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Conceptual Issues in Canonical Quantum Gravity and Cosmology

CLAUS KIEFER*
Institute for Theoretical Physics, University of Cologne,
Zülpicher Straße 77, 50937 Köln, Germany

Abstract: Existing approaches to quantum gravity exhibit plenty of startling conceptual issues. Here I restrict my attention to the canonical approach. Both classical and quantum canonical gravity are discussed. Most conceptual problems circle around the problem of time – the absence of any external time parameter. I then turn to quantum cosmology, where these and more problems can be discussed in a transparent way. I conclude with brief remarks on singularity avoidance, the arrow of time, and the interpretation of quantum theory in general.

Introduction

In his book *The Meaning of Relativity*, Albert Einstein emphasizes the following point [1]:

> Es widerstrebt dem wissenschaftlichen Verstande, ein Ding zu setzen, das zwar wirkt, aber auf das nicht gewirkt werden kann.[1]

This statement expresses our modern understanding of space and time. In contrast to Newtonian physics, no absolute fields exist in general relativity; space and time are fully dynamical. Spacetime acts on matter, but matter also acts on spacetime. The mutual relationship between both is described by the non-linear Einstein field equations. There is no fixed, absolute background any more; spacetime as described by the metric of general relativity is fully dynamical. This feature is called *background independence* and

* `kiefer@thp.uni-koeln.de`

[1] It is contrary to the scientific mode of understanding to postulate a thing that acts, but which cannot be acted upon.

is of central relevance for general relativity, but also for quantum gravity, since it is the dynamical fields that are subject to quantization. An absolute ('background') field can be characterized by the fact that there exist coordinates which assign universal values to all components of a field; such a field must not be quantized. A prominent example for a background field is the spacetime metric of special relativity, for which priviledged coordinates exist (the inertial coordinates) in which the metric assumes the standard form diag(-1,1,1,1).

In my contribution, I shall give a brief review of the central conceptual issues that arise in both classical and quantum gravity and that are all more or less directly connected with background independence and the related problem of time. Attention is restricted to the canonical approach, because there these issues are most transparent. A detailed exposition covering all aspects of quantum gravity can be found in my book [2]; an introduction into classical and quantum canonical gravity from a conceptual point of view can also be found in our essay [3].

A theory of quantum gravity seems to be needed because the singularity theorems predict that general relativity cannot be fundamentally complete. In particular, the origin of our Universe and the final fate of black holes do not seem to be comprehensible without quantum gravity. Unfortunately, no final theory exists to date, so discussing conceptual issues in quantum gravity means to discuss them in existing *approaches* to such a theory. However, one can put forward various arguments in support of the generality of these issues in most approaches. This should become clear from the following discussion.

What are the main approaches to quantum gravity? There exist presently two main classes:

- *Quantum general relativity*: this includes all approaches that arise from an application of formal quantization rules to general relativity. They can be subdivided further into:
  - *Covariant approaches* (such as perturbation theory, path integrals, and others), which are characterized by the fact that spacetime covariance plays a crucial role in some parts of the formalism, and
  - *Canonical approaches* (such as geometrodynamics, connection dynamics, loop dynamics, and others), in which a Hamiltonian formalism is being used.
- *String theory*: this is intended to be a unified quantum theory of all interactions, in which the quantized gravitational field can be distin-

guished as a separate field only in appropriate limits, for example, for energies lower than the fundamental string energy scale.

There also exist other even more fundamental approaches (such as the quantization of topology) which, however, have not been developed as far as the main approaches. In the following, I shall restrict myself to the canonical formalism. I start with classical canonical gravity, proceed then to quantum canonical gravity, and conclude with quantum cosmology where the conceptual issues of the full approach (and further issues) are exhibited clearly and explicitly.

Canonical classical gravity

The canonical formalism starts with the '3+1 decomposition' of general relativity [2]. Spacetime is assumed to be globally hyperbolic, that is, to be of the form $\mathbb{R} \times \Sigma$, where Σ denotes a three-dimensional manifold; spacetime is thus foliated into a set of spacelike hypersurfaces Σ_t. The dynamical variable is the three-dimensional metric, h_{ab}, which can be obtained as the metric that is induced by the spacetime metric $g_{\mu\nu}$ on each Σ_t. Instead of considering the three-metrics on each Σ_t, one can adopt the equivalent viewpoint and consider a t-dependent three-metric on the given manifold Σ. For each choice of Σ in the topological sense one obtains a different version of canonical gravity.

This lends itself to a more fundamental viewpoint: instead of starting with a four-dimensional spacetime, M, to be foliated, we assume that only Σ is given. Then, only *after* solving the field equations can we construct spacetime and interpret the time dependence of the metric h of Σ as being brought about by 'wafting' Σ through M via a one-parameter family of hypersurfaces Σ_t. The field equations can actually be divided into

- six dynamical evolution equations for h_{ab} and its canonical momentum π^{ab}, and

- four constraints, which are restrictions on the initial data, that is, restrictions on the allowed choices for h_{ab} and π^{ab} on an 'initial' hypersurface.

After having solved these equations, spacetime can be interpreted as a 'trajectory of spaces'. The origin of the constraints is the diffeomorphism invariance of general relativity. They have the explicit form

$$
\begin{aligned}
H[h,\pi] &= 2\kappa\, G_{ab\,cd}\pi^{ab}\pi^{cd} \; - \; (2\kappa)^{-1}\sqrt{h}({}^{(3)}\!R - 2\Lambda) \; + \; \sqrt{h}\rho \approx 0\,, & (1)\\
D^a[h,\pi] &= -2\nabla_b\pi^{ab} \; + \; \sqrt{h}j^a \approx 0\,, & (2)
\end{aligned}
$$

where $\kappa = 8\pi G/c^4$, Λ is the cosmological constant, ${}^{(3)}\!R$ is the three-dimensional Ricci scalar, and

$$
G_{ab\,cd} = \frac{1}{2\sqrt{h}}(h_{ac}h_{bd} + h_{ad}h_{bc} - h_{ab}h_{cd}) \qquad (3)
$$

is called the DeWitt metric; it plays the role of a metric on the space of all three-metrics. While (1) is called Hamiltonian constraint, the three constraints (2) are called momentum or diffeomorphism constraints.

There are two important theorems which connect the constraints with the evolution. The first one states:

> Constraints are preserved in time $\Longleftrightarrow$ energy–momentum tensor of matter has vanishing covariant divergence.

This can be compared with the corresponding situation in electrodynamics: the Gauss constraint $\nabla \mathbf{E} = 4\pi\rho$ is preserved in time if and only if electric charge is conserved in time. The second theorem states:

> Einstein's equations are the unique propagation law consistent with the constraints.

Again, this can be compared with the situation in electrodynamics: Maxwell's equations are the unique propagation law consistent with the Gauss constraint. Constraints and evolution equations are thus inextricably mixed.

A central conceptual issue in quantum gravity is the 'problem of time'. Part of this problem is already present in the classical theory. Namely, if we restrict ourselves for simplicity to *compact* three-spaces Σ, the total Hamiltonian is a combination of pure constraints; all of the evolution will therefore be generated by constraints and is thus, in a sense, pure gauge. How can this be reconciled with the usual interpretation of the Hamiltonian as a generator of time translations? The point is that the evolutions along different spacelike hypersurfaces are equivalent and lead to the same spacetime satisfying Einstein's equations. This freedom is expressed by the fact that the Hamiltonian generates both gauge transformations and time translations (hypersurface deformations). In other words, *no* external time parameter exists. All physical time parameters are to be constructed from within the system, that is, as a functional of the canonical variables; a priori there is no preferred choice of such an intrinsic time parameter. The absence of an extrinsic time and the non-preference of an intrinsic time is known as the *problem of time* in classical canonical gravity. As we shall see below, this leads in quantum gravity to stationary fundamental equations for a wave function which only depends on variables defined on the three-dimensional space Σ.

Above we have defined the DeWitt metric – the metric on the space of all three-metrics, see (3). Its signature is responsible for the structure of the kinetic term in the Hamiltonian constraint (1). As it turns out, the DeWitt-metric possesses an indefinite structure [2, 3]. It can be viewed at each spacepoint as a symmetric 6×6-matrix. This matrix can be diagonalized, and it is found thereby that the diagonal contains one minus and five plus signs; the DeWitt-metric is thus indefinite, and the kinetic term in (1) is indefinite, too. Due to the pointwise Lorentzian signature of $G^{ab\,cd}$, it is of a *hyper-Lorentzian structure* with infinitely many negative, null, and positive directions. This property will be of central relevance in the quantum theory.

Canonical quantum gravity

The classical constraints (1) and (2) can be implemented in the quantum theory in various ways; after all, the quantum theory can only be heuristically constructed and never be derived from the underlying classical theory. One possibility would be to try to solve the constraints classically and quantize only the remaining physical variables. However, this 'reduced quantization' leads to many problems; in particular, it is hard to perform in practice except for the simplest models [4, 2]. The alternative method is to implement the constraints à la Dirac as restrictions on physically allowed wave functionals. Replacing the canonical momenta by $-\mathrm{i}/\hbar$ times functional derivatives with respect to the three-metric, the Hamiltonian constraint becomes the *Wheeler–DeWitt equation*:[2]

$$\hat{H}\Psi \equiv \left(-2\kappa\hbar^2 G_{abcd}\frac{\delta^2}{\delta h_{ab}\delta h_{cd}} - (2\kappa)^{-1}\sqrt{h}\left({}^{(3)}R - 2\Lambda\right)\right)\Psi = 0. \qquad (4)$$

[2]For simplicity, we neglect here the non-gravitational fields, which occur in addition to the three-metric.

The kinetic term can only be interpreted as being formal, because the factor-ordering problem and the connected problem of regularizing the functional derivatives have not been addressed.

The quantization of the momentum constraints (2) leads to the equations

$$\hat{D}^a\Psi \equiv -2\nabla_b \frac{\hbar}{\mathrm{i}} \frac{\delta\Psi}{\delta h_{ab}} = 0, \tag{5}$$

which are called *quantum diffeomorphism (momentum) constraints*. They express the invariance of the wave functional under infinitesimal coordinate transformations on the three-dimensional space.

The 'problem of time', which was already discussed above in the context of the classical theory, is being enforced in the quantum theory. Namly, spacetime as such has completely disappeared! All that remains in the formalism is a wave functional which depends on the metric (and matter fields) on a three-dimensional manifold Σ. In retrospect, this is not surprising. A classical spacetime as a succession of three-dimensional hypersurfaces is fully analogous to a particle trajectory in mechanics (a succession of positions). In the same way that the particle trajectory vanishes in quantum mechanics, the spacetime vanishes in quantum gravity. This feature is independent of this particular quantization of Einstein's equations; it holds for any theory which at the classical level does not contain an external time parameter.

The absence of an external time and of spacetime does not necessarily mean that no sensible notions of time can be defined. Regarding the kinetic term in (4), one recognizes in view of (3) that is possesses an indefinite structure [2, 3]. The Wheeler–DeWitt equation thus has the form of a wave equation (more precisely, a local hyperbolic equation). Part of the three-metric thus comes with a positive sign in the kinetic term and can therefore be called *intrinsic time*, in full analogy to the time variable occurring in a standard wave equation. It turns out that it is just the scale part of the three-metric (the three-volume in simple models) which plays the role of intrinsic time.

A conceptual problem that is related to the problem of time, is the Hilbert-space problem: which inner product, if any, does one have to choose between wave functionals? From the point of view of the standard Schrödinger picture, one would like to employ the Schrödinger inner product (square integrable wave functionals). On the other hand, in view of the fact that (4) resembles more a wave equation, one would prefer a Klein–Gordon type of inner product, which is, however, indefinite. Every choice has its merits and its disadvantages, so it seems difficult to make a definite choice [2, 4].

What is an observable in quantum gravity? This question, too, is related with the problem of time. One would assume that all observables have to commute with both the Hamiltonian and momentum constraints. But this would mean that all observables would be constants of motion, because the total Hamiltonian is a sum of these constraints. Is therefore all change observed in Nature a pure illusion? The answer is no because we view the world from inside. As we shall see below, observers in the semi-classical approximation will have an approximate time variable at their disposal, which can be approximately identified with the standard time of non-relativistic physics. The timeless view of constants of motion would correspond to a hypothetical perspective from outside the world where everything (all branches of the wave functional) would be present at once.

So far we have restricted our discussion to quantum geometrodynamics. Of course, other canonical variables can be chosen. The most prominent choice at the moment is to choose holonomies and fluxes, leading to loop quantum gravity [5, 6]. While the details differ from quantum geometrodynamics, the timeless nature of the constraints remains, with all of the above consequences. A mathematically sound Hilbert-space structure can be constructed at least on the kinematical level, that is, before all the constraints are implemented; this inner product is of the Schrödinger type. For more details on loop quantum gravity, I refer to the literature [5, 2].

Quantum cosmology

Most of the conceptual issues in quantum gravity can be discussed in a transparent way in quantum cosmology. Quantum cosmology is the application of quantum theory to the Universe as a whole. That the whole cosmos must be fundamentally described in quantum terms follows from very general arguments, which are independent of gravity. All systems, except the most microscopic ones, are quantum entangled with their natural environment; this leads to their classical appearance – through a process called *decoherence* [7]. Since every environment has again an environment, this leads to the conclusion that the whole Universe must be described in quantum terms. It is only because the gravitational interaction dominates on large scales that we need a theory of quantum gravity to cope with quantum cosmology [2, 8, 9].

It is important to note that quantum effects in cosmology are not *a priori* restricted to the Planck scale, which reads

$$
\begin{aligned}
l_{\mathrm{P}} &= \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33}\,\mathrm{cm}, & (6)\\
t_{\mathrm{P}} &= \frac{l_{\mathrm{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\,\mathrm{s}, & (7)\\
m_{\mathrm{P}} &= \frac{\hbar}{l_{\mathrm{P}} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\,\mathrm{g} \approx 1.22 \times 10^{19}\,\mathrm{GeV}. & (8)
\end{aligned}
$$

The reason is that the *superposition principle*, which allows to form non-trivial quantum states, holds at any scale, not only the Planck scale.[3]

The simplest model of quantum cosmology is obtained if one quantizes directly a Friedmann–Lemaître universe; it is characterized by a scale factor, a, and we choose in addition a homogeneous massive scalar field, ϕ. The classical spacetime metric is of the form

$$
\mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\Omega_3^2, \tag{9}
$$

where $N(t)$ is the lapse function which encodes the invariance of the classical theory under reparametrizations of the time coordinate $t \to f(t)$. In the quantum theory, t disappears and only a and ϕ remain as the variables on which the wave function $\psi(a,\phi)$ depends.

The Wheeler–DeWitt equation then reads (with a convenient choice of units $2G/3\pi = 1$ and for the case of a closed universe)

$$
\frac{1}{2}\left(\frac{\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) - \frac{\hbar^2}{a^3}\frac{\partial^2}{\partial \phi^2} - a + \frac{\Lambda a^3}{3} + m^2 a^3 \phi^2\right)\psi(a,\phi) = 0. \tag{10}
$$

The factor ordering has been chosen in order to achieve covariance in the two-dimensional configuration space comprised of a and ϕ. The wave-nature of this equation is evident, and it is seen that the intrinsic time variable is given by the scale factor itself. The quantum diffeomorphism constraint is automatically satisfied by the ansatz (9). Solutions of the Wheeler–DeWitt equation in the context of quantum cosmology are often called ‘wave function of the universe’.

The new concept of time has far-reaching consequences: the classical and the quantum model exhibit two drastically different concepts of determinism, see Figure 1.

[3]This is, of course, an assumption, but an assumption which is applied in almost all discussions of quantum cosmology.

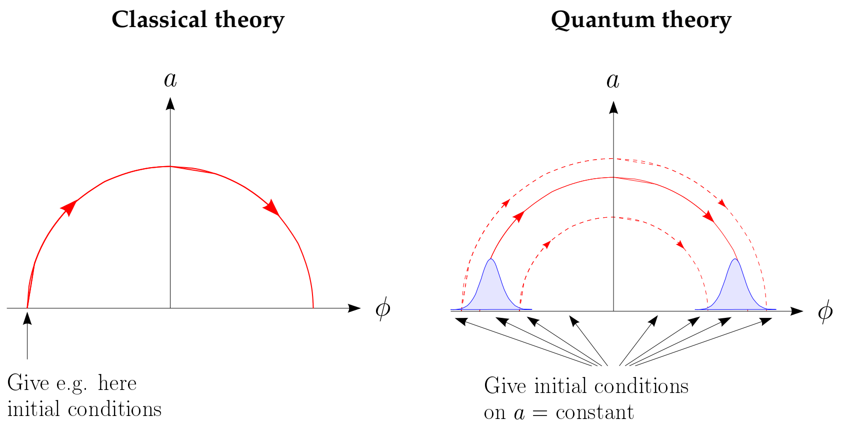

Figure 1. (a) Recollapsing part is deterministic successor of expanding part. (b) 'Recollapsing' wave packet must be present 'initially'.

Consider the case of a classically recollapsing universe. In the classical case (left) we have a trajectory in configuration space: although it can be parametrized in many ways, the important point is that it *can* be parametrized by some time parameter. Therefore, upon solving the classical equations of motion, the recollapsing part of the trajectory is the deterministic successor of the expanding part: the model universe expands, reaches a maximum point, and recollapses.

Not so for the quantum model. There is no classical trajectory and no classical time parameter and one must take the wave equation (10) as it stands. The wave function only distinguishes small a from large a, not earlier t from later t. There is thus no intrinsic difference between big bang and big crunch. If one wants to construct a wave packet following the classical trajectory as a narrow tube, one has to impose the presence of two packets as an initial condition at small a; if one chose only one packet, one would obtain a wave function which is spread out over configuration space and which does not resemble anything close to a narrow wave packet.

Wave packets are of crucial importance when studying the validity of the semiclassical approximation. In quantum mechanics, narrow wave packets remain narrow if the WKB approximation holds. In quantum cosmology, this issue has to be studied from the new viewpoint of the Wheeler–DeWitt equation (10). If the classical model describes a recollapsing universe, one has to impose in the quantum theory onto the wave function the restriction that it go to zero for $a \to \infty$; with respect to intrinsic time, this corresponds to a 'final condition'. Calculations show that it is then *not* possible to have narrow wave packets all along the classical trajectory; the packet disperses, see [2, 9] and the references therein. Again, this is a consequence of the novel concept of time.

But how do classical properties arise if wave packets necessarily disperse? The answer to this question is again decoherence [7]. In order to study this process, additional degrees of freedom must be introduced. They can then act as an 'environment' which becomes quantum entangled with a and ϕ, causing their classical appearance.

A straightforward way to introduce a large number of additional degrees of freedom is to consider small inhomogeneities superimposed on the homogeneous three-sphere of the closed Friedmann universe [10]. These inhomogeneities are described by small multipoles denoted by the set of variables $\{x_n\}$; they describe small gravitational waves and density perturbations. One then finds the more general Wheeler–DeWitt equation

$$\left(H_0 + \sum_n H_n(a, \phi, x_n)\right) \Psi(a, \phi, \{x_n\}) = 0, \tag{11}$$

where

$$\Psi(a, \phi, \{x_n\}) = \psi_0(a, \phi) \prod_n \psi_n(a, \phi, x_n),$$

and $H_0\psi_0 = 0$ is the original 'unperturbed' Wheeler–DeWitt equation (10). If ψ_0 is of WKB form, $\psi_0 \approx C \exp(\mathrm{i}S_0/\hbar)$ (with a slowly varying prefactor C), one gets [10, 11]

$$\mathrm{i}\hbar \frac{\partial \psi_n}{\partial t} \approx H_n \psi_n \tag{12}$$

with

$$\frac{\partial}{\partial t} \equiv \nabla S_0 \cdot \nabla\ . \tag{13}$$

The multipoles therefore obey separate Schrödinger equations with respect to some approximate time parameter t. This parameter is called '*WKB time*' – it controls the dynamics in this approximation and corresponds to the Friedmann time parameter of the classical model. This is the limit where the standard formalism of quantum theory with its Hilbert-space structure applies. A Hilbert space is needed to implement the probability interpretation of quantum theory, in particular, the conservation of probability with respect to external time t. Whether a Hilbert-space structure is needed in timeless quantum gravity, too, is thus an open issue.

This 'emergence of time' from timeless quantum gravity is one of the satisfactory features of quantum geometrodynamics. A corresponding recovery is not yet known in loop quantum gravity. An analogous feature can be discussed in Euclidean quantum gravity where the fundamental concept is a Euclidean path integral. A suggestion to find the quantum state of the Universe is encoded, for example, in the no-boundary condition of Hartle and Hawking [12, 2]. The time parameter t appears there in the limit where the saddle point approximation holds (corresponding to the WKB approximation) and where the saddle point gives a complex time in the Euclidean formulation – corresponding to the ordinary real time t as in (13).

In order to study the decoherence for a and ϕ, one has to solve the full Wheeler–DeWitt equation (11) and trace out all the multipoles from the resulting full quantum state. This gives a density matrix whose diagonal terms are suppressed in the generic case, which means that interferences between universes of different sizes can be neglected and the universe can be treated classically for most of its evolution [2, 9]. Moreover, one can also understand why and how interferences between the $\exp(\mathrm{i}S_0/\hbar)$- and $\exp(-\mathrm{i}S_0/\hbar)$-branches of a wave function become suppressed by decoherence. A calculation within a particular model leads, for example, to the following suppression term for the interference between these two branches [13]:

$$\exp\left(-\frac{\pi m H_0^2 a^3}{128\hbar}\right) \sim \exp\left(-10^{43}\right),$$

where the number arises for today's universe if some natural values are inserted for the Hubble parameter H_0 and the mass m of a fundamental Higgs field. One recognizes that today the universe behaves indeed very classically!

Once a classical background is established as an approximate concept, one can then address the quantum-to-classical transition for the relevant part of the multipoles itself on this background, which are the density fluctuations serving as the seeds for galaxy formation [14].

Using the above introduced concepts, one can also discuss the issues of singularity avoidance and arrow of time. Both issues can have to do with quantum effects far away from the Planck scale. As for the former example, classical models exist which exhibit a singularity at large scale factor, that is, far away from the Planck scale. For example, by choosing a scalar-field potential of the form

$$V(\phi) = V_0 \left(\sinh\left(|\phi|\right) - \frac{1}{\sinh\left(|\phi|\right)} \right) ,$$

one can obtain a 'big-brake singularity' – the universe suddenly stops its expansion in the future, while keeping both the scale factor and its time derivative finite, but leading to an infinite value for the deceleration. Discussing the corresponding quantum model, it was shown upon solving the Wheeler–DeWitt equation that *all normalizable solutions* vanish at the classical singularity, thus entailing complete singularity avoidance [15]. Singularity avoidance is also a central feature of loop quantum cosmology, which is discussed in another contribution to this volume [6].

As for the arrow of time, its origin can in principle be understood from quantum cosmology. The reason is that the Wheeler–DeWitt equation (10) is asymmetric with respect to intrinsic time a. Choosing a simple initial wave function which factorizes between the a and ϕ-part and the higher multipoles, the full solution is a wave function whose quantum entanglement between these two parts increases with a. Integrating out the multipoles leads to a density matrix whose impurity increases with a. This, in turn, leads to an increasing entanglement entropy which could be at the heart of the Second Law of thermodynamics [9]. An interesting consequence would be a formal reversal of the arrow of time at the region of the classical turning point [9, 16] – another quantum effect far from the Planck scale.

Last but not least, quantum cosmology has an important bearing on our understanding of quantum theory itself. Both quantum general relativity and string theory preserve the linear structure for the quantum states, that is, stick to the strict validity of the superposition principle. Since the Universe as a whole by definition contains all degrees of freedom, it must also describe all observers in quantum terms. The only interpretation known so far is the 'Everett interpretation', with decoherence as an essential part [7]. I thus want to conclude with the following quote from one of the pioneering papers on canonical quantum gravity [17]:

> Everett's view of the world is a very natural one to adopt in the quantum theory of gravity, where one is accustomed to speak without embarassment of the 'wave function of the universe.' It is possible that Everett's view is not only natural but essential.

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