NONPERTURBATIVE UNIFICATION OF TECHNICOLOR

G. Grunberg

Centre de Physique Théorique
Laboratoire Propre N° 14 du CNRS,
Ecole Polytechnique, 91128 - Palaiseau - Cedex -France.

ABSTRACT

Nonperturbative unification of technicolor (TC) with the standard model forces is considered in the scheme of Maiani, Parisi and Petronzio, where all interactions become strong at the GUT scale M_{GUT}. SU(4) with a single family of technifermions in the 6 emerges as the only plausible TC group. The low energy couplings are computed. A very flat behavior of the TC coupling is obtained, which could allow to raise up to M_{GUT} the scale of the new interactions responsible for the lightest pseudogoldstone bosons masses.

I. INTRODUCTION

In this talk, I describe an attempt [1] to bring together two apparently unrelated ideas, each with strong theoretical appeal, namely technicolor [2] (TC) and the nonperturbative unification scheme of Maiani, Parisi and Petronzio [3] (MPP). TC was introduced as an alternative to the elementary Higgs mechanism of $SU(2)_L \times U(1)_Y$ breaking (which is beset by the naturalness and gauge hierarchy problems). The symmetry breaking is now due to the vacuum condensation of new fermions (technifermions) bound by a new vector gauge force (TC) similar to QCD, but becoming strong at a scale $\Lambda_{TC} = 0(1\text{ TeV})$ (which could be explained by unification ideas [4]).

It is natural to assume technifermions carry the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ charges as ordinary fermions, and I shall concentrate below on the popular one technifamily model [5]:

$\begin{pmatrix} U^c \\ D^c \end{pmatrix}^i_L , \; (U^c)^i_R , \; (D^c)^i_R \; ; \; \begin{pmatrix} N \\ E \end{pmatrix}^i_L , \; (N^i)_R , \; (E^i)_R$ where $c: 1 \to 3$ is the color index, and i: $1 \to N$ is a TC index. The existence of a right handed technineutrino is assumed, in order to preserve the relation $M_W = M_Z \cos\theta_W$. The following two well-known problems afflict this model:

i) Since one technifamily contains 16 species of Weyl technifermions, there is a large global symmetry (in the absence of the standard model forces), resulting in a large number of pseudogoldstone bosons (there may be as many as 135 if the TC representation is real [5]), only 3 of them being "eaten" by the W and the Z. Even after switching on the standard model interactions, there still subsist two very light charged and two massless neutral pseudo's, which have not been observed so far.

ii) The TC condensates do not break the chiral $\psi \to \gamma_5\, \psi$ invariance of the ordinary (technisinglet) fermions, which thus stay massless.

II. NONPERTURBATIVE UNIFICATION

MPP assumed [3] the gauge interactions of the standard model are not asymptotically free above the Fermi scale $\Lambda_F \approx 250$ GeV (which requires $N \geq 5$ additional generations, on top of the three known ones), and become strong near a common high energy cut - off $M_{GUT} >> \Lambda_F$. If M_{GUT} is large enough, the low energy couplings will be close to their infrared (IR) stable fixed point value, namely zero, and therefore small. They will also be insensitive to the actual large values of the "fundamental" couplings at M_{GUT}. Thus, this scheme allows to predict them as a function of the number of generations, in term of a single free parameter (e.g:M_{GUT}), and without recourse to a specific grand unification group at M_{GUT}. As an example, consider the solution of the standard model one loop renormalization group (RG) equations:

$$\frac{1}{\alpha_i(\mu)} - \frac{1}{\alpha_i(M)} = \frac{\beta_o^i}{4\pi} \ln \frac{M^2}{\mu^2} \qquad (2.1)$$

If $\beta_o^i > 0$, the couplings will grow above $\mu \approx \Lambda_F$. Let us assume $\alpha_1(M) \approx \alpha_2(M) \approx \alpha_3(M) = 0(1) >> \alpha_i(\Lambda_F)$ at some scale $M = M_{GUT}$. Then the $1/\alpha_i(M)$ terms in eq. (2.1) can be neglected, giving:

$$\frac{1}{\alpha_i(\mu)} \approx \frac{\beta_0^i}{4\pi} \ln \frac{M_{GUT}^2}{\mu^2} \qquad (2.2)$$

from which one predicts, e.g. $tg^2\,\theta_W = \frac{\alpha_1}{\alpha_2} = \frac{\beta_0^2(N)}{\beta_0^1(N)}$ Phenomenologically acceptable values are obtained [3] for $5 \leq N \leq 7$. However, the requirement $N \geq 5$ appears ad-hoc in the MPP scheme, and furthermore, its original version with elementary Higgs scalars is still plagued by the gauge hierarchy problem. In the next section, I turn to a natural suggestion to adress both these questions.

III. INCLUDING TC IN THE MPP SCHEME

Let us assume 1) the N additional generations needed in the MPP scheme are just technifermion components in an N-dimensional representation of the TC group G_{TC} of a single technifermion S0(10) family: the one technifamily model is thus recovered, provided the standard model gauge group is extended to $G_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$. Conversely, one can show 1) that the requirement that TC be asymptotically free at one loop level, and that technifermions form a single S0(10) family belonging to an N-dimensional real representation of G_{TC} implies $N \geq 5$, if one also assumes that G_{TC} commutes with S0(10). Therefore, the single technifamily model naturally leads to a situation where the standard model interactions are not asymptotically free above Λ_{TC}, itself very suggestive of nonperturbative unification.

The next question is whether nonperturbative unification of TC is possible at all, since one would like to impose two conditions on β_{TC} (the TC β function) which are not obviously compatible, namely $\beta_{TC} < 0$ at low energies (a condition I assume necessary for the formation of a low energy technifermion condensate as the coupling grows strong in the IR region), but $\beta_{TC} > 0$ at high energy (for nonperturbative unification). I now show that this can be achieved provided $\beta_0 < 0$ but $\beta_1 > 0$, where β_0 and β_1 are respectively the one and two loop TC β function coefficients:

$$\frac{d\,\alpha_{TC}}{d\,\ln\mu^2} = \beta_{TC}\,(\alpha_{TC}) = \frac{\beta_0}{4\pi}\,\alpha_{TC}^2 + \frac{\beta_1}{(4\pi)^2}\,\alpha_{TC}^3 + \ldots$$

This assumption implies the existence of an IR fixed point $\alpha_{IR} = -\,4\pi \times \dfrac{\beta_0}{\beta_1}$ in the perturbative β_{TC}, stable against higher order corrections if α_{IR} is small enough. If there were no other interactions, starting from large values $\alpha_{TC}(M_{GUT}) >> \alpha_{IR}$ (such that $\beta_{TC} > 0$), and going down in energy, α_{TC} would monotonously decrease down to α_{IR}, and would never be able to move from the $\beta_{TC} > 0$ to the $\beta_{TC} < 0$ region. However, the presence of the standard model couplings, which contribute at the two loop level, drastically change the picture. We now have:

$$\beta_{TC}(\alpha_{TC}, \alpha_i) = \frac{\beta_0}{4\pi}\alpha_{TC}^2 + \frac{\beta_1}{(4\pi)^2}\alpha_{TC}^3 + \alpha_{TC}^2\sum_i \frac{\beta_1^i}{(4\pi)^2}\alpha_i + \dots \quad (i: 1\to 3)$$

Starting from large values of α_{TC}, α_i at M_{GUT}, and assuming the β functions are positive there, all four couplings will decrease with μ, until at some finite scale, α_{TC} reaches the values α_{IR}, while the α'_i s are still non zero. This is possible, since the true IR fixed point for the coupled system is at $\alpha_{TC} = \alpha_{IR}$, $\alpha_i = 0$ (I assume the standard model β functions are positive definite above Λ_{TC}). At this point, one gets

$$\beta(\alpha_{TC}, \alpha_i) = \alpha_{TC}^2\sum_i \frac{\beta_1^i}{(4\pi)^2}\alpha_i > 0$$ (the β_1^{i} ' s , representing the fermion loop

contribution, are always positive). Consequently, as μ is further decreased, α_{TC} will continue to decrease below α_{IR}. Since it must reached again its IR stable value α_{IR} in the limit $\mu \to 0$, α_{TC} necessarily goes through a minimum at some scale $\mu = \mu_0$. Then, as μ is further decreased in the "asymptotically free region" $\mu<\mu_0$ (where $\beta_{TC} < 0$), α_{TC} will continuously rise towards α_{IR} (while the α'_i s keep decreasing towards zero). Spontaneous chiral symmetry breaking is expected to take place at some scale $\Lambda_{TC} < \mu_0$, where α_{TC} becomes "strong enough", which, following current wisdom [6], I shall assume to mean

$\alpha_{TC}(\Lambda_{TC}) > \alpha_c \equiv \frac{\pi}{3} \times \frac{1}{C_2(R)}$, where $C_2(R)$ is the quadratic Casimir of the technifermion representation. The latter condition can be realized only if $\alpha_c < \alpha_{IR}$.

IV. A UNIQUE TC GROUP AND REPRESENTATION

One now has to search for G_{TC} with the constraint of having a single technifamily, ie 16 Weyl technifermions, in the fundamental representation, and such that $\beta_0 < 0$ and $\beta_1 > 0$.

Let us consider first $G_{TC} = SO(N)$ in the vector representation. Then

$$\beta_0 = -\frac{11}{3}N + \frac{54}{3}, \quad \beta_1 = -\frac{34}{3}N^2 + \frac{344}{3}N - 168 ,$$

which implies $N \geq 5$ (for $\beta_0<0$), and $N\leq 8$ (for $\beta_1>0$). Furthermore, N = 5 is excluded:

numerical results indicate that no minimum exists for α_{TC}, and β_{TC} always stays positive (the reason is that both α_{TC} and α_3 have very small, altough non-zero, IR fixed points of comparable magnitude in this case: $\alpha_{TC/IR} = 0.026$ and $\alpha_{3/IR} = 0.031$): SO(5) is too small a group to allow asymptotic freedom to be recovered in presence of 16 Weyl fermions. On the other hand, SO(7) and SO(8) are also excluded, this time because, being "too much asymptotically free", these groups cannot be nonperturbatively unified with low energy values of α_{TC} not too far above α_c. For instance, for N = 7, imposing $\alpha_{TC} \approx 1$ at M_{GUT}, one gets $\alpha_{TC} \geq 1$ in the whole energy range, ie a weak coupling regime is never reached, which suggests $\Lambda_{TC} = 0(M_{GUT})$ in this case. We are thus left only with SO(6) ≈ SU(4), for which one gets $\alpha_{IR} = 0.45$, a small enough value to make it reasonable that higher order terms in the perturbative β functions will not destabilize the two-loop fixed point, as long as non-perturbative effects (such as decoupling of technifermions) can be neglected. Note also in this model $\alpha_c = 0.42 < \alpha_{IR}$.

Turning to the other classical groups, one finds that G_{TC} = SU(N) with a single family in the N is excluded, since both β_0 and β_1 are negative for N > 2.G_{TC} = Sp(2N) is also excluded for N>1 for the same reason. Note the latter theories have pseudoreal TC representation, which has been shown [6)] to lead to phenomenologically unacceptable breaking of $SU(2)_L \times U(1)_y$.

V. LOW ENERGY COUPLINGS

The coupled RG equations for the $SU(4)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$ model (with a single technifamily in the 6 and three ordinary families) have been integrated, proceeding backward from M_{GUT} down to Λ_{TC}. In this range, one deals with 6 + 3 = 9 SO(10) families, whereas below Λ_{TC}, the TC sector decouples, and one deals effectively with the standard model interactions and three ordinary families. With large ultraviolet values of the couplings, low energy couplings depend only upon the ratio M_{GUT}/Λ_{TC}, which is adjusted to reproduce α_{em} (M_W) = 1/129 (for a given value of Λ_{TC}/M_W). Taking $M_{GUT}/\Lambda_{TC} = 4.4\times10^{10}$, one finds $\alpha_{TC}(\Lambda_{TC}) = 0.43$, $\alpha_3(\Lambda_{TC}) = 0.054$, $\alpha_2(\Lambda_{TC}) = 0.031$ and $\alpha_1(\Lambda_{TC})$ =

0.0105, with a very flat minimum $\alpha_{TC}(\mu_0) = 0.41$ at $\mu_0/\Lambda_{TC} \approx 10^5 - 10^6$ (depending upon the initial conditions). It is remarkable that α_{TC} hardly runs over most of the energy range, which, as we shall see, has interesting consequences for the pseudogoldstone bosons masses. When evolved below Λ_{TC}, these values yield:

$\sin^2\theta_W(M_W) = 0.239$, $\alpha_{em}(M_W) = 1/128$, $\alpha_3(M_W) = 0.063$ (if $\Lambda_{TC} = 10M_W$) ; or
$\sin^2\theta_W(M_W)=0.231$, $\alpha_{em}(M_W) = 1/129$, $\alpha_3(M_W) = 0.071$ (if $\Lambda_{TC} = 50\,M_W$).

VI. CONCLUSIONS AND DISCUSSION

Nonperturbative unification of TC with the standard model forces can be achieved, without recourse to a grand unification group at M_{GUT} (which presumably does not exist [7)]), and leads to a unique determination of the TC group and representation. One can also show[1)] this scheme gives interesting constraints on the number n of ordinary generations, in particular $n = 2$ appears necessary if one wants to have $M_{GUT} < M_{Planck}$ with $\Lambda_{TC} = 0(1\text{TeV})$. The present picture relies on the assumption of a "desert" between Λ_{TC} and $M_{GUT} = 0(10^{13}\text{ GeV})$. Additional interactions, which here are required to switch on only above M_{GUT}, are however necessary [2)] to raise the lightest pseudogoldstone boson masses, and to give masses to ordinary fermions. Even with such a high scale, these new interactions may still push the lightest pseudo's in the TeV range, owing to the very flat behavior of α_{TC}, which modifies the high energy behavior of the technifermion condensate, upon which these masses depend (see ref. 8)). The prospect for ordinary fermion masses seems less good, however. Of more immediate concern is the dynamics of chiral symmetry breaking and the calculability of the W mass in the present scheme. The crucial question is whether Λ_{TC} and M_W are really determined by the low energy behavior of α_{TC}, and are insensitive to the ultraviolet values of the couplings. This question is presently under study, and preliminary investigation indicate that an upper bound on the high-energy value of α_{TC} may be necessary[9)].

REFERENCES

1) G. Grunberg, Ecole Polytechnique preprint A 805 .1287 (to appear in Phys. Rev. D); Phys. Lett. B 203 (1988) 413.

2) S. Weinberg, Phys. Rev. D 13 (1976) 974; D 19 (1979) 1277; L. Susskind, Phys. Rev. D20 (1979) 2619. For reviews see E. Fahri and L. Susskind, Phys. Rep. 74 (1981) 217; R.K. Kaul, Rev. Mod. Phys. 55 (1983) 449.

3) L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B 136 (1978) 115; N. Cabbibo and G.R. Farrar, Phys. Lett. B 110 (1982) 107; G. Grunberg, Phys. Rev. Lett. 58 (1987) 1180; Ecole Polytechnique preprint A 710 .03 87 (1987).

4) H. Georgi, H.R. Quinn and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.

5) E. Fahri and L. Susskind, Phys. Rev. D 20 (1979) 3404; S. Dimopoulos, Nucl. Phys. B 168 (1980) 69.

6) See the review of M. Peskin, in: Recent advances in field theory and statistical mechanics, Les Houches Lectures (1982), eds J.B. Zuber and R. Stora (North Holland, Amsterdam, 1984).

7) P.H. Frampton, Phys. Rev. Lett. 43 (1979) 1912.

8) B. Holdom, Phys. Lett. 150B (1985) 301; T. Appelquist, D. Carrier, L.C.R. Wijewardhana and W. Zheng, Phys. Rev. Lett. 60 (1988) 1114, and references therein.

9) G. Grunberg and M. Mashaal, in preparation.