

String Cosmological Models with Bulk Viscosity in Lyra Geometry

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Abstract. Bianchi type-I cosmological model for a cloud of a string with bulk viscosity is investigated in Lyra geometry. To get a deterministic model of universe, we assumed a unique form of constant deceleration parameter (CDP). Also, the bulk viscosity is presented as a linear combination of two positive constants ξ_0 & ξ_1 with Hubble parameter i.e $\xi = \xi_0 + \xi_1 H$ and studied the results resemblance with the constant and variable bulk viscosity in three different cases. Also, we conclude that the displacement vector used in Lyra geometry is vanishing in late time. The physical acceptability of the model is investigated.

1. Introduction

In the last few decades, there has been considerable interest in alternative theories of gravitation. One of the most interesting among all is the scalar-tensor theory proposed by Lyra [1]. This bears a remarkable resemblance to Weyl's geometry [2]. In subsequent investigations, Sen [3] and Sen & Dunn [4] formulated a new scalar-tensor theory of gravitation and constructed an analogy of the Einstein field equations based on Lyra's geometry. He investigated that the static model with finite density in the context of Lyra manifold is similar to the static model in Einstein's general relativity. Halford [5] has developed a cosmological theory in Lyra's geometry, which introduced to the non-static perfect fluid model. Rao et al., [6] have presented Bianchi type II, VIII and IX string cosmological models with bulk viscosity in Lyra geometry. The cosmological models which have some relevance to the present work was carried out by Nayak et al., [7], Pradhan et al., [8-9], Ram et al., [10], kandalkar et al., [11], Dubey [12], and so forth.

As we know, Bulk viscosity plays a vital role in cosmology and contributes to the accelerated expansion of the universe known as the inflationary phase. The effects of viscosity on the evolution of cosmological models and the role of viscosity in avoiding the initial big bang singularity have been studied by several authors [13-15]. Nowadays, String cosmology has been a subject of considerable interest since cosmic strings are topologically stable in the early the universe and it arises during the phase transition after big-bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theory. The string theory is also useful to describe an event at the early stage of evolution of the universe in a lucid way. Cosmic strings play a significant role in the structure formation and evolution of the universe. The presence of string in the early universe has been explained by [16-18].



Motivated from the above studies, in this paper, we tried to investigate a Bianchi type-I universe filled with a bulk viscous fluid within the framework of Lyra geometry with time-dependent displacement vector field without assuming the barotropic equation of state for the matter field. The organization of the paper is as follows. In Section 2, we present the metric and Einstein's field equations. In Section 3, we deal with the solution of the field equations. We first show that the field equations are solvable for any arbitrary scale function. Thereafter, we obtain exact solutions of the field equation by assuming a power-law form of a scale factor and the bulk viscosity coefficient is directly proportional to the energy density of the matter. In Section 4, we discuss the physical and dynamical behaviours of the universe. Section 5 summarizes the main results presented in the paper.

2. Metric and Field Equations

We consider Bianchi type-I space-time in the form

$$ds^2 = -dt^2 + \sum A^2 dx^2, \quad (1)$$

where the metric potentials are functions of cosmic time t alone.

The field equations in the normal gauge for Lyra manifold as obtained by Sen (1957) is

$$R_i^j - \frac{1}{2} R g_i^j + \frac{3}{2} \varphi_i \varphi^j - \frac{3}{4} g_i^j \varphi_k \varphi^k = -T_i^j \quad (2)$$

Where, the geometrized unit $8\pi G=1, c=1$ and φ_i is the displacement field vector defined by

$$\varphi_i = (0,0,0,\beta(t)) \quad (3)$$

and other symbols have their usual meaning as in Riemannian geometry.

The energy momentum tensor for string cloud distribution is given by

$$T_j^i = (\rho + \bar{p})u_i u^j + \bar{p}g_j^i - \lambda x_i x^j, \quad (4)$$

where $\rho = \rho_p + \lambda$ is the energy density for a cloud of strings for particles attached to them, \bar{p} is the effective pressure, ρ_p is the rest energy density of particle, u^i is the four velocity vector of the particles, λ is the string tension density, x^j is a unit vector representing the direction of string so that $x^1 \neq 0, x^2 = x^3 = x^4 = 0$. The vectors u^i and x_i satisfy the conditions

$$u^i u_i = -x^i x_i = -1 \text{ and } u^i x_i = 0. \quad (5)$$

Choosing x^i as defined below

$$x^i = (A^{-1}, 0, 0, 0). \quad (6)$$

The effective pressure \bar{p} and isotropic pressure p are related by

$$\bar{p} = p - \xi \Theta. \quad (7)$$

Where ξ is the bulk viscous coefficient and Θ is the expansion scalar. In the particle density of the configuration is denoted by ρ_p , then

$$\rho = \rho_p + \lambda \quad (8)$$

The Einstein modified equations (2) together with equation (4) for the metric leads to the following form

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{3}{4}\beta^2 = -\bar{p} - \lambda, \quad (9)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{CA} + \frac{3}{4}\beta^2 = -\bar{p}, \quad (10)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 = -\bar{p} \quad (11)$$

$$\text{and} \quad \frac{A\dot{B}}{AB} + \frac{B\dot{C}}{BC} + \frac{C\dot{A}}{CA} - \frac{3}{4}\beta^2 = -\rho. \quad (12)$$

The energy conservation equation gives the following equation

$$\dot{\rho} + (\rho + \bar{p}) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) + \lambda \frac{\dot{A}}{A} = 0. \quad (13)$$

After a straightforward calculation of the conservation of R.H.S of equation (2), we get

$$\beta + \beta \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0. \quad (14)$$

As the energy conservation equation in Lyra's manifold is satisfied only on giving some special condition on the displacement vector β as shown above.

$$\text{The spatial volume is given by } V = ABC = a^3, \quad (15)$$

where $a(t)$ is a scale factor.

The expressions for scalar expansion and shear scalar are

$$\Theta = u_{;i}^i = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad (16)$$

$$\text{and } \sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{1}{2} \left[\left(\frac{\dot{A}}{A} \right)^2 + \left(\frac{\dot{B}}{B} \right)^2 + \left(\frac{\dot{C}}{C} \right)^2 \right] - \frac{1}{6} \Theta^2. \quad (17)$$

where $\sigma_i^j = \frac{1}{2} [u_{i;\alpha} p_j^\alpha + u_{j;\alpha} p_i^\alpha] - \frac{1}{3} \theta p_{ij}$.

Here the projection tensor p_{ij} has the form

$$p_{ij} = g_{ij} - u_i u_j \quad (18)$$

The Hubble parameter H and the mean anisotropy parameter are defined as

$$3H = \theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad (19)$$

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 \quad (20)$$

Where H_i , $i=1(2)3$ represents the directional Hubble parameter in the direction of the coordinate axis. For the metric (1), the directional Hubble parameters along different directions are defined as $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ and $H_z = \frac{\dot{C}}{C}$.

The deceleration parameter q which is defined as

$$q = -\frac{a\ddot{a}}{a^2}. \quad (21)$$

The deceleration parameter q indicates whether the universe inflates or not. The positive sign stands for decelerating model whereas the negative sign q indicates inflation.

3. Solution of the Field Equations

To get a stable solution to the highly non-linear five differential equations (9)-(13) containing the eight unknown variables $A, B, C, \beta, \rho, p, \lambda$ and ξ we assumed two additional conditions given below. To treat the model in a good manner here we assume that the shear tensor is proportional to expansion scalar Θ , which leads to the relation

$$A = (BC)^m \quad (22)$$

Where m is a positive constant.

Also, we assume that the deceleration parameter is considered as a constant

$$\text{i.e. } q = -1 + \frac{1}{n}. \quad (23)$$

Where n is a constant. It is obvious that the deceleration parameter is negative for $n < 0$ and $n > 1$ and is positive for $0 < n < 1$.

As the viscosity as a linear combination of constant and Hubble parameter which is expressed in

$$\xi = \xi_0 + \xi_1 H \quad (24)$$

Equation (23) immediately gives the average scale factor

$$a = (k_1 t + k_2)^n \quad (25)$$

with the help of (15), equation (25) reduces to

$$A = (k_1 t + k_2)^{\frac{3mn}{m+1}} \quad (26)$$

Equation (10)-(11), (22) and (26) together gives

$$B = (k_1 t + k_2)^{\frac{3n}{2(m+1)}} \exp \left(\frac{k_4 (k_1 t + k_2)^{1+3n}}{k_1 (3n-1)} \right) \quad (27)$$

$$C = (k_1 t + k_2)^{\frac{3n}{2(m+1)}} \exp\left(\frac{k_4(k_1 t + k_2)^{1+3n}}{k_1(1-3n)}\right) \quad (28)$$

Now the Hubble parameter H can be calculated using (19) and (26)-(28) as

$$H = \frac{nk_1}{k_1 t + k_2} \quad (29)$$

On integrating equation (14) we get

$$\beta = \beta_0 a^{-3} \quad (30)$$

where β_0 is integrating constant and represents the initial displacement vector.

$$\lambda = \frac{3nk_1^2(6m^2n-2m^2+3mn+m-3n+3)}{2(m+1)^2(k_1 t + k_2)^2} + \frac{3n(k_4 - k_1)}{(k_1 t + k_2)^{3n+1}} \quad (31)$$

$$\bar{p} = \frac{3nk_1^2(4m^2-3nm^2-3n-3m+2)}{4(m+1)^2(k_1 t + k_2)^2} + \frac{N}{(k_1 t + k_2)^{6n}}, \quad (32)$$

$$\text{where } N = -\frac{(3k_1^2 k_3^2 + 4k_4)}{4k_1^2}$$

$$\rho = \frac{9n^2 k_1(4m+k_1)}{4(m+1)^2(k_1 t + k_2)^2} + \frac{N}{(k_1 t + k_2)^{6n}}, \quad (33)$$

$$p = \bar{p} + \xi\theta$$

$$p = \frac{3nk_1^2(4m^2-3nm^2-3n-3m+2)}{4(m+1)^2(k_1 t + k_2)^2} + \frac{N}{(k_1 t + k_2)^{6n}} + \frac{3nk_1(\xi_0 k_1 t + \xi_0 \xi_1 k_1)}{(k_1 t + k_2)^2}, \quad (34)$$

$$\omega = \frac{\bar{p}}{\rho},$$

$$\omega = \frac{3nk_1^2(4m^2-3nm^2-3n-3m+2)(k_1 t + k_2)^{6n} + 4N(m+1)^2(k_1 t + k_2)^2}{9n^2 k_1(4m+k_1)(k_1 t + k_2)^{6n} + 4N(m+1)^2(k_1 t + k_2)^2}. \quad (35)$$

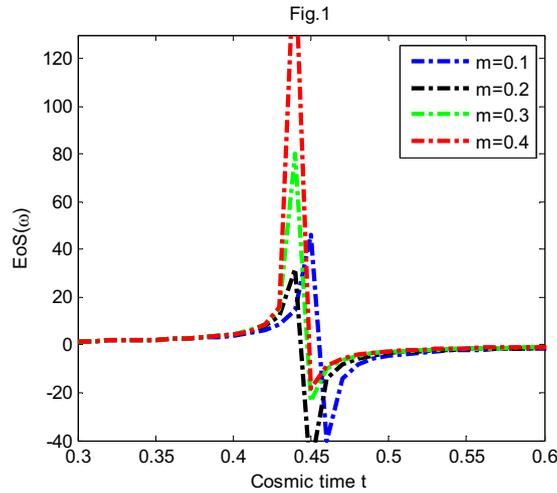


Fig.1: Variation of EoS versus cosmic time t for $n=1.2$, $k_1 = 1.16$, $k_2 = 0.3$, $k_3 = 0.3$ and $k_4 = 2.1$ with different m .

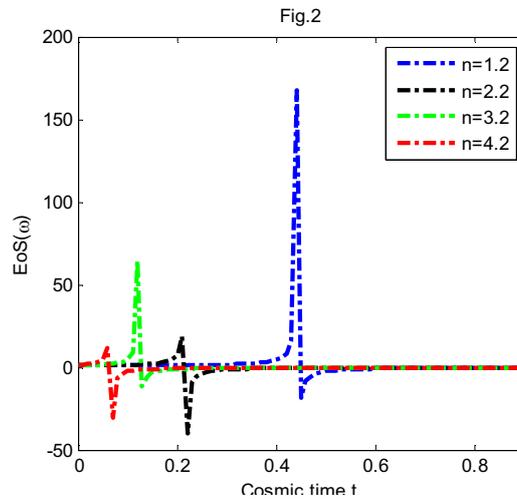


Fig. 2: Variation of equation of state versus cosmic time t for $m=0.4, k_1 = 1.16, k_2 = 0.2, k_3 = 0.3$ and $k_4 = 2.1$ with different n .

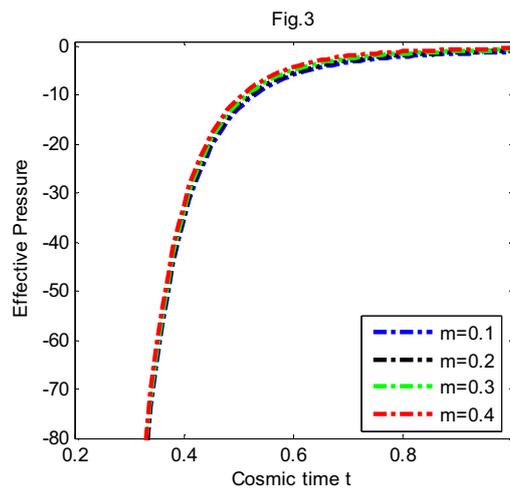


Fig.3: Variation of effective pressure versus cosmic time t for $n=1.2, k_1 = 1.16, k_2 = 0.2, k_3 = 0.3$ and $k_4 = 2.1$ with different m .

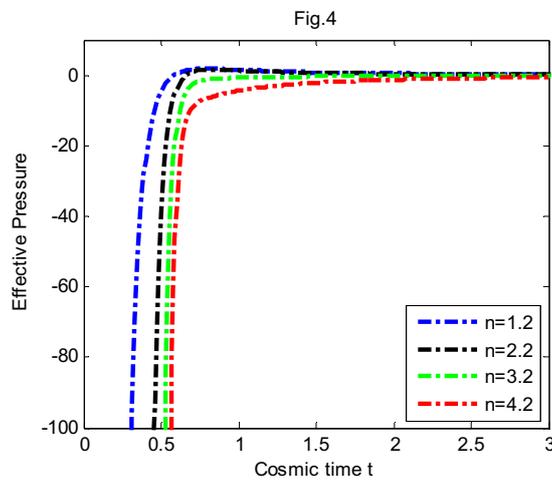


Fig.4: Variation of effective pressure versus cosmic time t for $m=0.4$, $k_1 = 1.16$, $k_2 = 0.2$, $k_3 = 0.3$, and $k_4 = 2.1$ with different n .

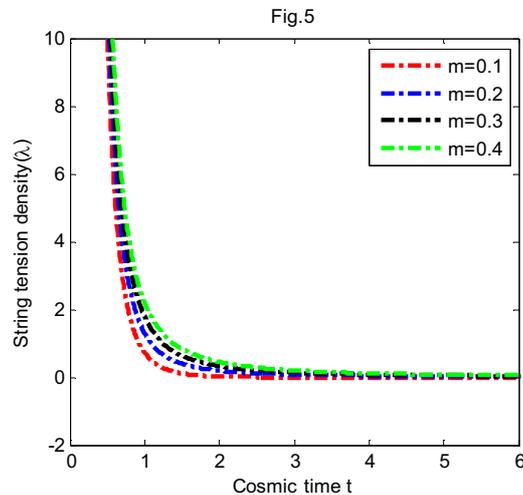


Fig.5: Variation of effective pressure versus cosmic time t for $n=1.2$, $k_1 = 1.16$, $k_2 = 0.2$, $k_3 = 0.3$ and $k_4 = 2.1$ with different m .

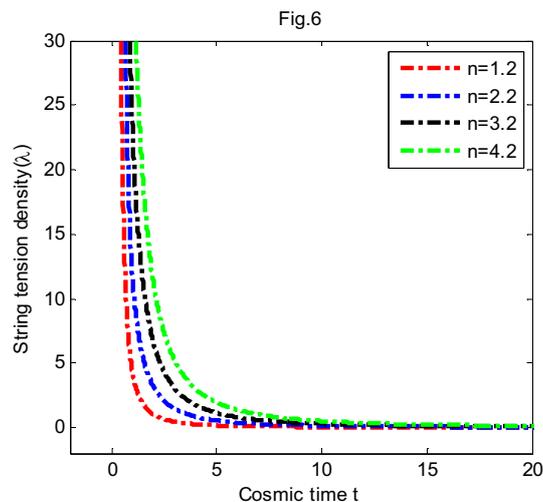


Fig.6: Variation of effective pressure versus cosmic time t for $m=0.4$, $k_1 = 1.16$, $k_2 = 0.2$, $k_3 = 0.3$ and $k_4 = 2.1$ with different n .

4. Results and discussion

In summary, we have considered Bianchi type-I space-time governed by the fluid with a combination of string cloud and viscous fluid distribution. For $0.41 < t < 0.44$ the equation of state parameter oscillates with a high pick in the positive side. But it remains negative for late time, which is consistent with the recent observational evidence. Later on, one can noticed that the effective pressure is negative with initial singularity and approaches to zero, which can be regarded as Big Rip singularity. Which matches to the results obtained by recent observation data i.e. supernova Ia. For $m < 1.2$, the effective pressure is negative for a finite future. As expected, we also obtained the energy density is positive with initial singularities

and approaches to zero later on. Here we conclude that both the assumed cases give nearly similar results. i.e., the model is independent of the choices of m and n . Finally, we concluded that the model represents an anisotropic universe. Moreover, the gauge function diverges initially and later it approaches to zero which is the result as expected. In view of above obtained results we may conclude that the model presented here can be acceptable at all. Also, the presence of bulk viscosity is to bring a change in perfect fluid model and it plays an important role in the early evolution of the universe.

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