Wormhole on the Lobachevsky Background

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Abstract

The exact spherical - symmetric static solution of Rosen - like equations of the bimetric theory is investigated. The background metric is not flat, but curved, with the Lobachevsky spatial sections and "cosmic time" c^2dt^2. There are two branches of the solution. The first is similar to the Shcwarzchild solution and turns to it when the Lobachevsky constant goes to ∞ but the second describes the traversible wormhole and has no Einstein's limit.

During many years, numerous attempts were made to extend the Einstein gravitation. One of these attempts is the well - known bimetric theories. At first, this type of models were considered by N. Rosen. He has treated some special cases when the background metric is a flat [1] or constant curvature metric [2]. If the background metric is flat (or Ricci - flat), the Rosen equations coincide with the Einstein ones. On the contrary, when the background metric is not flat, the equations obtained differ from the Einstein equations and nobody found spherically symmetric solutions to these equations.

Recently, Chernikov suggested considering a new interesting case [3] when the background metric

$$d\check{s}^2 = \check{g}_{ij}dx^idx^j$$

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describes the space - time that is a direct product of 3-dimensional Lobachevsky space and the "cosmic time" c^2dt^2:

$$d\check{s}^2 = c^2dt^2 - dr^2 - k^2\sinh^2\frac{r}{k}(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where the Lobachevsky constant k is the radius of the spatial curvature.

The Rosen - like equations are

$$R_{ij} = \check{R}_{ij}. \tag{2}$$

Here $R_{ij} = R^a_{aij}$ is the Ricci tensor of the physical metric g_{ij}, where the Riemann tensor

$$R^a_{kln} = \partial_k\Gamma^a_{ln} - \partial_l\Gamma^a_{kn} + \Gamma^a_{ks}\Gamma^s_{ln} - \Gamma^a_{ls}\Gamma^s_{kn};$$

$\check{R}_{ij}$ is the background Ricci tensor.

These equations are derived from the Rosen - like Lagrangian

$$L = \sqrt{-g}g^{mn}(P^a_{mb}P^b_{an} - P^a_{sa}P^s_{mn}),$$

where the affine - deformation tensor

$$P^i_{mn} = \check{\Gamma}^k_{mn} - \Gamma^k_{mn}$$

is the difference between the background connection coefficients $\check{\Gamma}^k_{mn}$ and the Christoffel symbols Γ^k_{mn}. Let us denote

$$\begin{aligned} d\Omega^2 &= d\theta^2 + \sin^2\theta d\phi^2, \\ \Lambda_1 &= \exp(2r_0/k)\frac{\sinh\frac{r-r_0}{k}}{\sinh\frac{r+r_0}{k}}, \\ \Lambda_2 &= \exp(2r_0/k)\frac{\cosh\frac{r-r_0}{k}}{\cosh\frac{r+r_0}{k}}. \end{aligned}$$

Chernikov found [3] the spherically symmetric solution

$$ds_1^2 = \Lambda_1c^2dt^2 - \Lambda_1^{-1}dr^2 - \exp(-2r_0/k)k^2\sinh^2\frac{r+r_0}{k}d\Omega^2. \tag{3}$$

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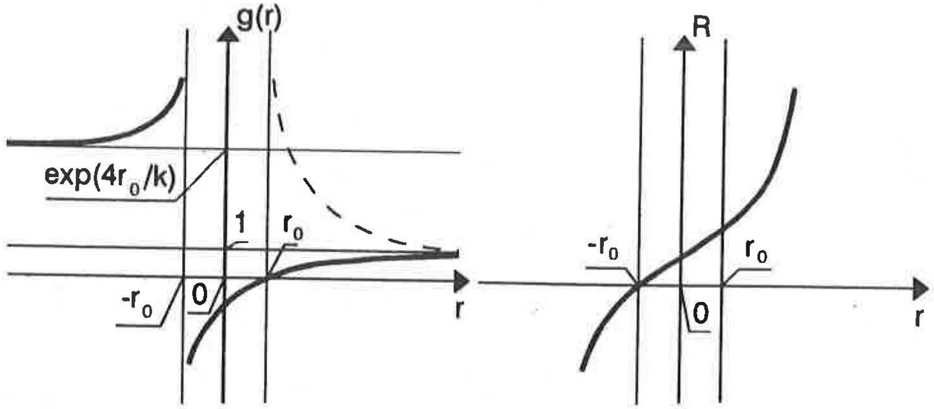
Here r_0 corresponds to the Shcwarzchild radius in the harmonic coordinates. This solution coincides with the well-known Shcwarzchild metric in the Fock representation

$$ds^2 = \frac{r - r_0}{r + r_0} c^2 dt^2 - \frac{r + r_0}{r - r_0} dr^2 - (r + r_0)^2 d\Omega^2, \tag{4}$$

if we put $k \to \infty$.



a) b)

Fig. 1. The metric function and the luminosity distance for the metric (3). The dashed line shows the value of g^{-1}.

However, this solution is not unique, and there is another branch of the solution [4] which has not Einstien's limit:

$$ds_2^2 = \Lambda_2 c^2 dt^2 - \Lambda_2^{-1} dr^2 - \exp(-2r_0/k) k^2 \cosh^2 \frac{r + r_0}{k} d\Omega^2. \tag{5}$$

All the spherically symmetric static metrics can be written as

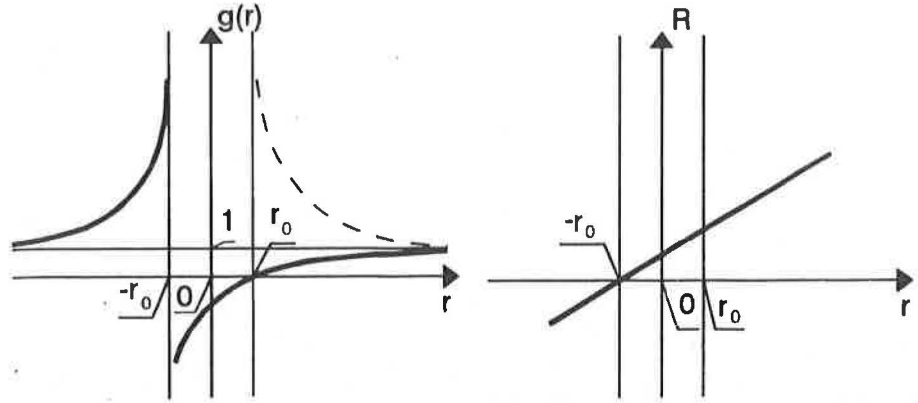
$$ds^2 = g c^2 dt^2 - \frac{1}{g} dr^2 - R^2 d\Omega^2.$$

The solution (3) corresponds to the choice of g and R as is shown in fig. 1.

The conventional Fock metric is shown in fig. 2, and the solution (5) is illustrated by fig. 3.

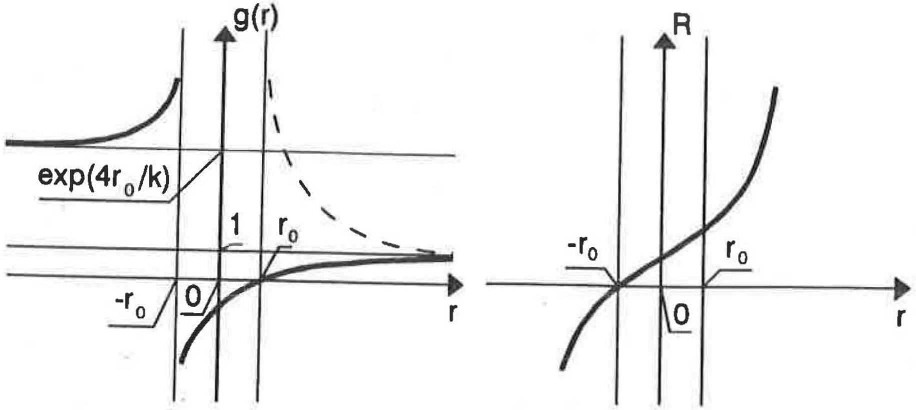
We can see that R in metrics (3) and (4) is a monotonic function of the radial coordinate r. Both these solutions have singularity at $r = -r_0$ and horizont at $r = r_0$.

On the contrary, the solution (5) is free from singularities. It is worth noting that in this case the quantity R has the minimum at $r = -r_0$ and g changes in the limits $1 < g < \exp 4r_0/k$. It means that the space - time described by this metric must be extended for negative values of r.



a) b)

Fig. 2. The metric function and the luminosity distance for the Fock metric (4). The dashed line shows the value of g^{-1}.



a) b)

Fig. 3. The metric function and the luminosity distance for the metric (5).

The total space - time can be considered as a solution of eq. (2) if the spatial sections of the background metric consist of two Lobachevsky spaces glued at $r = 0$ as shown in fig. 4.

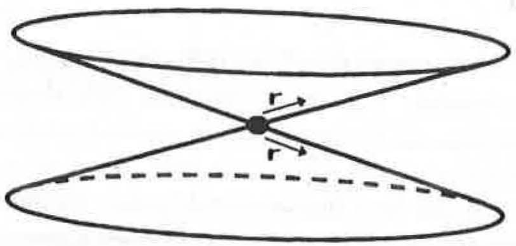


Fig. 4. The spatial section of the background metric embedded in fictitious high - dimensional space.

For interpretation of the region with negative r, we can do the following

coordinate and constant transformations:

$$\begin{aligned} \tilde r &= -r\exp(-\tfrac{2r_0}{k}); & \tilde t &= t\exp(\tfrac{2r_0}{k}); \\ \tilde k &= k\exp(-\tfrac{2r_0}{k}); & \tilde r_0 &= r_0\exp(-\tfrac{2r_0}{k}). \end{aligned}$$

Note that $\tilde r_0/\tilde k = r_0/k$, $\quad k > \tilde k$. The metric (5) takes the form

$$d\tilde s^2 = \tilde\Lambda_2 c^2 d\tilde t^2 - \tilde\Lambda_2^{-1} - \exp(\frac{2\tilde r_0}{\tilde k})\tilde k^2 \cosh^2\frac{\tilde r - \tilde r_0}{\tilde k} d\Omega^2,$$

where

$$\tilde\Lambda_2 = \exp(-\frac{2\tilde r_0}{\tilde k})\frac{\cosh\frac{\tilde r+\tilde r_0}{\tilde k}}{\cosh\frac{\tilde r-\tilde r_0}{\tilde k}}.$$

The constant - time slice of the solution is shown in fig. 5.

Now, we can see that our solution contains a traversible wormhole [5] at $r = -r_0$ connecting two infinite space - times $r > -r_0$ and $r < -r_0$. It consist of two asymptotically Lobachevskyan spaces. The scalar curvature takes different asymptotical values on these sheets. Moreover, while on the sheet with the biggest curvature we have attraction by the central source, but on the sheet with the lower curvature we have repulsion!

Concluding, this solution seems to be interesting since it is spherically symmetric and free from singularity.

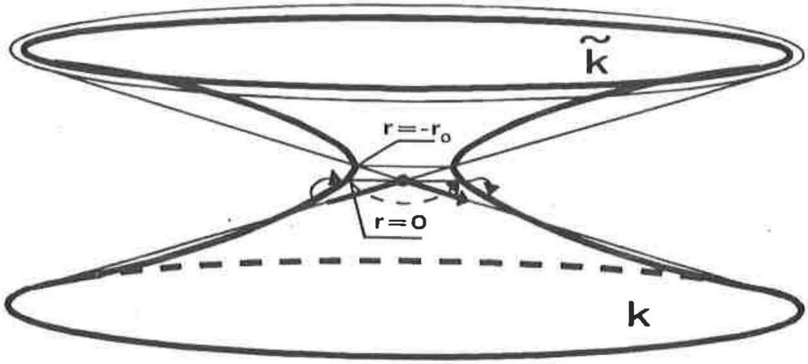


Fig. 5. The spatial section of the physical metric. The background metric is also shown. Continuous line in the background space corresponds to the discontinuity of the physical line at $r = 0$.

References

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