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To cite this article: Clayton Santos Mello *et al* 2016 *J. Phys.: Conf. Ser.* **706** 052008

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The pion electromagnetic structure with self-energy

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Abstract. We study the electromagnetic structure of the pion in terms of the quantum cromodynamic (QCD) model on the Breit-frame. We calculated the observables, such as the electromagnetic form factor. The priori to have a calculation covariant need to get the valence term of the eletromagnetic form factor. We use the usual formalism in quantum field theory (QFT) and light-front quantum field theory (LFQFT) in order to test the properties of form factor in nonperturbative QCD. In this particular case, the form factor can be obtained using the pion Light-Front (LF) wave function including self-energy from Lattice-QCD. Specifically, these calculations was performed in LF formalism. We consider a quark-antiquark vertex model having a quark self-energy. Also we can use other models to compare the pion electromagnetic form factor with different wave function and to observe the degree of agreement between them.

1. Introduction

The dynamics of the internal structure of hadrons affects their observable properties, and the electromagnetic form factor of hadrons is an example of such an observable [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. The interaction of a virtual photon with a meson probes its internal structure and dynamics through the meson form factor [11, 12, 13, 14]. The study of their form factors will thus allow us to extract information about the nonperturbative dynamics of its constituents [9, 14]. Specifically in this paper we treat only the pion meson internal structure. The theoretical prediction of electromagnetic form factor $F_\pi(q^2)$ at experimentally accessible q^2, below say 10 GeV^2, is a nontrivial task since the complex nonperturbative physics of confinement [1], dynamical chiral symmetry breaking (DCSB), and bound state structure are highly dependent on the modelling of the strong coupling regime that is not reachable using perturbative-QCD.

For our model we propose the inclusion of the self-energy of the quark at the quark-antiquark vertex $\Gamma_\pi(k)$ in the chiral limit [15], where the pion mass $m_\pi = 0$. This account was used only to define this vertex. Since the calculation of the form factor was conducted in the Breit-frame the vertex must be out of the chiral limit to satisfy the momentum conservation. The form factor was derived from the Feynman triangular diagram at impulse approximation. So we have only one pion elastic form factor [9].

The valence wave function of the pion in our model comes from the projection of the Bethe-Salpeter amplitude in the Light-Front (LF), and integrated in k^- (energy in LF). The four-vector momentum space in the LF, is defined as, k^μ, where $\mu = -, +, \perp$, energy, longitudinal and transversal momenta respectively.

In the configurations space we have x^μ, where $\mu = -, +, \perp$, longitudinal position, time and transversal direction respectively [16]. Here we use the valence wave function for the purpose of

showing only the pion form factor of valence whose is the square modulus of the wave function less the quark-photon vertex $\Gamma^\mu(k, P)$. It will be constructed from Ward-Takahashi identity (WTI) [17] in order to ensure the momentum conservation for the electromagnetic current J^μ. In order to minimize the zero-mode terms in LF [18] we calculate only the current component J^+. Since we have the electromagnetic current of the pion we can get the electromagnetic form factor, as explained in the following sections.

2. Pion and its constituent quarks

Dynamical chiral symmetry breaking (DCSB) is one of the most important properties of low energy QCD [19], and its breaking pattern has profound impact on phenomenological quantities, e.g. the appearance of pseudoscalar Goldstone bosons [20] and the non-degeneracy of chiral partners. The spontaneous breaking of chiral symmetry is a remarkable feature of QCD because it cannot be derived directly from the Lagrangian [19] it is related to the nontrivial structure of the QCD vacuum, characterised by strong condensates of quarks and gluons [15]. This is quite different from the explicit symmetry breaking, which is put in by hand through the finite quark masses, and appears in a similar way through the Higgs mechanism. There are two important consequences of the spontaneous breaking of chiral symmetry. The first one is that the valence quarks acquire a dynamical or constituent mass through their interactions with the collective excitations of the QCD vacuum that is much larger than the seed mass present in the Lagrangian. The second one is the appearance of a triplet of pseudoscalar mesons of low mass (π^+, π^-, π^0) which represent the associated Goldstone bosons [21].

The prominent role played by the pion as the Goldstone boson of spontaneously broken chiral symmetry has its impact on the low-energy structure of hadrons through pion cloud effects in the quark propagation [22]. In full QCD there are hadron contributions to the fully dressed quark-gluon vertex. These effects are generated by the inclusion of dynamical sea quarks in the quark-gluon interaction, and are therefore only present in the unquenched case. It is the aim of this paper to introduce these pion cloud effects into the quark propagator through quark mass function, and then all the way up into the meson Bethe-Salpeter amplitude (BSA) and the pion electromagnetic form factor.

3. Self-energy into pion-quark-antiquark vertex

Due to the inclusion of quark self-energy at vertex we have a pseudoscalar pion vertex it is given by [15]:

$$\Gamma_\pi(k) = i\gamma^5 M(k) \; ; \; \text{where } M(k) = m_0 - \frac{m^3}{k^2 - \lambda^2 - i\epsilon} \; , \tag{1}$$

where γ^5 is the Dirac matrix, k is the relative momentum between the constituent quarks of the pion. $M(k)$ is the quark mass function that has been obtained from Schwinger-Dyson equation (SDE) solutions [5, 23] who was able to fit the Lattice-QCD calculations [24, 25, 26]. The $m_0 = 0.014 \; GeV$ is the current quark mass and $m^3 = 0.189 \; GeV^3$ and $\lambda^2 = 0.639 \; GeV^2$ are the Lattice-QCD parameters [6]. The quark propagator also contains the quark mass function due to the presence of the self-energy in the legs into pion-quark-antiquark vertex. So we have to the quark propagator in the LF coordinates:

$$S(k) \;\; = \;\; \frac{\not{k} + M(k)}{k^2 - M^2(k) + i\epsilon} = \frac{\not{k}_{\text{on}} + M(k)}{k^2 - M^2(k) + i\epsilon} + \frac{\gamma^+}{2k^+} \; , \tag{2}$$

In the expression above, the $\not{k}_{\text{on}}$ subscript indicates the quark is on-shell.

And that has been obtained by the separation $\frac{\gamma^+}{2k^+}$ instant term in LF. The $k_{\mathrm{on}}^- = \frac{k_\perp^2 + M^2(k)}{k^+}$, it is on-shell energy. The four-vector in LF is defined for the usual coordinates as: $k^\pm = k^0 \pm k^3$ and $k_\perp = (k^1, k^2)$ the same for the Dirac matrices. The consequence of this is that the dot product $k^2 = k^+ k^- - k_\perp^2$ [16]. The increase $i\epsilon$ allows us to delocate the poles that contribute to integration on k^- via Cauchy's theorem [18]. This calculation makes the relative time between quarks to be eliminated [9].

3.1. Quark-photon vertex

For the Dirac structure of the electromagnetic current, we make use of Ward-Takahashi identity [17]. So we can extract the quark-photon vertex as follows:

$$
\begin{aligned}
q_\mu \Gamma^\mu(k; P, P') &= S^{-1}(P' - k) - S^{-1}(P - k) \\
\Gamma^\mu(k; P, P') &= \gamma^\mu + \Lambda^\mu(k; P, P') \ ,
\end{aligned}
\tag{3}
$$

where the correlation function, $\Lambda^\mu(k, P)$, is given by:

$$
\Lambda^\mu(k; P, P') = \frac{m^3 (2k - P' - P)^\mu}{[(P' - k)^2 - \lambda^2 + i\epsilon][(P - k)^2 - \lambda^2 + i\epsilon]} \ .
\tag{4}
$$

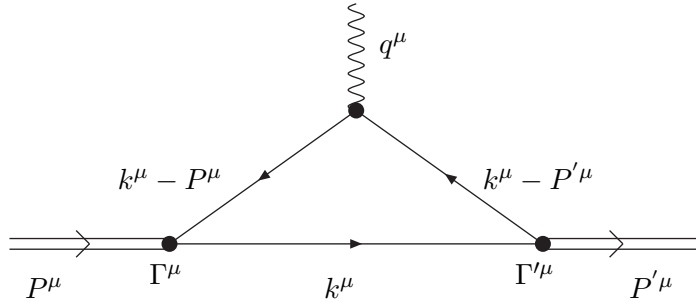From the Feynman triangular diagram, we obtain the pion electromagnetic current J^μ:

Figure 1: Triangular diagram in impulse approximation.

$$
\begin{aligned}
J^\mu &= i\frac{N^2 N_c}{f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} Tr\left[S(k)\Gamma_\pi(P' - k)S(P' - k)\Gamma^\mu(k; P, P')S(P - k)\Gamma_\pi(P - k)\right] \\
J^\mu &= i\frac{N^2 N_c}{f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{Tr\left[\mathcal{O}^\mu(k; P, P')\right]}{[k^2 - M^2(k) + i\epsilon]} M(P' - k)M(P - k) \\
&\qquad \frac{1}{[(P - k)^2 - M^2(P - k) + i\epsilon][(P' - k)^2 - M^2(P' - k) + i\epsilon]} \ .
\end{aligned}
\tag{5}
$$

Now we might writte the Dirac structure of this way:

$$
\begin{aligned}
\mathcal{O}^\mu(k; P, P') &= (\not{k} + M(k))\gamma^5 (\not{P}' - \not{k} + M(P' - k)) \Gamma^\mu(k; P, P') \\
&\quad (\not{P} - \not{k} + M(P - k))\gamma^5 \ ,
\end{aligned}
\tag{6}
$$

where $q = P' - P$, and using the Breit frame that is the transfered momentum in x-direction $q = (0, q^x, 0, 0)$. To the total initial momentum of the pion it is $P = (P^0, -q^x/2, 0, 0)$ and total final momentum is $P' = (P^0, q^x/2, 0, 0)$. We also made use of Drell-Yan condition, $q^+ = 0$, when

there no transfer momentum on longitudinal direction [18, 27, 28]. The Dirac trace structure, is writing as,

$$
\begin{aligned}
\mathcal{P}_n^+(k, P, P') &= -4m^6 \left[k^+ (a_n - b_n - c_n) + \left(b_n P^+ + c_n P'^+ \right) \right] \\
&+ 4m_0 m^3 \left[\left(c_n P'^+ + b_n P^+ \right) a_n + \left(-2k^+ + P^+ + P'^+ \right) b_n c_n \right] \\
&+ m_0 \left[4 \left(k_x^2 + m_0^2 \right) + 2 \left(P'^- P^+ + P^- P'^+ \right) + q^2 \right] \times \\
&\times a_n (2k^+ - P^+ - P'^+) m^3 \\
&+ a_n b_n c_n \left\{ \left[-4 \left[k_n^- (k^+ - P^+)(k^+ - P'^+) \right. \right. \right. \\
&+ \left. k_x^2 (P^+ + P'^+) \right] - 2k_x (P'^+ - P^+) q \\
&+ \left. k^+ (4k_x^2 + q^2) \right] + 4m_0^2 \left[k^+ - P^+ - P'^+ \right] \right\} \\
&+ \left\{ \left[-2k^+ P'^- a_n c_n + 2k_x q (c_n - b_n) a_n - 2k^+ P^- a_n b_n \right. \right. \\
&- 4k_x^2 (a_n (b_n + c_n) - b_n c_n) + 2k^+ b_n c_n (P'^- + P^-) \\
&- 4m_0^2 (a_n (b_n + c_n) + b_n c_n) \\
&+ \left. b_n c_n (-2P'^- P^+ - 2P^- P'^+ - q^2) \right] m^3 \\
&+ 2k_n^- \left[\left(-P'^+ a_n c_n - P^+ a_n b_n \right) + 2k^+ (a_n (b_n + c_n) - b_n c_n) \right. \\
&+ \left. (P^+ + P'^+) b_n c_n \right] m^3 - 4m^6 \left[m^3 - m_0 (a_n + b_n + c_n) \right] \\
&- \left. 4m_0 k_n^- k^+ a_n b_n c_n \right\} \frac{(2k^+ - P^+ - P'^+) m^3}{b_n c_n} \\
Tr[\mathcal{O}_n^+(k; P, P')] &= \frac{\mathcal{P}_n^+(k; P, P')}{a_n b_n c_n} \, .
\end{aligned}
\tag{7}
$$

Thus we have the electromagnetic current this way:

$$
\begin{aligned}
J^+ &= i \frac{N^2 N_c}{f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{m^6 Tr \left[\mathcal{O}^+(k; P, P') \right] a_n^2 b_n^2 c_n^2}{(a_n^2 (k^2 + i\epsilon) - (m_0 a_n - m^3)^2)) \, b_n c_n} \\
&\quad \frac{1}{(b_n^2 ((P - k)^2 + i\epsilon) - (m_0 b_n - m^3)^2))} \\
&\quad \frac{1}{(c_n^2 ((P' - k)^2 + i\epsilon) - (m_0 c_n - m^3)^2))} \\
&= i \frac{N^2 N_c}{f_\pi^2} \int \frac{d^4 k}{(2\pi)^4} \frac{m^6 \mathcal{P}^+(k; P, P')(y - \lambda^2 + i\epsilon)}{(y - y_1)(y - y_2)(y - y_3)} \\
&\quad \frac{1}{(z - z_1)(z - z_2)(z - z_3)(w - w_1)(w - w_2)(w - w_3)} \, .
\end{aligned}
\tag{8}
$$

Now we have written the quark mass function below, in order to obtain an expression for the pion electromagnetic form factor:

$$
\begin{aligned}
M(k) &= m_0 - \frac{m^3}{a_n} \, , \quad a_n = k^+ \left(k_n^- - \frac{f_a - i\epsilon}{k^+} \right) \, , \\
M(P - k) &= m_0 - \frac{m^3}{b_n} \, , \quad b_n = (P^+ - k^+) \left(P^- - k_n^- - \frac{f_b - i\epsilon}{(P^+ - k^+)} \right) \, , \\
M(P' - k) &= m_0 - \frac{m^3}{c_n} \, , \quad c_n = (P'^+ - k^+) \left(P'^- - k_n^- - \frac{f_c - i\epsilon}{(P'^+ - k^+)} \right) \, ,
\end{aligned}
\tag{9}
$$

where:

$$
\begin{aligned}
f_1 &= k_\perp^2 + Re[y_1] \ , \ f_2 = k_\perp^2 + Re[y_2] \ , \ f_3 = k_\perp^2 + Re[y_3] \ , \ f_a = k_\perp^2 + \lambda^2 \\
f_4 &= (P-k)_\perp^2 + Re[z_1] \ , \ f_5 = (P'-k)_\perp^2 + Re[z_2] \ , \ f_6 = (P-k)_\perp^2 + Re[z_3] \\
f_7 &= (P'-k)_\perp^2 + Re[w_1] \ , \ f_8 = (P-k)_\perp^2 + Re[w_2] \ , \\
f_9 &= (P'-k)_\perp^2 + Re[w_3] \ , \ f_b = (P-k)_\perp^2 + \lambda^2 \ , \ f_c = (P'-k)_\perp^2 + \lambda^2 \ .
\end{aligned} \tag{10}
$$

where $z_1 = w_1 = y_1$, $z_2 = w_2 = y_2$ and $z_3 = w_3 = y_3$ are the denominator roots of the electromagnetic current and they are derived of propagators when including the self-energy in legs at pion vertex. We identify the poles in k^- when we perform a change of variable $k^2 = y$, $(P-k)^2 = z$ and $(P'-k)^2 = w$, and the roots are:

$$
\begin{aligned}
e &= -6m^3 m_0 - 2\lambda^2 m_0^2 + m_0^4 \ , \\
f &= -18\lambda^2 m^3 m_0 - 6\lambda^4 m_0^2 + 18 m^3 m_0^3 + 6\lambda^2 m_0^4 - 2m_0^6 \ , \\
g &= 36\lambda^2 m^9 m_0 + 8\lambda^4 m^6 m_0^2 - 4m^9 m_0^3 - 4\lambda^2 m^6 m_0^4 \ , \\
d &= \sqrt[3]{2\lambda^6 - 27m^6 + 3\sqrt{3}\sqrt{-4\lambda^6 m^6 + 27m^{12} + g} + f} \ , \\
y_1 &= \frac{2}{3}\lambda^2 + \frac{m_0^2}{3} - \frac{\sqrt[3]{2}}{3d}\left(\lambda^4 + e\right) - \frac{d}{3\sqrt[3]{2}} - i\epsilon \ , \\
y_2 &= \frac{2}{3}\lambda^2 + \frac{m_0^2}{3} + \frac{(1+i\sqrt{3})}{3\sqrt[3]{4}d}\left(\lambda^4 + e\right) + \frac{(1-i\sqrt{3})d}{6\sqrt[3]{2}} - i\epsilon \ , \\
y_3 &= \frac{2}{3}\lambda^2 + \frac{m_0^2}{3} + \frac{(1-i\sqrt{3})}{3\sqrt[3]{4}d}\left(\lambda^4 + e\right) + \frac{(1+i\sqrt{3})d}{6\sqrt[3]{2}} - i\epsilon \ .
\end{aligned} \tag{11}
$$

The electromagnetic current, J^+, is also, writting like,

$$
J^+ = i\frac{N^2 N_c}{f_\pi^2} \int \frac{d^2 k_\perp dk^+ dk^-}{2(2\pi)^4} \frac{m^6 \mathcal{P}_n^+(k; P, P') a_n}{\mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_3 \mathcal{D}_4 \mathcal{D}_5 \mathcal{D}_6 \mathcal{D}_7 \mathcal{D}_8 \mathcal{D}_9} \ . \tag{12}
$$

here, are,

$$
\begin{aligned}
\mathcal{D}_1 &= k^+\left(k_n^- - \frac{f_1 - i\epsilon}{k^+}\right) ; \ \mathcal{D}_2 = k^+\left(k_n^- - \frac{f_2 - i\epsilon}{k^+}\right) ; \\
\mathcal{D}_3 &= k^+\left(k_n^- - \frac{f_3 - i\epsilon}{k^+}\right) ; \ \mathcal{D}_4 = \left(P^+ - k^+\right)\left(P^- - k_n^- - \frac{f_4 - i\epsilon}{P^+ - k^+}\right) ; \\
\mathcal{D}_5 &= \left(P'^+ - k^+\right)\left(P'^- - k_n^- - \frac{f_5 - i\epsilon}{P'^+ - k^+}\right) ; \ \mathcal{D}_6 = \left(P^+ - k^+\right)\left(P^- - k_n^- - \frac{f_6 - i\epsilon}{P^+ - k^+}\right) ; \\
\mathcal{D}_7 &= \left(P'^+ - k^+\right)\left(P'^- - k_n^- - \frac{f_7 - i\epsilon}{P'^+ - k^+}\right) ; \ \mathcal{D}_8 = \left(P^+ - k^+\right)\left(P^- - k_n^- - \frac{f_8 - i\epsilon}{P^+ - k^+}\right) ; \\
\mathcal{D}_9 &= \left(P'^+ - k^+\right)\left(P'^- - k_n^- - \frac{f_9 - i\epsilon}{P'^+ - k^+}\right) \ .
\end{aligned}
$$

We can identify nine propagators in electromagnetic current equation, J^+, with the following

poles:

$$
\begin{aligned}
k_1^- &= \frac{f_1}{k^+} - \frac{i\epsilon}{k^+}, \ k_2^- = \frac{f_2}{k^+} - \frac{i\epsilon}{k^+}, k_3^- = \frac{f_3}{k^+} - \frac{i\epsilon}{k^+}, \\
k_4^- &= P^- - \frac{f_4}{P^+ - k^+} + \frac{i\epsilon}{P^+ - k^+}, \ k_5^- = P'^- - \frac{f_5}{P'^+ - k^+} + \frac{i\epsilon}{P'^+ - k^+}, \\
k_6^- &= P^- - \frac{f_6}{P^+ - k^+} + \frac{i\epsilon}{P^+ - k^+}, k_7^- = P'^- - \frac{f_7}{P'^+ - k^+} + \frac{i\epsilon}{P'^+ - k^+}, \\
k_8^- &= P^- - \frac{f_8}{P^+ - k^+} + \frac{i\epsilon}{P^+ - k^+}, k_9^- = P'^- - \frac{f_9}{P'^+ - k^+} + \frac{i\epsilon}{P'^+ - k^+}.
\end{aligned} \tag{13}
$$

To valence range of the electromagnetic current, we verified the poles contribution, Fig. (2).
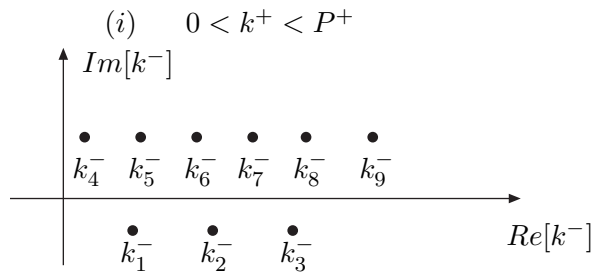

Figure 2: Poles position in Argand-Gauss plane for valence term of the electromagnetic current.

We have the valence contribuition term for the electromagnetic current, J^{+V}, which correspond the interval integration, $0 < k^+ < P^+$, in the light-front energy, k^-, (see the Fig.(2) above), is given below,

$$
J^{+V} = \frac{N^2 N_c}{f_\pi^2} \sum_{n=1}^3 \int \frac{d^2 k_\perp dk^+}{2(2\pi)^3} \frac{m^6 \mathcal{P}_n^+ \mathcal{D}_n}{k^+ \mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_3 \mathcal{D}_4 \mathcal{D}_5 \mathcal{D}_6 \mathcal{D}_7 \mathcal{D}_8 \mathcal{D}_9} \ , \tag{14}
$$

From the electromagnetic current, we can obtain the pion space-like electromagnetic form factor, with the expression below:

$$
\left\langle P'^+ \right| J^+ \left| P^+ \right\rangle = e(P'^+ + P^+) F_\pi(q^2) \ . \tag{15}
$$

where e is the elementary charge, and $F_\pi(q^2)$ is the electromagnetic form factor; the constant normalization N is obtained from the condition of charge, $F_\pi(q^2 = 0) = 1$, [7, 8, 18, 29, 30].

4. Numerical results

For analysis of our model we present the results we obtained for the electromagnetic form factor of the pion. In the Fig. (3) on the left we can see the form factor as a function of the square transfer momentum. And on right we find the same function multiplied by q^2. Also we compare our results with two other models, they are Light-front symetric vertex [30] and non-symetric vertex [29]. Since the technique used in our calculations is also based on these models have drawn analytical valence wave function to separate the instant terms in LF. We compared our model for the form factor with the experimental data according to the references [31, 32, 33, 34]. These data are for describing the structure of the pion at low energies. In this case q^2 for values lower than 10 $[GeV/c]^2$.
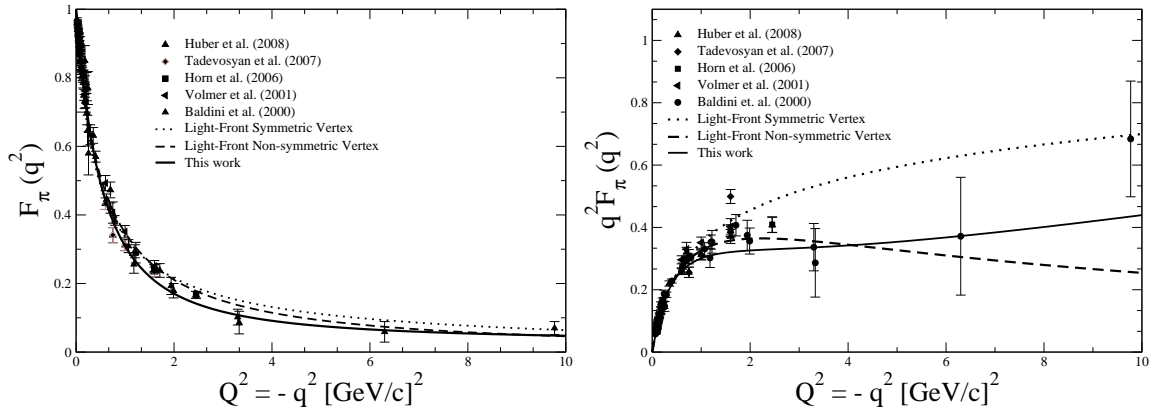
Figure 3: Pion electromagnetic form factor in terms of momentum transfer, q^2, with the light-front symmetric model, point line [30], dashed line, light-front non-symmetric vertex [29], solid line, present work, with the parameters from Lattice-QCD [6] and the experimental [35]. In this work, we use the following values parameters from the lattice-QCD, i.e, $\lambda^2 = 0.639 \; GeV^2$, $m_0 = 0.014 \; GeV$ and $m^3 = 0.189 \; GeV^3$.

5. Conclusion

In this paper, we show how to describe the electromagnetic structure of a particle pseudoscalar, pion. Through a constituent quark model, we obtained some of the pion observable, but using the formalism in light-front we see across terms that are not invariant under Lorentz transformations. In our case we do not find non-valence terms to form factor when it is in Breit-frame and on Drell-Yan condition.

We study the electromagnetic current in respect to the valence term which we can extract the wave function of two fermions system, the quark-antiquark pair that composes the pion structure. From the wave function at the center of mass of the system, technique that has identified valence wave function in our model, we get the observable of pion, like the electromagnetic form factor. The electromagnetic current is decribed by constituent quarks model with a self-energy at pion vertex as expected. The form factor decreases with increasing of the transfer momentum. In relation to the form factor multiplied by square transfer momentum, it increases with increasing of q^2. The quark mass function describes the system in which particles dynamically gain mass, according to the constituent quark model. From the contribution of the photon we can study the internal structure of the pion as pion electromagnetic structure.

References

[1] James P. Vary, Nucl.Phys.Proc.Suppl. **251-252** (2014) 155.
[2] B. L. G. Bakker, Few-Body Syst **49** (2011) 177.
[3] P. Maris and C. D. Roberts , J. Mod. Phys. **E 12**, 297 (2003).
[4] A. Bashir, L. Chang, I. C. Cloët, B. El-Bennich, Y-X. Liu, C. D. Roberts and P. C. Tandy ,Commun. Theor. Phys. **58**, 79 (2012).
[5] E. Rojas, J. P. B. C. de Melo, B. El-Bennich, O. Oliveira and T. Frederico, JHEP **1310**, 193 (2013) [arXiv:1306.3022 [hep-ph]].
[6] D. Dudal, M. S. Guimarães, L. F. Palhares and S. P. Sorella, arXiv:1303.7134 [hep-ph].
[7] J.P.B.C. de Melo, T. Frederico, E. Pace, G. Salme, Phys.Lett. **B581** (2004) 75.
[8] J.P.B.C. de Melo, T. Frederico, E Pace and G. Salme, Phys.Rev. **D73** (2006) 074013.
[9] J. P. B. C. de Melo and T. Frederico, Phy. Rev. **C55** (1997) 2043.
[10] P. Maris; C. D. Roberts and P. C. Tandy, Physics Letters **B 420**, 267273 (1998).
[11] Clayton S. Mello, Anacé N. da Silva, J.P.B.C. de Melo and T. Frederico, Few Body Syst. **56** (2015) 6-9, 509.

[12] J.P.B.C. de Melo, Anac N. da Silva, Clayton S. Mello and T. Frederico, Few Body Syst. **56** (2015) 6-9, 503.

[13] Clayton S. Mello, Jose P.Cruz Filho, Edson O. da Silva, Bruno El-Bennich, J.P.B.C. de Melo and Victo S. Filho, AIP Conf.Proc. **1520** (2013) 333.

[14] Clayton S. Mello, Anacé Nunes da Silva, J.P.B.C. de Melo and T. Frederico, Nucl.Phys.Proc.Suppl. **251-252** (2014) 62.

[15] C. D. Roberts, M. S. Bhagwat, A. Holl and S. V. Wright Eur. Phys. J. Special Topics **140**,(2007) 53.

[16] Harindranath, A., Pramana **vol. 55**, n. 1 and 2, (2000), 141. And hep-ph 9612244 v 230, jun (1998).

[17] Y. Takahashi, "On the generalized Ward identity", Nuovo Cim. **6** (1957) 371.

[18] J. P. B. C. de Melo, Covariância na Frente de Luz, Tese de Doutorado, IFUSP, (1998).

[19] Itzykzon, C and Zuber, J B, Quantum Field Theory., New york, **vol. 1**, (1980).

[20] Drell, Sidney D and Bjorken, James D, Relativistic Qauntum Mechanics, Editor McGraw-Hill Book Company, **Vol.1**, (1964).

[21] Alan D Martin e Francis Halzen, Jhon Wiley and Sons, (1984), New York, USA.

[22] Lei Chang; I. C. Cloët; J. J. Cobos-Martinez; C. D. Roberts; S. M. Schmidt and P. C. Tandy, PRL **110**,132001 (2013).

[23] S.R. Beane, W. Detmold, K. Orginos, M.J. Savage, Prog.Part.Nucl.Phys. **66** (2011) 1.

[24] Quantum Fields on a Lattice, I. Montvay and G. Münster, Cambridge University Press, (1997).

[25] George F. Sterman, Paul Stoler, Ann.Rev.Nucl.Part.Sci. **47** (1997) 193.

[26] A. H. Mueller, Nucl. Phys. **A654** (1999) 37C.

[27] Edson O. da Silva, J.P.B.C. de Melo, Bruno El-Bennich, Victo S. Filho, Phys.Rev. **C86** (2012) 038202.

[28] George H. S. Yabusaki, Ishtiaq Ahmed, M. Ali Paracha, J.P.B.C. de Melo, Bruno El-Bennich, Phys.Rev. **D92** (2015) 3, 034017.

[29] J. P. B. C. de Melo, H. W. L. Naus and T. Frederico, Phy. Rev. C**59** (1999) 2278.

[30] J. P. B. C. de Melo, T. Frederico, E. Pace, and G. Salmé, Nucl. Phys. A**707** (2002) 399; Braz. Jour. Phys. **33** (2003) 301.

[31] J. Volmer *et al.*, Phy. Rev. Lett. **86** 1713 (2001).

[32] T. Horn *et al.*, Phys. Rev. Lett. **97**, 192001 (2006).

[33] V. Tadevosyan *et al.*, Phys Rev. C **75**, 055205 (2007).

[34] G. M. Huber *et al.*, Phys. Rev. C **78**, 045203 (2008).

[35] R. Baldini, E. Pasqualucci, S. Dubnicka, P. Gauzzi, S. Pacetti and Y. Srivastava, Nucl. Phys. A **666** (2000) 38.