Extra Spacetime Dimensions and the LHC

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The last decade has seen an explosive revival of interest in extra spacetime dimensions. Inspired by developments in string theory, ingenious phenomenological models have been constructed in which gravity becomes strong at the scale of a few TeV, thereby solving the long-standing hierarchy problem of particle physics. Perhaps the most interesting aspect of these theories is the possibility of 'seeing' quantum gravity effects – including microscopic black holes – in experiments carried out at the TeV scale, of which the Large Hadron Collider (LHC) at CERN is the imminent one. Some of these ideas are reviewed in this article and the possibility of seeing signals for extra dimensions at the LHC are briefly discussed.

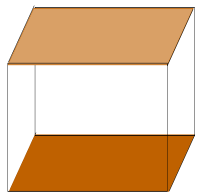
1. More Dimensions

From the earliest days one just has to look at a solid object – like the box depicted on the right – to get an idea of the three spatial dimensions normally described as length, breadth and height. Today this is usually the starting point of elementary mathematics textbooks, which then go on to say that every point in space can be described by three real numbers x, y and z, as Réne Descartés taught us three centuries ago. The overwhelming majority of humans are quite content with this simple description, but some of the more subtle minds have, over the ages, been attracted by the idea that there might be extra spatial dimensions beyond these canonical three. In fact, long before Descartés, the Hindu *Védas* enumerated ten *dishā* or directions, the Jewish *Kabbālah* talked of divine attributes being channelised to the earth through ten-dimensions called *sefirôt* and Mayan cosmogony visualised thirteen directions emanating like plumes from the cosmic serpent Kukulcán (Fig. 1).

Figure 1. Mayan sculpture of the 'plumed serpent' Kukulcán, spouting eleven feathers, purported to represent eleven directions. The remaining two directions are presumably pointed into and out of the plane of the picture

There is a dreamlike quality, however to these early speculations, and it may be contended that we are interpreting the ancient texts with the benefit of hindsight, whereas the original authors really meant something quite different. Since the precise meanings of the ancients have been lost with the extinction of the languages which they used, no one can really tell for sure. Be that as it may, even during the relatively recent times of Descartés, one had only to look across the English Channel to find the Cambridge philosopher Henry More (1614–1687) – whose language we *do* understand – speculating about the existence of a fourth space dimension. In a curious mixture of geometry and mysticism, harking back the days of Pythagoras and Plato, More claimed that the *spissitude* or fourth dimension of space represents the realm of spiritual things [1].

In a more modern context, and shorn of all mystic and spiritualistic trappings, extra space dimensions were first popularised, more or less as a mathematical curiosity, by the English polymath Charles Hinton in 1884. Hinton, who coined the word *tesseract* for a four-dimensional cube, tried to explain the three-dimensional section of a four-dimensional object by using the analogy of the time evolution of a three-dimensional object [2]. This inspired the novelist H.G. Wells to write that 'Everybody knows that time is only a kind of space' (*The Time Machine*, 1895). Wells was to prove a true prophet, for exactly ten years later, the true scientific foundation for the fourth dimension was laid by Albert Einstein's 1905 theory of Special Relativity, where space and time variables were, for the first time, allowed to *mix* non-trivially in moving frames of



reference. Nowadays, it is usual to denote the space and time variables by $x^0 = ct$, $x^1 = x$, $x^2 = y$ and $x^3 = z$, where c is the speed of light in vacuum. In this notation, when one considers the transformed coordinates in a frame of reference moving with a relative velocity v with respect to the previous one, they are related to the older coordinates by the set of linear equations

$$\begin{aligned} x'^0 &= \Lambda_{00}x^0 + \Lambda_{01}x^1 + \Lambda_{02}x^2 + \Lambda_{03}x^3 \\ x'^1 &= \Lambda_{10}x^0 + \Lambda_{11}x^1 + \Lambda_{12}x^2 + \Lambda_{13}x^3 \\ x'^2 &= \Lambda_{20}x^0 + \Lambda_{21}x^1 + \Lambda_{22}x^2 + \Lambda_{23}x^3 \\ x'^3 &= \Lambda_{30}x^0 + \Lambda_{31}x^1 + \Lambda_{32}x^2 + \Lambda_{33}x^3 \ , \end{aligned} \qquad (1)$$

where the coefficients $\Lambda_{\mu\nu}(\mu, \nu = 0, 1, 2, 3)$ are constants depending only on the ratio v/c. The new coordinates, therefore, are an admixture of *all* the previous ones – the new spatial coordinates x'^1, x'^2, x'^3 depend on the old time x^0, and the new time x'^0 depends on the old space coordinates x^1, x^2, x^3. In relativity, therefore time is indeed a sort of space, as Hinton and Wells had speculated.[1]

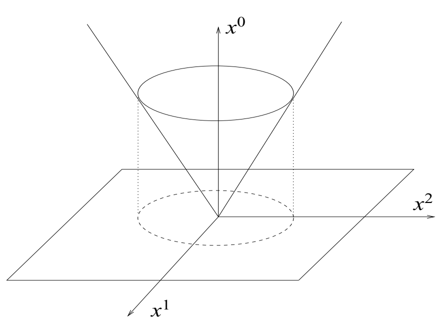


Figure 2. Three-dimensional projection of Minkowski space, with the x^3 $(= z)$ coordinate suppressed. Spherical (here circular) light wavefronts spreading out from the origin generate the so-called *light cone* (actually a hyper-cone). Events falling outside the light cone cannot be causally connected with an event at the origin

[1]Though time is still distinct because (*pace* Wells) we cannot go back in time.

It is, in fact, convenient to describe physics in terms of the geometric formulation (1911) of Hermann Minkowski, who invented the four-dimensional spacetime continuum (Fig. 2) which bears his name, and which is fundamental to all modern descriptions of relativity theory. In Minkowski space, the 'distance' δs between two neighbouring points at $\left(x^0, \vec{x}\right)$ and $\left(x^0 + \delta x^0, \vec{x} + \vec{\delta x}\right)$ is given by the pseudo[2]-Pythagorean formula

$$\delta s^2 = \left(\delta x^0\right)^2 - \left(\delta \vec{x}\right)^2 \qquad (2)$$

which means that light wavefronts emitted from the point $\left(x^0, \vec{x}\right)$ are just the spheres corresponding to $\delta s = 0$. One crucial demand of relativity is that the coefficients $\Lambda_{\mu\nu}$ should be such that $\delta s'^2 = \delta s^2$, which makes the transformation (1) between moving frames of reference rather like a rotation in the four-dimensional space. An obvious consequence of $\delta s = 0$ for light is that the path of a light ray satisfies

$$\left(\frac{\delta \vec{x}}{\delta x^0}\right)^2 = 1 \qquad (3)$$

which is the equation of a straight line in the three euclidean dimensions. This incorporates the ancient result that light travels in straight lines.

In 1914, a young Finnish relativist, Gunnar Nordstrom, tried adding a fifth (invisible) dimension to Minkowski space in a brilliant attempt [3] to unify Newtonian gravity with electromagnetic theory as formulated by Maxwell. Nordstrom formulated the Newtonian scalar potential Φ as the fifth component of the electromagnetic potential A_μ, and started with Maxwell's equations in five dimensions, hoping that its four-dimensional projection would yield both the usual Maxwell equations as well as Newton's law of gravitation. This theory failed to work, but one can only admire the courage and prescience of the young Finn who sought to thus unite two completely different forces in so ingenious a manner.

The correct four-dimensional theory of gravity, viz. Einstein's theory of General Relativity (GR), Fig. 3, appeared a year later, in 1915 – and though the basic idea are quite simple, it turned out to be far more mathematically complicated than Newton's simple theory, replacing the simple scalar gravitational potential Φ by a bunch of ten potential functions grouped into a

[2]'Pseudo' because of the negative sign.

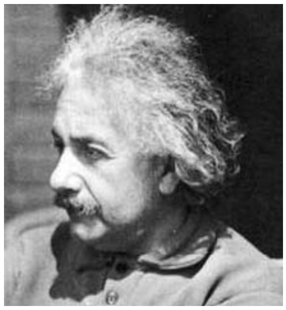


Figure 3. Albert Einstein's theory of Special Relativity extended the three-dimensional world in a scientific sense to four dimensions. His follow-up theory of General Relativity is still the best description we have of gravity in the classical sense

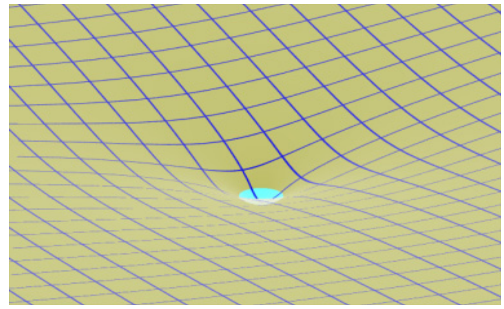


Figure 4. The 'rubber sheet analogy'. In the vicinity of the Sun, spacetime curves like a rubber sheet on which a weight has been placed, leading to the bending of light from distant stars, which follows the lines marked on the sheet. The curvature in this two-dimensional cartoon is greatly exaggerated

structure called a tensor, generally written as a matrix

$$\mathbb{G} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & g_{13} \\ g_{02} & g_{12} & g_{22} & g_{23} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{pmatrix} . \tag{4}$$

These ten functions $g_{\mu\nu} (\mu \leq \nu = 0, 1, 2, 3)$ determine the shape and curvature of the spacetime continuum, in which all material things are embedded. The path of light still follows the extremal property $\delta s = 0$, but now we have to re-define

$$\begin{aligned} \delta s^2 &= g_{00} \left(\delta x^0\right)^2 + g_{11} \left(\delta x^1\right)^2 + g_{22} \left(\delta x^2\right)^2 \\ &+ g_{33} \left(\delta x^3\right)^2 + 2 g_{01} \delta x^0 \delta x^1 + 2 g_{02} \delta x^0 \delta x^2 \\ &+ 2 g_{03} \delta x^0 \delta x^3 + 2 g_{12} \delta x^1 \delta x^2 + 2 g_{13} \delta x^1 \delta x^3 \\ &+ 2 g_{23} \delta x^2 \delta x^3 \end{aligned} \tag{5}$$

which reduces to the Minkowski form when $g_{00} = 1$, $g_{11} = g_{22} = g_{33} = -1$ and the rest vanish, i.e.

$$\mathbb{G} = \mathrm{diag.}\,(1, -1, -1, -1,) \ . \tag{6}$$

It is the presence of matter which makes these functions $g_{\mu\nu}$ deviate from their Minkowski values, which correspond to the case of Special Relativity, i.e. reference frames moving with uniform velocities relative to each other. A deviation from Minkowski space may, therefore, be identified with the presence of acceleration or equivalently, gravitational fields[3]. Einstein's field equations of GR, from which one relates the $g_{\mu\nu}$ to the matter-energy density in the universe are, therefore, the field equations of the gravitational field. Unlike the field equation $\nabla^2 \varphi = -4\pi G \rho$ of Newtonian gravity, however Einstein's equations are non-linear, though they do reduce to the linear law of Newton in the limit when the gravitational field becomes very weak. This limiting behaviour explains why Newton's law has proved so successful in explaining all terrestrial and astrophysical phenomena for 300 years. The only anomaly in Einstein's day happened to be a tiny discrepancy between the calculated and observed orbits of the planet Mercury, which being nearest to the Sun, feels the strongest gravitational attraction. This tiny discrepancy – at the level of one part in a hundred *million* – disappeared when GR was used instead of Newtonian gravity. In 1916, the German mathematician Karl Schwarzschild, while soldiering on the Russian front in World War I, discovered that the solution of Einstein's equations in the neighbourhood of an isolated massive spherical body such as the Sun leads to

$$\begin{aligned} \delta s^2 &= \left(1 - \frac{2 G_N M}{c^2 r}\right) \left(dx^0\right)^2 \\ &- \frac{\delta r^2}{1 - \frac{2 G_N M}{c^2 r}} - r^2 \left(\delta\theta^2 + \sin^2\theta \, \delta\varphi^2\right) , \end{aligned} \tag{7}$$

where M is the solar mass and (r, θ, φ) are spherical polar coordinates chosen with the centre of the Sun as origin. Clearly $\delta s = 0$ no longer leads to a straight line in the Euclidean coordinates, and this means that light must bend away from a straight line in the neighbourhood of the Sun. This is equivalent to a curva-

[3]That gravity manifests as an acceleration rather than a force, was discovered by Galileo long ago, and demonstrated in his famous experiment: dropping cannonballs from the Leaning Tower of Pisa.

ture effect in space, illustrated (as far as is possible) in Fig. 4. Three years later, Einstein and GR was proved correct when a British team led by Sir Arthur Eddington travelled to Principé Island in the South Atlantic where a total solar eclipse made it possible to measure the bending of light from distant stars in the vicinity of the Sun and its strong gravitational field. This result not only made Einstein world-famous, but also made the scientific community sit up and take notice of relativity, which had been considered arcane and akin to metaphysics by the more hard-headed specimens in the scientific community.

This success story of GR probably inspired the work of Theodore Kaluza, (Fig. 5) a young Polish teacher of mathematics, who, late in 1919, sent a paper [4] to Einstein in which he had shown that five-dimensional GR, with a circular fifth dimension of the kind introduced by Nordstrom, does indeed separate into four-dimensional GR plus Maxwell's equations in the limit when the circle shrinks to a point. This was, in effect, inverting the idea of Nordstrom: instead of a higher-dimensional Maxwell theory producing gravity in the four-dimensional theory, we now have a higher-dimensional theory of gravity which produces the Maxwell theory in the four-dimensional limit. Schematically we can write this as

$$\lim_{R\to 0} S^{(5)}_{\rm gravity} = S^{(4)}_{\rm gravity} + S^{(4)}_{\rm Maxwell}\ , \tag{8}$$

where S denotes an action integral[4], the subscript 'Maxwell' refers to electromagnetic theory and R is the radius of the extra dimension. In order to obtain this results, we require to identify some of the components of the 5×5 metric tensor with the electromagnetic potential, i.e. $g_{05} = A^0, g_{15} = A^1, g_{25} = A^2$ and $g_{35} = A^3$, while the remaining 4×4 block is the usual metric tensor, defining the gravitational fields. Given that fact that gravity and electromagnetism are different-looking phenomena, this is a simply amazing – and unexpected – result, often known as the Kaluza 'miracle', and it is hard to believe that so beautiful a theory can be anything but the real description of Nature.

The marriage of two such disparate theories is not, however, free from its own internal dissonances. We have to account for the fact that the strength of the gravitational interaction is terribly weak, compared to

Figure 5. Kaluza and Klein showed that electromagnetism can be unified with gravity if there is a fifth dimension

the strength of the electromagnetic interaction. Mathematically, this is expressed by the oft-quoted ratio $F_{\rm grav}/F_{\rm em} \sim 10^{-40}$, but one can get a better physical feeling for this disparity by the realisation that when one stands on the ground, the electrostatic repulsion between two layers of atoms, viz. the upper surface of the floor, and the other the lower surface of one's shoes – is enough to balance the gravitational attraction *of the entire earth* on one's body. To obtain this enormous ratio in the Kaluza theory where both interactions have a common origin, then, we require to set the only available free parameter R to an extreme value, viz. $R \sim 10^{-33}$ cm.[5] It follows (Section 3) that the masses of elementary particles in this theory are either zero or are proportional to $1/R$, which is equivalent to the enormous Planck energy scale of 10^{19} GeV. This is obviously contrary to experience, since we have several elementary particles with masses ranging between 1 eV to 100 GeV. Even otherwise, however, Kaluza's formulations was not without its internal flaws, and the great Einstein himself blew alternately hot and cold about it until a better version of the theory was developed (1926) by Oskar Klein [5](Fig. 5). The improved Kaluza-Klein theory had Einstein's full stamp of approval. Writing to the aged Hendrik Lorentz (1927), he wrote "It appears that the union of gravitation and Maxwell's theory is achieved in a completely satisfactory way by the five-dimensional theory". Despite Einstein's enthusiasm, however the mass problem was not solved, and nobody had the slightest inkling that a solution would be seventy years in coming. By 1930, Einstein himself had moved away from Kaluza-Klein theory and had started to develop the torsion-gravity version of unification which was to occupy him till the end of

[4] Experts will readily recognise these as the Einstein-Hilbert action $S^{(4)}_{\rm gravity} \sim \int d^4x\sqrt{-\det\mathbb{G}}\ \mathcal{R}^{(4)}$ and the Einstein-Maxwell action $S^{(4)}_{\rm Maxwell} \sim -\frac{1}{4}\int d^4x\sqrt{-\det\mathbb{G}}\ F_{\mu\nu}F^{\mu\nu}$.

[5] This is usually known as the *Planck length* ℓ_P.

his days.

The next quarter of a century kept physicists busy with the birth of quantum mechanics, World War II and the horrific discovery of nuclear weapons, and none of the stalwarts had much time for extra dimensions. In 1953, Wolfgang Pauli, one of the earliest GR aficionados, again turned to the Kaluza-Klein model, which he extended to *six* dimensions in an abortive attempt to explain the strong nuclear force. In doing so, he discovered an early version of what we would today call non-Abelian gauge theory, about a year and a half before the pioneering work of Yang and Mills and Shaw in this regard. However this theory required all the particles to be massless, and the interactions to be long-range, i.e. $\Phi \propto 1/r$ like gravitation and electromagnetism, whereas the strong interaction is definitely short-range, i.e. $\Phi \propto e^{-r/\lambda}/r$, where $\lambda \sim 10^{-13}$ cm. Pauli clearly believed this obstacle to be insuperable, as is clear from letters he wrote to Abraham Pais and to Yang himself [3].

During the middle years of the twentieth century, quantum field theory gradually established itself as the appropriate tool to describe fundamental interactions. Most of the physics ideas were firmly rooted in the four dimensions of Minkowski space. It was not until the (eventually Nobel-winning) invention of dimensional regularisation by Gerardus 't Hooft in 1971 that the scientific world woke up to the realisation that a quantum field theory which does not work in four dimensions may make perfect sense in other dimensions — and even have the amazing phenomenological success which the Standard Model (SM) of particle physics seems to have achieved. Kaluza-Klein theories then had a revival of sorts in the 1970s, with the advent of string theories, which live in higher dimensions, and necessarily carry many features of the Kaluza-Klein type, including the huge mass gap from zero mass to the Planck scale. In a string theory, the fundamental objects are tiny (around Planck length) one-dimensional objects called *strings*, whose different oscillation modes appear at low energies as particle-like fields like the photon, electron, quarks and so on. A string whose excitations are all boson fields[6] can be consistently defined only in 26 dimensions, fermions are found in the 'spectrum' of strings living in 10 dimensions and one can concoct string models which live in all sorts of dimensions between 4 and 11. Since the main focus in string theories has always been to study physics at the Planck scale, the problem of masses could be pushed under the carpet, claiming it to be a matter of detail — to be understood when we have the final theory. For this reason, there was little interest in Kaluza-Klein theory from the point of view of the particle physicist, and there was even less interest – even a certain amount of disbelief – in strings among particle physicists. In this adverse intellectual climate, pioneering – in fact, seminal – work [6] in a low-scale version of quantum gravity by Ignatios Antoniadis, alone (1990) and in collaboration with Karim Benakli and M. Quiros (1994), though published in reputed journals, made very little impact on the high-energy physics community at the time.

Figure 6. Pioneers of brane-world models. From L to R: Nima Arkani-Hamed, Savas Dimopoulos and Gia Dvali

It was Antoniadis, however, who got Savas Dimopoulos and Gia Dvali thinking about the possibility of extra dimensions with a phenomenological twist [7]. But it was not until March 1998, when Nima Arkani-Hamed joined up with Dimopoulos and Dvali to form the collaboration now known as ADD (Fig. 6), that extra dimensions of space were invoked in an elegant solution to the notorious *hierarchy problem* plaguing the SM of particle physics. It was only after their work [8] that the idea captured the imagination of the particle physics community. Within a month, the authors ADD had teamed up again with Antoniadis to embed their own ideas in a string theoretic framework [9], thereby setting off an explosion of interest in the area. A brilliant variation of the ADD ideas, introduced [10] by Lisa Randall and Raman Sundrum in 1999, has also gone a long way in promoting the concept as a whole. The origin of all these scientists is a triumph of the internationalisation of science: Arkani-Hamed hails from Iran, Antoniadis and Dimopoulos are from Greece and

[6]Boson fields have particle-like excitations which are *bosons*, i.e. there is no restriction on the number of such particles which can have the same quantum state. By contrast, fermion fields are equivalent to swarms of *fermions*, which obey the restriction that no two particles can occupy the same quantum state.

Dvali is from Georgia, though all of them are now working in the United States. Sundrum originates from the Australian component of the Indian diaspora, while Randall is American-bred with a typical Anglo-Saxon surname. In fact, original research in this area has come from every continent on the globe, except Antarctica. The author of this article, together with K. Sridhar (TIFR) and Prakash M. Mathews (Saha Institute), wrote some of the earliest papers [11] on this subject and was among the first to introduce it to the Indian scientific community.

A decade later, with close to 3,000 papers having been written on the subject, extra dimensions have achieved complete respectability as the most popular way – after supersymmetry – to go beyond the SM and conceive of new physics at higher energies. During the inauguration of the LHC on September 10, 2008, this theory even achieved widespread notoriety, with the media trumpeting a (proven false) doomsday theory that proton-on-proton collisions at the LHC would produce deadly black holes capable of swallowing up the earth and all its inhabitants with it.

The rest of this article is mostly devoted to explaining the basic ideas of the ADD and Randall-Sundrum (RS) models, in as non-technical a manner as clarity permits.

2. Compact Dimensions

The uninitiated reader is generally puzzled as to how there could ever be extra space dimensions. This article started with the statement that one has only to look at a solid object to conclude that space has just the three dimensions denoted traditionally by x, y and z, and no more. This is certainly true of dimensions of the non-compact type, i.e. those which stretch from $-\infty$ to $+\infty$. However as discovered by Nordstrom and Kaluza, there exists the possibility of invisible *compact* dimensions. To understand this idea, imagine a sheet of paper laid out flat on a table – clearly this is a two dimensional object, and both the dimensions have the same nature. Now imagine that the sheet is rolled up into a cylinder. This is still two-dimensional, but now one of the two dimensions has the topology[7] of a circle of radius, say R, i.e. it has become *compact*. R is known as the *compactification radius*. If now, the sheet is rolled up very tightly, so that $R \to 0$, then it will appear like a one-dimensional object to an observer whose optical resolving power is much larger than R. This is illustrated in Fig. 7, where the radius of the depicted cylinder decreases successively upwards, until the topmost cylinder looks like a line. We see, then that it is possible for a compact dimension to become invisible if it becomes very small. Obviously, if we can increase the magnifying power of our observation it will eventually be revealed. Moreover, if there can be one invisible compact dimension, there can be others too – in fact any number of them. Only experimental evidence can really establish or rule out the existence of such compact dimensions. The problem is that experiments rarely make such definitive statements. Typically a null experiment simply tells us that if there are compact dimensions, they must have size R less than such-and-such a value. Thus, there is always the non-falsifiable possibility of having tinier extra dimensions, which evade the experimental constraint.

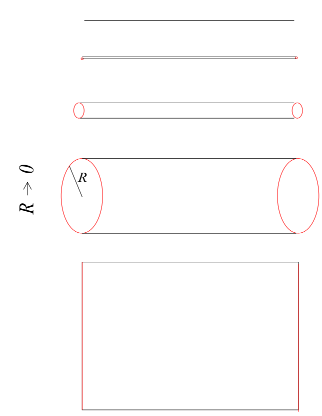


Figure 7. Compactifying two dimensions to one dimension

Stated baldly as we have done in the last paragraph, the idea of compact dimensions still appears quite bizarre. However compact dimensions may be found in any elementary textbook of quantum mechanics, and have actually been a mainstay of the quantum theory of solids for the last eight decades. The reader may recall that the wave-function of a particle in a box is described under two kinds of boundary conditions – vanishing boundary conditions, which generate standing waves, and *periodic boundary conditions*, which generate travelling waves. For an electron passing through a solid, obviously one requires a periodic boundary condition. Now, what is this periodic boundary condition?

[7]The word 'topology' is used quite deliberately, as meaning every shape which can be mapped continuously to a circle by a re-definition of coordinates using well-behaved functions. For example, a square or an ellipse has the topology of a circle, but an annulus does not.

It is the requirement that, come what may, the wave-function at a point $x = L$ will be identical with the wave-function at a point $x = 0$. There is no á priori reason to have this boundary condition[8] unless the points $x = 0$ and $x = L$ are one and the same. This makes the dimension compact, or rather the solid would be curved into a circular topology $\mathbb{S}_1$ with $2\pi R = L$, as in Fig. 8. In fact, as the periodic boundary condition is applied to all three dimensions, the typical solid studied in textbooks may be thought of as having the topology of a 3-torus $\left[\mathbb{S}_1\right]^3$. Conversely, a compact dimension may be thought of as just a periodic boundary condition in an extra degree of freedom (a.k.a. cöordinate!) of all the fields of the theory, with the compactification limit corresponding to a very small period after which the same values are repeated.

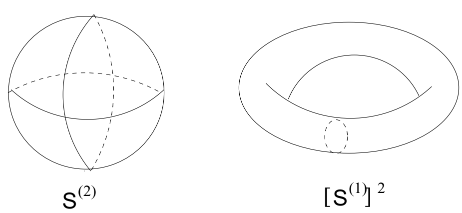


Figure 8. Compact manifolds in two dimensions: the sphere $\mathbb{S}^{(2)}$ and the two-torus $\left[\mathbb{S}^{(1)}\right]^2$

Of course, very little imagination is required to appreciate that the actual topology may not be that of a torus, but could well be that of any manifold such as a sphere or a sphere with handles, or something altogether more exotic, such as a folded or crumpled topology, or one with spikes. These would naturally correspond to mixed boundary conditions, with the period of each one being a function (or distribution) of the others. It is conceivable that one day we might learn a dynamical reason why these periodic boundary conditions develop or in other words, why some of the dimensions remain 'straight' while the others 'curl up' into tiny circles, or more complicated geometries. Till then, however we must be content with accepting compact dimensions as a phenomenological hypothesis, but not by any means an outlandish one.

[8]Most textbooks give a somewhat misleading argument that since only bulk properties matter, the wave-function over the surface does not matter, and can be taken as we wish. As early as 1912, Weyl had shown [12] that this statement is actually applicable only to the density-of-states function and specifically when the compactification takes place on a manifold.

The wave-function of a free particle propagating in five dimensions (x, y), where y is the fifth dimension in the form of a circle, will have the standard box-normalised form

$$\psi(x, y) = \frac{1}{\sqrt{\Omega_5}} e^{i(Et - \vec{p}.\vec{x} - p_5 y)}, \tag{9}$$

where Ω_5 is the volume of the five-dimensional box and p_5 is the component of momentum in the fifth (compact) direction. We choose units such that $\hbar = 1 = c$. This wave-function must satisfy the periodic boundary condition $\psi(x, y+2\pi R) = \psi(x, y)$ in the fifth direction, which means that

$$p_5 = \frac{n}{R}, \tag{10}$$

where n is an integer. The five-dimensional relativistic energy-momentum relation is then given by

$$\begin{aligned} E^2 &= M_0^2 + \vec{p}^2 + p_5^2 \\ &= \left(M_0^2 + \frac{n^2}{R^2}\right) + \vec{p}^2. \end{aligned} \tag{11}$$

For every value of n, therefore, we can conceive of a Kaluza-Klein mass

$$M_n^2 = M_0^2 + \frac{n^2}{R^2} \tag{12}$$

and a 'four-dimensional' energy-momentum relation $E^2 = \vec{p}^2 + M_n^2$. We observe that the fifth component of momentum looks like a discrete set of four-dimensional masses, which are proportional to $1/R$. The smaller is R, the higher is the first such mass $M_1 = M_0^2 + \frac{1}{R^2}$ and the others are, naturally, even higher. Thus, if $\frac{1}{R}$ is as high as the Planck mass, we shall not be able to observe any of these modes in the laboratory, which is equivalent to saying that in all experiments performed by us, we will have $p_5 = 0$, i.e. no momentum exchange with the fifth dimension.

What do the Kaluza-Klein masses M_n correspond to? To see this, let us imagine a multi-dimensional world, in which four of the dimensions are described by the usual Minkowski variables x^μ and the remaining D compact dimensions, $y_1, y_2, \ldots, y_D$ by a D-torus, i.e. a set of periodic boundary conditions under $y_i \to y_i + 2\pi R$. In the compactification limit, as $R \to 0$, we shall have $y_i \to 0$ for all $i = 1, D$. Now, we imagine a scalar 'bulk' field, $\Phi(x^\mu, y_i)$ spread over all five dimensions, and satisfying a five-dimensional Klein-Gordon equation

$$\left(\partial_t^2 - \nabla^2 - \partial_{y_1}^2 - \ldots - \partial_{y_D}^2 + M_0^2\right) \Phi(x^\mu, y_i) = 0 \,. \tag{13}$$

Because of the periodic boundary condition $\Phi(x^\mu, y_i) = \Phi(x^\mu, y_i + 2\pi R)$, we can expand the bulk scalar in a Fourier series

$$\Phi(x^\mu, y_i) = \sum_{\vec{n}=0}^{\infty} \Phi^{(\vec{n})}(x^\mu) e^{i\frac{\vec{n}.\vec{y}}{R}} \;, \tag{14}$$

where $\vec{n} = \{n_1, n_2, \ldots, n_D\}$ and $\vec{y} = \{y_1, y_2, \ldots, y_D\}$. Substituting this series into the bulk Klein-Gordon equation, we immediately obtain, for each four-dimensional Fourier coefficient $\Phi^{(\vec{n})}(x^\mu)$, the equation

$$\left(\partial_t^2 - \nabla^2 + \frac{\vec{n}^2}{R^2} + M_0^2\right) \Phi^{(n)}(x^\mu) = 0 \;, \tag{15}$$

which is simply the Klein-Gordon equation in four dimensions, with a mass

$$M_{\vec{n}} = \sqrt{M_0^2 + \frac{\vec{n}^2}{R^2}} \;. \tag{16}$$

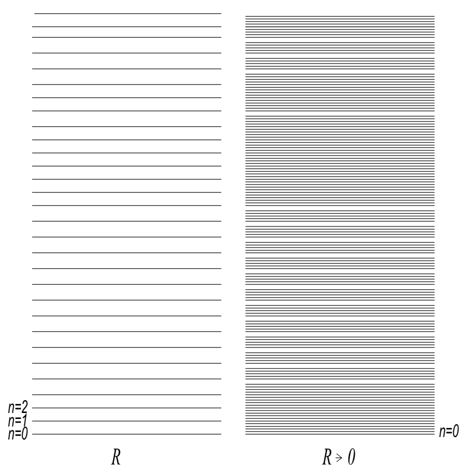


Figure 9. Illustrating a Kaluza-Klein tower of states with increasing mass. On the left, an intermediate value of R is assumed, while the right shows the case for a very small R. Note how the states become closely-spaced as $R \to 0$, forming a quasi-continuum. For this figure it is assumed that $M_0 = 0$

In the compactification limit $\vec{y} \to \vec{0}$, the bulk field $\Phi(x^\mu, y_i)$ reduces to

$$\Phi(x^\mu, y_i) = \sum_{\vec{n}=0}^{\infty} \Phi^{(\vec{n})}(x^\mu), \tag{17}$$

i.e. a sum of scalar fields with every increasing masses $M_{\vec{n}}$. This set of scalar fields, which will everywhere replace the bulk scalar field in the interaction Lagrangian when we take the compactification limit, is referred to as a *KK tower* of states (Fig. 9). The individual fields $\Phi^{(\vec{n})}(x^\mu)$ are called *KK modes*. Thus, for example, if we have a Yukawa interaction of this bulk scalar with a fermion field $\psi(x)$ which lives in four dimensions only, the interaction term in the action will look like

$$S_{\rm int} \;=\; \int d^4x d^D y \; g_Y^{(5)} \; \bar{\psi}(x)\psi(x)\Phi(x,\vec{y}) \, (2\pi R)^d \, \delta^D(\vec{y}) \tag{18}$$

where the delta function serves to confine the interaction term to the four dimensions. Clearly, integrating over the delta functions and using Eq. (17) reduces this to

$$\begin{aligned} S_{\rm int} &= \int d^4x \; g_Y^{(4)} \; \bar{\psi}(x)\psi(x) \sum_{\vec{n}=0}^{\infty} \Phi^{(\vec{n})}(x) \\ &= \sum_{\vec{n}=0}^{\infty} \int d^4x \; g_Y^{(4)} \; \bar{\psi}(x)\psi(x)\Phi^{(\vec{n})}(x) \;, \end{aligned} \tag{19}$$

which means that every KK excitation leads to a separate four-dimensional Yukawa term, with a coupling constant

$$g_Y^{(4)} = (2\pi R)^d \, g_Y^{(5)} \;. \tag{20}$$

While all these are technically possible, the trouble arises because of Eqs. 16 and 20. In Kaluza's original theory, where one extra dimension gives rise to the electromagnetic interaction, an analogue of Eq. 20 is applicable, i.e. the electronic charge is given by the relation $1/e = 2\pi R \frac{1}{\sqrt{16\pi G_N}}$, where $e \approx 0.3$ is the well-known electronic charge (in natural units) and G_N is the Newton constant. Recalling that $\sqrt{16\pi G_N} = 2/M_P$, we immediately obtain $1/e = \pi R M_P$, i.e. $1/R = \pi e M_P \approx M_P$. Invoking Eq. 16, this means that all the KK excitations – apart from the massless $\vec{n} = \vec{0}$ mode – will be tremendously heavy.

One can get around this argument, but at a heavy cost. If we are to have reasonable Kaluza-Klein masses, then we require $1/R \ll M_P$, i.e. $M_P R \gg 1$, which means that $e \ll 1$. In this case, the spin-1 exchange interaction obtained by Kaluza and Klein cannot be electromagnetism, but must be some fantastically weak force impossible to detect in the laboratory. This is not ruled out experimentally, nor can it ever be ruled out. However, given that Kaluza's original idea was to obtain electromagnetic theory out of a higher dimensional

GR theory, this appears, at first sight, to be a case of throwing away the baby with the bath water. This, in fact, was precisely Einstein's argument, which led to the original abandonment of Kaluza-Klein theory as a model of unification.

However, we now have to take into account the changes which have occurred in classical and quantum field theory since the early days of Kaluza. Not only have we discovered new forces (the strong and weak interactions) which cannot be derived from a higher dimensional GR theory without ascribing some absurdly artificial properties to the extended spacetime, but the weak interaction seems to be unified with electromagnetism. Today we believe that the electroweak and strong interactions are gauge theories, which are essentially a by-product of quantum mechanics. Thus, the requirement to generate electromagnetism out of a higher dimensional GR is no longer imperative, as it used to be in Kaluza's time. Ergo, we are no longer constrained to maintain small values of R and hence there is no longer a problem in having Kaluza-Klein states of intermediate mass.

Though the above fact was well-known from the 1970s, there was, quite naturally, resistance to give up as beautiful a construction as that of Kaluza and Klein, especially as that would mean removing the original motivation for higher-dimensional theories. The 1970s, however brought a new motive for more dimensions, because a relativistic string theory can be consistently developed only in 26 or 10 dimensions, depending on the boundary conditions. While string theory did borrow the idea of compactification from Kaluza-Klein theory, there was never any need to generate spin-1 gauge interactions from GR á lá Kaluza, since a vibrating string has enough spin-1 modes to be directly identified with the gauge interactions. Thus, though the ground for a Kaluza-Klein theory with accessible masses was already prepared by the time of the ADD revolution, there was no real reason to expect such masses. Thus, it was quite in accordance with one of the cardinal principles of science, viz. Occam's razor: *entities are not to be multiplied without necessity*, that no one thought it worthwhile to consider Kaluza-Klein masses of intermediate range.

3. Experimental Straightjackets

What had been largely abandoned by particle physicists and quantum field theorists continued to play a significant role in a then somewhat obscure branch of fundamental science – the so-called 'fifth force' experiments. These were the experiments devised to look for tiny deviations from Newtonian gravity which would signal corresponding deviations from Einstein gravity as well. Such deviations would be expected if there are extra compact dimensions, as well as in rival theories of gravitation, such as dilatonic gravity and the Brans-Dicke theory.

Why should one look for alternative theories of gravitation when the Einstein theory – and its Newtonian limit – work so well? The most striking reason to be dissatisfied with the Einstein theory lies in the so-called *cosmological constant problem* [13]. This is because Einstein's field equations of gravitation, which relate the gravitational potentials $\mathbb{G}$ to the energy-momentum tensor $\mathbb{T}$ can always be modified by replacing $\mathbb{T}$ by $\mathbb{T} - \Lambda\mathbb{G}$ (the negative sign is a convention). Here Λ is an unknown constant, called (by Einstein) the 'cosmological constant'. Noting that a uniform matter density ρ pervading all space would lead to $\mathbb{T} = -\rho\mathbb{G}$, it is easy to identify the cosmological constant with the vacuum energy density demanded in a *quantum* field theory, or to use a more fancy contemporary expression 'dark energy'. The vacuum energy is a purely quantum mechanical phenomenon, which can be traced ultimately to the *uncertainty principle*, one of the pivots on which the entire framework of quantum mechanics has been constructed. Since we have a superabundant wealth of evidence that the world is indeed quantum mechanical, it is hard to wish away the vacuum energy. In fact, adding this term is perfectly consistent with all of Einstein's initial assumptions, and hence, a good scientific procedure would be to keep this term and try to determine it from the observational data. If we do this, we obtain the result that, in units where $\hbar = 1 = c$

$$\Lambda \lesssim 1.3 \times 10^{-85}\ \mathrm{GeV}^2\ , \tag{21}$$

which means that the energy scale corresponding to the vacuum energy is around 1.1×10^{-33} GeV. This is amazingly small compared to the lightest known mass scale, that of the lightest neutrino, which is around 1 eV. Moreover, given that the vacuum energy will receive contributions from physics at all scales, including the Planck scale, there is no theoretical reason not to expect the vacuum energy to be of the order of the Planck scale, i.e. 10^{19} GeV. If we accept this argument, we must conclude that the observed vacuum energy is roughly 10^{-52} times smaller than the theoretical prediction. The cosmological constant problem lies in asking why the value of Λ is so tiny. The mind boggles at the smallness of the number 10^{-52} – better realised if we consider the fact that 10^{+52} is roughly the number of atoms contained in 10 million galaxies. One simple way out would be to postulate that there is some symmetry which forbids the writing of the $\Lambda\mathbb{G}$ term, but this cannot be invoked because that would mean that we cannot write $\mathbb{T} = \rho\mathbb{G}$ either, and that means that we would forbid the universe to have a uniform density. It would have to be a very strange symmetry indeed, which forbids this most symmetric of matter distributions! It is more practical to set $\Lambda \simeq 0$ by hand – for the moment, at least.

Once we accept that there are good reasons to look beyond the minimal theory of Einstein, the road to understanding gravity by doing experiments is clearly indicated. 'Fifth force' experiments perform precisely this task – they look for deviations from the inverse square law in highly sensitive (mostly terrestrial) experiments. Now it is easy to see that such deviations are directly predicted [14] if there are extra dimensions of the Kaluza-Klein type. To see this, let us consider, for simplicity, one extra dimension with the topology of a circle of radius R. We have seen that this leads to a scalar field having a four-dimensional projection $\Phi(x,0) = \sum_{n=0}^{\infty} \Phi^{(n)}(x)$, where each mode $\Phi^{(n)}(x)$ has mass $M_n = n/R$. Considering the Newtonian approximation, the gravitational potential will be just such a scalar field, which in the static limit with a point source m at the origin will satisfy the equations

$$\left(\nabla^2 + M_n^2\right) \Phi^{(n)}(\vec{x}) = -4\pi G_N \, m \, \delta^3(\vec{x}) \tag{22}$$

for each mode. This is the well-known Helmholtz equation with the solution $\Phi^{(n)}(x) = G_N m \, e^{-M_n r}/r$. Thus, the effective gravitational potential will be

$$\begin{aligned} \Phi(x,0) &= \sum_{n=0}^{\infty} G_N m \frac{e^{-nr/R}}{r} \\ &= \frac{G_N m}{r} \left[1 + \frac{e^{-r/R}}{1 - e^{-r/R}}\right] , \end{aligned} \tag{23}$$

summing the geometric series. In the limit $r \gg R$, the exponential term in the denominator can be neglected, so that we get the simple form

$$\Phi(x,0) = G_N m \left(\frac{1}{r} + \frac{e^{-r/R}}{r}\right) , \tag{24}$$

where clearly the deviation from the Newtonian $1/r$ form becomes significant only when r begins to be comparable to R.

The most sensitive verifications of the inverse square law are achieved through *torsion balance experiments* (Fig. 10). Starting from the pioneering work of Henry Cavendish in 1797 to the sensitive experiments of Baron Loránd von Eötvös during 1906–1909, to the state-of-the-art measurements of the Eöt-Wash group, currently in progress at the University of Washington, the basic idea is always the same. A pair of heavy objects are fixed to the two ends of a horizontal rod suspended from a fine wire[9]. The objects are then brought into the proximity of two other heavy objects, as shown in Fig. 10, so that the feeble gravitational force between them rotates the rod by a tiny angle. This is measured and the force of gravity is calculated from the angle of deviation. The Eöt-Wash experiment, led by Eric Adelberger, where the apparatus is mounted on a small hill which acts as the attracting mass, the simple rod is replaced by an ingenious arrangement of a cut-away cylinder and a disc with holes, so that the effect of pure $1/r^2$ forces cancels to zero. Thus, in the Eöt-Wash experiment, any twist in the wire becomes a fifth-force effect. Similar experiments have been performed or suggested – in an Australian mine shaft, in a bore hole in the Arctic icecap in Greenland and in a torsion balance to be mounted on a satellite and sent into outer space. Some of these experiments were also performed in the 1990s, in India, at the author's home institute [15].

What are the conclusions of all these experiments? In a nutshell, all the results are completely consistent with Newton's inverse square law of gravity and with the exact principle of equivalence, i.e. that gravity manifests as an acceleration rather than a force.[10] This means that, once the dust of several incorrect results reported in the 1990s has settled, all searches for the fifth force have yielded negative results. One can therefore use these null results to constrain fifth force theories, and in particular, to put bounds on the size of Kaluza-Klein dimensions. The best state-of-art results come from the Eöt-Wash experiment [16], where the fifth force is parameterised as

$$\Phi^{(V)}(r) = \alpha G_N m \frac{e^{-r/\lambda}}{r} , \tag{25}$$

where α and λ are strength and range parameters respectively. Obviously, a null result will rule out large values of both, and hence the Eöt-Wash results are presented as a forbidden region in the α-λ plane, as shown in Fig. 11. Comparing with Eq. 24, one immediately sees that Kaluza-Klein theories may be identified with $\alpha \approx 1$ and $\lambda = R$. The current data, then, require $R \lesssim 160\ \mu$m, while the experiment will eventually probe

[9] Quartz fibres are the best for this purpose.

[10] Elevated to a definition of gravity by Einstein.

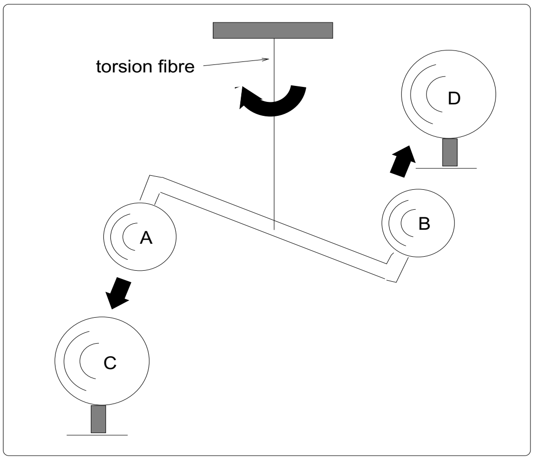


Figure 10. The basic torsion balance experiment. The masses A and B, affixed to the balance bar are attracted by the fixed masses C and D, as indicated by the arrows. The torque due to these forces is balanced by the elastic reaction of the fibre as shown. The bounding box indicates that the apparatus must be isolated from air currents and all kinds of noise, as the torsion effect is very weak

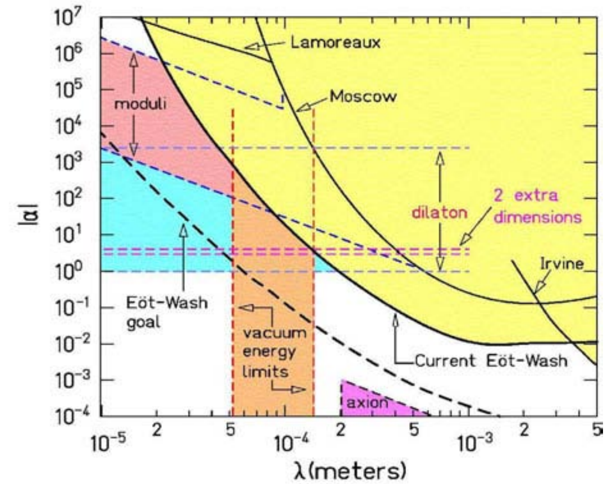


Figure 11. Results of various fifth force experiments ruling out parts of the α-λ plane. The solid boundary of the yellow-shaded region is the current result of the Eöt-Wash group, and the parallel dashed line represents the limit of sensitivity expected to be reached eventually at this experiment

as far as $R \approx 45\ \mu$m.

Once can say, therefore, that studies of gravity are consistent with the presence of extra compact dimensions so long as they are not larger than a hundred microns or so. However, we do have a wealth of microscopic probes which are smaller than this – notably atoms and molecules, which are at least a million times smaller. To such a minute particle, a 100-micron compact dimension would hardly be different from a non-compact one, and hence, we should observe their behaviour as if they live in $4+d$ dimensions. For example, an electron moving in the electric field of a nucleus would feel an electrostatic force proportional to $r^{-(2+d)}$, instead of just r^{-2}. Now this certainly does not happen, since it is well known that anything other than an inverse square law of force would lead to splitting of the Bohr energy levels of a hydrogen atom, giving different energies[11] to different values of the azimuthal quantum number ℓ. This would change the spectrum emitted considerably from what is observed in a classic Geissler tube experiment. Even high-school learning is enough, therefore, to conclude that the atomic electrons, at least, do not 'see' compact dimensions, which pushes R down to about a nanometre.

High energy experiments probe much smaller lengths than atomic spectra. This is because quantum mechanics tells us that each particle may be associated with a wave whose wavelength λ is inversely proportional to the particle momentum, i.e.

$$\lambda = \frac{h}{p} \;, \tag{26}$$

where p is the momentum and $h = 6.678 \times 10^{-27}$ erg-s is Planck's constant. It is fairly straightforward to show that the wavelength corresponding to an electron of energy 100 GeV — which is what was achieved at the now-defunct LEP machine (1991–2001) at CERN — is around 10^{-18} m, i.e. a billion times smaller than a nanometre. Even so, there were no indications that these electrons interacted in anything but the canonical four dimensions of Minkowski and Einstein. Precision results to the level of one in 100,000 were obtained from this machine, and these would certainly have changed if there were compact dimensions of any size comparable to the wavelength $\lambda \sim 10^{-18}$ m. It is more or less experimentally certain, therefore, that there are no compact dimensions of size $> 10^{-18}$ m, and hence, no

[11]In reality, all the ℓ values have the same energy, except for some small effects associated with the electron spin, a fact referred to as *accidental degeneracy* – this is intimately connected with an $SO(4)$ symmetry of the Hamiltonian in the specific case of an inverse square law.

Kaluza-Klein masses less than a few hundred GeV/c^2. This is a matter for little concern for string theories, since they are compatible with compact dimensions as small as 10^{-35} m, but it is disappointing for extra dimension enthusiasts who would like to see their ideas verified in the present generation of experiments.

The only way to rescue compact dimensions from the tight corner into which LEP and similar experiments have driven them is to somehow ensure that electrons and such probes remain confined to four dimensions only. The extra dimensions would then be seen only by gravity, which cannot be confined to any lower number of dimensions, since it is a manifestation of the spacetime itself. But why should such a confinement occur? The motivation for this was provided by ADD, in their classic 1998 paper, and it is to this that we now turn.

4. The Hierarchy Problem

In order to appreciate the motivation for the new class of extra dimensional models, it is necessary to introduce the reader to the hierarchy problem in the SM of elementary particle physics. In the SM, it is believed that all the elementary particles were initially massless as they separated out from pure radiation in the first moments after the Big Bang. As the nascent universe cooled below a certain critical temperature, these particles now acquired masses through their interactions with a coeval elementary scalar field $H(x)$, whose excitations are called Higgs bosons. This nice mechanism was discovered in the 1960s, but till date has not passed the test of experimental verification, which demands that we find the Higgs boson. Other articles in this volume discuss this issue, but, we may note, in passing, that the introduction of this elementary scalar is the simplest and neatest solution to the mass problem, and possibly the only one which is not seriously threatened by experimental data available at the moment. The search for the Higgs boson is, therefore the most urgent purpose for which the LHC has been constructed.

This Higgs field, which plays so crucial a role in this game, happens to be the only elementary scalar field in the SM. However, as early as 1976, it was known [17] that there is a technical problem with quantum theories containing elementary scalar fields. This arises from the self-interaction term λH^4 of the scalar field, which plays a crucial role in mass generation and, therefore, cannot be wished away. The problem arises as soon as we go beyond the classical level (in a perturbation theoretic approach) and try to compute quantum corrections to the mass of the Higgs boson. In the very first order, we immediately encounter 'infinities' proportional to the inverse size of the smallest length scale to which the theory is valid. For example, if this length scale is ℓ, then $\Delta M_H^2 \propto \lambda^2/\ell^2$. Now this is a well-known phenomenon[12] in quantum field theory, and is generally handled by arranging for a cancellation of this 'infinity' with an equally large negative term $\propto -\lambda^2/\ell^2$ in the 'bare' mass parameter in the Lagrangian. What is left, after the cancellation, is the physical mass of the Higgs boson, which is a measurable quantity, unlike the 'bare' mass parameter. Once the cancellation has taken place, we allow the 'smallest' length scale to go to zero, recovering, in the limit $\ell \to 0$, the original spacetime continuum. For elementary fermions and gauge bosons, this neat trick is enough to ensure that no further 'infinities' are encountered. For elementary scalars, on the other hand, it turns out that fresh 'infinities' arise when we compute the quantum corrections at the next order, i.e. at two loops. We can, of course, arrange another cancellation at the two-loop level, but this does not really help. For one thing, new 'infinities' would arise again at three-loops (and so on), and moreover, once the bare mass is tuned to give a cancellation at two loops, it cannot give a cancellation at one-loop. The one-loop mass would then become 'infinite', making nonsense of the perturbative approximation.

One can argue, of course, that the SM is not really valid upto zero length scales, since it ignores gravitational effects altogether. Especially, at the Planck length $\ell_P = \hbar c G_N^{-1/2} \sim 10^{-35}$ m, it is known that gravity becomes strong and will begin to dominate interparticle interactions. Taking this scale as the minimum cutoff length for the SM, then, we avoid the actual infinities, but this argument does not save the situation, since we end-up with a Higgs boson as heavy as $\hbar c \ell_P^{-1} \sim 10^{19}$ GeV. As the self-coupling λ is proportional to the mass of the Higgs boson, such a heavy Higgs boson would mean an effective value of $\lambda \sim 10^{38}$, which is quite absurd in a perturbative framework. All of this goes to show that there is an internal inconsistency in the SM, considered as a quantum field theory, with a hierarchy of scales between the electroweak scale at 10^{-18} m and a smaller scale such as the Planck scale at 10^{-35} m. This inconsistency is called the *hierarchy problem*, or equivalently, the *fine-tuning problem* (Fig. 12) . Such irreducible inconsistencies are known to be typical of an incomplete or effective theory — indicating that there is a more fundamental underlying theory,

[12] The technical name for these is *ultraviolet divergences* and the cancellation trick is called *mass renormalisation*.

of which the SM is either a part, or a low-energy limit. This conjecture is, in fact, the strongest argument for searching for new physics beyond the SM.

There are, in fact, two possible solutions to the hierarchy problem, once we accept that the SM requires to be augmented by the addition of new fields and/or interactions. One way to bypass the hierarchy problem is to introduce extra fields and couplings which generate new 'infinities' *cancelling* the existing ones (at every order) – so that there is no need to tune the 'bare' mass parameter. To ensure exact cancellation, we require a higher symmetry in the theory. Among the popular symmetries which do this job are *supersymmetry* i.e. symmetry between bosons and fermions, and the rather complicated gauge symmetries seen in the so-called *little Higgs* models. Once we have ensured such cancellations, one can make the cutoff length as small as we please, even as small as the Planck length, without endangering the perturbative framework. In a sense, this postpones the inevitable breakdown of the SM to an energy scale inaccessible except in the early universe just after the Big Bang.

The other alternative is to take the bull by the horns and place the cutoff length of the theory just below the currently-accessible scale of 10^{-18} m. In this latter case, a low mass ($\sim$ 100–200 GeV/c^2) Higgs boson arises out of weak cancellations, which are neither unnatural nor unstable against quantum fluctuations. However, this approach immediately calls for a new theory at the TeV level (i.e. at 10^{-19} m), since that scale will certainly be probed by the LHC. Extra dimensional theories, as introduced by ADD, belong to this class of solutions, the new theory being one in which gravity becomes strong at the TeV scale, thereby invalidating the SM.

At the present jucture, on the eve of the LHC run, there is no *scientific* reason to prefer any one solution of the hierarchy problem over another solution. Of course, physicists have their preferences – prejudices, if you like – but here one person's philosophy is as good as another's. Almost all these models are elegant in conception but ugly in execution. This means that each starts with a simple and attractive idea, but in trying to explain the wealth of experimental data, one has to introduce extra assumptions, which are neither so simple, nor attractive. It is, of course, possible, that all this is happening because we are reasoning with insufficient information – like a newspaper speculating about a crime. The truth may turn out to be vastly more complex, with all or many of these ideas forming a part of it, just as the six blind men in the story had each grasped a piece of the elephant without realising what the whole beast looked like. Even this statement is a speculation, however and only the LHC data will show us, at least dimly, where we really stand.

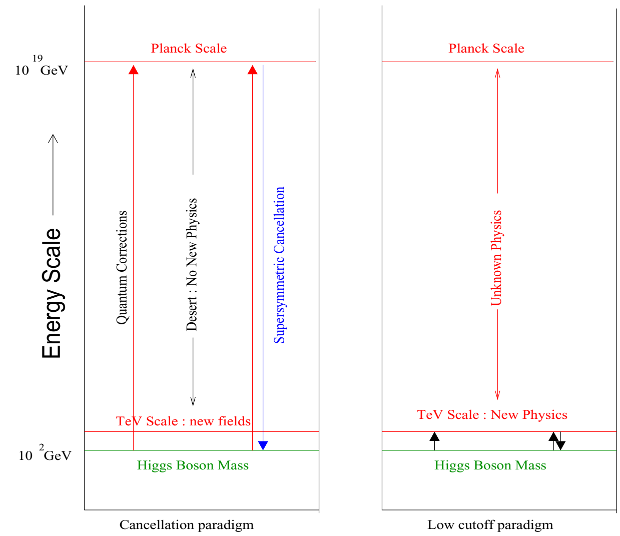


Figure 12. Illustrating the hierarchy problem and its solutions. If there is no new physics between the SM at a few hundred GeV and the Planck scale at 10^{19} GeV, quantum corrections drive the mass of the Higgs boson to the Planck scale. In cancellation-type solutions, new fields entering around a TeV cancel these quantum corrections. In low-cutoff solutions, new physics enters at the TeV scale and any cancellations which may still occur are small

5. The ADD Construction

In March 1998, the first paper [8] of ADD came out. The basic idea – like all great ideas – was startlingly simple. In a Kaluza-Klein theory, one starts by describing higher-dimensional gravity through an action integral of the form

$$S^{(4+d)} = \frac{1}{16\pi G_N^{(4+d)}} \int d^4x \; d^dy \; \mathcal{L}^{(4+d)} \; , \tag{27}$$

where $G_N^{(4+d)}$ is the actual gravitational coupling constant and $\mathcal{L}^{(4+d)}$ represents the Lagrangian density of the gravitational field. The coordinates $y_i (i = 1, d)$ represent the compact dimensions. As these shrink to a point, the action reduces to the form

$$S^{(4)} = \frac{\Omega_d}{16\pi G_N^{(4+d)}} \int d^4x \, \left[\mathcal{L}^{(4)} + \ldots\right] \; , \tag{28}$$

where Ω_d is the volume of the compact space formed by these d cooordinates and the dots represent very weak interactions which will not concern us any further in this article. The appearance of the ordinary four dimensional Lagrangian density $\mathcal{L}^{(4)}$ is what we have earlier referred to as the Kaluza miracle. Comparing Eq. 28 with the standard form

$$S^{(4)} = \frac{1}{16\pi G_N} \int d^4x \; \mathcal{L}^{(4)} \; , \tag{29}$$

where G_N is the usual Newton constant of gravitation, leads to the identification

$$G_N = \frac{G_N^{(4+d)}}{\Omega_d} \; . \tag{30}$$

In terms of the Planck length $\ell_P = \hbar c G_N^{-1/2}$, defined earlier, and its higher dimensional equivalent $\widetilde{\ell}_P \equiv \ell_P^{(4+d)}$, this leads to the relation

$$\widetilde{\ell}_P^{2+d} = A \, \ell_P^2 \; R^d \; , \tag{31}$$

where A is a constant not dramatically different from unity. If we choose $R \sim \ell_P$, as Kaluza did, then we have $\widetilde{\ell}_P \sim \ell_P$, as Kaluza found. However, if we choose R larger, we will immediately increase the length $\widetilde{\ell}_P$. For example, if we choose $d = 6$ and $R \sim 10^{-18}$ m, as allowed by LEP experiments, we get $\widetilde{\ell}_P \sim 10^{-22}$ m, which is much much larger than ℓ_P. At the length scale of 10^{-22} m, therefore, the SM will certainly break down, since we would be well inside the compact dimensions and gravity is as strong as the electroweak and other interactions at the effective Planck scale $\widetilde{\ell}_P$. Taking this length scale as the level of granularity for the SM does help in reducing the acuteness of the hierarchy problem, but it still implies Higgs boson masses of around 10^6 GeV, and a self-coupling $\lambda \sim 10^7$, showing that the problem is still there.

The observation that the hierarchy problem is ameliorated, but not solved, by a Kaluza-Klein formulation of the SM was the key observation of ADD. However, in order to actually *solve* the hierarchy problem, one has to make R larger still. To bring $\widetilde{\ell}_P \sim 10^{-19}$ m – at which stage the Higgs boson mass remains close to the expected range and λ remains perturbative – once has to choose $R \sim 10^{-14}$ m, which is about the size of a large nucleus. As we have seen there are good arguments to rule out extra dimensions of this size, unless there is a mechanism by which *they are inaccessible to matter*, such as electrons, nuclei, etc. There is, in fact, nothing crucial in this argument that was not known to Kaluza, or for that matter, the physicists of the 1970s, but they did not have the motivation to solve the hierarchy problem by expanding the Planck length to the limits of observation. Having such a strong motivation, however, ADD proceeded to create just such a mechanism, constructing a (somewhat contrived) quantum field theory in $4+d$ dimensions with a 4-dimensional domain wall[13] on which all the SM fields can be trapped. In this trapped condition, none of the SM particles can access the extra d dimensions, thus validating all the spectroscopic observations from Ångstrom to Bohr to LEP. However, gravity which is a measure of the topography of spacetime itself, cannot be thus confined, and hence, one can easily apply the arguments following Eq. 31 to this model.

Unbeknownst to themselves, ADD at that point had, in essence, rediscovered a suggestion made way back in 1983 by Misha Shaposhnikov and Valery Rubakov. In their paper [18], entitled "Do we live inside a domain wall?" the two Russian scientists had speculated that the SM fields live inside just such a narrow and deep potential well as constructed by ADD. However, they were looking for a solution to the cosmological constant problem and not to the hierarchy problem, and for reasons best known to themselves, they did not pursue the idea any further. Neither did their idea gain much currency. In fact, the idea that the universe might be a four-dimensional kink in a higher dimensional world had been suggested even earlier – in 1982 – by Keiichi Akama [19], but this did not attract any attention at all until much later, when ADD had made the concept famous. Even the original paper of ADD, which is wordy and imprecise, may have shared the fate of its precursors, had it not been for a new addition to the team. This was Ignatios Antoniadis, (Fig. 13) then at Paris, whom we have mentioned before as having pursued ideas about low-scale effects in string theory almost a decade before the ADD collaboration. In April 1998, a month after the first ADD paper, Antoniadis and ADD got together to write a paper [9] which has shaped the field of brane world physics since. This replaces the domain wall constructed so artificially by ADD with a *D-brane* – a kind of spacetime kink occurring naturally in most string theories, which had been discovered by Joseph Polchinski (Fig.13) only a

[13]A domain wall is the boundary between two different phases, such as the surface of a bubble inside a liquid or a liquid droplet suspended in air.

few years earlier. Thus, we have a very good reason for expecting the SM fields – including electrons, photons, nuclei and all the known things – to be trapped in four dimensions. If so, then the only constraints on the size of the extra dimensions come from pure gravity measurments, and these, as we have seen are pretty loose, permitting sizes as large as $R \lesssim 0.16$ mm. Even with $d = 2$, this means that $\widetilde{\ell}_P$ can be large enough to remove the hierarchy problem.

What is a D-brane? To understand this, we should first note that in a string theory, the fundamental objects are tiny one-dimensional objects, which move in a space of 10 (or 26) dimensions. These 'strings' may be open or closed, depending on whether they have free end-points, or whether they form closed loops. These strings interact among themselves, joining together and breaking up, or even forming large conglomerates. At large length scales, the strings appear like point particles, and the interactions of the strings look like the scattering of point particles among themselves. The advantage of such a theory is, of course, that one never has to take the pointlike limit of zero size, so that $1/\ell$-type singularities never appear. It can be shown that the different oscillation modes of a single string appear like different particles, so that a single string can, depending on which mode is being excited, appear as a scalar, a vector boson, or a fermion, at low resolution. Even more exciting is the fact that one of the oscillation modes of a closed string appears like a spin-2 particle, which can be identified with a graviton, the quantum of the gravitational field. A string theory is, therefore a theory of gravity as well as a theory which lives in higher dimensions, and it is natural to embed the ADD construction in such a theory.

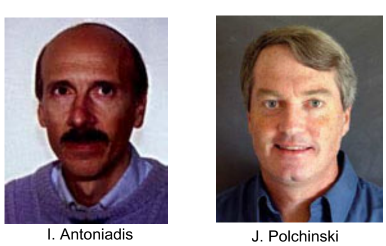


Figure 13. Brane world pioneers: Ignatios Antoniadis of CERN, Geneva and Joseph Polchinski of the Kavli Institute, University of California at Santa Barbara

One of the interesting possibilities in an interacting string theory is that under certain circumstances, massive condensates of strings may form in a lower dimension, just as a bunch of atoms (which normally move in three dimensions) can bond together to form a flat two-dimensional plate. Such a lower dimensional object in string theory is called a D-brane, and it can be treated as a dynamical object in itself, just as a plate can be treated as an object in itself [20]. However the most important property of a D-brane is that it acts like a sticky membrane for open strings, whose open ends get stuck to the D-brane[14]. Thus, all the interactions of open strings will be confined to the neighbourhood of the D-brane, within a thickness comparable to the (tiny) length of the strings, and will appear, at low resolution, to be confined to the lower-dimensional space marked by the D-brane. As we have seen that the SM fields can be identified with different oscilation modes of open strings, this offers a natural and elegant mechanism for confining SM interactions to a lower dimension. This mechanism will not work for closed strings, since there are no ends to stick to the D-brane, and hence closed strings will be free to propagate in all the 10 (or 26) dimensions. As gravitons appear among the excitations of closed strings, this means that gravity propagates in the entire spacetime.

The exact construction of the ADD-Antoniadis model is, therefore, as follows. We assume that the fundamental underlying theory is a fermionic string theory, valid in 10 (or it can be 11) dimensions, of which 6 (or 7) are compact ones, henceforth referred to as the *bulk*. For d (we shall see presently that $d \geq 2$) of these compact dimensions, the radius of compactification R is large, maybe as large as 100 μm. Hence it is possible to have a large Planck length $\widetilde{\ell}_P$ in the bulk. In addition to this, we assume that there is a D_3 brane, extending to infinity along all its three spatial directions, which we identify with the observed universe. All the SM fields correspond to oscillation modes of open strings which have one or both ends confined on the D_3 brane. This means that they are confined within a thickness δ which is indicative of the length of the strings. Closed strings are free to propagate in the brane or the bulk at will. This model is sketched in Fig. 14. The black region indicates the D_3 brane which is our universe, or rather a cutaway portion of it, since the actual brane extends to infinity in all directions. The perpendicular line penetrating the brane represents the d compact directions with large radius R, the black and white dots

[14]The name D-brane originates from this property: 'brane' is short for membrane and D refers to the fact that the ends of the string will be forced to satisfy a Dirichlet boundary condition.

being identified. This is a schematic way of indicating a d-torus, which is sketched in Fig. 8 for $d = 2$. The red squiggly lines represent strings, with the open ones stuck to the brane, while the closed ones are depicted in the bulk, away from the brane. The SM fields are, therefore confined to the box of thickness δ drawn around the brane.

One of the most elegant things about the ADD construction is that it gives us an explanation of why the gravitational interaction is so weak compared to the electroweak and other interactions. This is because electroweak interactions correspond to the interactions of open strings which are bound to the D_3 brane, and hence are closely packed together – or in the language of quantum mechanics, have overlapping wave functions. On the contrary gravity corresponds to interactions of closed strings which are free to roam around in the bulk and only occasionally cross the brane. It is only when these rare crossings occur that gravitational interactions of matter are seen. Again in the language of quantum mechanics, this means that the wavefunction of the graviton is spread over the entire bulk and has a very tiny overlap with the wavefunctions of SM particles on the brane. The gravitational interaction is, therefore suppressed by a factor governed by the brane-to-bulk size ratio, i.e. δ/R. If we take $\delta \sim 10^{-19}$ m and $R \sim 10^{-4}$ m, we will get a suppression factor of 10^{-15} which is about the ratio of the gravitational force to electroweak forces. Loosely, therefore, we may say that most of the gravitational influence of a given source is spread out through the bulk, and we on the brane, measure only a minuscule fraction of it.

d	$\widetilde{\ell}_P$(m)	$\widetilde{M}_P$	R(m)	M_1
2	10^{-20}	10 TeV	10^{-4}	10^{-3} eV
3	10^{-19}	1 TeV	10^{-8}	10 eV
4	10^{-19}	1 TeV	10^{-11}	10 keV
5	10^{-19}	1 TeV	10^{-13}	MeV
6	10^{-19}	1 TeV	10^{-14}	10 MeV
7	10^{-19}	1 TeV	10^{-15}	100 MeV

As we have seen, setting $\delta \sim 10^{-18}$ m ensures that this model is consistent with all precision tests showing that the SM fields interact in three spatial dimensions only. Setting $R \lesssim 0.1$ mm makes everything consistent with gravity experiments of the Eöt-Wash type. This means that the effective Planck length can be made as large as $\widetilde{\ell}_P \sim \delta \sim 10^{-19}$ m, by choosing R suitably, according to the formula given in Eq. 31. Taking $A \sim 1$, we obtain $R \sim 10^{-19+32/d}$ m, which is exhibited in the table below for $d = 2$–7. The choice $d = 1$ is omitted, as it leads to $R \sim 10^{14}$ m, which is patently absurd since it is as large as the solar system.

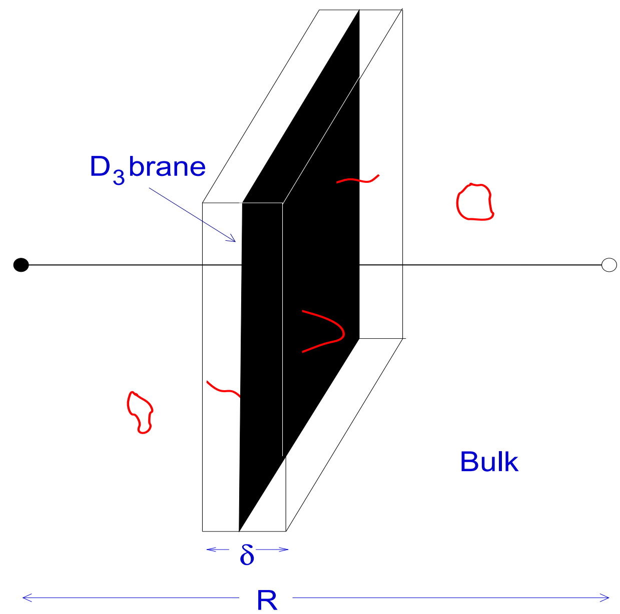


Figure 14. Sketch explaining the ADD model with a D_3-brane, shown as a dark surface

The second and third column in the above table refer to the effective Planck length and Planck mass in the bulk, i.e. the level at which the SM begins to fail. This has been kept at a TeV (= 1000 GeV), for $d \geq 3$, so that there is no chance of a hierarchy problem appearing. For $d = 2$, the same value of $\widetilde{\ell}_P$ would lead to R around a millimetre, which is ruled out by the Eöt-Wash data. However, taking $\widetilde{\ell}_P$ an order of magnitude smaller, which means cutting of the SM at 10 TeV instead of 1 TeV, leads to an acceptable value of R, as shown. The fourth column represents the required radius of compactification, and it is easy to see that such small dimensions are not likely to be probed soon in gravity experiments of the Eöt-Wash nature. In fact, as d increases, the required R rapidly shrinks to the nuclear size, i.e. a femtometre. The fifth and final column respresents the spacing between the masses of Kaluza-Klein excitations, which is proportional to $1/R$, and it can be seen that it is always very small compared to the energies (around a 100–1000 GeV) at modern colliders such as the LHC. This is an important result and it leads to the most exciting feature of the ADD model, viz. the possibility that it would lead to observable signatures at existing and upcoming collider experiments.

How do these signatures come about? We must note that all experiments take place on the brane, being part of the observable universe. Considered from the point of view of an observer located on the brane, the ADD model is mostly the SM (with a cutoff at $\widetilde{\ell}_P \sim 10^{-19}$ m) in addition to very weak gravitational interactions which occur whenever the bulk graviton crosses the brane. If we could do the experiment in all the $4+d$ dimensions, we would observe the bulk graviton having an interaction strength $\widetilde{G}_N = 2\widetilde{\ell}_P^2$, which is almost of electroweak strength, since $\widetilde{\ell}_P \gg \ell_P$. However, we ourselves are bound to the brane and hence must look at the interaction from a four-dimensional perspective. On the brane, Eq. 17 shows that the massless bulk graviton field will reduce to a tower of four-dimensional graviton fields or *KK modes*, each having mass $M_{\vec{n}}$ given by Eq. 16 with $M_0 = 0$. This means that we have a dense quasi-continuum of masses all the way up to the cutoff scale $\widetilde{M}_P$. We may recall that each massive mode corresponds to having a certain amount of momentum in the bulk, as shown by Eqs. 10–12. Now each mode will interact like a single graviton in four-dimensional Einstein gravity, analogous to the interaction shown in Eq. 19, and with the usual coupling $G_N = 2/\ell_P$ as indicated by Eq. 20. This is so weak an interaction that the probability for a single KK mode to be produced is too small to see even a single such event in the entire decade-long run of the LHC. Should such an event occur, however the single graviton mode is likely to fly off undetected since it will hardly interact at all with the matter in the detectors. Thus, one would detect the other partners in the reaction, and conclude that some energy and momentum has gone missing.[15] The real point of departure for the ADD model is that this should happen in the *same* way for *every* KK mode. This means that the total probability for such an event to happen will get multiplied by the total number of KK modes available, which must be given by the machine energy E divided by the KK mass spacing, which is around $1/R$. The table of mass spacings shown above immediately tells us that for $E \sim 10^3$ GeV – typical at the LHC – this factor varies between 10^{15} and 10^4 as d goes from 2–7. This huge number of KK modes is enough to offset the low probability of producing a single KK mode, and we end up with a decent probability of seeing events with a substantial missing energy and momentum at the LHC, especially for missing energy goes to create the mass of the KK modes, i.e. a momentum component in the bulk. This corresponds to gravitons interacting on the brane and flying off into the bulk, which is the behaviour expected of the closed strings whose excitations the gravitons correspond to.

[15]Such missing energy and momentum signals have been traditionally used to infer the presence of the weakly-interacting neutrinos, and are suggested as signals of other theories with weakly-interacting particles, such as supersymmetry.

If the above discussion seems too abstract, let us fix our ideas by considering a particular process at the LHC. This was first studied (among many other processes) by Gian Giudice and his collaborators at CERN, and almost immediately followed by similar studies by Michael Peskin and collaborators at Stanford, and by Tao Han, Joe Lykken and Ren-zie Zhang at Fermilab. All these papers [21] were made public about six months after the ADD-Antoniadis paper, within a single eventful week in November 1998, and they provided the immediate trigger for the explosion of interest that followed. The process we concentrate on here is that by which the protons in the LHC beams collide to create a photon and a KK mode of the graviton. A typical Feynman graph for this process is shown on the left of Fig. 15.

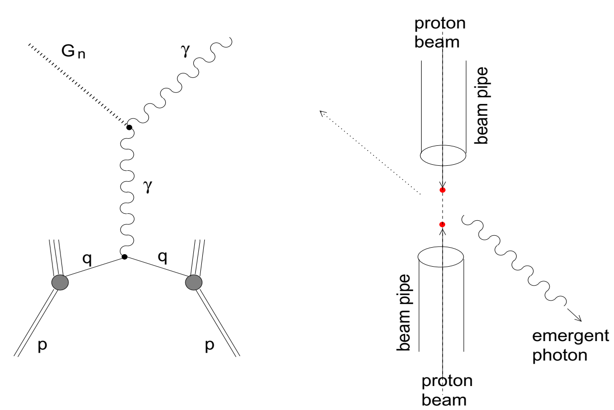


Figure 15. Single photon production in association with an invisible KK mode of the graviton in proton-proton (pp) collisions at the LHC. A typical Feynman diagram is shown on the left, with q denoting a quark or antiquark in the proton, and G_n indicating a KK mode. The three little lines indicate the rest of the proton, which continues down the beam pipe and is lost. The actual event topology is shown on the right. With a dotted line showing the hypothetical path taken by the unobserved graviton

In this graph, at the moment of collision of the two protons, a quark from one proton and an anti-quark from the other get annihilated, producing a virtual photon, which immediately decays into a real photon and a KK mode of the graviton. Any of the modes can be produced in this way, so long as there is enough energy in the initial states to be converted into the mass of the KK mode, viz. $M_{\vec{n}}$. Now the KK mode will, as explained above, go undetected, so that the event topology will appear as sketched on the right of Fig. 15 – a single photon in the final state, with nothing visibly balancing its momentum in the transverse direction. Thus, if the cross-section (a measure of the probability of the reaction to occur) for creation of the $\vec{n}$-th mode is given by $\sigma_{\vec{n}}$, the total cross-section for seeing a single photon of this kind will be a sum over all accessible modes, i.e. $\sigma_\gamma = \sum_{\vec{n}} \sigma_{\vec{n}}$, where the sum commences from $\vec{n} = \vec{0}$ and gets cut off when $M_{\vec{n}}$ becomes greater than the machine energy.

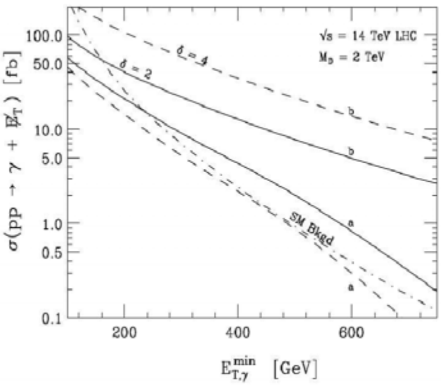


Figure 16. Single photon production at the LHC as predicted by Giudice et al [21]. To connect with the text, read d for δ and $\widetilde{M}_P$ for M_D. The symbol $\not{E}_T$ denotes undetected energy/momentum

In Fig. 16 we reproduce a graph from the work of Giudice *et al* [21], in which the cross-section for single photon production at the LHC is predicted. For a general idea, one needs to focus only on the curves marked **a**, where the solid line denotes $d = 2$ and the dashed line denotes $d = 4$. The dot-dashed horizontal line is an estimate of the SM background. This goes to show that this signal will be clearly observable only if $d = 2$ and just barely if $d = 3$. However, that should not be taken very seriously, since this is just one out of many processes, and there are others which are more viable from a phenomenological point of view.

The period immediately after November 1998 was a busy period for the high energy community. Once it had been established that one could not only solve the hierarchy problem, but that there was a possibility of observing quantum gravity effects in the laboratory, the ADD model really caught the fancy of scientists around the world. The first paper by Mathews, Sridhar and the author [11] appeared towards the end of November 1998 – within two weeks of the Giudice *et al* paper. Over the next few years, each person or group tried their hand at applying the ideas of ADD in their own area of expertise, from cosmology to supernovae to practically every conceivable process at real, upcoming and even proposed scattering experiments. As one wit remarked, large extra dimensions were being used to explain everything except the extinction of the dinosaurs.

After the initial euphoria, however, saner reflection prevailed. It turned out that a direct string-theoretic realisation of a spacetime with a D_3 brane, some large compact dimensions, and some small compact dimensions, was not so easy to construct. String theorists, confident about the physics of strings of length around 10^{-35} m, were wary of the much larger strings of length around 10^{-19} m proposed in the ADD model. There were technical problems associated with the cutting off of KK modes in virtual processes and in the calculation of quantum corrections involving graviton loops, but one could say that such technical problems plague most new theories. However the biggest blow to the ADD idea came from the realisation that it does not really solve the hierarchy problem – it simple re-formulates it! Let us see how this happens.

The easiest way to see this is to recall the origin of the hierarchy problem in the SM – it comes from the self-interactions of the Higgs scalar field. But gravitons have self-interactions too! Thus the mass of every KK mode gets a quantum correction which would be as large as the cutoff of the theory, i.e. $\widetilde{M}_P$. Since these masses are inversely related to the size R of the large compact dimensions, this means that the increase in mass corresponds to a shrinkage in the bulk. However, a shrunken bulk implies a corresponding shrinkage in $\widetilde{\ell}_P$, which leads to an even higher value of the cutoff $\widetilde{M}_P$. Re-evaluating the quantum corrections with this higher cutoff drives the masses of the light KK modes still higher and causes further shrinkage of the bulk. The process continues to bootstrap in this fashion, until stability is reached when $R \sim \widetilde{\ell}_P \sim \ell_P$ and all the graviton masses are of order M_P. This is going back to Kaluza and the original reason for abandonment of the idea of extra dimensions. Moreover the cutoff for the SM is now M_P and hence the hierarchy problem is restored in full glory. The only way to prevent this dismal scenario is to find a mechanism which allows some of the compact dimensions to remain large, while the others remain small. This can be achieved in a supersymmetric theory — but in supersymmetry, one can solve the hierarchy problem for the Higgs boson directly.

All that the ADD model has achieved, therefore, is to replace the hierarchy problem in the scalar mass by a hierarchy problem in the graviton masses, i.e. a hierarchy problem in the size of the compact dimensions. The initial u.s.p. for the model is, therefore lost. Nevertheless, it is not difficult to invoke supersymmetry, or some such idea, to save the situation, since the underlying theory is a string theory and hence necessarily supersymmetric. Nowadays, most scientists are contented with a pragmatic approach – to take the ADD model as a phenomenological possibility, assuming that the stability of the compact dimensions is achieved in some unknown way. This is similar to the way in which the SM has found universal acceptance, in spite of having the hierarchy problem in scalar masses.

6. Black Holes and Doomsday Predictions

Even though it is flawed as a solution to the hierarchy problem, the ADD model still gives rise to one of the most exciting – and bizarre – predictions ever seen in high energy physics. This is the suggestion that if this is a true picture of the world, then proton-proton collisions at the LHC would give rise to tiny *black holes*. Thus, just as the realm of string theory and quantum gravity is brought into laboratory experiments, so does the black hole – that bizarre solution of Einstein's gravitational equations, hitherto thought to be a by-product of dying stars – enter into the laboratory.

What is a black hole? If we glance at Eq. 7 it will become apparent that something bizarre must happen if the size of a massive spherical object r is decreased below the Schwarzschild radius $r_S = 2G_N M/c^2$, for at this value the radial and temporal coordinates exchange sign. This leadsto exactly the expected behaviour – motion along the radial coordinate can be unidirectional only, i.e. towards the singularity at the centre $r = 0$. Hence, once inside the distance $r = r_S$, known as the *horizon*, it is impossible for anything material – even light – to escape from this object, which is accordingly called a black hole.

If we set the mass M to the mass of a proton, then the corresponding Schwarzschild radius becomes the Planck length $\ell_P \sim 10^{-35}$ m. This, in fact, is one way to define the Planck scale. This means that so long as the proton does not come into contact with any matter at distances of this order, no black holes will form. To bring protons so close to matter, they must be accelerated to Planck energy, i.e. 10^{19} GeV, which was achievable only in the early universe, moments after the Big Bang. This could have resulted in the formation of primordial black holes, some of which may have 'evaporated' and some of which may have accreted matter to become giant black holes sitting at the centre of galaxies and quasars. Since laboratory energies, even at the vaunted LHC, are no larger than 10^4 GeV, there is no question, in four-dimensional Einstein gravity, of producing such black holes.

Large extra dimensions change the scenario completely. For now the effective Planck scale is brought down to $\widetilde{\ell}_P$, which means that the Schwarzschild radius of the proton is as large as 10^{-19} m. Protons will approach this close if their energies are of order 10^3 GeV, which will certainly be the case at the LHC. Accordingly, we expect two such protons to coalesce [22], forming a micro black hole of mass $2M_p$. If such a micro black hole is stable, it could then draw in nearby protons and grow in mass as well as horizon size, enabling it to eventually swallow up the LHC machine, the LHC tunnel, France, Switzerland, Europe and eventually the whole earth with all its inhabitants. Even the Moon would be shivered into bits and gradually sucked into the maw of this rapacious monster.

This apocalyptic vision is not correct, however, and that is because we live in a quantum world, rather than a classical one. In quantum theory, the vacuum surrounding any black hole is not an emptiness, but a bubbling ocean of virtual particle-antiparticle pairs, such as pairs of electron and positron. Such pairs are perennially being created out of the vacuum, using some of the enormous vacuum energy, and then annihilate again, returning their energy to the vacuum. Stephen Hawking, the world-famous Cambridge physicist, showed in the 1970s, that in the neighbourhood of a black hole, however, strange things begin to [23]. It can be shown that in free space an antiparticle travels backwards in time, as it were, and near a black hole, the role of time is played by the radial coordinate r. Thus, when a virtual electron-positron pair is created near a black hole horizon, the electron is drawn in, falling towards the origin with ever-increasing speed, while the positron shoots

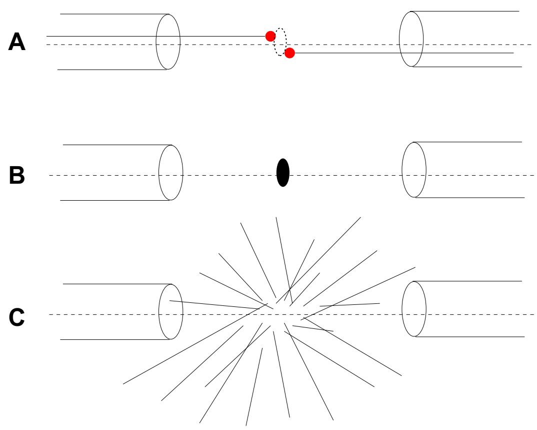


Figure 17. Micro black hole at the LHC. **A**: Two protons approach each other with impact parameter within the Schwarzschild radius. **B**: The protons coalesce into a black hole. **C**: The black hole decays by Hawking radiation, spraying particles in all directions

out, with ever-increasing speed outwards. This accelerated motion of a charged particle like the positron causes radiation, known as *Hawking radiation.* Where does the energy of this radiation come from? Eventually from the black hole itself – at the cost of its mass. Thus, if a black hole is small enough, it can eventually lose all its mass through Hawking radiation, or 'evaporate'. This is the reason why it is thought that most of the primordial black holes have disappeared.

Micro black holes at the LHC will also lose energy by Hawking radiation. The time scale is easy to calculate. It is given by the time taken by interactions to cross the black hole, i.e. $r_S/c \sim 10^{-29}$ s. Such a black hole will, for all effective purposes, be stillborn, since it will decay long before even the nearest protons in the beam (typically separated by about 10^{-6} m) can reach it. There is absolutely no danger, therefore, of such a black hole accreting any mass and growing. The world as we know it is still a safe place.

There is still a catch in the above argument, and that is the fact that it is assumed that the micro black holes at the LHC would be produced at rest. However, there is always a velocity spread in the beam so that some protons have more momentum than others. Collisions of such protons would lead to fast-moving black holes, which live much longer due to a relativistic effect called time dilation, discovered long ago by Einstein. One requires, therefore, to carry out a careful study and see if there is any chance that even a single micro black hole may be produced with a long enough lifetime to start accreting mass in the LHC experiment. After all even one accreting black hole is enough to destroy the earth. A detailed and careful analysis of this has been performed recently by Steve Giddings and Michelangelo Mangano at CERN [24], and their conclusion is that this probability is small enough to be virtually zero[16]. Moreover, any black hole with a long enough time-dilated lifetime would be a high-speed one which would pass through the earth without interacting with any matter and eventually decay harmlessly outside the earth.

No only do black holes pose no danger to us, but they also provide a unique signature of any ADD-type model [22]. For when a micro black hole decays, it will result in a spray of all sorts of particles without prejudice, which would form a near isotropic distribution of hits in the detector, as indicated in Fig. 17. At its peak, the LHC could be producing ten million black holes per year, each with this kind of spectacular signal. There is really no way in which this can be missed, so it may well be black holes which provide the first evidence that the world has more than four dimensions.

Black hole signatures are interesting and the arguments of Professors Giddings and Mangano are reassuring, but it is even more reassuring to think that the ADD model is just an idea, and not even the best one in its own genre. A much better solution of the hierarchy problem using extra dimensions was suggested by Randall and Sundrum, and it is now to their ideas that we turn.

7. The Randall-Sundrum Model

The key concept in the model of Randall and Sundrum (Fig. 18) is that of *naturalness.* This was introduced by Paul Dirac in the 1930s and is an important issue in any quantum theory. In classical mechanics, we do not bother if one parameter is very small and another is very big. Thus, the fact that a grain of dust is very much smaller than a mountain and that an elephant is much bigger than a flea does not cause any eyebrows to be raised. In the strange world of quantum mechanics, however, this happy situation is no longer true. This is

[16]We are happy to live with many such dangerous but low probabilities. For example there is a tiny probability that all the molecules of air in the room will, through random motions, collect in a corner and leave us asphyxiated; there is a tiny — well not so tiny really — probability that a piece of cosmic debris will strike the earth and cause it to break up; and so on.

because of the *completeness* property of quantum states – every state is equivalent to a linear combination of other states[17]. Thus, bizarre as it may sound, a grain of quantum dust has a component of quantum mountain in it and a quantum flea has a component of quantum elephant in it. In some experiments, therefore, which happen to probe just that component, the grain of dust will appear as big as a mountain, and similarly the flea would appear as an elephant. In more technical language, any number which is very small or very large (depending on what it is being compared with) is unstable under quantum corrections, and will tend to stabilise only when the two numbers in question are equal or nearly equal. We have already seen two examples of this happening: the small mass of the Higgs boson is driven to the Planck mass scale by quantum corrections, and the large compact dimensions of ADD are driven to the tiny Planck length, again by quantum corrections. It was Dirac who declared [25] that this kind of equality which leads to quantum stability is *natural*, and that large or small numbers in a quantum theory are *unnatural*.

We have just argued that both the SM and the ADD model share the same feature of unnaturalness, which is what the hieracrhy problem is all about. However, quantum mechanics notwithstanding, the enormous difference in strength between gravity and electroweak interactions is a fact, and cannot be wished away. In the summer of 1999, about a year after the original papers of ADD, Lisa Randall of Princeton University and Raman Sundrum, then at Boston University, proposed a model [10] with just one extra dimension which was able to create this huge difference without using any large fundamental numbers. Their ingenious construction now goes by the name RS model.

The RS model assumes that there is a single extra dimension which has the topology of a circle folded along a diameter – $\mathbb{S}^{(1)}/\mathbb{Z}_2$ for the experts. For obvious reasons, such a space is called an *orbifold* – this one being the simplest of a whole class of objects which go by this name. This is pictured in Fig. 19, where we also note that at the two extreme points lie two D_3 branes, denoted as the *visible* (black) and the *invisible* (blue) branes respectively. No explanation is given or sought why this spatial dimension should be contorted in this fashion, the reason presumably lying in the underlying string theory[18]. The coordinate along the extra dimension is parametrised by an angle ϕ which is clearly limited to the range $0 \leq \phi \leq \pi$. RS then proceed to solve the five-dimensional field equations of Einstein gravity in this configuration. The solution can be done if and only if one imposes boundary conditions on the branes, which are equivalent to choosing four-dimensional cosmological constants – one positive and one negative – on the two branes, as well as a carefully-matched negative cosmological constant in the five-dimensional bulk[19], satisfying the relations

$$\Lambda_i = -\Lambda_v = -\Lambda_5 \ . \tag{32}$$

Here $\Lambda_{i,v,5}$ denote the cosmological constants on the invisible brane, the visible brane and the bulk, in that order. There is a good deal of fine-tuning in these choices of cosmological constants, since even a small deviation cannot lead to a solution of the Einstein equations. However, this may be turned to our advantage by claiming that this is these are the equilibrium values reached after the Big Bang, assuming that the universe has always satisfied some solution of Einstein's equations. Once, however, the choices are made, RS obtain a solution for the line element

$$\delta s^2 = e^{-\mathcal{K}R\phi}\left[\left(\delta x^0\right)^2 - \left(\vec{x}\right)^2\right] - R^2\ \delta\phi^2 \ , \tag{33}$$

where $\mathcal{K}$ is a constant which may be interpreted as the curvature of the fifth dimension, and is related

Figure 18. Lisa Randall and Raman Sundrum, whose proposal of warped extra dimensions provides the best solution of the hierarchy problem within extra dimensional models

[17] Provided those states are linearly independent.

[18] The RS construction has proved to be very difficult to embed in any of the standard string theories, and nowadays is visualised more as a phenomenological construct. Whatever be the underlying theory, however, it must have compact dimensions and D branes, which indicates that it must be some kind of string-like theory.

[19] This makes it a five-dimensional anti-de Sitter space, generally denoted AdS_5.

to the five-dimensional Planck scale $\widetilde{M}_P$ and the five-dimensional cosmological constant Λ_5 by

$$\mathcal{K} \simeq \widetilde{M}_P^3/M_P^2 = -\Lambda_5/(24M_P^2) \ . \tag{34}$$

On the 'invisible' brane, where $\phi = 0$, this reduces to a purely Minkwoski form, noting that the length around the fifth direction is just $y = R\phi$. However, as we proceed towards the 'visible' brane at $\pi = 0$, the usual spacetime part undergoes an exponential contraction, though the fifth dimension is unchanged. Such an asymmetric contraction is normally understood as a warping and hence the exponential $e^{-\mathcal{K}R\phi}$ is referred to as a *warp factor*. A sketch of this warping effect is shown in Fig. 20. Clearly the warping will be maximum on the 'visible' brane, at $\phi = \pi$, which is identified with the observed universe.

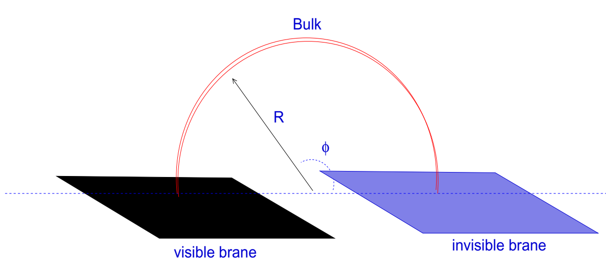


Figure 19. Sketch of the RS construction. The double red line indicates the $\mathbb{S}^{(1)}/\mathbb{Z}_2$ extra dimension. The black region marked 'visible brane' corresponds to the known universe, while the blue region marked 'invisible brane' corresponds to a 'shadow world' of strong gravity

What does all this have to do with the hierarchy problem? The answer is that we start by assuming all interactions to have the same strength, gravity as well as the electroweak interaction, and this is characterised by $1/\widetilde{\ell}^2$, where $\widetilde{\ell} \sim 10^{-19}$ m. However, gravity at this strength is an effect native to the invisible brane, where it has this strength. On the visible brane, we only see that amount of gravitational interaction which reaches us across the bulk, and this arrives after a drastic reduction of the length scale

$$\widetilde{\ell} \to e^{-\pi\mathcal{K}R}\widetilde{\ell} \ . \tag{35}$$

The very reasonable choice $\mathcal{K}R \simeq 11.73$ takes the right side of the above equation to $\ell_P \sim 10^{-35}$ m. On the other hand, electroweak interactions are native to the visible brane – where they are confined, as in the ADD case – and are of the typical strength $1/\widetilde{\ell}_P^2$. This means is that we have been able to generate the enormous difference in strength between the gravitational and electroweak strengths without having recourse to any unnaturally large or small numbers. In more picturesque language, gravity is weak not because of any inherent weakness, but because we see it shining on us very dimly across a highly opaque higher dimension, which allows only a tiny fraction of the force to get through.

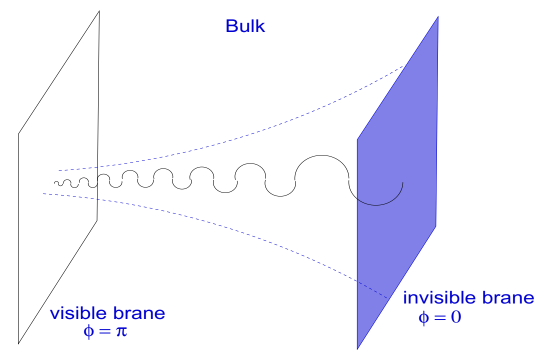


Figure 20. Warping effect in the RS model. All length scales get damped as we proceed from the invisible to the visible brane, and the graviton wavefunction damps out proportionately. Of course, the sketch is only illustrative, since the actual damping is exponential and the separation between the branes is miniscule

What is the size R of the extra dimension? A glance at Eq. 34 shows that it would be natural to choose $\widetilde{M}_P$ as well as $\mathcal{K}$ in the ballpark of M_P, which would automatically make $R \sim \ell_P$, since $\mathcal{K}R \sim 10$. All these sizes are, of course, approximate within an order of magnitude, but the important points to note are that (a) there is no unnatural hierarchy of sizes and (b) the extra dimension is really small – practically as small as originally envisaged by Kaluza and Klein. Thus, like the original KK model and the ADD Model, the RS Model also starts with two basic length scales, one much larger than the other, viz. the effective electroweak length $\widetilde{\ell}_P \sim 10^{-19}$ m, and the Planck length $\ell_P \sim 10^{-35}$ m. What this model really achieves, therefore, is to keep these two length scales apart in such a way that they cannot influence each other through quantum corrections, while generating their large ratio by exponentiating a relatively small number. This is no mean achievement, given the difficulty of the original problem.

There are many it ad hoc things in the RS model. There is the question of what happened to the other 5 compact dimensions, if the whole is embedded in a string theory of 10 dimensions. Why do they not develop similar warping? Secondly, why does the compact dimension get orbifolded into $\mathbb{S}^{(1)}/\mathbb{Z}_2$ — this must have a dynamical origin. Perhaps it was a simple circle or a non-compact dimension just after the Big Bang, and then it developed this peculiar topology as time evolved. In the absence of any model for this, we can only speculate. The other major question has to do with how the cosmological constant on the branes and on the bulk got aligned in the way they have to be for a RS solution to exist. Moreover, if the visible brane has a negative cosmological constant, why do we not see its effects in cosmology? Since, as we have seen, we have little or no understanding of the whole business of the cosmological constant, one can only hope that when we do begin to understand this, some of the mysteries of the RS model will be unravelled as well.[20]

One of the consequences of the warping effect is that the KK tower of gravitons acquires masses around a few hundred GeV – unlike the ADD case, where the masses range from very small values to the cutoff scale. This is because, the extra dimension being small (comparable to ℓ_P) the KK tower would normally have a mass gap of the order of the Planck mass, as in the original Kaluza-Klein theory. However, this large mass, when seen on the visible brane, appears with the warp factor $e^{-\pi\mathcal{K}R}$, and is reduced to the level of the electroweak scale, i.e. a few hundred GeV. When the details are worked out, the actual mass parameter turns out to be [26]

$$m_0 = \mathcal{K}e^{-\pi\mathcal{K}R} \tag{36}$$

and the masses of the heavy KK modes of the graviton are given by

$$M_n = m_0\,\xi_n \qquad (n = 1, 2, \ldots). \tag{37}$$

The ξ_n are the successive zeroes of the Bessel function $J_1(x)$ of order unity, which appears in diffraction theory, and start from $\xi_1 = 1.22\pi$, $\xi_2 = 2.33\pi$, and so on.

[20] In this context one may make the somewhat cynical comment that the RS model succeeds in pushing the hierarchy problem — which we have just begun to start comprehending — on to the cosmological constant problem, which is still as big a mystery as it ever was.

Not only does the warp factor render the KK graviton modes massive, but it also makes the interactions of each KK mode much stronger. This is because the usual interaction, proportional to ℓ_P^2 gets 'warped up' according to Eq. 35, and becomes $\widetilde{\ell}_P^2$. This can eventually be parametrised in terms of a coupling constant

$$c_0 = \frac{\mathcal{K}}{M_P}, \tag{38}$$

which is a fraction ~ 0.1 and hence comparable with the electroweak coupling $g \simeq 0.6$. The KK modes of the graviton, therefore, will behave very much like weakly-interacting massive particles, being produced at the LHC if there is enough energy and having very short lifetimes, so that they appear to decay practically immediately, i.e. at the interaction vertex itself [27].

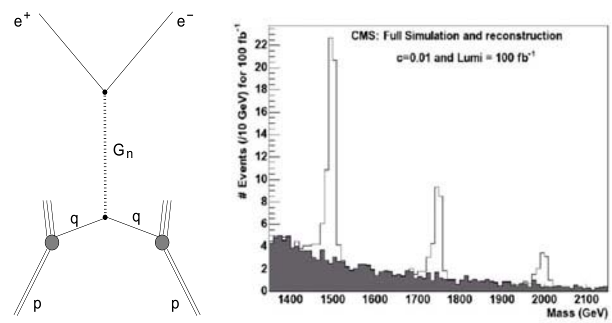


Figure 21. Di-electron signal for an RS graviton resonance. One of the Feynman diagrams is shown on the left. One the right is shown the results of a simulation using the CMS detector [28], for $c_0 = 0.01$ and with $m_0 = 400$ GeV

At the LHC, therefore, massive graviton KK modes are likely to appear as resonances in basic processes, such as the production of electron-positron pairs, or $\mu^+\mu^-$ pairs, or a pair of hadronic jets. This is illustrated in Fig. 21. On the left, a typical Feynman diagram for the production of an electron-positron (e^+e^-) pair is shown. On the right is shown the results of a simulation of this process by the CMS collaboration at the LHC [28]. On the horizontal axis, marked "Mass" is plotted the 'invariant mass' of the e^+e^- pair, i.e. the quantity

$$M_{e^+e^-} = (E_+ + E_-)^2 - (\vec{p}_+ + \vec{p}_-)^2 \ , \tag{39}$$

where $E_\mp$ and $\vec{p}_\mp$ refer to the energy and momentum of the electron and the positron respectively. The shaded histogram represents the expectations from the SM, while the peaks represent the expectations in the RS model.

As expected in a quantum resonance phenomenon, the probability of interaction shoots up when this invariant mass $M_{e^+e^-}$ matches with the mass of a real graviton mode. Three distinct peaks are predicted (for this choice of parameters), and a clinching proof that these are indeed RS graviton modes would be if the corresponding masses were found to be in the ratio $\xi_1 : \xi_2 : \xi_3$. Of course, this graph does not tell us the whole story, and there are many possible variations. For example, it is entirely possible that the coupling c_0 will be larger, in which case the resonant peaks will be shorter and fatter, and as $c_0 \to 0.1$ will simply represent small excesses over the SM histogram. It is also possible that the value of m_0 may be so large that all the graviton resonances lie beyond the kinematic reach of the LHC. In such cases we would have to look for other effects, such as those involving virtual graviton modes, to look for signals of warped gravity. Studies of this nature abound in the literature, but it would be beyond the aim of this article to take up a detailed discussion of this very interesting topic.

8. Modulus Stabilisation and the Radion

In the previous section we have discussed the RS model and shown how it provides an elegant solution of the hierarchy problem by never bringing a large and a small number together in such a way that they can be influenced by each other. In doing so, we have glossed over a major element of fine tuning in this model. Like the ADD model, this lies in the size R of the extra dimension, but here it is not the quantum stability of this size that is in question, as it is close to the Planck length anyway. What is in question is why the product $\mathcal{K}R = 11.73$ precisely, and what would happen if it varied a little. Since the warp factor $e^{-\pi\mathcal{K}R}$ is responsible for creating the factor of 10^{16} between the TeV scale and the Planck scale, clearly that ratio will be sensitive to small changes in $\mathcal{K}R$. In fact, even the choice $\mathcal{K}R = 11.0$ would make the warp factor a whole order of magnitude too small, and, conversely, choosing $\mathcal{K}R = 12.5$ would make it an order of magnitude too large. In a string theory – or any underlying theory – the size of the extra dimension R should be a dynamical variable – in string theoretic parlance, a *modulus*. Such a modulus should show time variation, and this would appear as a time variation in M_P, i.e. in the gravitational constant G_N. Given the fact that all astrophysical evidence indicates that G_N has shown no measurable variation since the Big Bang, it is clear that the modulus R must be remarkably stable, i.e. the distance between the observed universe of the visible brane, and the shadow world of the invisible brane must be remarkably constant, and have the just-so value $\mathcal{K}R = 11.73$. This is indubitably a case of fine-tuning.

The original work of Randall and Sundrum did not address the question of stabilisation of the R modulus at all. In fact, RS went on to construct a model variant where $R \to \infty$, where there is no question of fine-tuning. However, it was soon realised that the original model, fine-tuning and all, was much more relevant for high energy physics than its successor. The issue of modulus stabilisation had, therefore to be addressed seriously.

Two solutions to the modulus stabilisation problem were proposed within a few months of the original Randall and Sundrum suggestion. The first one – which has proved more popular – was by Walter Goldberger and Mark Wise (GW) from CalTech [29], and it used the high energy theorist's favourite tool, viz. a scalar field, to provide the stability effect. We shall discuss this presently. In the next spring, a supersymmetric solution was proposed by Jonathan Bagger and his collaborators, from Johns Hopkins University [30]. Though elegant, this solution never became popular, probably because the high energy community, from the beginning, has viewed extra dimensions as a *alternative* to supersymmetry as a solution to the hierarchy problem.[21]

What was the simple and attractive solution proposed by Goldberger and Wise? Like Yukawa and Higgs and Lindé before them, they had recourse to postulating the existence of a scalar field $B(x, \phi)$ – in this case, one which lives in the full five dimensional spacetime (x, ϕ). By dint of choosing B^4-type self interactions of this field in the bulk, and extra self-interactions of the form $\lambda(B^2 - v^2)^2$ on the two branes, they were able to show that the modulus $\mathcal{K}R$ is trapped in a deep potential with a minimum which can be set to 11.73 without much fine tuning at any stage. A sketch of this potential is shown in Fig. 22. The steep walls flanking the minimum show that it would take a major disturbance of the entire universe to pull the modulus out of the

[21] This is not really a logical stand, because string theories, on which brane world models are predicated, contain both supersymmetry *and* extra dimensions. However, like many collective prejudices similarly divorced from logic, it seems to have stood the test of time.

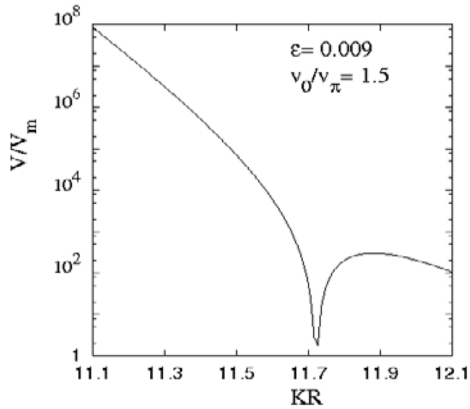


Figure 22. Illustrating the steep minimum in the Goldberger-Wise potential at the minimum $\mathcal{K}R = 11.73$. Note that the vertical axis is plotted on a logarithmic scale

minimum and set it rolling. In the absence of any such disturbance, the modulus is stable.

One important consequence of having a bulk field of this form is that fact that the warp factor $e^{-\pi\mathcal{K}R}$ on the visible brane, i.e. our universe, is no longer just a constant, but may be parametrised as $e^{-\pi[11.73+T(x)]}$, where $T(x)$ is a dynamical field. The values assumed by $T(x)$ must be very small, as indicated by the steep walls of the potential well in Fig. 22, but since they are exponentiated, they appear as a normal scalar field $\Phi(x)$ on the visible brane. For obvious reasons, this field is called a *radion*. It turns out that there are no serious theoretical constraints on the mass of this radion, and hence it can be chosen light or heavy as we wish – within reason, that is to say. Thus, it is theoretically equally possible to have a 1 GeV radion, or a 10,000 GeV radion. If the radion is light, i.e. within the kinematic access of the LHC, then it may prove to be a very distinct signal for an RS kind of model, stabilised by a GW-type mechanism.

Technically, however, there is a problem in identifying a radion signal at the LHC or any other machine. This is because a real radion couples to matter in a manner identical with the Higgs boson. The signals for both are, therefore, identical, and it will be difficult to tell whether such signals are due to a Higgs boson or a radion being produced. Of course, if both are produced, and detected, then some of the detailed behaviour in decays of these different scalars can be exploited to distinguish them. This is particularly true when the signal involves virtual quantum states, or in the language of quantum field theory, loop-level effects [31]. However all this is upset by that fact that both radion and Higgs boson are quantum mechanical states with identical quantum numbers, which permits them to form mixed states which correspond exactly to neither. Detection of such mixed states may be easy, but identification of the components of Higgs boson and radion in them is a non-trivial matter, and would require the collection of a lot of data at the LHC or any other machine before it can even be attempted [32].

9. Different Strokes

In the decade or so since extra dimensions became fashionable again, there have been many attempts to devise alternative models, using some of the basic ideas of ADD and RS, but innovating more. It would be tedious and long-winded to attempt a comrehensive listing of all the new ideas that have been suggested. However mentioning a few of the ideas may give a flavour of the kind of thinking that has been going on in this context.

There have not been all that many modifications of the ADD model, which is very simple. There have been attempts to change the compactification scheme, the simplest idea being to assume different radii of compactification for different dimensions [33]. The fact that the LHC may operate at the actual scale where gravity is strong has inspired an attempt to write down a toy string theory and work out possible signals for it. The use of supersymmetry to stabilise the large size/Flora of the bulk has been suggested [30], but has not found very many takers till date.

One of the most ingenious ideas suggested in this context is that of *dimensional deconstruction* [34]. This takes note of the fact that the observable feature of the ADD and similar models is the tower of KK modes. The proposers of this theory point out that if the world is purely four-dimensional and at a very small length scale, there is a somewhat complicated gauge theory involving replication of a particular gauge symmetry many times, then, at larger length scales this may well appear as a uniformly-spaces set of spin-2 states, which could be confused with a KK tower of gravitons. In that case, if one sees such repeated states at the HC, for example, one would jump to the conclusion that there are extra dimensions, even though the world is purely four-dimensional. This is an ingenious idea, but it does require one to postulate a very complicated gauge symmetry at small length scales. Such complicated symmetries do arise in string theories – but if we are to believe string theories, we might as well believe in extra dimensions too!

The RS model has inspired more variants, starting with the second paper of Randall and Sundrum themselves. Apart from obvious extensions like incresing the number of warped dimensins, there have been attempts to explain the observable universe as a D_3-brane which is an intersection of higher dimensional branes – this complicated construction solves some long-standing problems in the physics of flavour. This is a bit like the traditional parable of using a sledgehammer to crack a nut, but that is a criticism that may be applied, in some ways, to the RS model itself.

Perhaps the most interesting alternative idea to come out of the RS model was the idea of having an extra dimension in the form of an $\mathbb{S}^{(1)}/\mathbb{Z}_2$ orbifold and dispensing with the two D_3 branes altogether [35]. In this case, of course, *all* the SM fields live in the bulk, and, at large lenth scales, we would see each of them as a KK tower. This scenario is called a *universal extra dimension* or UED. More details may be found in the article by Dobrescu in this volume.

The other idea which has generated a lot of recent activity is that of *quantum holography.* In 1997, about six months before the first ADD paper, Juan Maldacena at Harvard had conjectured that in a model with D-branes, a theory of gravity in the bulk may appear on the brane as a theory involving gauge interactions [36]. By changing from one set of variables to another, using what is called a *duality transformation*, one theory transforms into the other[22]. Thus, a perfect fluid on the brane may appear as a particular type of black hole in the bulk theory, and similarly, there are black hole solutions of quantum gravity in the bulk which look like a theory of strong interactions on the brane. In RS-type models, the nature of quantum gravity in the bulk happens to be exactly of the type (AdS) required for the Maldacena conjecture to work, so there have been suggestions that at the LHC (i.e. on the visible brane) we may find an effective theory of composite quarks and leptons[23], which is actuallly the Maldacena dual of fundamental theory of quantum gravity in the bulk [37]. Ingenious and exciting as these ideas are, they generally lack falsifiability, since practially any theory on the brane can have an exotic gravitational counterpart in the bulk, which our brane-bound experimental equipment will not be able to test.

Most of these ideas beyond the basic ADD and RS models have to do with gaining a deeper understanding of theories with extra dimensions, or of the SM itself. Hard-headed particle physicists, especially experimental physicists, have not, therefore, shown much enthusiasm for these. Thus, apart from a few efforts, the bulk of phenomenological and experimental studies of extra dimensions have to do with just four kinds of new physics:

- Towers of invisible ADD gravitons.
- Heavy RS graviton resonances.
- A relatively light scalar radion.
- KK modes in UED models.

It remains to be seen if there are hints of more exotic new physics of the kind described above at the LHC. For this we may have to wait a few years till enough data are collected to infer backwards and pin down the nature of the new physics.

10. Valediction

At the LHC, the effort of nations and the toil of thousands is being poured into the most important quest of all – the quest for understanding the innermost working of nature. The first step would be to understand the origin of mass in the visible universe, for which it is essential to discover the Higgs boson. The next step would be to discover why the Higgs boson mass is stable, i.e. to find the correct solution to the hierarchy problem. Next after that we would seek to discover the nature of the dark matter component of the universe. It may be mentioned in passing that 95% of the universe consists of dark matter and vacuum energy – all invisible. This is a humbling thought, but, to the determined thinker, it provides an extra impetus to seek out the real nature of the universe. Extra dimensions are a part of this quest, a small, but essential piece of the jigsaw puzzle which, when solved, would unite the whole universe and all its workings into a single theory. Of this end, one cannot describe it better than in the

[22]This is technically called the AdS-CFT correspondence, where AdS stands for *Anti-de Sitter* (cosmology) and CFT stands for *Conformal Field Theory.*

[23]This goes by the name *technicolour.*

words of Jalal-ud-din Rumi, the thirteenth century Sufi poet [38]:

My place is placeless, my trace is traceless,
no body, no soul, I am from the soul of souls.
I have chased out duality, lived the two worlds as one.
One I seek, one I know, one I see, one I call –
The First and the Last, the Outer and the Inner.

On this note we conclude our story.

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