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NEUTRINO MASSES AND MIXING ANGLES: A TRIBUTE TO GUIDO ALTARELLI

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Abstract

I present a personal recollection of Guido Altarelli, focused on his contribution to the problem of neutrino masses and mixing angles. I recap the main ideas in model building, illustrating how the subject evolved from the study of continuous-abelian to discrete-nonabelian flavour symmetries, emphasising the point of view of Guido on the subject. I conclude by commenting the present status of the field.

1 Guido vision on neutrinos

I first met Guido in 1989, on the eve of LEP start, and this marked the beginning of an intense collaboration lasted 25 years. In 1989 I was a graduate student at the University of Geneva, while Guido was a leader of the CERN theory group, fully committed to the LEP program and to its demanding activity. I was soon captured by Guido real passion for physics, for his deep perspective in many areas of our field and by his stunning ability in synthesising deep concepts with few well-chosen words. I remember the enthusiasm and energy with which he dragged me into the world of neutrinos when neutrino oscillations were discovered by Superkamiokande in 1998. He was eager to participate in the fascinating adventure launched by the new data and I had the fortune and the privilege of being at his side.

As for other aspects of particle physics, Guido had his own vision about neutrinos. Neutrino masses are very small, much smaller than the other fermion masses and this can be naturally explained by the violation of the total lepton number L at a very large scale M. In Guido's words [1] [1)]: *"Given that neutrino masses are certainly extremely small, it is really difficult from the theory point of view to avoid the conclusion that L conservation must be violated. In fact, in terms of lepton number violation the smallness of neutrino masses can be explained as inversely proportional to the very large scale where L is violated, of order M_{GUT} or even M_{Pl}."* On dimensional grounds we have:

$$m_\nu \approx \sqrt{\Delta m^2_{atm}} \approx \frac{(e.w.\ scale)^2}{M} \quad , \tag{1}$$

leading to the estimate $M \approx 10^{14\div15}$ GeV, not far from M_{GUT}. Guido considered this [2)] *"The most impressive numerology that comes out from neutrinos."* It is reasonable that the relation (1) arises from the seesaw mechanism, the simplest realisation of which requires a set of heavy right-handed neutrinos. To Guido these were strong indications in favour of a grand unified theory (GUT) [3)]: *"We consider that the existence of right-handed neutrinos ν^c is quite plausible because all GUT groups larger than SU(5) require them. In particular the fact that ν^c completes the representation 16 of SO(10): $16 = \bar{5} + 10 + 1$, so that all fermions of each family are contained in a single representation of the unifying group, is too impressive not to be significant."* Guido believed that GUTs had to be incorporated in our picture of particle physics [2)]: *"GUTs are the most attractive conjecture for the large scale picture of particle physics. GUT is not the Standard Model (SM), is beyond the SM, but is the most standard physics beyond the SM. Most of us think that there should be something like a*

[1]Sentences quoted here from works done in collaboration have all been written by Guido.

GUT." Once the idea of heavy right-handed neutrinos is accepted, we get as a bonus an elegant explanation of the observed baryon asymmetry: *"Another big plus of neutrinos is the elegant picture of baryogenesis through leptogenesis (after LEP has disfavoured baryogenesis at the weak scale)."* Of course, once embedded in a GUT, neutrinos pose the problem of a consistent description of quarks and lepton masses and mixing angles in a less flexible setting. Guido regarded this challenge as a big opportunity. If dominated by the seesaw relation, $m_\nu = -m_D^{\nu T} M^{-1} m_D^\nu$, neutrino masses are potentially linked to the other charged fermion masses. Back in 1998 the quark sector was already reasonably well-known, but a baseline model for quark masses and mixing angles was missing. Neutrino masses and the large atmospheric mixing angle indicated by the data were interesting new inputs which, especially in a constrained framework as the one provided by GUTs, could have brought a new insight into the flavour puzzle.

2 Lepton mixing angles and GUTs

In GUTs particle classification is greatly clarified. Quarks and leptons of the same generation belong to few multiplets of the grand unified group, a single representation being sufficient in the case of SO(10). Charge quantisation and gauge anomaly cancelation, which look miraculous within the SM, are thus neatly explained. Being members of the same gauge multiplets, quarks and leptons lose their fundamental distinction in GUTs and we should justify why in this context the lepton mixing angles are so different from the quark ones. An appealing explanation of this property can be found within SU(5) GUTs. In a minimal formulation of the SU(5) GUT, matter fields are described by three copies of the $10 = (q, u^c, e^c)$ and $\bar{5} = (l, d^c)$ representations, while the Higgs fields φ and $\bar{\varphi}$ transform as 5 and $\bar{5}$, respectively. Fermion masses are described by the Yukawa interactions:

$$\mathcal{L}_Y = 10\ y_u\ 10\ \varphi + \bar{5}\ y_d\ 10\ \bar{\varphi} + \frac{1}{M}\bar{5}\ w\ \bar{5}\ \varphi\varphi + ... \tag{2}$$

where $y_{u,d}$ and w are matrices in generation space. After electroweak symmetry breaking the first term describes up-quark masses, the last one is the grand unified version of the Weinberg operator and gives rise to neutrino masses of the type given in eq. (1). The second term describes at the same time down-quark masses and charged lepton masses, which are equal at the GUT scale in this approximations. Corrections to this relation are provided by additional contributions denoted by dots. It would be desirable that the matrices $y_{u,d}$ and w had entries of the same order of magnitude, with no built-in structure. Starting from anarchical matrices $y_{u,d}$, we can easily produce the hierarchy observed in the charged fermion sector by a rescaling of the matter fields:

$$10 \to F_{10}\ 10 \quad , \qquad\qquad \bar{5} \to F_{\bar{5}}\ \bar{5} \quad . \tag{3}$$

Here $F_{10,\bar{5}}$ are diagonal matrices of the type

$$F_X = \left(\begin{array}{ccc} \epsilon'_X & 0 & 0 \\ 0 & \epsilon_X & 0 \\ 0 & 0 & 1 \end{array}\right) \qquad\qquad (1 \geq \epsilon_X \geq \epsilon'_X) \quad . \tag{4}$$

For instance, after rescaling the 10 representations, the effective matrix of Yukawa couplings for the up quarks becomes

$$\mathcal{Y}_u = F_{10}\ y_u\ F_{10} \quad , \tag{5}$$

which is hierarchical and nearly diagonal if $1 \gg \epsilon_{10} \gg \epsilon'_{10}$. By adjusting the suppression factors ϵ_{10} and ϵ'_{10} we can reproduce the quark masses and generate small contributions to the quark mixing angles. Such a mechanism is rather generic in model building. The rescaling matrices F_X can arise in a variety of frameworks such as models with an abelian flavour symmetry, models with an extra dimension and models with partial compositeness or specific conformal dynamics [4].

Since the mass hierarchy in the down-quark and charged-lepton sectors is much less pronounced than in the up-quark sector, we need a milder rescaling from $F_{\bar{5}}$. As a useful reference we can choose

$$F_{\bar{5}} = \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \quad , \tag{6}$$

which corresponds to the so-called anarchy scenario [5]. In this case we find

$$m_u : m_c : m_t \approx m_d^2 : m_s^2 : m_b^2 \approx m_e^2 : m_\mu^2 : m_\tau^2 \tag{7}$$

which is approximately correct. Moreover, at the leading order we have

$$\mathcal{Y}_e = \mathcal{Y}_d^T \quad , \tag{8}$$

where both $\mathcal{Y}_e$ and $\mathcal{Y}_d$ are lopsided matrices since only F_{10} is operating. The relation (8) should be corrected since it leads to wrong mass equalities for the first two generations. The required corrections are sizeable, but not huge and (8) can still be valid at the level of orders of magnitude. In the limit where (8) is exact, it predicts a small contribution to the quark left-handed mixing and a large contribution to the lepton left-handed mixing, which is exactly what we observe. For the right-handed components a large (small) mixing for quarks (leptons) is predicted, which however is not observable at low energies.

The neutrino mass matrix is $m_\nu \propto F_{\bar{5}} w F_{\bar{5}}\ v^2/M$. When (6) holds neutrino mass ratios and mixing angles are generated from the random, order-one, matrix elements of w, which is consistent with the data to first approximation. Actually, the discovery of $\theta_{13} \approx 0.15$ and the first hints for a non-maximal atmospheric mixing angle have strengthened the case for anarchy. However within the extreme choice in eq. (6) there is no preference for the type of neutrino mass ordering and no explanation of the smallness of $\sin^2\theta_{13}$ and $\Delta m^2_{sol}/\Delta m^2_{atm}$. Guido thought that (6) could be replaced by a more generic possibility, such as

$$F_{\bar{5}} = \left(\begin{array}{ccc} \lambda^{Q_1} & 0 & 0 \\ 0 & \lambda^{Q_2} & 0 \\ 0 & 0 & 1 \end{array} \right) \quad . \tag{9}$$

Here λ is an expansion parameter, typically smaller than 0.5 and $Q_{1,2}$ are two positive charges, $Q_1 \geq Q_2 \geq 0$. Anarchy is reproduced when $Q_{1,2} = 0$. It is not surprising that we found several examples with Q_1 non vanishing where a small θ_{13} is more easily reproduced than in anarchy [6)]. In all the more successful examples the normal ordering of neutrino masses is preferred. First hints of such a preference are currently shown by several neutrino oscillation experiments.

Though rather appealing at first sight, this approach has clear limitations. The most severe one is that the entries of the matrices $y_{u,d}$ and w are independent order-one parameters. Predictions for the various physical quantities can only be formulated in terms of distributions, assuming some statistical distribution for the unknown matrix elements of $y_{u,d}$ and w. Models in this class typically predict flat distributions for the CP violating phases. Thus features such as the closeness of the Dirac CP phase to the maximal value are purely accidental in this framework. It is not possible to go beyond order-of magnitude estimates, whereas today we have precise data and we would like to have models whose predictions can be tested at the level of accuracy reached by the present experiments.

3 More symmetry?

More predictive frameworks typically require more symmetries. Early model building has been largely influenced by some features of lepton mixing angles such as the smallness of θ_{13}, the closeness of the atmospheric angle to the maximal value and, more recently, the indication of a maximal Dirac CP phase. Some form of quark-lepton complementarity has also been invoked. If one or more of these features are not accidental, they can help us in searching for some fundamental principle that rules the flavour sector. Several symmetric patterns of lepton mixing angles have been suggested in the past, such as the tribimaximal (TB) mixing or the bimaximal (BM) mixing:

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \quad , \qquad U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad . \tag{10}$$

They incorporate some of the above-mentioned features. Today we know that these patterns need sizeable corrections, but they can still be adopted as first order approximations to the true lepton mixing matrix U_{PMNS}. Following this approach we can regard U_{PMNS} as an expansion around a leading order matrix U^0_{PMNS}, which can coincide with U_{TB}, U_{BM} or some other symmetrical form:

$$U_{PMNS} = U^0_{PMNS} + ... \tag{11}$$

where dots stand for corrections. It is not difficult to identify flavour symmetries leading to U^0_{PMNS}. For example discrete flavour symmetries showed very efficient in reproducing U_{TB}, U_{BM} or other leading order patterns. These constructions require small non-abelian permutation groups, such as A_4 and S_4. In the so-called direct approach we can predict the three mixing angles and the CP violating phase, while neutrino masses are only constrained within extended ranges and are fitted by adjusting the free parameters [7].

Before a non-vanishing θ_{13} was established, the TB mixing nicely agreed with the data. In most of the models based on discrete symmetries, deviations from U_{TB} were expected to be small, not to spoil the agreement between the predicted and measured value of the solar mixing angle, which was known to a good accuracy. In particular the angle θ_{13} was expected not to exceed few degrees, a prediction that turn out to be wrong. While working with Guido on this topic, I was very excited about the neat prediction offered by our A_4 model. Guido, much wiser and forward-looking than me, wanted to add the following comment in our paper [8]: *"Special models are those where some symmetry or dynamical feature assures in a natural way the near vanishing of θ_{13} and/or of $\theta_{23} - \pi/4$. Normal models are conceptually more economical and much simpler to construct. We expect that experiment will eventually find that θ_{13} is not too small and that θ_{23} is sizably not maximal."*

Indeed $\theta_{13} \approx 9^0$ is much larger than the value predicted by the simplest schemes leading to TB mixing at the leading order. On the one side this supports models based on anarchy and its variants, which were expecting θ_{13} of about that size. On the other hand this result does not rule out models based on non-abelian symmetries, in particular discrete symmetries. For instance through a rotation in the 23 or 13 neutrino sectors, the TB mixing is modified into a pattern with non-vanishing θ_{13} and non-maximal θ_{23}, while the solar angle is unchanged, to first approximation. Similarly the BM mixing can be altered by a rotation in the 12 charged lepton sector, bringing the solar angle to the experimentally allowed range and generating a non-vanishing θ_{13}. These modifications can be obtained by relaxing the symmetry requirements and lead to testable sum rules among the three mixing angles and the Dirac CP violating phase. Alternatively we can look for other leading order mixing patterns, closer to the data, by scanning the set of discrete groups. There are infinitely many discrete groups and a full classification of all the related lepton mixing patterns exists now [9]. Mixing angles close to the observed ones can be obtained by appealing to sufficiently large groups, but in all cases the Dirac CP phase is trivial, which is disfavored by the present data. Finally, we can combine discrete flavour symmetries with the CP symmetry and analyse the possible symmetry breaking patterns. In this case, even choosing small discrete groups one can reproduce realistic mixing angles and a non trivial Dirac phase.

One of the weak points of the approach is that there are no predictions for neutrino masses in models based on discrete symmetries. Moreover there is no hint for such symmetries from quarks. Large hierarchies and small mixing angles do not seem to require discrete groups. Extension to GUTs are possible and there are many existence proofs, but they look rather complicated. To summarize in the words of Guido [10] : *"In conclusion, one could have imagined that neutrinos would bring a decisive boost towards the formulation of a comprehensive understanding of fermion masses and mixings. In reality it is frustrating that no real illumination was sparked on the problem of flavor. We can reproduce in many different ways the observations, in a wide range that goes from anarchy to discrete flavor symmetries but we have not yet been able to single out a unique and convincing baseline for the understanding of fermion masses and mixings. In spite of many interesting ideas and the formulation of many elegant models the mysteries of the flavor structure of the three generations of fermions have not been much unveiled."*

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