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Conformal Constraints and the Evolution of the Nonsinglet Meson Distribution Amplitude*

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Using conformal Ward identities, the influence of the conformal symmetry breaking on the evolution of the flavor-nonsinglet quark-antiquark distribution amplitude is investigated in dimensional regularization and a minimal subtracted scheme. It is found that a conformal consistency relation provides some simplification for construction of the distribution amplitude. Both the evolution in next-to-leading approximation and a closed expression for the $O(\alpha_s)$ correction to the eigenfunctions of the evolution kernel in the $\beta = 0$ case can thus be determined analytically.

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I. INTRODUCTION

For different exclusive quantum chromodynamic (QCD) processes it has been proven that the scattering amplitude at large momentum transfer Q^2 factors as a convolution of process-independent distribution amplitudes, with a process-dependent perturbatively computable hard scattering amplitude [1–4]. Furthermore, the leading order perturbative QCD analysis has been performed for a large number of such processes including mesons and baryons. For processes including only pseudoscalar flavor-nonsinglet mesons, such as the electromagnetic form factor and the transition form factor, the next-to-leading order was performed both partially [5] and seminumerically [6]. The critical point of these analyses is the determination of the distribution amplitude in next-to-leading order.

The Q^2 evolution of the flavor-nonsinglet distribution function $\phi(x, Q^2)$ of a pseudoscalar meson is controlled by [7]

$$Q^2 \frac{d}{dQ^2} \phi(x, Q^2) = \int_0^1 dy V(x, y; \alpha_s(Q^2)) \phi(y, Q^2). \tag{1}$$

Here x is the fraction of the longitudinal meson momentum which is carried by the valence quark, and $\alpha_s = g^2/(4\pi)$ is the QCD fine structure constant. The evolution kernel $V(x, y; \alpha_s) = (\alpha_s/2\pi)V^{(0)}(x, y) + (\alpha_s/2\pi)^2 V^{(1)}(x, y) + \cdots$ has been computed perturbatively in one- and two-loop approximation by using the dimensional regularization in the modified minimal subtracted ($\overline{\text{MS}}$) scheme. The complicated two-loop computation was performed by different authors in light-cone gauge [8] and in covariant gauge [9]. The results agree with each other, however, they are in conflict with conformal symmetry prediction for the eigenfunctions of $V(x, y; \alpha_s)$ [10].

The symmetry prediction was obtained by postulating conformal symmetry for the operator product expansion at short distances. The prediction should be valid for a non-trivial fixpoint g^* so that $\beta(g^*) = 0$. The predicted eigenfunctions

$$P_k(x) \propto \frac{d^k}{dx^k} (x(1-x))^{k+1-\gamma_k(\alpha_s)/2} \tag{2}$$

are generalizations of the polynomials $(1-x)xC_k^{3/2}(2x-1)$, where $C_k^{3/2}$ denotes the Gegenbauer polynomials of order 3/2. The eigenvalues of the evolution kernel are $\gamma_k/2$ where γ_k coincides with the forward nonsinglet anomalous dimensions for deep inelastic scattering. In scalar $[\phi^3]_6$ theory in six dimensions for MS or $\overline{\text{MS}}$ schemes, the eigenfunctions of the explicit computed two-loop evolution kernel possess the predicted form[†]; but this is not the case in QCD. The conclusion of the analysis in Ref. [10] was that the conformal symmetry in gauge field theories is broken by an unknown source.

In this paper the conformal symmetry breaking in massless gauge theory will be analyzed with the help of conformal Ward identities. Thus, it is more convenient to treat with so-called conformal operators instead of directly with the amplitude $\phi(x, Q^2)$. The relation between $\phi(x, Q^2)$ and the expectation values of the conformal operators, and the construction of $\phi(x, Q^2)$ in leading approximation will be reviewed in Sec. II.

In Sec. III the special conformal Ward identities for Green functions of conformal operators will be derived in a dimensional regularization and $\overline{\text{MS}}$ scheme. For technical simplification the analysis is restricted to the abelian case. However, since for $\beta = 0$ the symmetry breaking part of $V^{(1)}(x, y)$ does not contain the Casimir operator C_A of the adjoint representation of $SU_c(3)$, this treatment is sufficient to explain the two-loop results in QCD. All sources of the conformal symmetry breaking will be identified, including violation by renormalization of the coupling constant. Furthermore, we will obtain, from a conformal consistency relation, the influence of special conformal anomaly on the off-diagonal anomalous dimension matrix of the conformal operators.

In Sec. IV the special conformal anomaly matrix is computed in one-loop order. The two-loop approximation of the off-diagonal anomalous dimension matrix follows from the mentioned consistency relation.

In Sec. V the evolution of $\phi(x, Q^2)$ will be constructed in dependence of the forward anomalous dimensions γ_k and the special conformal anomaly matrix. With the help of the results in Sec. IV, the simplicity of the developed formalism is demonstrated for the next-to-leading order approximation. This new analytical result could not be obtained directly from the evolution equation (1). Furthermore, correction to the eigenfunctions of $V(x, y; \alpha_s)$ for $\beta = 0$ the $O(\alpha_s)$ will be given in a "closed form."

[†]The analysis showed also that the prediction is not valid in any renormalization scheme.

II. DISTRIBUTION AMPLITUDE IN LEADING ORDER

The leading order solution of the evolution equation (1) can be directly constructed with the help of the moments [1,2]:

$$\int_0^1 dx C_k^{\frac{3}{2}}(2x-1)\phi(x;Q^2) = < 0|O_{kk}(\mu^2)|P >^{red}_{|\mu^2=Q^2} . \tag{3}$$

Here $< 0|O_{kk}(\mu^2)|P >^{red} = < 0|O_{kl}(\tilde{n},\mu^2)|P > /(\tilde{n}P)^{l+1}$ denotes reduced expectation values of local gauge invariant operators

$$O_{kl}(\tilde{n}) =: \overline{\psi}(0)\lambda\Gamma C_k^{\frac{3}{2}}\left(\frac{\tilde{n}\overleftrightarrow{D}}{\tilde{n}\partial_+}\right)(i\tilde{n}\partial_+)^l\psi(0) :, \tag{4}$$

where $\partial_+^\nu = \overrightarrow{\partial}^\nu + \overleftarrow{\partial}^\nu$ and $\overleftrightarrow{D}^\nu = \overrightarrow{\partial}^\nu - \overleftarrow{\partial}^\nu + 2igA^\nu$, which are sandwiched between vacuum and one meson state with momentum P. The flavor matrix λ and the spin matrix $\Gamma = \gamma^5\tilde{n}\gamma$ correspond to a specific flavor-nonsinglet pseudoscalar meson. The arbitrary light-cone vector $\tilde{n}_\nu$ picks out the leading twist two contributions, which are given by spin $(l+1)$ tensors. Therefore, because of Lorentz-invariance, there is no mixing under renormalization with respect to the index l. Furthermore, the renormalization of the operators induces the renormalization point μ, which is set equal to the factorization scale Q.

In one-loop order the operators are multiplicatively renormalizable [2,11] so that the evolution of the moments can be easily determined. This property was investigated in Refs. [11,13] and it comes from the fact that the operators form an infinite irreducible representation of the collinear conformal algebra in the free field theory, which is the subalgebra O(2,1) of the full conformal algebra O(4,2) [12]. Therefore, each "tower" of operators is labelled by the conformal spin k. However, it can be seen in Sec. III that conformal symmetry is violated on the one-loop level. A further renormalization property, which holds true in each order of perturbation theory, comes from the Poincaré-invariance of the theory. The operator O_{kl} mixes with only the operators $O_{k'l}$, for which $k' \leq k$ is valid (see, e.g., Ref. [10]). Thus, the general form of the renormalization group equation is

$$\mu\frac{d}{d\mu}O_{kl}(\mu^2) = \sum_{k'=0}^{k}\gamma_{kk'}(\alpha_s(\mu^2))O_{k'l}(\mu^2), \tag{5}$$

where the anomalous dimension matrix $\gamma_{kk'} = (\alpha_s/2\pi)\gamma_k^{(0)}\delta_{kk'} + (\alpha_s/2\pi)^2\gamma_{kk'}^{(1)} + \cdots$ is diagonal in one-loop order. Note that $\gamma_{kk'}$ is directly related to the evolution kernel,

$$\gamma_{kk'}(\alpha_s) = 2\int_0^1 \int_0^1 dxdy C_k^{\frac{3}{2}}(2x-1)V(x,y;\alpha_s)\frac{(1-y)y}{N_{k'}}C_{k'}^{\frac{3}{2}}(2y-1), \quad (6)$$

where $N_k = (k+1)(k+2)/(8k+12)$ is a normalization factor and $(1-x)xC_k^{3/2}(2x-1)$ are just the eigenfunctions of $V^{(0)}(x,y)$. Because of the triangularity of $\hat{\gamma} := \{\gamma_{kk'}\}$, the eigenvalues of $V(x,y;\alpha_s)$ are also beyond the one-loop order given by $\gamma_{kk}/2$.

Let us now construct the Q^2 dependence of $\phi(x,Q^2)$. Using the completeness relation of Gegenbauer polynomials (see, e.g., [14])

$$\sum_{k=0}^{\infty} \frac{(1-x)x}{N_k} C_k^{\frac{3}{2}}(2x-1)C_k^{\frac{3}{2}}(2y-1) = \delta(x-y), \; N_k = \frac{(k+1)(k+2)}{8k+12}, \quad (7)$$

one finds from Eq. (3) the following conformal spin expansion of the distribution amplitude [10,15]:

$$\phi(x,Q^2) = \sum_{k=0}^{\infty} \frac{(1-x)x}{N_k} C_k^{\frac{3}{2}}(2x-1) < 0|O_{kk}(\mu^2)|P >^{red}_{|\mu^2=Q^2} . \quad (8)$$

The Q^2 dependence of the reduced matrix elements are given by the simple leading order solution of Eq. (5),

$$O_{kl}(Q^2) = \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{\frac{\gamma_k^{(0)}}{\beta_0}} O_{kl}(Q_0^2), \quad \alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{Q^2}{\Lambda^2}\right)}. \quad (9)$$

Here Q_0 is an appropriate reference momentum, $\Lambda \sim 0.3$ GeV is the QCD scale parameter,

$$\gamma_k^{(0)} = C_F\left(3 + \frac{2}{(k+1)(k+2)} - 4\sum_{i=1}^{k+1}\frac{1}{i}\right), \quad (10)$$

where $C_F = 4/3$ and $\beta_0 = (11/3)C_A - (2/3)n_f$ with n_f is the number of quark flavors and $C_A = 3$. Inserting the solution (9) in the representation (8) one gets the leading order solution of the evolution equation (1):

$$\phi(x,Q^2) = \sum_{k=0}^{\infty} \frac{(1-x)x}{N_k} C_k^{\frac{3}{2}}(2x-1) \left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{\frac{\gamma_k^{(0)}}{\beta_0}} < 0|O_{kk}(Q_0^2)|P >^{red} . \quad (11)$$

Because of Eq. (10) the partial waves for $k > 0$ will be suppressed with increasing Q^2 so that $\phi^{as}(x) = 6(1-x)x$ $< 0|O_{00}|P >^{red}$ is the asymptotic distribution amplitude.

Let us remark that the nonperturbative input $< 0|O_{kk}(Q_0^2)|P >^{red}$ can be computed from Eq. (3), i.e.,

$$< 0|O_{kk}(Q_0^2)|P >^{red} = \int_0^1 dx C_k^{\frac{3}{2}}(2x-1)\phi(x;Q_0^2). \quad (12)$$

A theoretical estimation of the first two moments for a reference momentum square of $Q_0^2 \sim 1$ GeV is more or less possible from sum rules [16], instanton vacuum model [17], and lattice QCD [18].

III. CONFORMAL CONSTRAINTS

The simple form of the leading order approximation of $\phi(x, Q^2)$ will be changed in the next-to-leading order by the mixing of the operators O_{kl}. In this section it will be shown that this mixing is completely induced by special conformal symmetry breaking. In the first step the Ward identities of special conformal transformation (CWI) will be derived in abelian gauge theory in the $\overline{\mathrm{MS}}$ scheme (see introduction). In the second step a consistency relation will be written down from which the off-diagonal part of the anomalous dimension matrix $\hat{\gamma}$ can be computed. An analogous treatment for scalar $[\phi^3]_6$ theory in six dimensions was given in detail in Ref. [19]; therefore only a few steps will be pointed out.

As mentioned, it is sufficient to treat with the colinear conformal algebra. The corresponding transformation laws for the fields follow from the special conformal transformation laws by contraction with the dual light-cone vector $\tilde{n}^*$, which satisfies the normalization condition $\tilde{n}^*\tilde{n} = 1$ [20]:

$$\delta^c_- \psi(x) = \rho\left(2\tilde{n}^* x\left(d_\psi + x\frac{\partial}{\partial x}\right) + 2\tilde{n}^{*\mu}\Sigma_{\mu\nu}x^\nu - x^2\tilde{n}^{*\mu}\frac{\partial}{\partial x^\mu}\right)\psi(x), \tag{13}$$

$$\delta^c_- A_\alpha(x) = \rho\left(2\tilde{n}^* x\left(d_A + x\frac{\partial}{\partial x}\right) + 2\tilde{n}^{*\mu}\Sigma_{\mu\nu}x^\nu - x^2\tilde{n}^{*\mu}\frac{\partial}{\partial x^\mu}\right)A_\alpha(x). \tag{14}$$

Here ρ is a small parameter, $\Sigma_{\mu\nu}$ is the spin operator,

$$\Sigma_{\mu\nu}\psi = \frac{1}{4}[\gamma_\mu, \gamma_\nu]\psi, \quad \Sigma_{\mu\nu}A_\alpha = g_{\mu\alpha}A_\nu - g_{\nu\alpha}A_\mu, \tag{15}$$

and $d_i, i = \{\psi, A\}$, are the scaling dimensions of the fields. To include the conformal symmetry breaking by renormalization of the fields in the representation of the conformal algebra, one has to choose d_ψ and d_A as the sum of canonical and anomalous dimension. Taking the canonical dimension for space-time dimension $n = 4 - 2\epsilon$, then

$$d_\psi(\epsilon, g) = 3/2 - \epsilon + \gamma_\psi(g), \qquad d_A(\epsilon, g) = 1 - \epsilon + \gamma_A(g), \tag{16}$$

where $\gamma_i = \mu\frac{d}{d\mu}\ln\sqrt{z_i(\epsilon, \mu^2)}, i = \{\psi, A\}$, are the anomalous dimensions and $z_i(\epsilon) = 1 + z_i^{[1]}/\epsilon + z_i^{[2]}/\epsilon^2 + \cdots$ are the renormalization factors of the fields computed in $\overline{\mathrm{MS}}$ scheme [21]. In this scheme the relations $\gamma_i = -g\frac{\partial}{\partial g}z_i^{[1]}(g)/2$ are valid [22].

A simple treatment for the derivation of CWI for Green functions of the operator O_{kl} is allowed in the functional integral approach. Using dimensional regularization, the regulator of the uv divergencies is manifest in the renormalized action by the choice of the non-integer space-time dimension $n = 4 - 2\epsilon$, $\epsilon > 0$ [19,23]:

$$[S] = \int d^n x \mathcal{L}(x), \quad \mathcal{L} = i z_\psi \bar{\psi} \gamma^\nu D_\nu \psi - \frac{1}{4} z_A F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial A)^2,$$

where $D_\nu = \partial_\nu + ig\overline{\mu}^\epsilon A_\nu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$‡. Here $\overline{\mu} = \mu[\exp\{\gamma_E\}/(4\pi)]^{1/2}$, where γ_E is the Euler constant, is chosen to satisfy the $\overline{\text{MS}}$ convention. Using the invariance of the generating functional for renormalized disconnected Green functions of the operators O_{kl},

$$Z_{kl}(\bar{\eta}, \eta, J) = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A\, [O_{kl}] e^{i[S] + i\int d^n x (\bar{\eta}(x)\psi(x) + \bar{\psi}(x)\eta(x) + J_\nu(x) A^\nu(x))}, \tag{17}$$

with respect to the special conformal transformations (13) and (14), and differentiating the resulting identity with respect to the sources $\eta, \bar{\eta}$, and J_μ, one gets the following result:

$$< [O_{kl}](\delta^c_- X) > = - < (\delta^c_- [O_{kl}]) X > - < [O_{kl}](\delta^c_- [S]) X > . \tag{18}$$

Here $< A >$ denotes the vacuum expectation value of the time-ordered product $\mathrm{T} A \exp i[S]$, $[O_{kl}]$ is the renormalized operator insertion, and X symbolizes the product of elementary fields at different space-time points. The left-hand side of the identity (18) is simply given by a differential operator [see Eqs. (13) and (14)] acting on the renormalized Green functions $< [O_{kl}] X >$, and therefore both sides of Eq. (18) are finite.

Next let us compute the right-hand side of the CWI (18) beginning with the variation $\delta^c_- [O_{kl}]$. The renormalized operator insertion can be expressed as

$$[O_{kl}] = \sum_{k'=0}^{k} Z_{kk'}(\epsilon, g) O_{k'l}, \tag{19}$$

where $O_{k'l}$ is defined in (4) with $\overleftrightarrow{D}^\nu = \overrightarrow{\partial}^\nu - \overleftarrow{\partial}^\nu + 2ig\overline{\mu}^\epsilon A^\nu$. Using Eq. (19), the calculation of $\delta^c_- [O_{kl}]$ is straightforward and provides a conformal non-covariant and divergent expression

‡Without explicit notation it will be treated with different flavor fields to allow the definition of flavor-nonsinglet operators.

$$\delta_{-}^{c}[O_{kl}] = -i\rho\sum_{k'=0}^{k}\left\{\hat{Z}(\epsilon,g)\left(\hat{a}(l)+2(\gamma_\psi(g)-\epsilon)\hat{b}(l)\right)\hat{Z}^{-1}(\epsilon,g)\right\}_{kk'}[O_{k'l-1}]$$
$$+\rho\frac{\bar{\beta}(\epsilon,g)}{g}\int d^n x 2(\tilde{n}^* x)A_\mu\frac{\delta}{\delta A_\mu}[O_{kl}], \tag{20}$$

where the matrices $\hat{a}(l)$ and $\hat{b}(l)$ are defined by the elements [19]:

$$a_{kk'}(l) = 2(k-l)(k+l+3)\delta_{kk'},$$
$$b_{kk'}(l) = \begin{cases} 2(l+k'+3)\delta_{kk'} - 2(2k'+3) & \text{if } k-k' \geq 0 \text{ and even} \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

The first term on the right-hand side in Eq. (20) has the same form as in scalar $[\phi^3]_6$ theory. In addition, a further term appears, which is caused by the gauge field and is proportional to[§] $\bar{\beta}/g$. Note that $\bar{\beta}(\epsilon,g) = \mu\frac{d}{d\mu}g(\mu)$ is divided in $\bar{\beta}(\epsilon,g) = -\epsilon g + \beta(g)$, where $\beta(g)$ is explicitly independent of ϵ.

The second term on the right-hand side of Eq. (18) contains the operator product $[O_{kl}]i(\delta_{-}^{c}[S])$. At first let us consider the variation $i\delta_{-}^{c}[S]$, which provides a combination of renormalized operators,

$$i\delta_{-}^{c}[S] = \rho\frac{\bar{\beta}(\epsilon,g)}{g}[\Delta_{-}^{\beta}] + \rho\sigma(g)[\Delta_{-}^{\sigma}] + \rho[\Delta_{-}^{gf}], \tag{22}$$

where $\sigma = \mu\frac{d}{d\mu}\ln\xi(\mu)$. Each of the first two terms on the right-hand side of this equation is proportional to a renormalization group coefficient. The third term is caused by the gauge-fixing term [23,24] and has no influence on physical quantities. The renormalized operator insertions are defined by

$$[\Delta_{-}^{\beta}] = i\int d^n x 2(\tilde{n}^* x) g\frac{\partial}{\partial g}\mathcal{L}(x) - i2z_A\int d^n x(\tilde{n}^* A)\partial A, \tag{23}$$
$$[\Delta_{-}^{\sigma}] = i\int d^n x 2(\tilde{n}^* x)\xi\frac{\partial}{\partial\xi}\mathcal{L}(x) + \frac{i}{\xi}\int d^n x(\tilde{n}^* A)\partial A, \tag{24}$$
$$[\Delta_{-}^{gf}] = i\frac{n}{\xi}\int d^n x(\tilde{n}^* A)\partial A. \tag{25}$$

In the gauge-dependent sector of the theory this decomposition of $i\delta_{-}^{c}[S]$ disagrees with that in Ref. [19]. However, with the standard BRS treatment it was proved in [25] that each operator defined in Eqs. (23)–(25) is renormalized.

[§] Here the well-known relation $\gamma_A - \epsilon = \bar{\beta}/g$ was used. This interpretation follows from the analogous derivation of the dilation Ward identities and the comparison with the renormalization group equation.

Generally, products of renormalized operators can provide uv divergencies. Because of the product $[O_{kl}]i(\delta^c_-[S])$ the second term on the right-hand side of Eq. (18) is not finite. However, the renormalization of $[O_{kl}]i(\delta^c_-[S])$ is simplified in Landau gauge, i.e., $\xi \to 0$. In this gauge only the product $[\Delta^\beta_-][O_{kl}]$ appearing in $[O_{kl}]i(\delta^c_-[S])$ provides uv divergencies [25].

As mentioned above, the right-hand side of the CWI (18) is finite so that the uv divergencies in the first and second term must cancel each other. However, since $\bar{\beta} = -\epsilon g + \beta$, a finite part remains. To extract this part we introduce the (minimal) subtractive renormalization

$$\left[\left(\Delta^\beta_- + \int d^n x 2(\tilde{n}^* x) A_\mu \frac{\delta}{\delta A_\mu}\right) O_{kl}\right] = \left([\Delta^\beta_-] + \int d^n x 2(\tilde{n}^* x) A_\mu \frac{\delta}{\delta A_\mu}\right)[O_{kl}] - i \sum_{k'=0}^{k} \hat{Z}^*_{kk'}(\epsilon, g, l)[O_{k'l-1}], \tag{26}$$

where $\hat{Z}^*(\epsilon, g, l) = \hat{Z}^{[1]*}(g,l)/\epsilon + \hat{Z}^{[2]*}(g,l)/\epsilon^2 + \cdots$ is dependent on l.

Inserting Eqs. (20) and (22) into Eq. (18) and using Eq. (26), one gets in the limit $\epsilon \to 0$ and for $\xi \to 0$

$$\begin{aligned} < [O_{kl}\delta^c_- X/\rho > &= i \sum_{k'=0}^{k} \{\hat{a}(l) + \hat{\gamma}^c(g,l)\}_{kk'} < [O_{k'l-1}]X > \\ &- \frac{\beta}{g} < \left[\left(\Delta^\beta_- + \int d^n x 2(\tilde{n}^* x) A_\mu \frac{\delta}{\delta A_\mu}\right) O_{kl}\right] X > \\ &- \sigma < [\Delta^\sigma_- O_{kl}]X > - < [\Delta^{gf}_- O_{kl}]X > . \end{aligned} \tag{27}$$

We denote $\hat{\gamma}^c(g,l) = \lim_{\epsilon\to 0}\left(\hat{Z}(\epsilon,g)\left(\hat{a}(l) + 2(\epsilon - \gamma_\psi(\epsilon,g))\hat{b}(l)\right)\hat{Z}^{-1}(\epsilon,g) - \frac{\bar{\beta}(\epsilon,g)}{g}\hat{Z}^*(\epsilon,g,l) - \hat{a}(l)\right)$ as the special conformal anomaly matrix. Since $\hat{\gamma}^c$ must be finite and $\hat{Z} = \hat{1} + \hat{Z}^{[1]}/\epsilon + O(\epsilon^{-2}), \hat{Z}^{-1} = \hat{1} - \hat{Z}^{[1]}/\epsilon + O(\epsilon^{-2})$, and $\hat{Z}^* = \hat{Z}^{*[1]}/\epsilon + O(\epsilon^{-2})$ it follows in the limit $\epsilon \to 0$

$$\hat{\gamma}^c(g,l) = -2\hat{b}(l)\gamma_\psi(g) + 2[\hat{Z}^{[1]}(g), \hat{b}(l)] + \hat{Z}^{*[1]}(g,l). \tag{28}$$

Note that $\hat{Z}^{[1]}$ is induced by the ϵ term of the canonical dimension of the field ψ and in the $\overline{\text{MS}}$ scheme it can be completely expressed by $\hat{\gamma} + 2\gamma_\psi \hat{1}$ [22]. Thus, the new information contained in $\hat{\gamma}^c$ comes from $\hat{Z}^{*[1]}$, which is induced by the ϵ term of the $\bar{\beta}$ function and must be computed from Eq. (26).

A consistency relation for the anomalies can be derived from the commutator rule between the special conformal variation δ^c_- and dilation δ^d_-, i.e., $[\delta^d, \delta^c_-] = \rho\delta^c_-$, and the conformal Ward identities. Thereby, δ^d_-/ρ acts on $< [O_{kl}]X >$ as $\mu\partial_\mu + N_\psi\gamma_\psi + N_A\gamma_A$ (N_ψ and N_A denote the number of ψ and A fields in X). The result

$$\left[\hat{a}(l) + \hat{\gamma}^c(g,l) + 2\frac{\beta(g)}{g}\hat{b}(l), \hat{\gamma}(g)\right] = 0 \tag{29}$$

has the same form as in ϕ^3 theory and can be used to determine the off-diagonal part $\hat{\gamma}^{ND}$ of $\hat{\gamma}$. Since $\hat{a}(l)$ is a diagonal matrix and independent of g [see Eq. (21)] the diagonality of $\hat{\gamma}^{(0)}$ follows immediately. For the same reason, $\hat{\gamma}^{ND}$ in m-loop order is determined by the $(m-1)$-loop approximation of $\hat{\gamma}^c$, β, and $\hat{\gamma} = \hat{\gamma}^D + \hat{\gamma}^{ND}$, where $\hat{\gamma}^D$ is the diagonal part of $\hat{\gamma}$. This recurrence relation and the diagonality of $\hat{\gamma}^{(0)}$ provide

$$\begin{aligned}\hat{\gamma}^{ND}(g) &= -\frac{\mathcal{G}}{\hat{1}+\mathcal{G}}\hat{\gamma}^D(g)\\ &= -\mathcal{G}\hat{\gamma}^D(g) + \mathcal{G}^2\hat{\gamma}^D(g) - \cdots,\end{aligned} \tag{30}$$

where the operator $\mathcal{G}$ is defined by**

$$\mathcal{G}\hat{A} := \begin{cases} \frac{\left[\hat{\gamma}^c(l) + 2\frac{\beta}{g}\hat{b}(l), \hat{A}\right]_{kk'}}{2(k-k')(k+k'+3)} & \text{if } k-k' > 0 \\ 0 & \text{otherwise.}\end{cases} \tag{31}$$

From these equations we learn that the off-diagonal matrix $\hat{\gamma}^{ND}$ in m-loop order is completely determined by the $(m-1)$-loop approximation of the special conformal anomaly matrix $\hat{\gamma}^c$, the β function and the diagonal matrix $\hat{\gamma}^D$. For instance, in two-loop order one finds with $\beta/g = -(\alpha/4\pi)\beta_0 + O(\alpha^2)$, where $\alpha = g^2/(4\pi)$,

$$\gamma_{kk'}^{ND(1)} = \frac{\gamma_k^{(0)} - \gamma_{k'}^{(0)}}{2(k-k')(k+k'+3)}\left(\gamma_{kk'}^{c(0)} - \beta_0 b_{kk'}\right). \tag{32}$$

IV. COMPUTATION OF $\hat{\gamma}^{C(0)}$ AND THE OFF-DIAGONAL PART OF $\hat{\gamma}^{(1)}$

First we will compute the one-loop approximation of the special conformal anomaly matrix $\hat{\gamma}^c$ in Landau gauge. Since in $\overline{\text{MS}}$ scheme $\hat{\gamma}(g) + 2\gamma_\psi(g) = -g\frac{\partial}{\partial g}\hat{Z}^{[1]}(g)$ holds true and $\gamma_\psi^{(0)} = 0$ for $\xi = 0$ it follows from the definition (28)

$$\hat{\gamma}^{c(0)}(l) = -[\hat{\gamma}^{(0)}, \hat{b}(l)] + \hat{Z}^{*[1](0)}(l). \tag{33}$$

Corresponding to Eq. (26) $\hat{Z}^{*[1](0)}$ can be computed from the uv divergence of the two-point 1-PI Green function

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**The spin $l+1$ and the gauge parameter ξ dependence of $\hat{\gamma}^c$ must cancel out since the left-hand side on Eq. (30), i.e., $\hat{\gamma}^{ND}$, is independent of both.

$$\Gamma\left(p_1,p_2|\left([\Delta_-^\beta]+\int d^nx 2(\tilde{n}^*x)A_\nu\frac{\delta}{\delta A_\nu}\right)[O_{kl}]\right)=\Gamma\left(p_1,p_2|\Delta_-^\beta O_{kl}\right)$$
$$+\Gamma\left(p_1,p_2|\int d^nx 2(\tilde{n}^*x)A_\nu\frac{\delta}{\delta A_\nu}O_{kl}\right)+O(\alpha^2). \tag{34}$$

In Landau gauge Δ_-^β is simply given by $\Delta_-^\beta = -g\bar{\mu}^\epsilon \int d^nx 2(\tilde{n}^*x)\bar{\psi}\gamma^\nu A_\nu\psi$, and so it generates a derivative on each external vertex with respect to the incoming momentum (see Fig. 1). In the second term on the right-hand side of Eq. (34), $\int d^nx 2(\tilde{n}^*x)A_\nu\frac{\delta}{\delta A_\nu}$ generates a derivative in momentum space acting on the operator vertex (Gegenbauer polynomials). This derivative can be expressed by a derivative with respect to one external momentum and a modified operator insertion (see Fig. 2).

The sum of Figs. 1 and 2a represents the one-loop contributions with respect to $p_+ = p_1 + p_2$ differentiated unrenormalized 1-PI Green function $\Gamma(p_1,p_2|O_{kl})$. Thus, it contains the following uv divergence [19]:

$$I^a_{kl} = i\frac{\alpha}{2\pi\epsilon}\gamma_k^{(0)}2\tilde{n}^*_\nu\frac{\partial}{\partial p_{+\nu}}C_k^{\frac{3}{2}}\left(\frac{\tilde{n}p_-}{\tilde{n}p_+}\right)(\tilde{n}p_+)^l\lambda\Gamma$$
$$= i\frac{\alpha}{2\pi\epsilon}\gamma_k^{(0)}\sum_{k'=0}^{k} b_{kk'}(l)C_{k'}^{\frac{3}{2}}\left(\frac{\tilde{n}p_-}{\tilde{n}p_+}\right)(\tilde{n}p_+)^{l-1}\lambda\Gamma, \tag{35}$$

where Γ and λ are spin and flavor matrices, respectively. Corresponding to definition (26) this uv divergence provides the following contribution to $\hat{Z}^{*[1]}$:

$$\hat{Z}^{*[1]a}(l,g) = \frac{\alpha}{2\pi}\hat{\gamma}^{(0)}\hat{b}(l) + O(\alpha^2). \tag{36}$$

The remaining graphs in Fig. 2b are given by the Feynman integral

$$G^b_{kl} = i\frac{\alpha C_F}{2\pi}\int\frac{d^nk}{i\pi^2}(2\pi\bar{\mu})^{2\epsilon}\left[\frac{\gamma_\alpha(p_1-k)^\mu\gamma_\mu\tilde{n}^\nu\gamma_\nu}{(p_1-k)^2k^2}+\frac{\tilde{n}^\nu\gamma_\nu(p_2+k)^\mu\gamma_\mu\gamma_\alpha}{(p_2+k)^2k^2}\right]$$
$$\times\left[\tilde{n}^\alpha-\frac{k^\alpha\tilde{n}k}{k^2}\right](\tilde{n}k)_+^{-2}C_k^{\frac{3}{2}}\left(\frac{\tilde{n}p_-+2\tilde{n}k}{\tilde{n}p_+}\right)(\tilde{n}p_+)^l\lambda\Gamma, \tag{37}$$

where C_F =1 in abelian gauge theories. Here the following "+" definition was introduced

$$x_+^{-2}F(x) = \frac{F(x)-F(0)-F'(0)}{x^2}. \tag{38}$$

This Feynman integral can be simply computed if one replaces the Gegenbauer polynomials by $\delta(2x-1-(\tilde{n}p_-+2\tilde{n}k)/\tilde{n}p_+)$. The integration provides the uv divergence

$$I^b_{kl} = i\frac{\alpha}{2\pi\epsilon}\sum_{k'=0}^{k-2} w_{kk'} C_{k'}^{\frac{3}{2}}\left(\frac{\tilde{n}p_-}{\tilde{n}p_+}\right)(\tilde{n}p_+)^{l-1}\lambda\Gamma, \tag{39}$$

where

$$w_{kk'} = \int_0^1\int_0^1 dxdy C_k^{\frac{3}{2}}(2x-1)[w(x,y)]_+ \frac{(1-y)y}{N_{k'}} C_{k'}^{\frac{3}{2}}(2y-1), \tag{40}$$

$$[w(x,y)]_+ = w(x,y) - \delta(x-y)\int_0^1 dz w(z,y) + \frac{d}{dx}\delta(x-y)\int_0^1 dz(z-y)w(z,y),$$

$$w(x,y) = -C_F\theta(y-x)\frac{x}{y}\frac{2}{(x-y)^2} + \begin{Bmatrix} x\to 1-x \\ y \to 1-y \end{Bmatrix}. \tag{41}$$

Thus, the corresponding part of $\hat{Z}^{*[1]}$ is

$$\hat{Z}^{*[1]b}(g) = \frac{\alpha}{2\pi}\hat{w} + O(\alpha^2), \quad \text{where} \quad \hat{w} := \{w_{kk'}\}. \tag{42}$$

With (36) and (42) one gets $\hat{Z}^{*[1]} = \hat{Z}^{*[1]a} + \hat{Z}^{*[1]b} = (\alpha/2\pi)(\hat{\gamma}^{(0)}\hat{b} + \hat{w}) + O(\alpha^2)$, and so from Eq. (33) it follows

$$\hat{\gamma}^{c(0)}(l) = \hat{b}(l)\hat{\gamma}^{(0)} + \hat{w}. \tag{43}$$

The term proportional to $\hat{b}$ has just the same form as in scalar theory. Additionally, in gauge field theory, the conformal invariance is broken by the matrix $\hat{w}$. This breaking comes from the graphs in Fig. 2b and is produced by the renormalized gauge field of the covariant derivative in O_{kl} and is induced by the $-\epsilon g$ term of $\bar{\beta}$.

With the help of Eq. (32) the off-diagonal part of the anomalous dimension matrix $\hat{\gamma}^{ND}$ can be computed in two-loop order from the one-loop approximation (43) of the conformal anomaly matrix $\hat{\gamma}^c$,

$$\gamma_{kk'}^{ND(1)} = \frac{\gamma_k^{(0)} - \gamma_{k'}^{(0)}}{2(k-k')(k+k'+3)}\left(b_{kk'}\gamma_{k'}^{(0)} + w_{kk'} - \beta_0 b_{kk'}\right), \tag{44}$$

where $\gamma_k^{(0)}$ and $b_{kk'}$ are defined in Eqs. (10) and (21), respectively. Furthermore, $w_{kk'}$ is defined by the integral (40). Performing the integration provides, after some algebra

$$w_{kk'} = C_F\begin{cases} -4(2k'+3)(k-k')(k+k'+3) & \text{if } k-k'>0 \\ \times\left[\frac{A_{kk'}-\psi(k+1)+\psi(0)}{(k'+1)(k'+2)} + \frac{2A_{kk'}}{(k-k')(k+k'+3)}\right] & \text{and even} \\ 0 & \text{otherwise} \end{cases} \tag{45}$$

with

$$A_{kk'} = \psi\left(\frac{k+k'+2}{2}\right) - \psi\left(\frac{k-k'-2}{2}\right) + 2\psi(k-k'-1) - \psi(k+1) - \psi(0), \qquad \psi(z) = \frac{d}{dz}\ln(\Gamma(z+1)).$$

Although the result (44) was derived in abelian gauge field theory, in this loop approximation it should also be valid for QCD (if one takes the correct values of C_F and β_0). In fact, with the help of the representation (40) it can be analytically proven that Eq. (44) is in agreement with the explicit computed two-loop approximation of the evolution kernel [8,9].

V. SOLUTION OF THE EVOLUTION EQUATION

A. General formalism

It was pointed out in Sec. II that the Q^2 dependence of the distribution amplitude can be obtained in leading order by solving the renormalization group equation (5) of local operators O_{kl}. In principal the same treatment is also possible beyond the leading order; however, the renormalization group equation (5) must be diagonalized.

As mentioned above, $\hat{\gamma}$ posseses the eigenvalues $\gamma_k = \gamma_{kk}$. Therefore, the renormalization group analysis provides for multiplicative renormalized operators $\tilde{O}_{kl}$

$$\tilde{O}_{kl}(\mu^2) = \exp\left\{\frac{1}{2}\int_{\mu_0^2}^{\mu^2}\frac{dt}{t}\gamma_k(g(t))\right\}\tilde{O}_{kl}(\mu_0^2). \tag{46}$$

Furthermore, the operators O_{kl} can be completely expressed by

$$O_{kl}(\mu^2) = \sum_{k'=0}^{k} B_{kk'}(g(\mu^2))\tilde{O}_{k'l}(\mu^2). \tag{47}$$

Consequently, one finds from the renormalization group equation (5) that the matrix $\hat{B}$ satisfies the differential equation

$$[\hat{B}(g), \hat{\gamma}^D(g)] + \beta(g)\frac{\partial}{\partial g}\hat{B}(g) = \hat{\gamma}^{ND}(g)\hat{B}(g). \tag{48}$$

To fix the solution of this equation one has to choose an initial condition. It will be seen below that it is very convenient to introduce $\hat{B}(g(\mu_0^2)) = \hat{1}$ for a reference point μ_0 so that $O_{kl}(\mu_0^2) = \tilde{O}_{kl}(\mu_0^2)$.

Assuming that $\hat{\gamma}^{ND}$ is small, Eq. (48) can then be solved perturbatively. The ansatz $\hat{B} = \sum_{i=0}^{\infty}\hat{B}^{(i)}$, where $\hat{B}^{(0)} = \hat{1}$, provides an inhomogeneous differential equation for the matrix $\hat{B}^{(i)}$ with the source $\hat{\gamma}^{ND}\hat{B}^{(i-1)}$. Summing up the solutions of these recurrence relations provides

$$\hat{B}(g) = \frac{\hat{1}}{\hat{1} - \mathcal{L}\hat{\gamma}^{ND}(g)}$$
$$= \hat{1} + \mathcal{L}\hat{\gamma}^{ND}(g) + \mathcal{L}\left(\hat{\gamma}^{ND}\mathcal{L}\hat{\gamma}^{ND}\right)(g) + \cdots, \tag{49}$$

where the integral operator $\mathcal{L}$ acts on a triangular and off-diagonal matrix as

$$\mathcal{L}\gamma^{ND}_{kk'}(g) = \int_{g_0}^{g} \frac{dg'}{\beta(g')}\gamma^{ND}_{kk'}(g')\exp\left\{\int_{g'}^{g}\frac{dg''}{\beta(g'')}\left(\gamma_k(g'') - \gamma_{k'}(g'')\right)\right\}, \tag{50}$$

with $g_0 = g(\mu_0^2)$. Furthermore, with the help of Eq. (30), the off-diagonal matrix $\hat{\gamma}^{ND}$ can be expressed by $\hat{\gamma}^c + 2(\beta/g)\hat{b}$ and $\hat{\gamma}^D$, consequently,

$$\hat{B}(g) = \frac{\hat{1}}{\hat{1} + \mathcal{L}\left(\frac{\mathcal{G}}{\hat{1}+\mathcal{G}}\hat{\gamma}^D(g)\right)}, \quad \mathcal{G}\hat{A} := \begin{cases} \frac{\left[\hat{\gamma}^c(l) + 2\frac{\beta}{g}\hat{b}(l), \hat{A}\right]_{kk'}}{2(k-k')(k+k'+3)} & \text{if } k - k' > 0 \\ 0 & \text{otherwise.}\end{cases} \tag{51}$$

Taking into account the obtained renormalization group equation solution, it is now possible to determine the evolution of the distribution amplitude. Inserting Eq. (47) in Eq. (8) provides the following expansion:

$$\phi\left(x, Q^2\right) = \sum_{n=0}^{\infty}\phi_n(x, \alpha_s(Q^2))e^{\frac{1}{2}\int_{Q_0^2}^{Q^2}\frac{dt}{t}\gamma_n(g(t))} < 0|O_{nn}(Q_0^2)|P >^{red}, \tag{52}$$

where

$$\phi_n(x, \alpha_s) = \sum_{k=n}^{\infty}\frac{(1-x)x}{N_k}C_k^{\frac{3}{2}}(2x-1)B_{kn}(\alpha_s). \tag{53}$$

Here B_{kn} is given in Eq. (51). Obviously, beyond the leading order the Gegenbauer polynomials are generalized to Q^2 dependent (non-polynomial) functions $\phi_n(x, \alpha_s(Q^2))$. As mentioned the matrix $\hat{B}$, and therefore also $\phi_n(x, \alpha_s)$, are completely determined by conformal symmetry breaking, i.e., by $\hat{\gamma}^c + 2(\beta/g)\hat{b}$, and by the eigenvalues γ_n and by an initial condition for $\hat{B}$. This condition was chosen so that $\phi_n(x, \alpha_s(Q_0^2)) = (1-x)xC_n^{3/2}(2x-1)/N_n$ for a reference momentum square Q_0^2 holds true. Thus, as in leading order, the non-perturbative input $< 0|O_{nn}(Q_0^2)|P >^{red}$ can be simply computed from Eq. (12).

B. Next-to-leading approximation

The formalism developed in the previous section provides some simplification for the computation of the Q^2 dependence of $\phi(x, Q^2)$. For instance, the inputs for the next-to-leading approximation are γ_n and β in two-loop order, which are both well known, and $\hat{\gamma}^c$ in one-loop order. The crucial point of the direct construction from the evolution equation (1) is that $\hat{\gamma}^{ND(1)}$, and therefore also $\hat{B}^{(1)}$, could not be analytically determined from the kernel $V^{(1)}(x,y)$; only a numerical calculation of the first few matrix elements of $\hat{B}^{(1)}$ was possible. Now, the corresponding approximation of $\hat{B}$ can be obtained from Eq. (51), $B_{kn} = \delta_{kn} - \{\mathcal{L}\mathcal{G}\gamma^D\}_{kn} + \cdots$, and inserting in Eq. (53) provides the following correction for the functions

$$
\begin{aligned}
\phi_n(x,\alpha_s(Q^2)) &= \frac{(1-x)x}{N_n} C_n^{\frac{3}{2}}(2x-1) + \frac{\alpha_s(Q^2)}{2\pi}\phi_n^{(1)}(x,Q^2) + \cdots \\
\phi_n^{(1)}(x,Q^2) &= -\sum_{k=n+2}^{\infty} \frac{(1-x)x}{N_k} C_k^{\frac{3}{2}}(2x-1)\left[1-\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right)^{1+\frac{\gamma_k^{(0)}-\gamma_n^{(0)}}{\beta_0}}\right] \\
&\quad \times \frac{\gamma_k^{(0)}-\gamma_n^{(0)}}{\gamma_k^{(0)}-\gamma_n^{(0)}+\beta_0}\, \frac{b_{kn}(\gamma_n^{(0)}-\beta_0)+w_{kn}}{2(k-n)(k+n+3)},
\end{aligned}
\tag{54}
$$

where $\gamma_n^{(0)}$ is given in Eq. (10) and the matrix elements b_{kn} and w_{kn} are defined in Eqs. (21) and (45), respectively. Substituting this expression for the partial waves in Eq. (52), one gets the next-to-leading order approximation of $\phi(x,Q^2)$, where the two-loop approximation of γ_n can be found in Ref. [26].

C. The $\beta = 0$ case

From the theoretical point of view, it is interesting to discuss the $\beta = 0$ case. In this case α_s is independent of Q^2 and the evolution of the distribution amplitude is simplified to

$$
\phi\left(x,Q^2\right) = \sum_{n=0}^{\infty} \phi_n^{ef}(x,\alpha_s)\left(\frac{Q^2}{Q_0^2}\right)^{\gamma_n/2} < 0|O_{nn}(Q_0^2)|P >^{red},
\tag{55}
$$

where $\phi_n^{ef}(x,\alpha_s)$ are the eigenfunctions of the kernel $V(x,y;\alpha_s)$. At first it will be given a general treatment to obtain these eigenfunctions in a "closed form," analogous to the representation of the predicted functions (2). After this the $O(\alpha_s)$ corrections will be computed.

From Eqs. (1) and (52) it is easy to see that for $\beta = 0$ the eigenfunctions are formally given in Eq. (53), where $\hat{B}$ is defined in Eq. (51). However, the operators $\mathcal{L}$ and $\mathcal{G}$ are now simplified to $\mathcal{L}A_{kn} = -A_{kn}/(\gamma_k - \gamma_n)$ and $\mathcal{G}A_{kn} = [\hat{\gamma}^c, \hat{A}]_{kn}/(2(k-n)(k+n+3))$, respectively. Thus, with some algebra it can be shown that $\hat{B}$ is now independent of $\hat{\gamma}^D$ and simply expressed by the special conformal anomaly matrix:

$$\hat{B}(g) = \frac{\hat{1}}{\hat{1} + \mathcal{J}\hat{\gamma}^c(g)} = \hat{1} - \mathcal{J}\hat{\gamma}^c(g) + \mathcal{J}(\hat{\gamma}^c(g)\mathcal{J}\hat{\gamma}^c(g)) + \cdots, \tag{56}$$

where the operator $\mathcal{J}$ is defined by

$$\mathcal{J}\hat{A} := \begin{cases} \frac{A_{kn}}{2(k-n)(k+n+3)} & \text{if } k-n>0 \\ 0 & \text{otherwise.} \end{cases} \tag{57}$$

Therefore, the eigenfunctions can be written as

$$\phi_n^{ef}(x,\alpha_s) = \sum_{k=n}^{\infty} \frac{(1-x)x}{N_k} C_k^{\frac{3}{2}}(2x-1) B_{kn}, \quad B_{kn} = \left(\frac{\hat{1}}{\hat{1} + \mathcal{J}\hat{\gamma}^c(g)} \right)_{kn}. \tag{58}$$

From the representation (58) we will now derive a differential equation, which can be used for determination of the functions ϕ_n^{ef}. Using the eigenvalue equation [14]

$$\left((1-x)xC_k^{\frac{3}{2}}(2x-1)\right)'' = -(k+1)(k+2)C_k^{\frac{3}{2}}(2x-1), \tag{59}$$

and the property $(k-n)(k+n+3)B_{kn} = -\{\hat{\gamma}^c\hat{B}\}_{kn}/2$, which follows from Eq. (56), it is straightforward to derive the following equation:

$$(1-x)x\frac{d^2}{dx^2}\phi_n^{ef}(x,\alpha_s) + (n+1)(n+2)\phi_n^{ef}(x,\alpha_s) = \frac{1}{2}\int_0^1 dy \gamma^c(x,y,\alpha_s)\phi_n(y,\alpha_s), \tag{60}$$

where

$$\gamma^c(x,y,\alpha_s) = \sum_{\substack{k,k'=0 \\ k>k'}}^{\infty} \frac{(1-x)x}{N_k} C_k^{\frac{3}{2}}(2x-1)\gamma_{kk'}^c(\alpha_s)C_{k'}^{\frac{3}{2}}(2y-1). \tag{61}$$

The representation (58) suggests that the second-order differential equation (60) must be supplemented by the constraints

$$\phi_n^{ef}(0,\alpha_s) = \phi_n^{ef}(1,\alpha_s) = 0, \quad \int_0^1 dx C_n^{\frac{3}{2}}(2x-1)\phi_n(x,\alpha_s) = 1. \tag{62}$$

Now let us compute the $O(\alpha_s)$ correction to the eigenfunctions. From Eq. (60) we see that in this approximation $\phi_n^{ef}(x,\alpha_s)$ are completely determined by the leading order of $\hat{\gamma}^c = (\alpha_s/2\pi)(\hat{b}\hat{\gamma}^{(0)} + \hat{w}) + \cdots$. It is also known that the term $\hat{b}\hat{\gamma}^{(0)}$ provides the predicted eigenfunctions (2) [10,27]. To treat the "additional" conformal symmetry breaking term $\hat{w}$ we make the ansatz

$$\phi_n^{ef}(x,\alpha_s) = \frac{(1-x)x}{N_n} C_n^{\frac{3}{2}}(2x-1) + \frac{\alpha_s}{2\pi}\phi_n^{(1)}(x) + \cdots,$$
$$\phi_n^{(1)}(x) = -\frac{1}{2}\gamma_n^{(0)} \frac{d}{d\rho} \frac{((1-x)x)^{1+\rho}}{N_n} C_n^{3/2+\rho}(2x-1)_{|\rho=0} + \Delta\phi_n^{(1)}(x), \tag{63}$$

where $\Delta\phi_n^{(1)}(x)$ is completely determined by $\hat{w}$. Inserting this ansatz in Eq. (60) and using Eq. (40) provides for the following inhomogeneous equation $\Delta\phi_n^{(1)}(x)$:

$$(1-x)x\frac{d^2}{dx^2}\Delta\phi_n^{(1)}(x) + (n+1)(n+2)\Delta\phi_n^{(1)}(x) = \frac{1}{2}\int_0^1 dy[w(x,y)]_+ \frac{(1-y)y}{N_n} C_n^{\frac{3}{2}}(2y-1), \tag{64}$$

where $[w(x,y)]_+$ is defined in Eq. (41). It is simple to prove that the wanted solution can be written as

$$\Delta\phi_n^{(1)}(x) = \int_0^1 dy[g(x,y)]_+ \frac{(1-y)y}{N_n} C_n^{\frac{3}{2}}(2y-1) + n_n \frac{(1-x)x}{N_n} C_n^{\frac{3}{2}}(2x-1),$$
$$[g(x,y)]_+ = g(x,y) - \delta(x-y)\int_0^1 dz g(z,y),$$
$$g(x,y) = C_F\theta(y-x)\frac{\ln\left(1-\frac{x}{y}\right)}{(x-y)} + \left\{\begin{matrix} x\to 1-x \\ y\to 1-y \end{matrix}\right\}, \tag{65}$$

where the constraint $\Delta\phi_n^{(1)}(0) = \Delta\phi_n^{(1)}(1) = 0$ was used. Furthermore, the normalization term n_n is determined by the requirement that the nth moment of $\phi_n^{(1)}(x)$ vanishes. The computation of the integral (65) provides the following representation for the eigenfunctions

$$\phi_n^{ef}(x,\alpha_s) = (-1)^n \frac{2(3+2n)}{(n+1)!}\frac{d^n}{dx^n} x^{1+n}(1-x)^{1+n}\left(1+\frac{\alpha_s}{2\pi}C_F F_n(x) + O(\alpha_s^2)\right) \tag{66}$$

where

$$\begin{aligned} F_n(x) = {} & \frac{\gamma_n^{(0)}}{C_F}\left(-\frac{1}{2}\ln(x(1-x)) + \psi(1+n) - \psi(3+2n)\right) \\ & +\frac{1}{2}\ln^2\left(\frac{1-x}{x}\right) + \left(\frac{2}{n+2}-1\right)\left(\frac{\ln(1-x)}{x} + \frac{\ln(x)}{1-x}\right) \\ & +\sum_{i=2}^{1+n}\left(\frac{1}{n+2}-\frac{1}{i}\right)\frac{\ln(1-x)+\sum_{l=1}^{i-1}\frac{x^l}{l}}{x^i} \\ & +\sum_{i=2}^{1+n}\left(\frac{1}{n+2}-\frac{1}{i}\right)\frac{\ln(x)+\sum_{l=1}^{i-1}\frac{(1-x)^l}{l}}{(1-x)^i} \\ & +2\psi'(n) - \frac{\pi^2}{2} + 2\frac{\psi(n+1)-\psi(0)}{2+n} + 2\frac{\psi(n)-\psi(0)}{1+n}. \end{aligned} \tag{67}$$

The term in $F_n(x)$ that is proportional to $\gamma_n^{(0)}$ agrees with the predicted eigenfunction (2). The remaining part of $F_n(x)$ is therefore induced by the "additional" conformal symmetry breaking in gauge field theory. Note that $F_n(x)$ is only logarithmically divergent at the endpoints $(x=0, x=1)$ so that the eigenfunctions vanish at this points.

VI. CONCLUSIONS

Based on the conformal spin expansion and a conformal consistency relation I have determine the evolution of the flavor-nonsinglet meson distribution amplitude, which is controlled by conformal anomalies, i.e., dilatation and special conformal anomalies. The evolution of the coefficients of this expansion is completely controlled by the forward anomalous dimensions γ_k, and the Q^2 dependence of the "conformal partial waves" is induced by the special conformal symmetry breaking.

An important feature of the conformal consistency relation is that information which usually comes from an n-loop order calculation can also be obtained from the $(n-1)$-loop order approximation of the special conformal anomaly matrix. For instance, the one-loop calculation of this anomaly matrix allowed us to show that the explicit computed two-loop evolution kernel is in agreement with conformal symmetry breaking (an analytically consistent check for the two-loop eigenvalues of this kernel with the computed Lipatov-Altarelli-Parisi kernel was given in [28]). Moreover, in this formalism it is also much easier to compute the evolution of the distribution amplitude than to directly solve the evolution equation. Thus, in this paper the $O(\alpha_s)$ correction of the "conformal partial waves" and of the eigenfunctions of the evolution kernel for the $\beta = 0$ case could be analytically determined.

Some open questions about the conformal symmetry breaking in gauge field theories remain. Does the special conformal anomaly matrix contain more information than the anomalous dimension matrix at the same loop level? In other words, is the special conformal anomaly completely induced by the violation of dilatation? That means that the eigenfunctions and the evolution of the distribution amplitude would be determined only by the forward anomalous dimensions [see, e.g., the predicted eigenfunctions (2)]. It was not possible to decide this question from the definition of the special conformal anomaly matrix. However, an answer for the $\beta = 0$ case could be found from the treatment given in [10], which is based on conformal symmetry arguments and the conformal operator product expansion in light-cone gauge[††]. But, as mentioned, the prediction obtained for the eigenfunctions does not hold true. It can be seen in this paper that the reason for this conflict is an additional symmetry breaking term which arrives from the

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[††]This choice is necessary to define the distribution amplitude in terms of a bi-local operator, which can be expanded with respect to local operators.

gauge invariance of the theory (covariant derivative of the conformal operators). Thus, we guess that the conformal symmetry arguments used in [10] do not hold true in light-cone gauge. This point could be clarified by the conformal Ward identities derived in light-cone gauge.

Finally, let us give a short outlook. The analytical results obtained for the evolution of the distribution amplitude allow a detailed analysis of the $O(\alpha_s)$ correction for both the flavor-nonsinglet meson electromagnetic form factor and the transition form factor. In this case it would be helpful to know the closed expression for the eigenfunctions, which is just the $\beta_0 \to 0$ limit of the partial waves.

With the formalism developed in this paper it also seems possible to determine the distribution amplitude in next-next-to-leading order. Assuming that the tree-loop order of the forward anomalous dimensions γ_k are known, the additional information for the construction in this order can be obtained from a two-loop calculation of the special conformal anomaly matrix, which is a little bit more complicated than the two-loop calculation in Ref. [8,9]. Furthermore, it should be possible to attack the flavor-singlet case in next-to-leading order. It is expected that the corresponding analysis of conformal symmetry breaking provides a straightforward generalization of the conformal consistency relation (29). Then, as in the flavor-nonsinglet case, the two-loop contribution of the forward anomalous dimensions are known so that the one-loop calculation of the conformal anomaly matrices provides the necessary information to construct the next-to-leading approximation of the quark and gluon distribution functions.

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IX. CAPTIONS

Figure 1:

In one-loop order the operator insertion $[\Delta_-^\beta]$ generates a derivative on each vertex with respect to the corresponding external momentum.

Figure 2:

The one-loop graphs of the operator insertion $\int d^n x 2(\tilde{n}^* x) A \frac{\delta}{\delta A}[O_{kl}]$ can be divided in graphs which are differentiated with respect to the external momenta (Figure 2a) and in two other graphs which are defined by the Feynman-integral in Eq. (37) (Figure 2b).

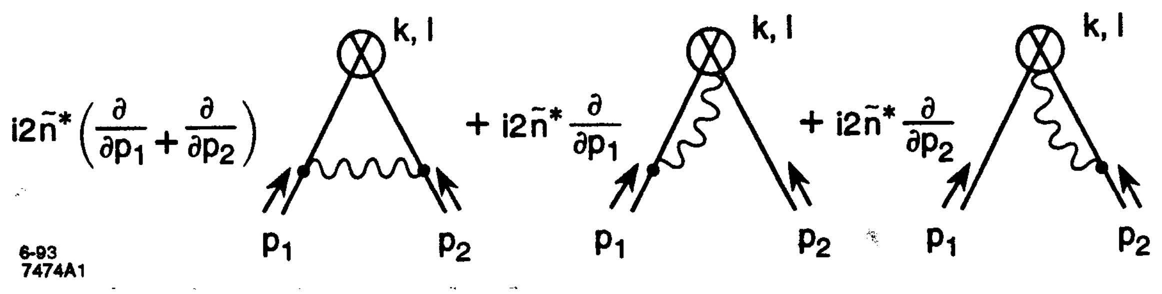

Fig. 1

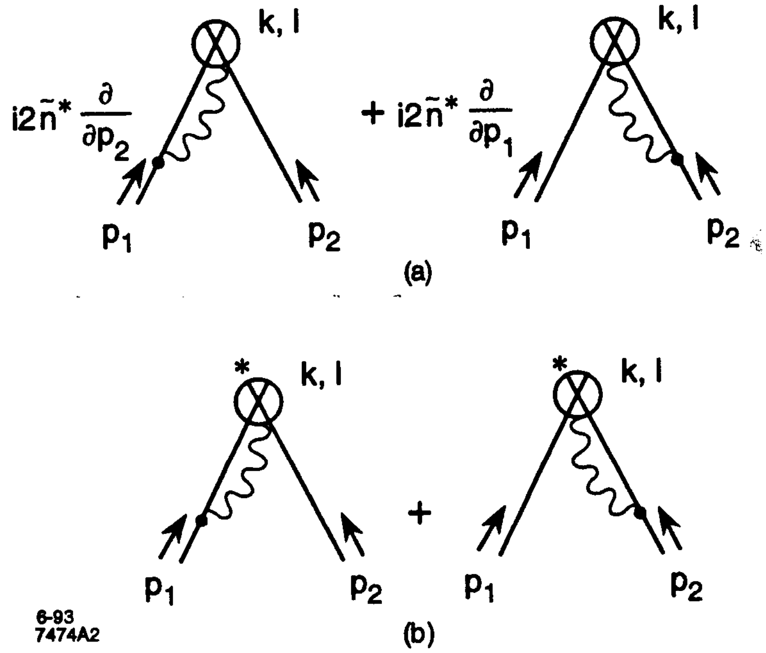

Fig. 2