NEW PARAMETERS FOR NUCLEAR CHARGE RADIUS FORMULAS

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Parameters of widely used nuclear rms charge radius formulas have been refitted based on the latest experimental data for about 900 nuclei. It has been seen that the new parameters in the formulas give better results than the previous ones. Besides, an $N^{1/3}$-dependent formula has been proposed and discussed. This formula gives effective results for rms charge radius. The standard deviation in all formulas with new parameters are concentrated between -0.1 and 0.1. In other words, for about 90% of nuclei, the differences of charge radii from experimental values are lower than 0.1 fm.

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1. Introduction

One of the most fundamental properties of atomic nuclei is the nuclear charge radius [1]. It plays a key role in studying the characters of nucleus and testing theoretical models of nuclei as well as in studying astrophysics and atomic physics. The study of nuclear charge radii can serve to impose strong constraints on the saturation properties of nuclear forces [2]. Nuclear charge radius contributes to our knowledge of the shell structure of nuclei and it provides direct information for the Coulomb energy of nuclei. Therefore, it is important for nuclear mass formulae. The nuclear charge radii data are already well-known for many nuclei, especially for the nuclei closing to the β stability line [3–5]. Ever since the relativistic ion beams started to be used in the field of nuclear physics [6], the research subjects of the nuclear structure physics are extended to the region of exotic nuclei which lie far from β stability line, near the neutron and proton drip-lines. The developments in the measurement techniques for charge radii of nuclei provide more accurate experimental results which can be used to improve model parameters. Because of this, experimental and theoretical nuclear charge radii studies are one of the important topics in nuclear physics.

Nuclear radius of a nucleus is related to the distributions of its density. As is well known, the charge distributions in neutrons and protons are different. This makes the charge density distributions of nucleus different from those of the matter and proton density. Indeed, the charge density distribution determines the radius of atomic nucleus. The root-mean-square (rms) charge radius, $R = \langle r^2 \rangle^{1/2}$, of a nucleus can be measured via the experimental methods of electromagnetic interaction between the nucleus and electrons or muons. There are mainly four methods for the measurements: elastic electron scattering (e^-), transition energies in muonic atoms (μ^-), K_αX-ray isotope shift (KIS) and optical isotope shift (OIS) [7]. The first three have been employed only for stable isotopes. However, the latest one has been performed on nuclei far from stability, because the development in the optical laser spectroscopy methods gives the opportunity to carry out measurements of radioactive atoms with lifetimes less then 1 ms. Measurements of transitions in muonic atoms and elastic electron scattering experiments give information on charge radii R while K_αX-ray and optical isotope shifts give information on their isotopic changes $\delta\langle r^2 \rangle$ [8]. The results of two different methods based on evaluation of data [3, 4] and new data obtained from laser spectroscopy have been combined into a single rms nuclear charge radii data which include over 900 isotopes [5].

From the theoretical side, a number of microscopic and macroscopic theories can be used for calculation of rms charge radius. The relativistic mean field (RMF) theory [9–11] gives much better results for both spherical and deformed nuclei. In particular, relativistic continuum Hartree–Bogoliubov theory which is an extended version of RMF theory is reliable for describing the nuclear properties near the drip-lines. There are different estimated phenomenological rms charge radius formulas which give similar results. Their reliability crucially depends on the test of their estimations for newly acquired rms charge radii data that is experimentally measured with high accuracy. The volume or radius of the nucleus is naturally proportional to the nuclear mass number. However, the conventional A-dependent rms charge radius formula is not globally valid for all nuclei in which there is a significant difference between proton and neutron numbers. Also, the experimental data indicate that the $R_0/A^{1/3}$ ratio is not constant [12]. It is seen from the developed formula that the Z or isospin dependent formula describe nuclei much better.

Recently, artificial neural networks [13] have been employed for estimating rms nuclear charge radii [14]. Experimental data have been used in the network. By using this method, the charge radii have been generated by great accuracy. Furthermore, according to the results from the network, a new simple formula has been estimated by least-square fitting. This simple formula also describes charge radii well.

In order to obtain the global description of the charge radius, empirical formula is fitted to the experimental charge radius data. Therefore, using this estimated formula can be useful for evaluating unmeasured charge radius data. Especially, the formula is crucial for obtaining the charge radii of the nuclei lying in the region far from β stability valley. Therefore, new parameters of existing nuclear rms charge radius formulas have been obtained by using the latest experimental data and compared with the previous ones. Furthermore, an $N^{1/3}$-dependent formula has been proposed and discussed.

2. Method

In the present study, latest experimental data [5] have been used for applying least-square fitting based on the Levenberg–Marquardt method [15] in order to obtain new parameters for widely used rms charge radius formulas taken from Ref. [16]. These formulas have been listed in Table I. This fitting procedure has been practised on recent experimental rms charge radii data for 898 nuclei. All the nuclei from $A = 2$ to $A = 248$ have been considered and no standard deviation uncertainty threshold has been used.

3. Result and discussions

In Table I, new parameters obtained from this work for nuclear charge radii formulas by considering experimental charge radii of 898 nuclei have been presented together with their root-mean-square deviation (RMSD) values. Also, calculated RMSD values by using the parameters of previous study [16] have been listed for comparison. The RMSD is defined by

$$\mathrm{RMSD} = \sqrt{\sum_{i=1}^{N} \frac{\left(R^i_{\mathrm{exp}} - R^i_{\mathrm{est}}\right)^2}{N}}\,, \tag{1}$$

TABLE I

The new parameters of nuclear charge radii formulas which have been obtained by least-square fitting to the experimental data taken from [5].

Formula	Parameters [16]	RMSD	New parameters	RMSD
$R_c = r_A A^{1/3}$	$r_A = 1.223$ fm	1.372	$r_A = 0.951$ fm	0.118
$R_c = r_Z Z^{1/3}$	$r_Z = 1.631$ fm	1.359	$r_Z = 1.271$ fm	0.099
$R_c = \sqrt{5/3}\left(\left(r_p Z^{1/3}\right)^2 + 0.64\right)^{1/2}$	$r_p = 1.242$ fm	1.340	$r_p = 0.961$ fm	0.075
$R_c = r_A\left(1 - b\frac{N-Z}{A}\right)A^{1/3}$	$r_A = 1.269$ fm; $b = 0.252$	1.341	$r_A = 0.996$ fm; $b = 0.278$	0.095
$R_c = r_A\left(1 - b\frac{N-Z}{A} + c\frac{1}{A}\right)A^{1/3}$	$r_A = 1.235$ fm; $b = 0.177$; $c = 1.960$	1.354	$r_A = 0.966$ fm; $b = 0.182$; $c = 1.652$	0.081
$R_c = r_Z\left(1 + b\frac{N-N/Z}{Z}\right)Z^{1/3}$	$r_Z = 1.631$ fm; $b = 0.062$	1.873	$r_Z = 1.245$ fm; $b = 0.015$	0.099

where N is the total number of experimental data and it can be used for determining the overall agreement of the new parameters of nuclear charge radii formulas and experimental data. In Eq. (1), R_{exp} and R_{est} denotes experimental and estimated rms charge radii, respectively. In Fig. 1, the

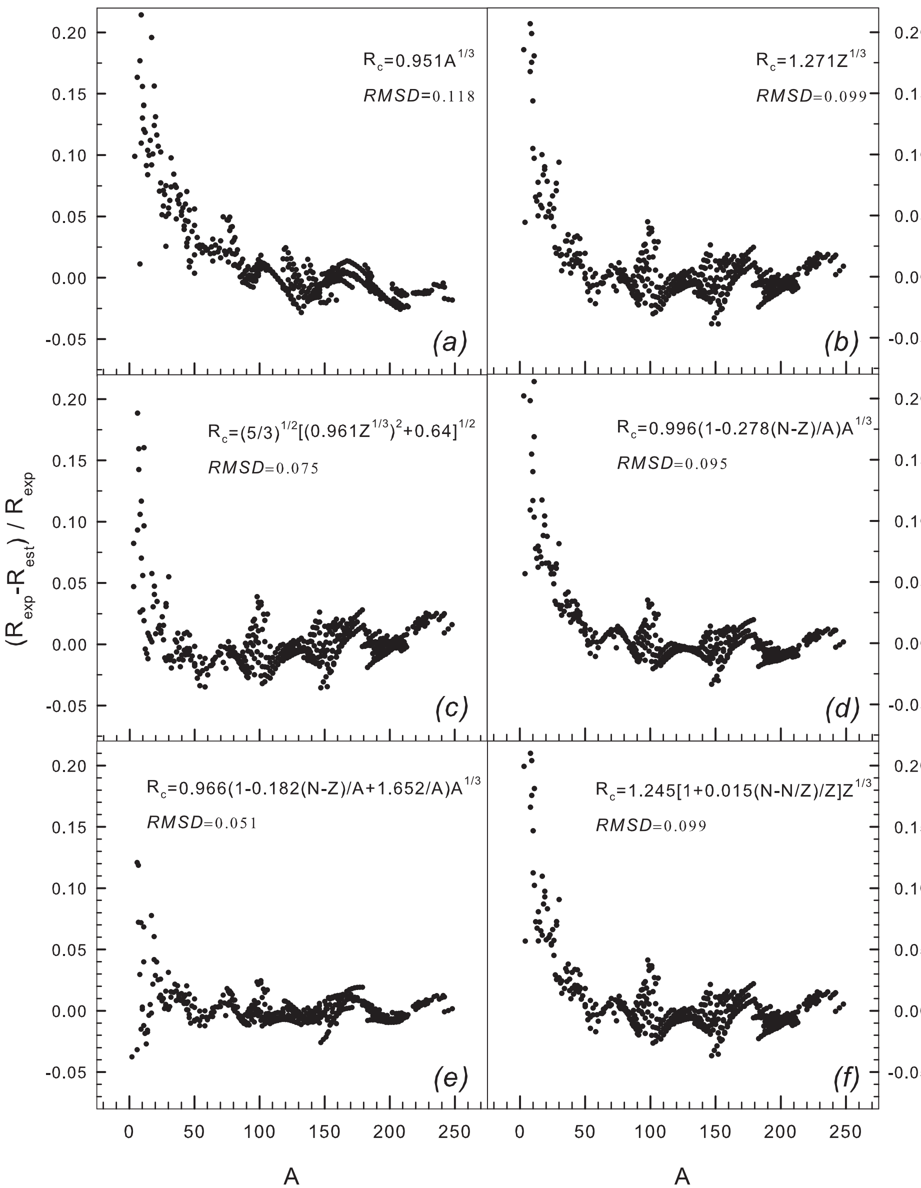

Fig. 1. The relative differences between the experimental (exp) and estimated (est) rms charge radii for several isotopes ranging from $A = 2$ to 248.

relative deviations of estimated charge radii results with the new parameters are shown. It is clear from Fig. 1 that the deviation from experimental rms charge radius are larger in the region below mass number 40 ($A \leq 40$). The maximum deviations from experimental values for different formulas are between 0.3 and 0.9 for $A \leq 40$ region, whereas these deviations are about 0.2 for $A > 40$. This point indicates that the region ($A \leq 40$) should be considered separately. However, in this work, without any regional restriction we have performed fitting procedure for all nuclei in order to obtain global description of charge radius. The minimum relative deviations have been seen in Fig. 1 (e) belonging to the fifth formula. In the case of this formula, the value of RMSD is 0.051. According to the results, the new parameters give about 10 to 30 times better RMSD values than the previous ones [16]. There is another point for consideration. In the region of small A ($A \leq 40$), the experimental values are generally larger than the estimated values except the fifth formula, whereas in the $A > 40$ region, the deviations are equally spaced in the positive and negative values of the y-axis.

In the previous conventional works, atomic number (Z) or mass number (A) of the nuclei had been taken into account in the formulas. In this study, the simple single term estimations on rms charge radius have also been performed without any restriction of the power of the formula to 1/3. $R_c = r_A A^\beta$, $R_c = r_Z Z^\beta$ and $R_c = r_N N^\beta$ nuclear charge radii formulas have been considered to obtain the parameters of simple A, Z and N-dependent nuclear charge radii formulas. These new parameters and their rms deviations are listed in Table II. As can be understood from this table, N-dependent charge radius gives comparable results to A-dependent and Z-dependent ones. Thus, it is concluded that the N-dependent formula is also useful for describing nuclear rms charge radius.

TABLE II

Simple single term nuclear rms charge radius formulae (A-, Z- and N-dependent) which have been obtained by least-square fitting to the experimental data taken from Ref. [5].

Formula	Parameters	RMSD
$R_c = r_A A^\beta$	$r_A = 1.169$ fm; $\beta = 0.291$	0.072
$R_c = r_Z Z^\beta$	$r_Z = 1.399$ fm; $\beta = 0.310$	0.087
$R_c = r_N N^\beta$	$r_N = 1.473$ fm; $\beta = 0.275$	0.103

As known, the charge radius dependence on the neutron number helps to reveal even a slight influence of different nuclear parameters, *e.g.* deformation, moments, nucleon pairing energy [5]. In the case of inspecting data along the isotopic series in which Z-dependent formula does not work, the neutron number dependence becomes important. The N-dependence of rms charge radii had been discussed previously in [8]. So we have also employed the data for obtaining simple $N^{1/3}$-dependent formula. The formula $R_c = r_N N^{1/3}$ has been considered for fitting. The novelty estimated parameter is $r_N = 1.140$. The RMSD value of this formula is 0.171. This value is not a good one when it is compared with those of the others given in Table I. Because of this, $N^{1/3}$-dependent rms charge radius formula as including contribution of asymmetry has been considered. This formula is given by

$$R_c = r_N \left(1 - b\frac{N-Z}{N}\right) N^{1/3} . \tag{2}$$

Equation (2) is similar to the $R_c = r_A(1 - b\frac{N-Z}{A})A^{1/3}$ [17] given in Table I. There can be found only one difference between these formulas. In Eq. (2), N is taken into account instead of A. It should be noted that along the isotopic chain of an element proton number Z remains constant while mass number A varies, as related only with the variation of neutron number N. This means that taking into account N-dependence instead of A-dependence in nuclear charge radii, the formula could give reliable results. The estimated parameters r_N and b for Eq. (2) are 1.262 and 0.349, respectively. These parameters give considerably good results as can be seen in Fig. 2. The RMSD value of this formula is 0.091. It is smaller than those of the formula $R_c = r_A(1 - b\frac{N-Z}{A})A^{1/3}$ which is 0.095. As indicated before, the RMSD values of $R_c = r_A A^{1/3}$ and $R_c = r_N N^{1/3}$ are 0.118 and 0.171. From these arguments, it can be clearly say that asymmetry contri-

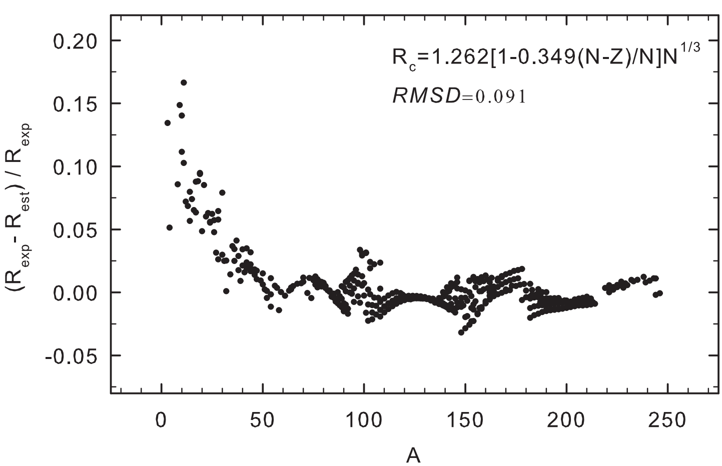

Fig. 2. The same as in Fig. 1, but for the new estimated formula with $N^{1/3}$-dependence.

butions to $A^{1/3}$ and $N^{1/3}$-dependent nuclear charge radii formulas improve results. It should be noted, however, that asymmetry contribution provides larger improvement in $N^{1/3}$-dependent charge radii formula than those of $A^{1/3}$-dependent one.

Besides, the neutron halo nuclei are highly related to the neutron number. Therefore, this formula can be considered in the case of studying neutron halos. We have also shown the deviations from experimental values according to the N and Z number of the nuclei in Fig. 3. In this figure, the rms charge radii deviations of 27 nuclei are larger than 0.2 fm. The rms charge radii deviations of 49, 332, 483 and 15 nuclei are between 0.2–0.1, 0.1–0, 0– −0.1 and −0.1–0.2 fm, respectively. This indicates that the charge radii deviations concentrate between −0.1 and 0.1. The errors are approximately 2%. The differences between calculated results from all formula with the new parameters and experimental data are rarely found larger than 0.2 fm, while they are lower than 0.1 fm for most of the nuclei (90%).

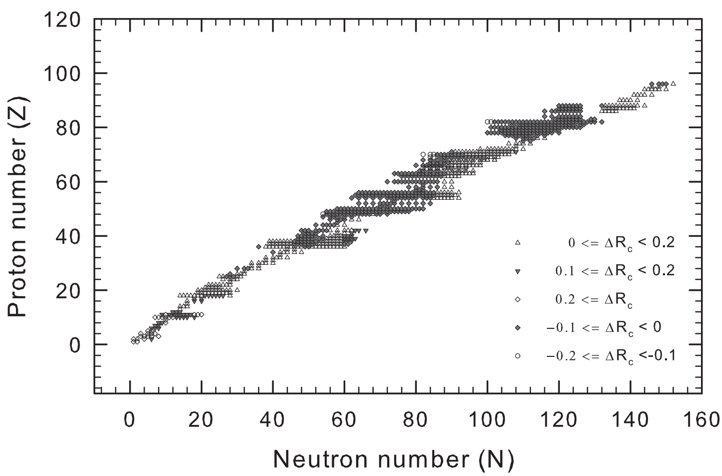

Fig. 3. The differences between the experimental and calculated rms nuclear charge radii by using the new estimated formula ΔR_c for several nuclei as functions of neutron and proton numbers. Unit is in [fm].

4. Summary

The well-known rms charge radius formulas have been refitted by using least-square fitting procedure. By using the latest experimental data the new parameters of these charge radius formulas have been derived. A new $N^{1/3}$-dependent formula has been presented and found as a reliable one.

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