cdot \vec{f}_0(\mathrm{P}, \lambda) \qquad [7]$$

where $\vec{f}_0(\mathrm{P}, \lambda)$ belongs to the functions defined by

$$\vec{f}_k(\mathrm{P}, \lambda) \equiv -\lambda\vec{\nabla}_{\mathrm{P}} \int_V \left[\left(\frac{\lambda}{r}\right)^{1-k} \exp(\frac{-r}{\lambda})\right] \frac{dV}{\lambda^3} = \int_S \left(\frac{\lambda}{r}\right)^{1-k} \exp(\frac{-r}{\lambda})\,\vec{n}\,\frac{dS}{\lambda^2} \qquad [8]$$

Here $r$ is the distance from P to the current infinitesimal element considered in the integral and $\vec{n}$ is the unit vector normal to the surface element $dS$ of the ground. The geological structure of the area, under and around the point of measurement, is characterized [15] by an Oligocen graywacke ("Macigno") extending over a distance of at least 1 to 2 km in all directions. The density of the rock is about 2.4 g/cm$^3$. Other geological layers surrounding the "macigno" formation, though of different composition, have similar density. The use of eq. [8] seems therefore to be adequate, as long as the charge $q$ of the fifth force is *approximately* proportional to the mass.

The evaluation, as a function of $\lambda$, of the source integrals of eq. [8] has been performed by means of a mathematical model of the soil*, giving the average height in pixels of $25 \times 25$ m$^2$. In the closest region, the surface integral $\vec{f_0}$ has also been evaluated[1] using the level lines of a 1:10000 map (in steps of 10 m in height). No attempt having been made to take into account smaller details, as the building itself, we assume that this description is reliable only for $\lambda > 50$m. The results obtained with the two procedures agree, within a few percent, in the region of values of $\lambda$ where the latter is significant.

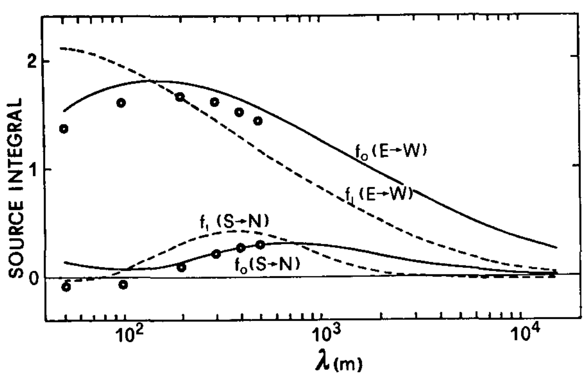


*Fig. 6: – Values of the E→W and S→N components of the vector functions $\vec{f_0}(\lambda)$ and $\vec{f_1}(\lambda)$ of eq. [4], evaluated on the basis of the mathematical model of the soil. The results are expected to be relatively less accurate for large values of $\lambda$, due to the uncertainties in the composition of the earth crust at depths larger than a few km. Values of $\vec{f_0}$ obtained in ref. 1] from the level lines of a map at the scale 1:10000, are also shown as open circles. At both extremes of the interval, the S→N component becomes very small (and relatively unstable with respect to calculation parameters) due to cancellation effects.*

* These data, deduced from the general map of Italy at the scale 1:25000, have been supplied by Istituto Geografico Militare. They cover a region of about $40 \times 40$ km$^2$ around the place of the measurement.

Horizontal components of the field $\vec{f_0}$ in the East-→West and in the North-→South directions, as a function of the assumed range $\lambda$, are shown in fig. 6. Assuming $q = B$, as in ref. 2], one would finally obtain the limits to $|\alpha_1\lambda_1|$ shown in fig. 7. These limits are comparable to those obtained by Fitch *et al* [16]] (at the 3 standard deviation level) for $\lambda < 3$ km, and about a factor 2 better at $\lambda = 10$ km (upper limit of $\lambda$ considered there). Limits from Stubbs *et al*[17]] (at the $3\sigma$ level) are comparable with ours at the upper limit of their interval ($\lambda = 1$ km) and somewhat better at lower values of $\lambda$. Within this model, our results are not consistent with those obtained by Thieberger at Palisades [5]], which would imply, *e.g.*, $|\alpha\lambda| = (1.2 \pm 0.4)$ m for $\lambda$ <100 m, and $|\alpha\lambda| = (\approx 3 \pm 1)$ m for $\lambda$ =1000 m.
Our measurement is not particularly suitable to test Boynton's [12]] assumption $q = N - Z$, for which better constraints have been assigned, *e.g.*, by Adelberger *et al* [18]]. It turns out to be rather sensitive, however, for the "Cosmon" model of Peccei, Solà and Wetterich [3]]. According to these authors, the fifth force could be mediated by a scalar particle (the *Cosmon*) to be identified with the Goldstone boson related to the spontaneous break-up of the dilatation symmetry. The atomic charge of their (purely scalar) force can be approximated with a linearized expression [19]] of the form $q_0/\mu = 1 + c_B\ [B/\mu + \gamma\ (N - Z)/\mu]$ (where $\gamma \approx -1/30$). The two terms depending on the composition would partially cancel in the case of differential measurements of refs. 17] and 18], while they have the same sign for Thieberger's experiment [5]], for the Princeton experiment[16]] and in our case. Limits to $|c_B\alpha_0\lambda_0|$, deduced from our results in the frame of the model of ref. 19], are also shown in Fig. 7.

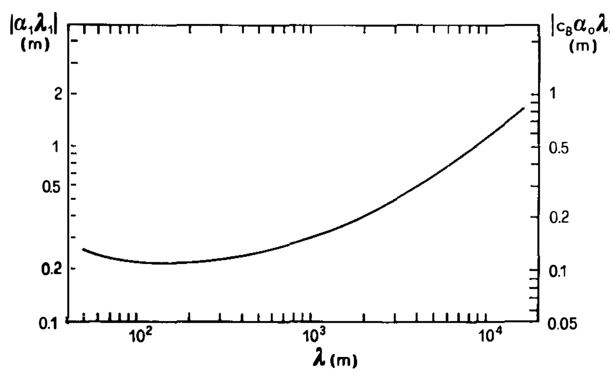


*Fig. 7: – Limits to the strength of the fifth force, as a function of the range $\lambda$, for a vector interaction coupled to B (left-hand scale) and for the "Cosmon model" of ref 19] (right-hand scale), as deduced from the present results.*

The more general expression [6] with $\alpha_0 \neq 0$ and $\alpha_1 \neq 0$ can hardly be tested on the only basis of the present results. However, one may exploit the boundaries on the difference $\delta\lambda/\overline{\lambda} = (\lambda_0 - \lambda_1)/(\lambda_0 + \lambda_1)$ as done by Stacey *et al* [10] on the basis of geophysical measurements, to obtain constraints on the composition dependent part of the force, similar to those reported in the same paper [10] for the composition–averaged strengths.
For $\delta\lambda/\overline{\lambda} \ll 1$ we obtain:

$$\begin{aligned}\Delta\vec{a}' = G\,\rho_g\{\overline{\lambda}[\alpha_0 \cdot \Delta(q^{(0)}/\mu)\cdot < q^{(0)}/\mu >_g + \alpha_1 \cdot \Delta(q^{(1)}/\mu)\cdot < q^{(1)}/\mu >_g] \cdot \vec{f_0}(P,\lambda) \\ + \delta\lambda[\alpha_0 \cdot \Delta(q^{(0)}/\mu)\cdot < q^{(0)}/\mu >_g - \alpha_1 \cdot \Delta(q^{(1)}/\mu)\cdot < q^{(1)}/\mu >_g] \cdot \vec{f_1}(P,\lambda)\}\end{aligned} \qquad [9]$$

From the calculated values of the vector functions $\vec{f_0}$, $\vec{f_1}$ shown in fig. 6, bounds to the relevant constants can be derived for any definite assumption on the charges. As an example, we consider here the particular choice corresponding to the combined effect of a scalar "cosmon" force as assumed in ref. 19], and of the vector force of ref. 2]. Our experimental limits would give, in this case

$$|\; A\;\overline{\lambda}\;\vec{f_0}(\overline{\lambda}) + D\;\delta\lambda\;\vec{f_1}(\overline{\lambda})\;| < 0.26\text{ m} \qquad [10]$$

where $A = \alpha_0 \Delta q^{(0)}/\Delta q^{(1)} + \alpha_1$, $D = \alpha_0 \Delta q^{(0)}/\Delta q^{(1)} - \alpha_1$, and $\Delta q^{(0)}/\Delta q^{(1)} \approx c_B\;(1 + 30.5\gamma) \approx -0.66$ with the values of $c_B$ and $\gamma$ given in ref. 19].
Due to the vector character of $\vec{f_0}$ and $\vec{f_1}$, for each value of $\lambda_1$ the allowed region for the parameters is limited by an ellipse in the $A$, $D$ plane. However, as it is clear from fig. 6, only the linear combination of $A$ and $D$ corresponding to the E→W component of the force is significantly constrained by the present results. Moreover, in this particular model, the scalar and the vector term contribute with the same sign to the quantity $A$, while they tend to cancel each other in $D$, and the second term of eq. [9] can be neglected, as long as $|\;\delta\lambda/\overline{\lambda}\;| \ll 1$. For the quantity $|A\overline{\lambda}|$ we obtain therefore the same limits we find (see Fig. 7) for $|\alpha_1\lambda_1|$ in the case of a pure vector force coupled to barion number.
Therefore, also in this model, it is not possible to explain the contrast of our results with those of ref. 5, at least for the values of $\overline{\lambda}$ up to some km. For larger values of the ranges, our negative result could perhaps be explained on the basis of the rather uniform distribution of masses (compared to that at Palisades[5,20]) in the deep underground, as it is suggested by the regular regional behavior of gravity. However, a significant limit to the strength of the fifth force would be provided, in this case, by the available results of free-fall experiments[21].

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