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DISCUSSION

OPPENHEIMER: I think that this reminds one of Oehme's analysis of the structure singularities.

DISPERSION RELATIONS AND THE CAUSALITY CONCEPT (*)

J. M. Lozano and M. Moshinsky

Instituto de Física, Universidad de México, México, D.F.

(presented by J. M. Lozano)

It is well known that the S-wave scattering [1, 2, 3] $S(\kappa)$ by a cut potential of radius a leads to a scattering function that is an analytic function of κ in the upper half I_+ of the complex plane of the wave number κ, except possibly, for poles on the positive imaginary axis that corresponds to bound states. Besides, an elementary discussion of the square well potential of range a, shows that

$$S(\kappa)\exp(2i\kappa a)\rightarrow 1 \qquad (1)$$

when κ is in I_+ and $|\kappa|\rightarrow\infty$. A general proof of this property for an arbitrary cut potential of range can be given using the Born approximation.

As a consequence of the properties of $S(\kappa)$ given in the previous paragraph, we see that in the absence of bound states, the following integral vanishes

$$\int_c[2i(\kappa^2-k^2)]^{-1}\{\exp[i\kappa(r-r_0)]-S(\kappa)\times \times\exp[i\kappa(r+r_0)]\}d\kappa=0 \qquad (2)$$

where c is the contour of Fig. 1, and $r\geq r_0\geq a$. The proof of Eq. (2) is immediate if we complete the contour c by the dotted contour of Fig. 1, and use Eq. (1)

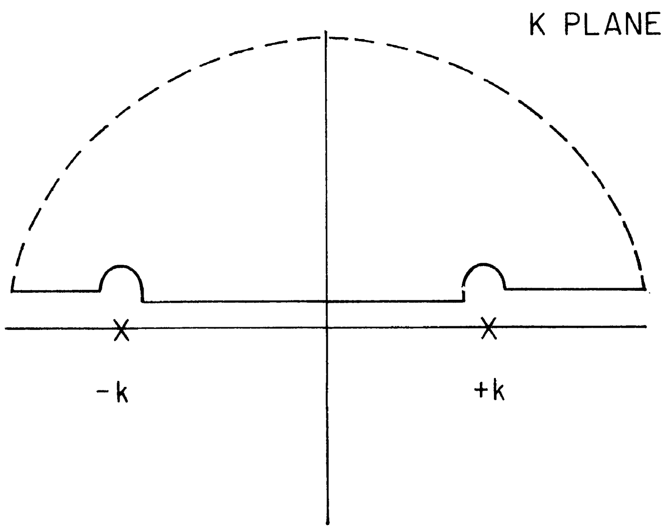

Fig. 1 Contour for the κ integration.

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and the Cauchy theorem. In particular, if $r = r_0 = a$ we get from Eq. (2) the dispersion relations

$$\mathrm{Im}\left[S(k)\exp(2ika)\right] = -\frac{2k}{\pi}\mathrm{P}\int_0^\infty \frac{\mathrm{Re}\left[S(\kappa)\exp(2i\kappa a)\right]}{\kappa^2 - k^2}\,d\kappa \qquad (3a)$$

$$\mathrm{Re}\left[S(k)\exp(2ika)\right] = \frac{2}{\pi}\mathrm{P}\int_0^\infty \frac{\mathrm{Im}\left[S(\kappa)\exp(2i\kappa a)\right]}{\kappa^2 - k^2}\,d\kappa \qquad (3b)$$

where Im and Re stand for imaginary and real part, respectively, and P indicates the principal value.

For non-cut potentials, Martin [3] and others have shown that the properties of $S(\kappa)$ of the first paragraph no longer hold. For example, it is possible to construct potentials [3] with poles of $S(\kappa)$ in I_+ outside the imaginary axis, and also to have poles on the imaginary axis that do not correspond to bound states. It follows, therefore, that for non-cut potentials, $S(\kappa)$ no longer has the analytic properties that make it satisfy Eq. (2) or the dispersion relations (3).

We could ask ourselves what would be the generalization of Eq. (2) to arbitrary potentials, so as to obtain a new function, which we will denote by the name of dispersion function, that satisfies dispersion relations.

We obtain the dispersion function using essentially a causality principle formulated as follows: the time dependent Green function for the scattering by an arbitrary potential should be bounded for all times. As a consequence of this causality principle we can see that the dispersion function is the Laplace transform of the time dependent Green function and that it satisfies an equation which reduces to Eq. (2) in the particular case of cut potentials of radius a.

We can show directly from the Schrödinger equation that the dispersion function is analytic in I_+ except possibly for poles on the imaginary axis associated with bound states. Furthermore, using the Born approximation, we can show that the dispersion function tends to zero as a function of κ, when κ is in I_+ and $\kappa| \to |\infty$. From these properties of the dispersion function, an equation that is a generalization of Eq. (2) follows immediately.

We discuss the particular case of scattering by the Eckart potential [4, 5] which gives a very simple $S(\kappa)$. We obtain explicitly the dispersion function and the time dependent Green function of this problem, and express the latter in terms of basic interaction Green (BIG) functions associated with the poles and zeros of $S(\kappa)$. We can show that the BIG functions of physical significance are those associated with the poles of $S(\kappa)$ that are also poles of the dispersion function. From the asymptotic form of the BIG functions, when $t \to \infty$ we obtain restrictions on the analytic behavior of the dispersion functions that are similar to those obtained from the causality principle. Using the properties of the BIG functions, we briefly discuss the significance of the complex poles and zeros of $S(\kappa)$ comparing it with the results of our previous papers [6,7] and with the recent analysis of Beck and Nussenzweig [8].

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